Comparing apples and oranges: gravity lessons from QFT

Charlotte Sleight

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

PPSMC Dec 12, 2014.

Four fundamental forces

Electromagnetic

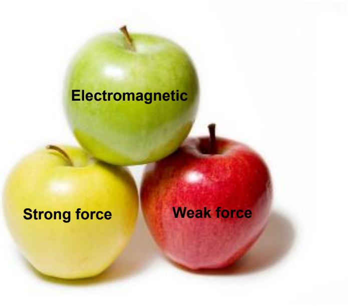
Gravity

Strong force

Weak force

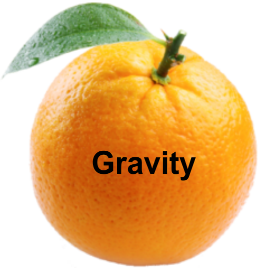
Theory Fruit Bowl

Standard Model



small scales + low mass

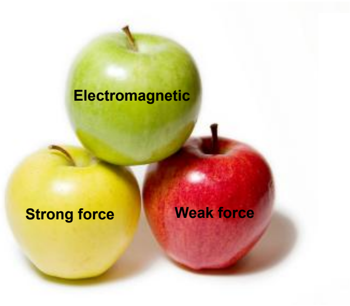
General Relativity



large scales + high mass

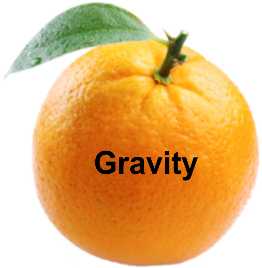
Theory Fruit Bowl

Standard Model



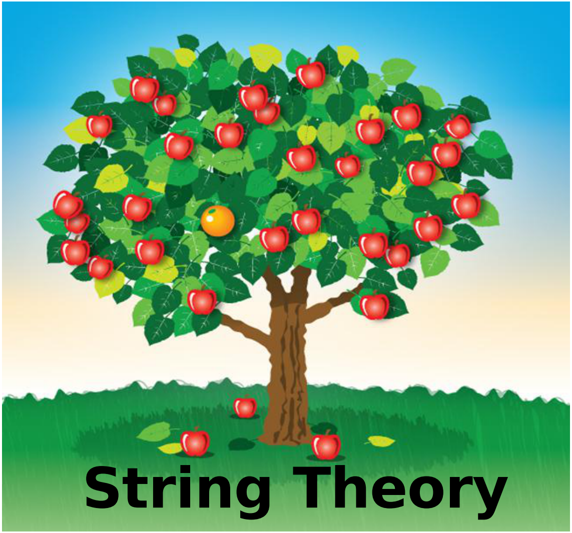
small scales + low mass

General Relativity



large scales + high mass

Problem: Cannot compare apples and oranges.

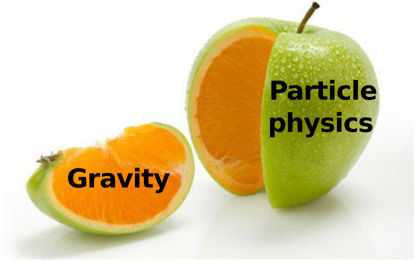

String Theory

Holography

• String theory provides a another framework in which we can **compare apples and oranges**.

• Leads us to conjecture that certain **QFTs** can be equivalently described by **gravity theories** in one higher dimension, and vice versa.

• **Difficult** problems in one theory become **simpler** from point of view of holographic dual theory.



Outline

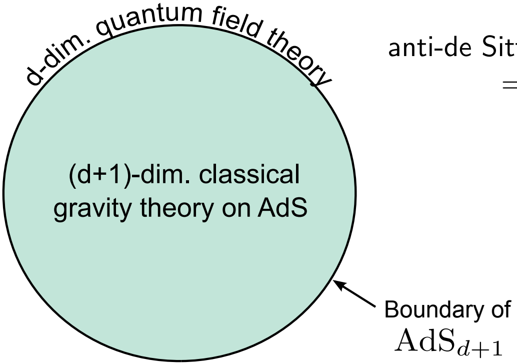
- Equivalences between QFTs and gravity theories on **anti-de Sitter space**.

- **Learning** about gravity theories **from** QFT.

- **Example:** Learning about gravity **interactions** from QFT.

Gravity on anti-de Sitter space

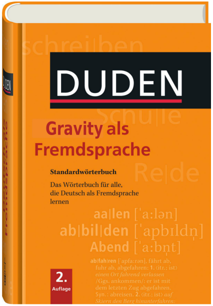
A QFT in d-**dimensional flat space** can be equivalently described by a classical gravity theory in $(d+1)$-**dimensional anti-de Sitter space**.



anti-de Sitter space
$=$ Lorentzian hyperbolic space

Field theory to gravity dictionary

Small excerpt:



Field theory		Gravity theory
Operator, $\mathcal{O}_{\mu_1 \ldots \mu_r}$.	$\longleftrightarrow$	Field, $\phi_{\mu_1 \ldots \mu_r}$.
Conserved operator. e.g. EM tensor: $\partial^\mu T_{\mu\nu} = 0$,	$\longleftrightarrow$	Gauge field. metric fluctuation, $h_{\mu\nu}$.

Calculating stuff:

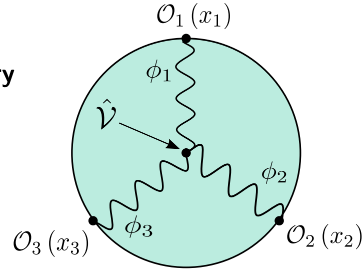
Correlation function.	$\longleftrightarrow$	Witten diagrams. (Feynman diagram in AdS.)
$\vdots$		$\vdots$
Physical quantity.	$\longleftrightarrow$	Physical quantity.

An example:

Field theory three-point function using **gravity**

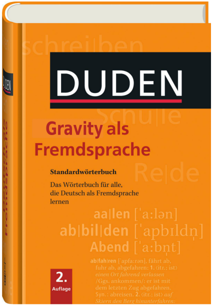
We calculate $\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_3(x_3)\rangle$ via **gravity** Witten diagram.

1. **Field theory** operators $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$ excite their corresponding **gravity** fields ϕ_1, ϕ_2 and ϕ_3.
2. ϕ_1, ϕ_2 and ϕ_3 propagate from the **boundary** into the **interior** of AdS space.
   (Propagators in Feynman diagram.)
3. ϕ_1, ϕ_2 and ϕ_3 **interact** in a cubic vertex $\hat{\mathcal{V}}$.
   (Vertex joins propagators.)
4. **Integrate** over all interaction points.



three-point Witten diagram

Dictionary opens up possibilities to learn:



1. If can calculate on **both** sides ...

   $\longrightarrow$ good **test** of relation between gravity + QFT.

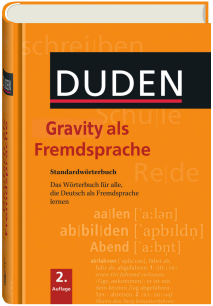
2. If **can't** compute in **QFT** but **can** in **gravity**...

   $\longrightarrow$ Make **predictions** for **QFT** using **gravity**.

3. If **can't** compute in **gravity** but **can** in **QFT**...

   $\longrightarrow$ Make **predictions** for **gravity** using **QFT**.

Dictionary opens up possibilities to <u>learn</u>:



1. If can calculate on **both** sides ...

   $\longrightarrow$ good **test** of relation between gravity + QFT.

2. If **can't** compute in **QFT** but **can** in **gravity**...

   $\longrightarrow$ Make **predictions** for **QFT** using **gravity**.

3. If **can't** compute in **gravity** but **can** in **QFT**...

   $\longrightarrow$ Make **predictions** for **gravity** using **QFT**.

Effective example:

QFT on the boundary of a gravity theory is non-interacting.

$\longrightarrow$ Can compute **anything**, in principle.

$\longrightarrow$ Can therefore hope to learn lots of things about the **gravity theory** though studies of the **QFT**.

$\longrightarrow$ In this case, the gravity theory has gauge fields of spins **greater than two:** they are **higher-spin gravity theories.**

Higher-spin gravity theories

- Defined on anti-de Sitter space.
- **Field content:**

Ordinary gravity:	spin-2,	$g_{\mu\nu}$,	$\delta g_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$.
Higher-spin gravity:	spin-2,	$g_{\mu\nu}$,	$\delta g_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$.
	spin-3,	$\varphi_{\mu\nu\rho}$,	$\delta \varphi_{\mu\nu\rho} = \partial_{(\mu} \xi_{\nu\rho)}$.
	⋮		
	spin-s,	$\varphi_{\mu_1 \dots \mu_s}$,	$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_s)}$.
	⋮		

$+$ real scalar field, ϕ.

Why study higher-spin gravity theories?

They're highly symmetric: huge amount of symmetry generated by infinite tower of higher-spin gauge fields.

$\longrightarrow$ The symmetry is believed to hold beyond the Planck scale.

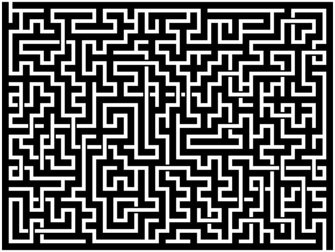
$\longrightarrow$ Opportunity to use symmetry as guiding principle in understanding quantum gravity.

However ...

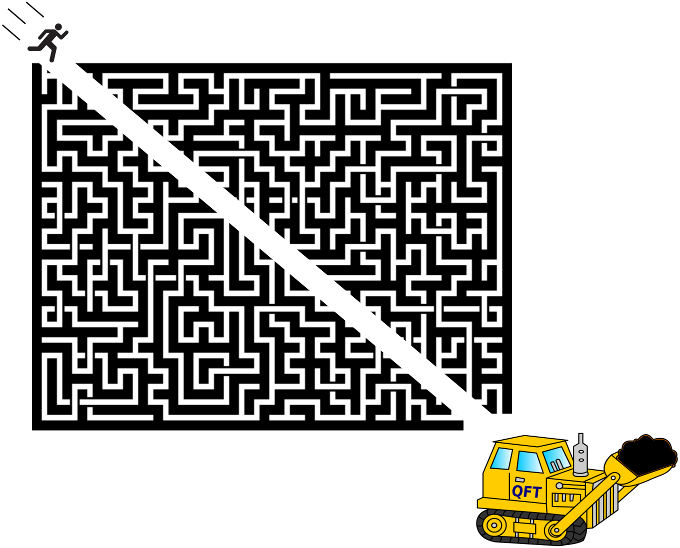
- No complete non-linear action known.
- Only know action up to cubic order in fields

$$S\left[\varphi\right] = \sum_{s=0}^{\infty} \int d^{d+1}x \left[\mathcal{L}^{(2)} \left[\varphi_{\mu_1 \ldots \mu_s}\right] + \mathcal{L}^{(3)} \left[\varphi_{\mu_1 \ldots \mu_s}\right] + ..??\right].$$

- Finding higher order interactions within higher-spin theory itself is very hard!



Look to the easy dual field theory for help!



Look to the easy dual field theory for help!

Dictionary:

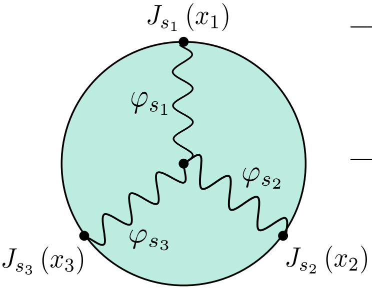
Spin-s gauge field $\varphi_s \quad \longleftrightarrow \quad$ **spin-s operator** J_s**,** $\quad \partial \cdot J_s = 0.$

(Higher-spin gravity on AdS) (In the QFT on boundary of AdS)

Look to the easy dual field theory for help!

To compute $\langle J_{s_1} J_{s_2} J_{s_3} \rangle$ using gravity Witten diagram ...

... need to know cubic interaction of higher-spin gauge fields φ_{s_1}, φ_{s_2} and φ_{s_3}!



$\longrightarrow$ Field theory correlation functions know about higher-spin interactions!

$\longrightarrow$ And correlation functions are easy to compute in a non-interacting field theory!

Idea:

Study field theory correlation functions of more than three operators

$\longrightarrow$ learn about higher order interactions of higher-spin fields!

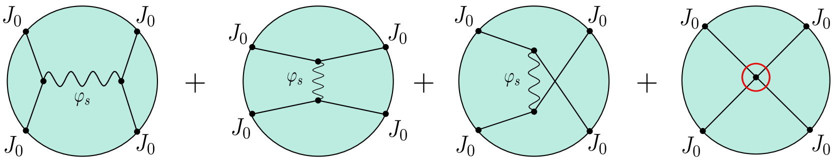
Simplest example:

Finding the quartic interaction of real scalar ϕ in higher-spin theory.

Dictionary: Real scalar ϕ in high-spin theory translated to scalar operator J_0 in field theory.

• To find quartic interaction of ϕ, consider the four-point function $\langle J_0 J_0 J_0 J_0 \rangle$. This is easy to compute in the trivial QFT.

• Corresponding higher-spin gravity calculation of $\langle J_0 J_0 J_0 J_0 \rangle$:

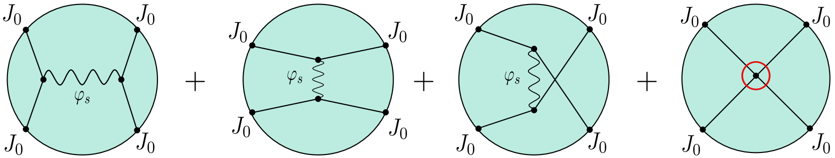


... and sum over all spins, s.

How to extract the quartic vertex?

1. Calculate $\langle J_0 J_0 J_0 J_0 \rangle$ in the QFT – easy.

2. Consider the higher-spin gravity calculation of $\langle J_0 J_0 J_0 J_0 \rangle$:



- Can compute first three diagrams [1412.0016].

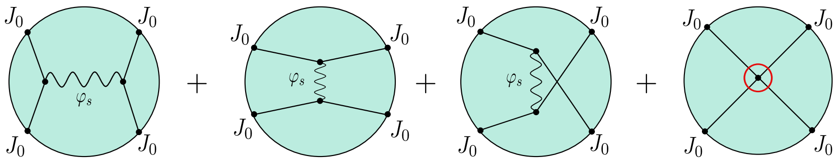
Ingredients: (1). Cubic vertices between φ_s and ϕ – already known.

(2). Propagators for φ_s and ϕ.

- Sum contribution from each spin $s = 0, 2, 3, \ldots$

How to extract the quartic vertex?

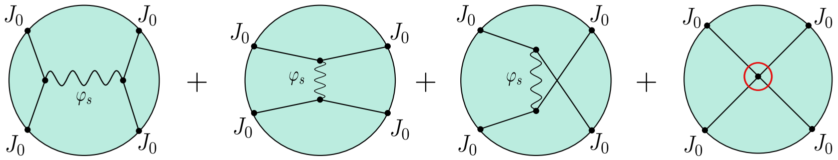
3. Make ansatz for scalar quartic vertex, and compute last diagram.



4. Finally, compare with QFT computation of $\langle J_0 J_0 J_0 J_0 \rangle$, and determine the required form of quartic interaction ansatz for the two calculations to agree. In progress...

How to extract the quartic vertex?

3. Make ansatz for scalar quartic vertex, and compute last diagram.



4. Finally, compare with QFT computation of $\langle J_0 J_0 J_0 J_0 \rangle$, and determine the required form of quartic interaction ansatz for the two calculations to agree. In progress...

Next step: extend to quartic interactions of higher-spin fields, and beyond quartic order.

Summary

• Through holography we can learn new things about gravity theories by studying QFT.

• Particularly effective when the QFT dual to gravity theory is very easy.

• Example: learning about interactions in higher-spin gravity from QFT correlation functions.

$\longrightarrow$

We translated an apple into an orange.

⟶

We translated an apple into an orange.

Merry Christmas!