Bound States in Quantum Field Theory

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Contents

Large N–Bound states

- Witten: Baryons as bound states of N heavy quarks
- gain expansion parameter $1/N$ by considering $SU(N)$
- need heavy quarks because description non–relativistic
- $M \sim N$
- naturally leads to corrections of the form $1/N$

Graviton Bound States (Dvali, Gomez; 1112.3359)

- consider black hole as bound state of N soft gravitons ($\lambda \sim r_g$) with source radius $R = r_g = 2G_N M$
- interpret GR as EFT of graviton on flat spacetime, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

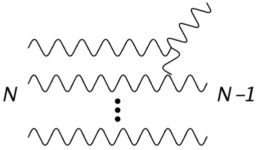
Scaling Relations

$$M = \sqrt{N} M_p, \quad r_g = \sqrt{N} L_p, \quad \alpha = 1/N \tag{1}$$

- e.g. for solar mass black hole: $N \sim 10^{71}$
- coupling weak, but large collective effect $\alpha N = 1$

Hawking Radiation and Information Paradox

- known results recovered as $N \to \infty$



$$\Gamma \sim 1/r_g + \mathcal{O}(1/N) \tag{2}$$

- information paradox: non-unitary evolution of black hole
  $\to$ pure state evolves into mixture
- $1/N$ corrections could resolve it
- back-reaction gets order one after Page time $\sim N$
- can recover information from black hole
  $\to$ purification of Hawking radiation

Basic Idea (Hofmann, Rug; 1403.3224)

- develop method to describe generic bound states of N constituents in QFT
- represent $|\mathscr{B}\rangle$ in terms of the constituents

$$|\mathscr{B}\rangle = \frac{1}{\Gamma_{\mathscr{B}}} \int \mathrm{d}^4 P \, B(P) \int \frac{\mathrm{d}^4 x}{(2\pi)^4} \, \mathrm{e}^{\mathrm{i}P\cdot x} J(x) |\Omega\rangle \tag{3}$$

- current J has the same quantum numbers as the true state $\Rightarrow$ same field content as the bound state
- wave function $B(P)$ contains information about the momentum distribution

Current

- current should have the same quantum numbers as the full state
- it should have the same symmetries as well

Examples:

- black hole (made out of N gravitons h):

$$J^{\mathscr{B}}(x) = h^N(r) \tag{4}$$

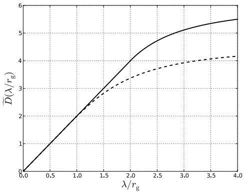
- ρ–meson (contains u–quark and d–quark):

$$J^{\rho}_{\mu}(x) = \frac{1}{2}\Big(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\Big)(x) \tag{5}$$

Black Hole Observables

- use Wick's theorem to evaluate observables $\langle \mathscr{B} | O | \mathscr{B} \rangle$
- normal ordered part does not vanish in the non–trivial vacuum $\Rightarrow$Non-perturbative effects (condensates)
- light-cone constituent distribution:

$$\mathscr{D}(r) = \int \mathrm{d}^3 k \, \mathrm{e}^{-\mathrm{i} k \cdot r} \langle \mathscr{B} | n(\mathbf{k}) | \mathscr{B} \rangle \tag{6}$$



Black Hole Observables

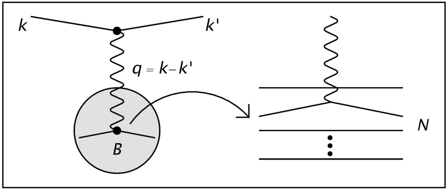
- energy density: $\mathscr{E}_{\mu\nu}(x) = \langle \mathscr{B} | T_{\mu\nu}(x) | \mathscr{B} \rangle$
- evaluation using in $N, M \to \infty$, N/M fixed limit leads to

$$M^2 = \frac{\langle h^{2(N-1)} \rangle}{\langle h^{2(N-2)} \rangle} N^2 \tag{7}$$

- same scaling as Witten gets for baryons
- leads to $1/N$–corrections

Scattering (Gruending, Mueller, Hofmann, Rug; 1407.1051)

- $\langle \mathscr{B}'\Phi' | \mathscr{B}\Phi \rangle$ in tree approximation



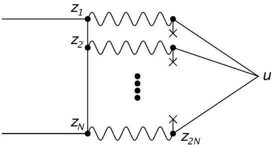
$$S = \int \mathrm{d}^4 x \left[\frac{1}{2} h_{\mu\nu} \varepsilon^{\mu\nu}_{\alpha\beta} h^{\alpha\beta} + \frac{1}{M_\mathrm{P}} h_{\mu\nu} T^{\mu\nu} \right] \quad (8)$$

Scattering

- factorization in perturbative and non–perturbative part
- $r_g^{-2} \ll q^2 \ll M_p^2$: EFT description valid, but resolution of bound state possible
-
$$k'^0 \frac{\mathrm{d}\sigma}{\mathrm{d}^3 k'} \sim \mathscr{D}(r) \tag{9}$$
- parton distribution can be measured by scattering experiments

Black Hole Formation (Gruending, Mueller, Hofmann, Rug; to appear)

- consider transplanckian scattering with $M = \sqrt{s}$
- impact parameter smaller than Schwarzschild radius



Amplitude: $\langle \mathscr{B} | a^\dagger(k_1) a^\dagger(k_2) \rangle \sim f(N) e^{-N}$

- perturbative and non-perturbative part factorize

Other Classical Space–times (Gruending, Mueller, Hofmann, Rug; to appear)

- the same method can be used for different classical space–times
- for example: Anti–de Sitter, de Sitter
- corresponding current has to respect all the symmetries
- can recover the spectrum of inflationary perturbations
- evaluate energy momentum tensor of dS

ρ–meson (Gruending, Mueller, Hofmann, Rug; to appear)

- can use the same method for hadrons like the ρ– meson

$$J^{\rho}_{\mu}(x) = \frac{1}{2}\Big(\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d\Big)(x) \tag{10}$$

- similar to QCD sum rules one gets expression for observables in terms of quark and gluon condensates

$$M_{\rho} = \int d^3x \langle\rho|T_{00}(x)|\rho\rangle \tag{11}$$

- can calculate observables directly, does not have to compare to spectral density function
- instead one has to model the wave function of the ρ– meson

Summary

Summary:

- description of generic bound state in QFT
- simple for black hole since N is large
- $1/N$ corrections as solution to information paradox
- scaling $M \sim N$, embedding of observables in scattering experiments
- black hole production: $\mathscr{A} \sim e^{-N}$

Outlook

Outlook:

- computation of $1/N$ corrections
- RG evolution
- black hole merger
- hadronic states
- AdS/CFT correspondence

Thank You for Your Attention