The electroweak fit at NNLO and prospects for LHC and ILC

68% and 95% CL contours w/o $M_W$ and $\kappa_V$ measurement
- Present SM fit
- Prospect for LHC
- Prospect for ILC/GigaZ

$\kappa_V$ private LHC average $\pm 1\sigma$

$\Lambda = \frac{4\pi V}{\sqrt{1-\kappa_V^2}}$

$M_W$ world comb. $\pm 1\sigma$
The idea of electroweak fits

- Electroweak interactions are the puzzling part of the Standard Model:
  - parity (and CP) violation
  - non-trivial mixing of gauge groups SU(2)×U(1)
  - massive gauge bosons (Higgs mechanism)

- Want to test the electroweak sector with ultimate precision

The electroweak coupling sector is given by three parameters:

- g, g' (coupling constants of SU(2), U(1))
- Vacuum expectation value of the Higgs field (v)
- (the weak mixing angle is then given requiring the photon to be a massless vector field)

In practice can use the three most precise parameters:

$\alpha (\Delta \alpha/\alpha = 3 \cdot 10^{-10})$, $G_F (\Delta G_F/G_F = 5 \cdot 10^{-7})$, $m_Z (\Delta m_Z/m_Z = 2 \cdot 10^{-5})$
The idea of electroweak fits (ii)

- If one measures more observables than the minimum one can test the model

- However the situation is more complicated:
  - known (and unknown) particles run in loops
  - the observables therefore depend on all other parameters of the model ($m_H$, $m_t$, $\alpha_s$...)
  - this feature was already used successfully to predict $m_t$ in the early 90ies and $m_H$ until two years ago
  - now that all SM parameters are known experimentally this can be used to test BSM models (which was possible, of course, already before)
The observables

- Coupling sector in SM fixed by 3 parameters
  - Take most precise: \( \alpha(\Delta\alpha/\alpha = 3 \cdot 10^{-10}) \), \( G_F(\Delta G_F/G_F = 5 \cdot 10^{-7}) \), \( m_Z(\Delta m_Z/m_Z = 2 \cdot 10^{-5}) \)

- If more observables are measured the model can be tested
  - Z-fermion couplings:
    - axial-vector coupling: \( g_{Af} = I_3^f \)
    - vector coupling: \( g_{Vf} = g_{Af} \left( 1 - 4|Q_f| \sin^2 \theta_W \right) \)
  - W-Z mass relation:
    \[ \sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2} \]
  - Z-partial widths:
    \[ \Gamma_f \propto g_{A,f}^2 + g_{V,f}^2 \quad \Gamma_{\text{had}} \propto \Gamma_{\text{had}}^0 \left( 1 + \frac{\alpha_s}{\pi} + \ldots \right) \]
  - Asymmetries at LEP and SLC:
    \[ A_f = \frac{2g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2} \rightarrow \frac{g_V}{g_A} \Rightarrow \sin^2 \theta_W \]
The observables(ii)

Loop corrections make the observables dependent on all other parameters of the model

- On the Z-pole resonant diagrams dominate $\rightarrow$ loop corrections can be parametrised by 2 form-factors

\[ g_{A,f} = \sqrt{1 + \Delta \rho_f I_3^f} \]

\[ \sin^2 \theta_{\text{eff}}^f = (1 + \Delta \kappa_f) \sin^2 \theta \left( \frac{g_V}{g_A} \right) \]

\[ m_W^2 = \frac{m_Z^2}{2} \left( 1 + \sqrt{1 - \frac{\sqrt{8\pi\alpha(1 + \Delta r)}}{G_F m_Z^2}} \right) \]

- e.g. $\Delta \rho$:

\[ \Delta \rho = \frac{3 G_F}{4\pi^2 \sqrt{2}} \left( \frac{m_t^2}{2} - m_W^2 \frac{s^2}{c^2} \ln \frac{m_H}{m_Z} \right) + \ldots \]
The data

Data used for electroweak fits:
- Z mass and couplings from LEP1/SLD (1989-2000)
- W mass (and width) from LEP2/Tevatron
- Top-quark mass from Tevatron/LHC
- (other fermion masses play a very minor role)
- Higgs mass from LHC
- Low energy $e^+e^-$ data to fix running of $\alpha$
- $G_F$ from muon lifetime measurements

Some electroweak low energy data like neutrino scattering or atomic parity violation have been used in the fits but where finally discarded because of ununderstood effects.
LEP and SLC

LEP:
- $e^+e^-$ collider at CERN (1989-2000) in the now LHC tunnel
- 4 experiments: ALEPH, DELPHI, L3 and OPAL
- 1989-1995:
  - running at or near the Z-pole ($\sqrt{s}\approx91$ GeV)
  - $4\cdot10^6$ Zs/experiment for Z properties
- 1996-2000:
  - Running at 161 GeV<$\sqrt{s}$<208 GeV
  - 750 pb$^{-1}$/experiment for W properties and Higgs searches

SLC:
- Linear $e^+e^-$ collider at SLAC, running 1992-1998 at the Z-pole
- One experiment: SLD, Low luminosity ($\approx500000$Zs)
- However beam polarisation <80%
Z-lineshape at LEP1

- Precise scan around Z pole gives mass, total and partial widths
- Experimental challenge: understand event selection, luminosity and beam energy to unprecedented accuracy
- Results presented in terms of minimally correlated variables:

\[
\begin{align*}
  m_Z \\
  \Gamma_Z \\
  \sigma_0^{\text{had}} &= \frac{12\pi}{m_Z} \frac{\Gamma_l \Gamma_{\text{had}}}{\Gamma_Z^2} \\
  R_l &= \frac{\Gamma_{\text{had}}}{\Gamma_l}
\end{align*}
\]

- \(\Gamma_l\) sensitive to Z axial coupling
- \(\Gamma_{\text{had}}\) measures \(\alpha_s\)
- \(\Gamma_{\text{inv}} = \Gamma_Z - \Gamma_{\text{had}} - 3\Gamma_l\) constrains number of light neutrinos to 3
Asymmetries at LEP/SLC

- Asymmetries measure interference of vector and axial vector coupling
  \[ A_f = \frac{2g_V f g_{Af}}{g_V^2 + g_{Af}^2} \rightarrow \sin^2 \theta_{\text{eff}} \]

- Forward backward asymmetry
  \[ A_{FB}^f = \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4} A_e A_f \]

- Left right asymmetry (with polarised beams)
  \[ A_{LR} = \frac{1}{P} \frac{N_L - N_R}{N_L + N_R} = A_e \]

- \( \tau \)-polarisation
  \[ P_\tau (\cos \theta) = \frac{A_{\tau} (1 + \cos^2 \theta) + 2A_e \cos \theta}{(1 + \cos^2 \theta) + 2A_{\tau} A_e \cos \theta} \]

- All asymmetries basically sensitive to \( \sin^2 \theta_{\text{eff}} \)
Sensitivity of the $A$s on $\sin^2\theta_{\text{eff}}$

$A_f = \frac{2g_{V_f}g_{A_f}}{g_{V_f}^2 + g_{A_f}^2}$

$g_{V_f} = g_{A_f} \left(1 - 4|Q_f|\sin^2\theta_{\text{eff}}^f\right)$
Heavy quark partial width

- The ratio of the b- and c-partial width to the hadronic width can be measured using flavour tagging ($R_b$, $R_c$)

- Especially $R_b$ is interesting since all loop corrections cancel apart from vertex corrections involving heavy particles

- They now agree well with the SM prediction but some intermediate slight deviations have already been interpreted as a proof of SUSY
Measurement of the W-mass

- $m_W$ measured at LEP2 and the Tevatron
- Tevatron now dominates the measurement
- No LHC results available yet

CDF II preliminary $\int L \, dt \approx 200 \text{ pb}^{-1}$

$M_W = (80493 \pm 48_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 86/48$

Mass of the W Boson

- Measurement
  - CDF-0/I
  - DØ-I
  - DØ-II $^{(1.0 \text{ fb}^{-1})}$
  - CDF-II $^{(2.2 \text{ fb}^{-1})}$
  - DØ-II $^{(4.3 \text{ fb}^{-1})}$
  - Tevatron Run-0/I/II
  - LEP-2
  - World Average

- $M_W$ [MeV]
  - $80432 \pm 79$
  - $80478 \pm 83$
  - $80402 \pm 43$
  - $80387 \pm 19$
  - $80369 \pm 26$
  - $80387 \pm 16$
  - $80376 \pm 33$
  - $80385 \pm 15$

March 2012
Measurement of the top mass

$m_t$ now measured with similar precision at LHC and Tevatron

- More precise measurements released by D0 and CMS in the meantime
- However theory error might be somewhat underestimated
- In Gfitter we assume an additional theory error of 0.5GeV

$$m_t = 173.34 \pm 0.76$$
Measurement of the Higgs

- Discovery of the Higgs was essential for electroweak fits since now the Standard Model is fully defined.
- Mass precision already far better than ever needed in electroweak fits.

Private average: \( m_H = 125.14 \pm 0.24 \text{GeV} \)
The theory

\[ \sin^2 \theta_{\text{eff}} \] is known to full 2-loop in \( O(\alpha^2) \) and \( O(\alpha \alpha_s) \) and leading 3- and 4-loop corrections.

\( m_W \) is known to the same precision.

The Z partial widths are calculated to full 2-loop order for fermionic corrections and higher orders to match \( \sin^2 \theta_{\text{eff}} \).

Final state QED and QCD radiation is known to 4-loop for massless fermions and 3-loop for massive fermions.

\( \Gamma_W \) is only known to 1-loop but it only plays a minor role in the fits.

In all cases estimated theory uncertainties are significantly smaller than experimental uncertainties but are taken into account in the fits.
The theory (ii)

New since last round:

* the full fermionic 2-loop corrections to the $Z$ partial widths have been calculated by A. Freitas (JHEP 1404, 070)
* this led to very small changes in the fit results confirming the old theory-error estimate
* an early version of this calculation contained a bug which caused a significant change in $R_b$ and has been corrected

Theory errors used in the fits:

<table>
<thead>
<tr>
<th>observable</th>
<th>theo error</th>
<th>exp error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_W$</td>
<td>4 MeV</td>
<td>15 MeV</td>
</tr>
<tr>
<td>$\sin^2 \theta^\ell_{\text{eff}}$</td>
<td>$0.5 \cdot 10^{-4}$</td>
<td>$1.6 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>0.5 MeV</td>
<td>2.3 MeV</td>
</tr>
<tr>
<td>$\sigma^0_{\text{had}}$</td>
<td>6 pb</td>
<td>37 pb</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$1.5 \cdot 10^{-4}$</td>
<td>$6.6 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$m_t$</td>
<td>0.5 GeV</td>
<td>0.76 GeV</td>
</tr>
</tbody>
</table>
The running of $\alpha$

- The fine structure constant runs by about 7% from $Q^2=0$, where it is precisely measured to the Z-scale.

- The running can be written as
  \[
  \alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)} \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s)
  \]

- $\Delta\alpha_{\text{lep}}(m_Z^2)$, $\Delta\alpha_{\text{top}}(m_Z^2)$ can be calculated perturbatively with negligible error.

- $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$ is obtained from a mixture of perturbative QCD and $e^+e^- \rightarrow$ hadrons data at low energy.

- We use $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2) = 0.02757 \pm 0.00010$

  M. Davier et al., arxiv:1010.4180
Fit codes

- LEPEWWG and others used theory predictions from ZFITTER (D. Bardin et al.) in their fits
- TopaZ (G. Passarino) discontinued now but very important to validate ZFITTER
- GAP (J. Erler) used for PDG review
- Gfitter (M. Baak et al.) several publications in the last years
- Several codes extending the fits to BSM, especially SUSY

The fits of the different groups agree very well and it's mostly personal preference which one is shown

Some differences in treatment of theory errors which just start to matter
Fit codes (ii)

- In this talk I will mainly restrict myself to results from the Gfitter collaboration

- Data are fitted with the full covariance matrix as anywhere else

- Theory errors are treated as Gaussian distributions now, minimal differences to old Rfit treatment but more stable fits
History of Electroweak Fits

- $m_t$ predictions from loop effects since 1990
- Official LEPEW fit since 1993
- After the top was discovered, also a prediction of $m_H$ became possible

The Higgs-mass precision was always much worse because of the quadratic dependence

The fits have always been able to predict $m_t$ and $m_H$ correctly!
Best test of compatibility of data with model is a fit to all data and a look at the pulls.

The data agree well with the prediction:

$$\chi^2/\text{ndf} = 17.8/14 \Rightarrow \text{Prob} = 21\%$$

The only unconstrained free parameter is the strong coupling constant:

$$\alpha_s(m_Z^2) = 0.1196 \pm 0.0028\,(\text{exp}) \pm 0.0009\,(\text{theo})$$

comparing well with the world average:

$$\alpha_s(m_Z^2) = 0.1185 \pm 0.0006$$
Predictions of observables

- After the Higgs discovery precise predictions of all observables are possible
- The good agreement between the measurement and the prediction could already be seen in the pull plot
# Fit results in detail

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input value</th>
<th>Free in fit</th>
<th>Fit Result</th>
<th>w/o exp. input in line</th>
<th>w/o exp. input in line, no theo. unc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H$ [GeV]</td>
<td>125.14 ± 0.24</td>
<td>yes</td>
<td>125.14 ± 0.24</td>
<td>93^{+25}_{-21}</td>
<td>93^{+24}_{-20}</td>
</tr>
<tr>
<td>$M_W$ [GeV]</td>
<td>80.385 ± 0.015</td>
<td>–</td>
<td>80.364 ± 0.007</td>
<td>80.358 ± 0.008</td>
<td>80.358 ± 0.006</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>2.085 ± 0.042</td>
<td>–</td>
<td>2.091 ± 0.001</td>
<td>2.091 ± 0.001</td>
<td>2.091 ± 0.001</td>
</tr>
<tr>
<td>$M_Z$ [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>yes</td>
<td>91.1880 ± 0.0021</td>
<td>91.1200 ± 0.011</td>
<td>91.1200 ± 0.010</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>–</td>
<td>2.4950 ± 0.0014</td>
<td>2.4946 ± 0.0016</td>
<td>2.4945 ± 0.0016</td>
</tr>
<tr>
<td>$\sigma^0_{\text{had}}$ [nb]</td>
<td>41.540 ± 0.037</td>
<td>–</td>
<td>41.484 ± 0.015</td>
<td>41.475 ± 0.016</td>
<td>41.474 ± 0.015</td>
</tr>
<tr>
<td>$R_{\ell}^j$</td>
<td>20.767 ± 0.025</td>
<td>–</td>
<td>20.743 ± 0.017</td>
<td>20.722 ± 0.026</td>
<td>20.721 ± 0.026</td>
</tr>
<tr>
<td>$A_{FB}^{0,\ell}$</td>
<td>0.0171 ± 0.0010</td>
<td>–</td>
<td>0.01626 ± 0.0001</td>
<td>0.01625 ± 0.0001</td>
<td>0.01625 ± 0.0001</td>
</tr>
<tr>
<td>$A^{(*)}_{\ell}$</td>
<td>0.1499 ± 0.0018</td>
<td>–</td>
<td>0.1472 ± 0.0005</td>
<td>0.1472 ± 0.0005</td>
<td>0.1472 ± 0.0004</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}^{Q_{FB}}$</td>
<td>0.2324 ± 0.0012</td>
<td>–</td>
<td>0.23150 ± 0.00006</td>
<td>0.23149 ± 0.00007</td>
<td>0.23150 ± 0.00005</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.670 ± 0.027</td>
<td>–</td>
<td>0.6680 ± 0.00022</td>
<td>0.6680 ± 0.00022</td>
<td>0.6680 ± 0.00016</td>
</tr>
<tr>
<td>$A_b$</td>
<td>0.923 ± 0.020</td>
<td>–</td>
<td>0.93463 ± 0.00004</td>
<td>0.93463 ± 0.00004</td>
<td>0.93463 ± 0.00003</td>
</tr>
<tr>
<td>$A_{FB}^{0,c}$</td>
<td>0.0707 ± 0.0035</td>
<td>–</td>
<td>0.0738 ± 0.0003</td>
<td>0.0738 ± 0.0003</td>
<td>0.0738 ± 0.0002</td>
</tr>
<tr>
<td>$A_{FB}^{0,b}$</td>
<td>0.0992 ± 0.0016</td>
<td>–</td>
<td>0.1032 ± 0.0004</td>
<td>0.1034 ± 0.0004</td>
<td>0.1033 ± 0.0003</td>
</tr>
<tr>
<td>$R_{c}^{0}$</td>
<td>0.1721 ± 0.0030</td>
<td>–</td>
<td>0.17226^{+0.00009}_{-0.00008}</td>
<td>0.17226 ± 0.00008</td>
<td>0.17226 ± 0.00006</td>
</tr>
<tr>
<td>$R_{b}^{0}$</td>
<td>0.21629 ± 0.00066</td>
<td>–</td>
<td>0.21578 ± 0.00011</td>
<td>0.21577 ± 0.00011</td>
<td>0.21577 ± 0.00004</td>
</tr>
<tr>
<td>$m_c$ [GeV]</td>
<td>1.27^{+0.07}_{-0.11}</td>
<td>yes</td>
<td>1.27^{+0.07}_{-0.11}</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$m_b$ [GeV]</td>
<td>4.20^{+0.17}_{-0.07}</td>
<td>yes</td>
<td>4.20^{+0.17}_{-0.07}</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>173.34 ± 0.76</td>
<td>yes</td>
<td>173.81 ± 0.85 (∇)</td>
<td>177.0^{+2.3}_{-2.4} (∇)</td>
<td>177.0 ± 2.3</td>
</tr>
<tr>
<td>$\Delta \alpha^{(5)}_{\text{had}}$ $(M_Z^2)^{(\Delta)}$</td>
<td>2757 ± 10</td>
<td>yes</td>
<td>2756 ± 10</td>
<td>2723 ± 44</td>
<td>2722 ± 42</td>
</tr>
<tr>
<td>$\alpha_s(M_Z^2)$</td>
<td>–</td>
<td>yes</td>
<td>0.1196 ± 0.0030</td>
<td>0.1196 ± 0.0030</td>
<td>0.1196 ± 0.0028</td>
</tr>
</tbody>
</table>
$m_H$ and the $\sin^2\theta$ saga

- The two most precise $\sin^2\theta_{\text{eff}}$ measurements ($A_{LR}^{b}(\text{SLD}), A_{FB}^{b}(\text{LEP})$) don't agree very well.

- The prediction including $m_H$ agrees well with the average.

- The agreement with the two single measurements is similar.
Indirect predictions

- Before the Higgs discovery the electroweak fit was used to predict the Higgs mass.
- The prediction agrees well with the measured mass.
- The error of the prediction is a factor 100 larger than the present LHC Higgs mass error.
Prediction of \( m_t \) and \( m_H \)

- Precision mainly from \( m_W \) and \( \sin^2 \theta_{\text{eff}} \)
- Partial widths play only minor role
Indirect predictions (ii)

- Also 2D indirect predictions are possible
- In all cases the predictions agree well with the measurement
- For the Born level quantities the predictions are more precise than the measurements
- The discovery of the Higgs was essential for that
Oblique parameters

The STU parameters were designed to ease BSM analyses:
- $T$ absorbs the isospin breaking contributions ($\Delta \rho$)
- $S$ takes the remainder in $\Delta \kappa$
- $U$ takes what is still left in $\Delta r$ ($U=0$ in many models)

![Plot showing fit contours for $U=0$]
STU without $m_H$

- Without $m_H$, new physics effects could often be compensated by a change in the Higgs mass.
- This resulted often in weak limits.
- However, few new studies available since many models no longer valid.

![Sequential Fourth Fermion Generation](image)

明清:1107.0975
Little Higgs models solve hierarchy problem making it a pseudo-Goldstone boson of a new gauge group.
This introduces new gauge bosons and a top-partner of same spin.
A new T-parity prevents large disagreement with electroweak precision observables and provides a dark matter candidate.
Nevertheless precision data can severely constrain these models.
Littlest Higgs exclusion

EWPO only

+LHC Higgs couplings

$R = \frac{\lambda_1}{\lambda_2}$ = ratio of Yukawa couplings in top-sector

$f$: breaking scale of new symmetry

Reuter/Tonini/de Vries
arXiv:1310.2918

$R$ vs $f$ [GeV]

MPI Colloquium 11.11.14
Klaus Mönig - Electroweak Fits
Constraints to Higgs couplings

The loops containing a Higgs scale with the Higgs couplings

In a simple model also used by the LHC all fermion couplings are scaled by $\kappa_F$ and W,Z couplings scaled by $\kappa_V$

This results in a non-renormalisable model and the loop corrections must be cut off by a parameter $\Lambda = \frac{\lambda}{\sqrt{|1 - \kappa_V^2|}}$

(usually used: $\lambda = 4\pi v \approx 3$ TeV)

Electroweak precision data are sensitive to $\kappa_V$:

$$S = \frac{1}{12\pi} (1 - \kappa_V^2) \ln \frac{\Lambda^2}{m_H^2}, \quad T = \frac{9}{4 \cos^2 \theta} S, \quad U = 0$$

The deviation can get arbitrarily large when one sends the cut-off scale to infinity

On the contrary direct LHC measurements don't depend on the cut-off scale, but need other assumptions
Higgs coupling constraints (ii)

- At the moment electroweak precision data improve $\kappa_V$ constraints by about a factor 2 for $\lambda \approx 4\pi v$

- Constraints get stronger for larger $\lambda$ while LHC direct searches are more sensitive to lower values

- Of course the electroweak fits assume that there is no other new physics at high energy
A note on $\alpha_s$

- From the electroweak fit one obtains

$$\alpha_s(m_Z^2) = 0.1196 \pm 0.0028_{\text{exp}} \pm 0.0006 R_{V,A} \pm 0.0006 \Gamma_i \pm 0.0002 \sigma^h_{\text{had}} \pm 0.0030$$

- This value assumes that the SM is valid in the calculation of the (partial) widths

- A more model independent value can be obtained from the STU fit:

$$\alpha_s(m_Z^2) = 0.1204 \pm 0.0031$$

- Both values are in good agreement with each other and with the world average:

$$\alpha_s(m_Z^2) = 0.1185 \pm 0.0006$$
Future prospects

- The Tevatron will finalise a few analyses
- The LHC will take a factor 100 more data where the analyses can be done up to a different level of pile-up
- Around 2030 we might get the ILC which may run in the GigaZ mode sometimes later
- Our theory colleagues will continue to calculate
LHC/Tevatron prospects

- **W-mass:**
  - Current Tevatron measurement should end up in a 10MeV uncertainty
  - With improvements in the PDFs etc. LHC + Tevatron may end up with 8 MeV

- **Top-mass:**
  - We assume that the present 0.76 GeV error can be reduced to 0.6 GeV
  - With further theoretical understanding the 0.5 GeV theory error could be halved

- **Higgs-mass**
  - The current error of 0.4 GeV is expected to go down to 0.1 GeV
  - This is however completely irrelevant for the electroweak fits
ILC prospects

Top mass:
- The ILC can measure $m_t$ with a threshold scan with much smaller theory uncertainties
- $\Delta m_t = 100\text{MeV}$ should be possible

W mass:
- $\Delta m_W = 5\text{MeV}$ may be possible from a threshold scan

Z parameters:
- $\sin^2 \theta$ can be measured with a precision of $1.3 \cdot 10^{-5}$ from the left-right asymmetry with polarised beams at GigaZ (factor $>10$ improvement)
- The Z mass cannot be improved (beam energy!)
- $R_l$ can be improved to $4 \cdot 10^{-3}$ (factor $6 \rightarrow \alpha_s$)
- Other width related observables are difficult to predict (beamstrahlung!), no improvement assumed at the moment
Other prospects

Due to new measurements of $e^+e^- \rightarrow \text{hadrons}$ at low energy colliders and new determinations of $\alpha_s$ from the lattice the error on $\Delta \alpha_{\text{had}}^5 (m_Z^2)$ is assumed to go down from $10 \cdot 10^{-5}$ to $4.7 \cdot 10^{-5}$.

The present theoretical uncertainties $\delta_{\text{theo}} m_W = 4 \text{ MeV}$ and $\delta_{\text{theo}} \sin^2 \theta_{\text{eff}} = 4.7 \cdot 10^{-5}$ are assumed to be reduced to $1 \text{ MeV}$ and $1 \cdot 10^{-5}$.

This requires electroweak 3-loop calculations, however they are only needed after GigaZ.
### Summary on input assumptions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experimental input $[\pm 1\sigma_{\text{exp}}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
</tr>
<tr>
<td>$M_H$ [GeV]</td>
<td>0.4</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>15</td>
</tr>
<tr>
<td>$M_Z$ [MeV]</td>
<td>2.1</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}^{\ell} [10^{-5}]$</td>
<td>16</td>
</tr>
<tr>
<td>$\Delta \alpha_{\text{had}}^5 (M_Z^2) [10^{-5}]$</td>
<td>10</td>
</tr>
<tr>
<td>$R_l^0 [10^{-3}]$</td>
<td>25</td>
</tr>
</tbody>
</table>
Future Blueband Plot

- Higgs prediction can be improved by a factor 5 at ILC
- This allows stringent consistency tests of the Standard Model
- LHC only gives a moderate improvement of a factor <2

![Graph showing Higgs prediction improvements at ILC and LHC](image_url)
2D plots

- A similar picture one gets looking at 2D distributions
- From all observables $\sin^2 \theta$ is by far the most important one
- Already with today's precision the top mass is not the limiting observable
Improvements in STU

- Large improvement along the long diagonal
- Nevertheless larger correlation due to the dominance of $\sin^2\theta$

68% and 95% CL fit contours for $U=0$

$(SM_{\text{ref}}: M_H=125 \text{ GeV}, m_t=173 \text{ GeV})$

- Present uncertainties
- Prospects for LHC
- Prospects for ILC/GigaZ

SM Prediction

$M_H = 125.14 \pm 0.24 \text{ GeV}$

$m_t = 173.34 \pm 0.91 \text{ GeV}$
Improvements in Higgs couplings

- At the end of LHC the direct measurements are more precise.
- After ILC the two are competitive again.
- Of course the ILC direct Higgs measurements are more model independent than at LHC.
Global picture of improvements

- At LHC more weight is given to $m_W$, the rest of the picture remains unchanged.
- At ILC the prediction is dominated by $\sin^2\theta$, giving slightly less weight to $m_t$.
- After the ILC measurements $m_Z$ and $\Delta\alpha_{\text{had}}^5(m_Z^2)$ (after the improvement) play an equally important role as $\sin^2\theta$.
- Further improvement is thus only possible if also these two quantities can be improved.
- Due to the improvement in $R_l$, $\alpha_s$ can be measured with a precision of 0.0006 (exp) equally precise as the current world average (the theory error from $\Gamma_i$ will vanish due to the precise measurement of $\sin^2\theta$).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta_{\text{meas}}$</th>
<th>$\delta_{\text{fit}}^{\text{tot}}$</th>
<th>$\delta_{\text{fit}}^{\text{theo}}$</th>
<th>$\delta_{\text{fit}}^{\text{exp}}$</th>
<th>$\delta M_W$</th>
<th>$\delta M_Z$</th>
<th>$\delta m_t$</th>
<th>$\delta \sin^2\theta^f_{\text{eff}}$</th>
<th>$\delta \Delta\alpha_{\text{had}}$</th>
<th>$\delta \alpha_S$</th>
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<td>+10</td>
<td>+31</td>
<td>+28</td>
<td>+5</td>
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<td>$M_W$ [MeV]</td>
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<tr>
<td>$m_t$ [GeV]</td>
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<td>0.6</td>
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<td>2.3</td>
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<td>6.6</td>
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<td>4.5</td>
<td>3.7</td>
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<tr>
<td>$\Delta\alpha_{\text{had}}$</td>
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<td>44</td>
<td>13</td>
<td>42</td>
<td>31</td>
<td>6</td>
<td>10</td>
<td>41</td>
<td>-</td>
<td>2</td>
</tr>
</tbody>
</table>

**Present uncertainties**

| $M_H$ [GeV]     | < 0.1                  | +21                                  | -18                                 | +4                                  | +20         | -14        | +6          | +8              | +18            | +3          | +5          |
| $M_W$ [MeV]     | 8                      | 5.5                                 | 1.8                                 | 5.2                                 | -           | 2.5        | 3.5         | 4.8              | 0.8            | 2.6          | 2.6          |
| $M_Z$ [MeV]     | 2.1                    | 7.2                                 | 1.4                                 | 7.0                                 | 6.0         | -          | 2.8         | 5.9              | 0.8            | 1.9          | 1.9          |
| $m_t$ [GeV]     | 0.6                    | 1.5                                 | 0.2                                 | 1.5                                 | 1.3         | 0.4        | -           | 1.2              | 0.2            | 0.2          | 0.2          |
| $\sin^2\theta^f_{\text{eff}}$ | 16                    | 3.0                                 | 1.1                                 | 2.8                                 | 2.5         | 1.1        | 1.4         | -1.5             | 1.5            | 0.9          | 0.9          |
| $\Delta\alpha_{\text{had}}$ | 4.7                  | 36                                  | 6                                   | 36                                  | 25          | 9          | 12          | 35              | -              | 5           | 5           |

**LHC prospects**

| $M_H$ [GeV]     | < 0.1                  | +7.4                                 | -7.0                                 | +2.5                                | +6.9        | +3.9       | +4.3        | +0.9            | +3.3           | +4.3        | +0.3        |
| $M_W$ [MeV]     | 5                      | 2.3                                 | 1.3                                 | 1.9                                 | -           | 1.7        | 0.3         | 1.3              | 0.7            | 0.3          | 0.3          |
| $M_Z$ [MeV]     | 2.1                    | 2.7                                 | 1.0                                 | 2.6                                 | 2.5         | -          | 0.4         | 1.3              | 1.9            | 0.2          | 0.2          |
| $m_t$ [GeV]     | 0.1                    | 0.8                                 | 0.2                                 | 0.7                                 | 0.6         | 0.5        | -           | 0.3              | 0.4            | 0.2          | 0.2          |
| $\sin^2\theta^f_{\text{eff}}$ | 1.3                  | 2.3                                 | 1.0                                 | 2.0                                 | 1.7         | 1.2        | 0.2         | -1.5             | 1.5            | 0.1          | 0.1          |
| $\Delta\alpha_{\text{had}}$ | 4.7                 | 6.4                                 | 3.0                                 | 5.6                                 | 2.7         | 4.1        | 0.8         | 3.9              | -              | 0.2          | 0.2          |

**ILC/GigaZ prospects**

$^{(\circ)}$ In units of $10^{-5}$.  

| $M_H$ [GeV]     | < 0.1                  | +7.4                                 | -7.0                                 | +2.5                                | +6.9        | +3.9       | +4.3        | +0.9            | +3.3           | +4.3        | +0.3        |
| $M_W$ [MeV]     | 5                      | 2.3                                 | 1.3                                 | 1.9                                 | -           | 1.7        | 0.3         | 1.3              | 0.7            | 0.3          | 0.3          |
| $M_Z$ [MeV]     | 2.1                    | 2.7                                 | 1.0                                 | 2.6                                 | 2.5         | -          | 0.4         | 1.3              | 1.9            | 0.2          | 0.2          |
| $m_t$ [GeV]     | 0.1                    | 0.8                                 | 0.2                                 | 0.7                                 | 0.6         | 0.5        | -           | 0.3              | 0.4            | 0.2          | 0.2          |
| $\sin^2\theta^f_{\text{eff}}$ | 1.3                  | 2.3                                 | 1.0                                 | 2.0                                 | 1.7         | 1.2        | 0.2         | -1.5             | 1.5            | 0.1          | 0.1          |
| $\Delta\alpha_{\text{had}}$ | 4.7                 | 6.4                                 | 3.0                                 | 5.6                                 | 2.7         | 4.1        | 0.8         | 3.9              | -              | 0.2          | 0.2          |
Conclusions

- As usual the Standard Model of electroweak interactions describes the data well.
- This remains true after the Higgs discovery when absolute predictions without ambiguities are possible.
- Higgs couplings to vector bosons can be constrained at the moment better than direct measurements if the cut-off parameter is above $4\pi v \approx 3$ TeV.
- Further improvements can be expected from LHC running and especially from GigaZ running at the ILC.
- This would give about a factor 5 on indirect predictions of other parameters which can be compared with precise direct measurements of Higgs couplings.