Proton driven plasma wakefields

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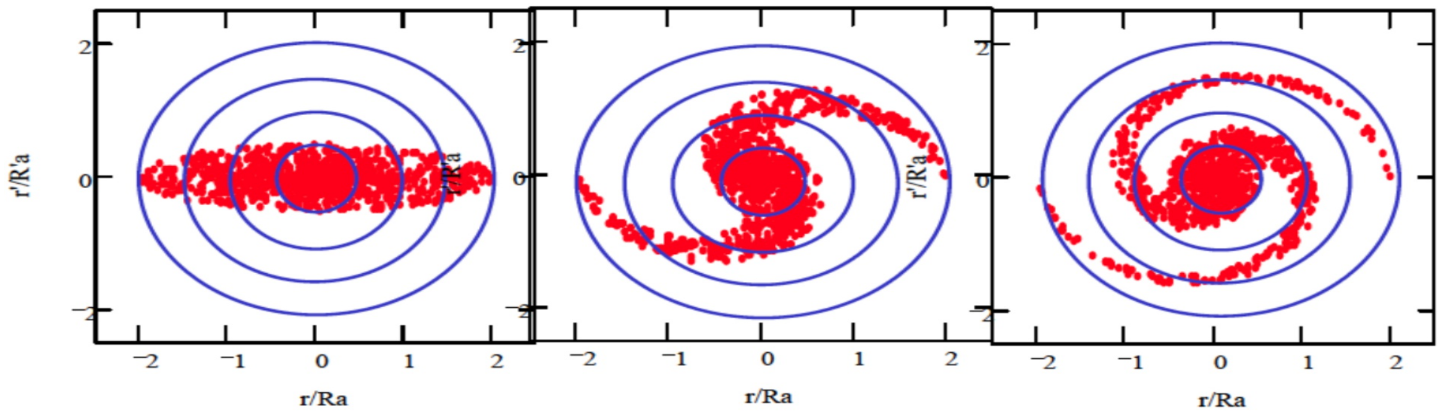
Outline

1. Proton beams
2. Linear plasma wakefields
3. Proton beam self modulation and plasma acceleration

Proton beams

Beam is characterized using phase space

- Ensemble of particles with $P_z >> P_r$ → particles travel in z direction
- Each particle has 3 coordinates and momenta P_z P_r and P_Φ
  - → Complete System is described with a Hamiltonian and phase space
  - → Beam can be described as a phase space distribution $f_p(z,r,\Phi,p_z,p_r,p_\Phi,...,t)$



- Due to large γ interaction between particles can often be neglected
- Tracking the phase space gives beam evolution .. but there is another useful way

Emittance

(Normalised RMS) emittance ε is a beam quality parameter

Normalised emittance:

$$\epsilon_n^2 = (\beta\gamma)^2 [\langle r^2\rangle\langle(\frac{dr}{dz})^2\rangle - \langle(r\frac{dr}{dz})^2\rangle] = (\beta\gamma)^2 \epsilon_{rms}^2$$

- < x > means averaging the quanitity x over the particle distribution in the phase space
- γ is the usual gamma factor of the particles (energy spread is neglected)
- Define $\sqrt{\langle r^2\rangle}$ as radius R of beam!
- dr/dz is the “slope” the particles have with the main propagation direction z

- Emittance is a conserved quantity and can be used to simplify beam propagation

Proton beams

Proton beam radius evolves like a single particle trajectory

- No Interaction of each particle with each other!
- No External Forces (beam drifting in free space)
- Beam Radius obeys equation of motion:

$$R'' = \frac{\epsilon^2}{R^3}$$

- ' Derivative is d/dz ! (z = c t)
- This is the equation of a single particle with angular momentum with solution:

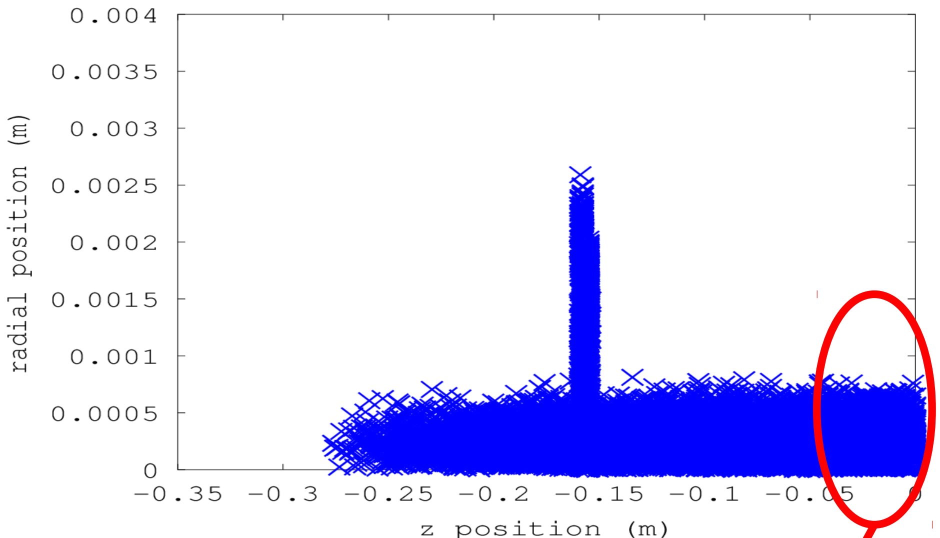
$$R(z) = \sqrt{R_0 + R_0'(z - z_0)^2 + \frac{\epsilon^2}{R_0^2}(z - z_0)^2}$$

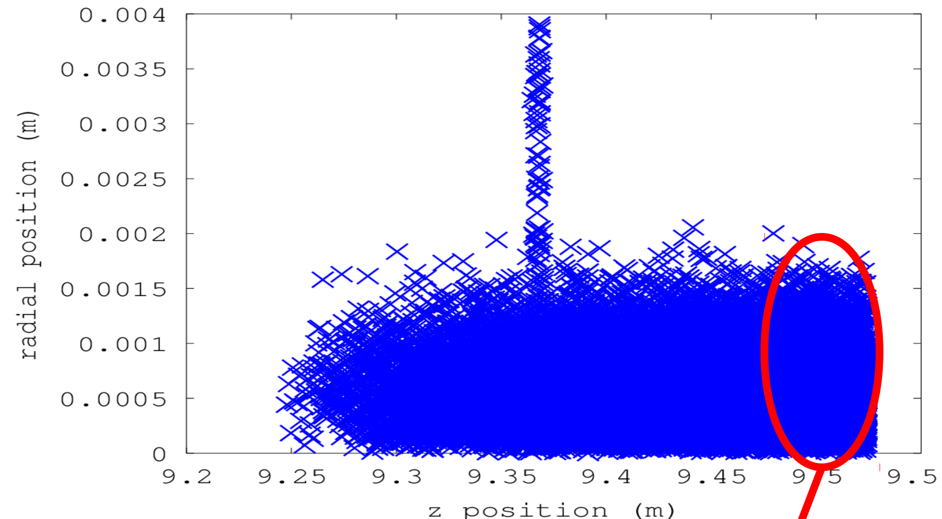
- At a waist ($R_0' = 0$) we have:

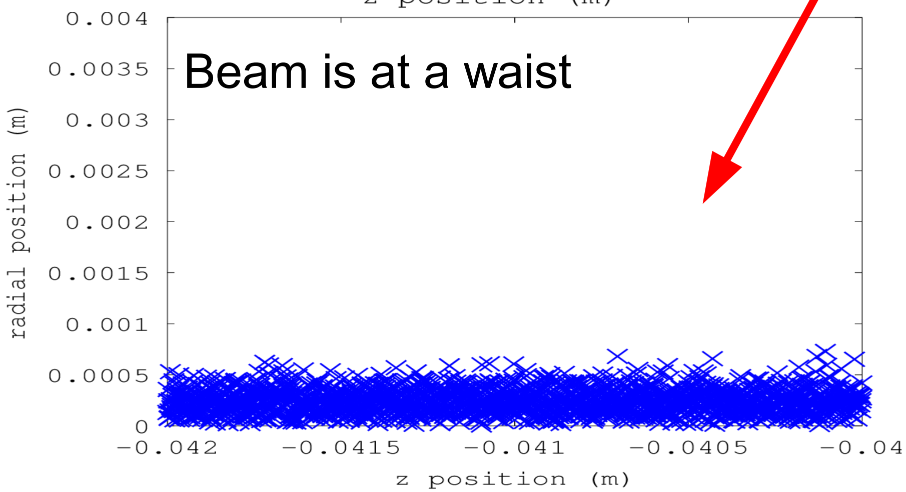
$$R(z) = R_0 \sqrt{1 + \left(\frac{z - z_0}{\beta_w}\right)^2} ; \beta_w = \frac{R_0^2}{\epsilon}$$

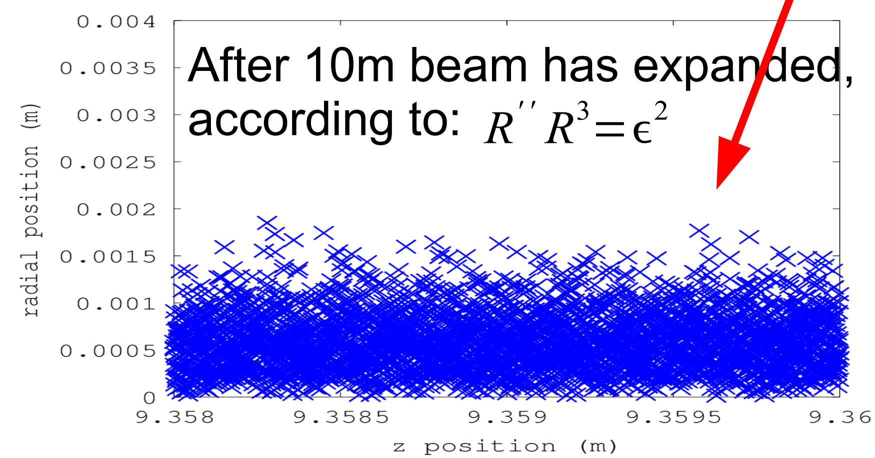
Proton beam evolution

Proton beam radius grows in free space, no substructure









Linear plasma wakefields

Response of a linear infinite plasma is a harmonic oscillator

- Consider Infinite extended cold (electrons and ions do not move) plasma
- Consider fixed ion background
- Electron density n_0 → plasma frequency $\omega_{pe}=\sqrt{\frac{e^2 n_0}{\epsilon_0 c}}$

- Consider incoming beam density n_b as a function (ultrarelativistic beam) of

$$n_b=f(z-ct)\,g(r)=f_b(\xi)\,g_b(r)$$

- Look at plasma density perturbation n_{pe} to this incoming charge density in r and ζ (z-ct) coordinates!

$$n_{pe}=f_{res}(\xi)\,g_{res}(r)$$

Linear plasma wakefields

Response of a linear infinite plasma is a harmonic oscillator

ζ dependent part of plasma response is determined by:

$$\frac{d^2 n_{pe}}{d\xi^2} + k_{pe}^2 n_{pe} = -k_{pe}^2 n_b$$

$$k_{pe} = \frac{\omega_{pe}}{c}$$

External bunch density

General solution is given by:

$$n_{pe} = \int_{\infty}^{\xi} \sin(k_{pe}(\xi - \xi')) f_b(\xi') d\xi' \times g_{res}(r)$$

r dependent part of plasma response is given by:

$$g_{res}(r) = \int_0^r r' f(r') I_0(k_{pe} r') K_0(k_{pe} r) dr' + \int_r^{\infty} r' f(r') I_0(k_{pe} r) K_0(k_{pe} r') dr'$$

Linear plasma wakefields

Plasma density perturbation creates electric fields that act on bunch

$$\vec{\nabla}\vec{E}=\frac{-n_{pe}}{\epsilon_0}$$

- Accelerating and deaccelerating fields periodic in ζ

$$E_z \sim \int_{\infty}^{\xi} \cos(k_{pe}(\xi-\xi'))\, f_b(\xi')\, d\xi' \times \hat{g}(r)$$

- Focussing and defocussing fields also periodic ζ:

$$E_r \sim \int_{\infty}^{\xi} \sin(k_{pe}(\xi-\xi'))\, f_b(\xi')\, d\xi' \times \hat{g}(r)$$

- Beam evolves under the influence of these self induced fields!

→ Self modulation instability

Proton beam self modulation

Beam radius equation with flat top beam

$$R'' - \frac{\epsilon^2}{R^3} = C \frac{I_2(k_p R)}{R} \int_\infty^\xi d\xi' \sin(k_p(\xi - \xi')) f(\xi') \frac{K_1(k_p R(\xi'))}{R(\xi')}$$

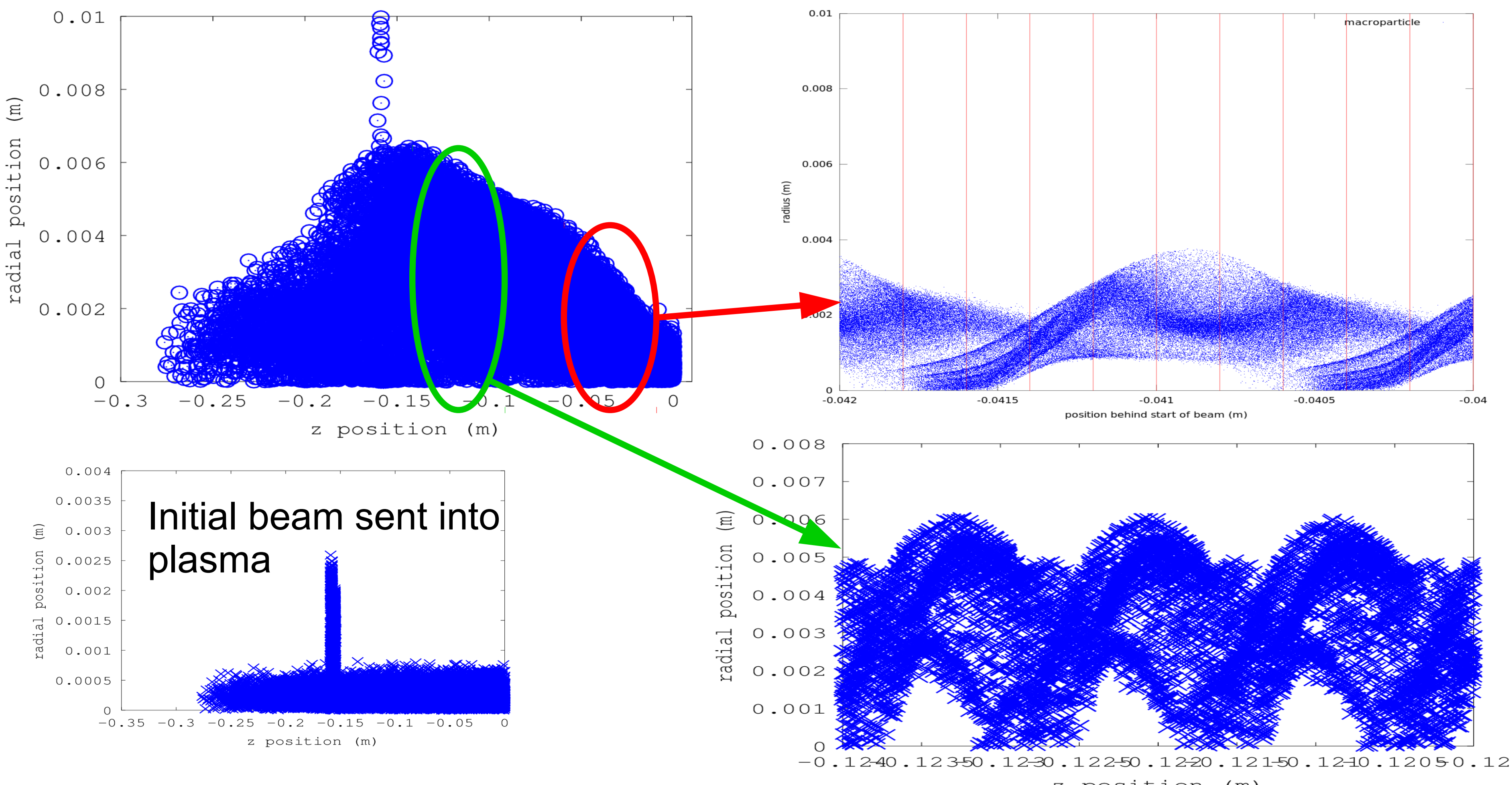
Normal Beam evolution without external force

External force due to plasma response (flat top beam, $g(r) \sim \theta(r_0 - r)$)

- Focussing and defocussing forces act on bunch radius!
- Simulations show self modulation instability of a bunch which is much longer than the plasma wavelength!

Self modulation

Proton beam self modulates in plasma, structure recognizable



Plasma acceleration

Self modulated proton bunch is a resonant driver for harmonic oscillator

- Self modulation period is on the order of plasma wavelength (resonance frequency!)
- Off axis protons do not contribute to wakefields (plasma is only ~1mm)

$$\frac{d^2 n_{pe}}{d\xi^2} + k_{pe}^2 n_{pe} = -k_{pe}^2 n_{mod}$$

Modulated beam drives now plasma density perturbation

- Electric field in longitudinal direction is proportional to

$$E_z \sim \int_{\infty}^{\xi} \sin\left(k_{pe}(\xi - \xi')\right) n_{mod}(\xi')\, d\xi'$$

→ large acceleration for trailing witness bunch possible (resonant driving of plasma with self modulated bunch)

Thanks !