Recent developments in pQCD beyond MLLA

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Outline

- Jets in Quantum Chromodynamics:
  - production of jets $\longrightarrow$ problems
  - Feynman diagrams, probabilistic interpretation of cascading processes
  - infrared and collinear singularities $\longrightarrow$ some resummation approaches in pQCD
  - coherence effects and some tools in pQCD

- Results
  - Single inclusive differential one-particle distribution as a function of $k_\perp$ $\frac{d\sigma}{d\ln k_\perp}$ in MLLA and Next-to-MLLA; comparison with CDF p-p data
  - Gluon to quark multiplicity ratio in a sub-jet with jet axis from color current

- Conclusions

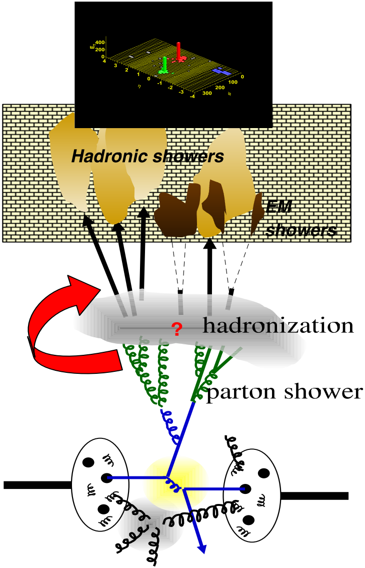
References

Phys. Rev. D **78** (2008) 014019; Phys.Rev.Lett. **100** (2008) 052002; Phys. Rev. D **78** (2008) 034010.

Part I

Jets in Quantum Chromodynamics

Production of jets



- Partonic cascade: traited in pQCD
  - planar gauge: tree amplitudes ⇒ parton shower picture (probabilistic interpretation)
- Hadronization: advocates for Local Parton Hadron Duality Hypothesis (LPHD)
  - partonic distributions ≃ hadronic distributions: factor $\mathcal{K}^{ch}$
  - "limiting spectrum:" $Q_0 \sim \Lambda_{QCD}$

Problems

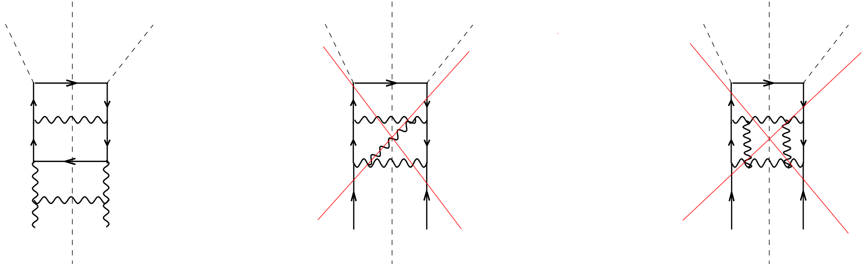
- $\alpha_s(Q^2 \to \Lambda_{QCD}^2) \to \infty$

- Infrared and collinear singularities $\Rightarrow$ problems on the convergence of the resumed series through their large logarithmic contributions

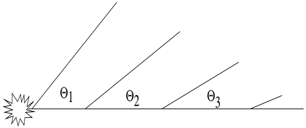
- Hadronization (Local Parton Hadron Duality)

• Exclusive Processes: resummation of all Feynman digrams at a given order in α_s

• Inclusive processes (jet calculus): resummation of all leading, sub-leading logarithmic contributions (...) coming from the emission of soft and/or collinear gluons in jets: "planar gauge" $\Rightarrow$

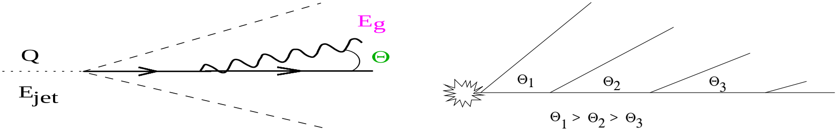


amplitude:



"probabilistic branching processes"

Infrared and collinear singularites



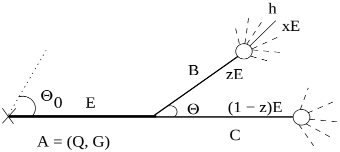
- collinear singularities: $\Theta \to \Theta_{min} \Rightarrow$ divergence in $\log \Theta \Rightarrow$ resummation over all collinear logs: $\Theta > \Theta_{min} = Q_0/E_g$
- infrared singularities: energy of the gluon $\ll$ energy of the parent $\Rightarrow \log(1/x)$; with $x = E_g/E_{jet} \ll 1$ (energy fraction of the jet carried away by one parton)
- in general: infrared $+$ collinear
- condition of "Angular Ordering"$\Leftrightarrow$"$k_\perp$-ordering" in DGLAP evolution

Resummation schemes

- DLA: $\alpha_s \log(1/x) \log \Theta$ ($\alpha_s \log^2 \sim 1 \Rightarrow \log \sim \alpha_s^{-1/2}$): resummation soft and collinear gluons
  - main ingredient to the estimation of inclusive observables in jets
  - neglects the energy balance
- Single Logs (SL): $\alpha_s \log \Theta$
  - collinear gluons
  - running of $\alpha_s(k_\perp \to Q_0) \ldots$ "$\Rightarrow \beta \times \alpha_s^n \log^n \Theta$"
- MLLA: $\underbrace{\alpha_s \log\log}_{\mathcal{O}(1)} + \underbrace{\alpha_s \log}_{\mathcal{O}(\sqrt{\alpha_s})}$: the SL corrections to DLA
  - "restore" the energy balance
  - take into account the running of $\alpha_s(k_\perp)$
- Next-to-MLLA: $\underbrace{\alpha_s \log\log}_{\mathcal{O}(1)} + \underbrace{\alpha_s \log}_{\mathcal{O}(\sqrt{\alpha_s})} + \underbrace{\alpha_s \log\log^{-1}}_{\mathcal{O}(\alpha_s)}$
  - improve the restoration of the energie balance
  - and allow to increase the range in "x" ($k_\perp \approx x E_{jet} \Theta$)

Some tools in pQCD

- Exact solution of approached integro-differential equations: MLLA evolution equations at $x \ll 1$
  - for the one-particle inclusive distributions: $D = \frac{\delta}{\delta u} Z(u)$
  - for n-particle correlations (iterative solutions): $D^{(n)} = \frac{\delta^n}{\delta u_1 \ldots \delta u_n} Z(u)$
- From $\frac{\partial^n}{\partial u^n}$ over $Z \rightarrow$ MLLA Master Equation:



$$\frac{d}{d \ln \Theta} Z_A\left(p, \Theta; \{u\}\right) = \frac{1}{2} \sum_{B,C} \int_0^1 dz\, \Phi_A^{B[C]}(z)\, \frac{\alpha_s\left(k_\perp^2\right)}{\pi} \left(Z_B\left(zp, \Theta; \{u\}\right) Z_C\left((1-z)p, \Theta; \{u\}\right) - Z_A\left(p, \Theta; \{u\}\right)\right)$$

- Solution of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) partonic evolution equations in Mellin's space (valid for $x \sim 1$) (we use it in certain integrals)

  - $\frac{d}{d \ln Q^2} D(x, Q^2) = \int_x^1 \frac{dz}{z} \frac{\alpha_s(Q^2)}{2\pi} \Phi(z)\, D\left(\frac{x}{z}, Q^2\right)$

  - $\underbrace{D(j, Q^2)}_{\text{Known!}} = \int_x^1 dx\, x^{j-1} D(x, Q^2)$

- Derivatives $\frac{d^n}{dj^n} D(j, Q^2) = \int_x^1 dx\, x^{j-1} \ln^n x\, D(x, Q^2),\ n = 1, 2, \ldots$

Part II

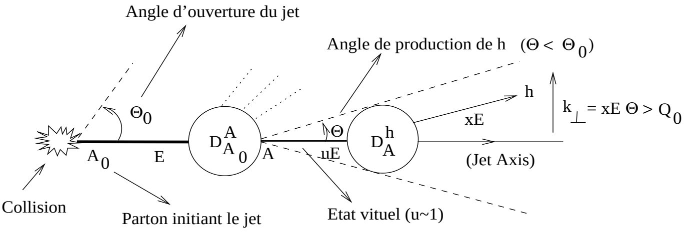
Main results

Charged hadrons distribution ($k_\perp$) in MLLA and NMLLA

- Single inclusive differential one-particle distribution as a function of $k_\perp \frac{d\sigma}{d\ln k_\perp}$ in MLLA and Next-to-MLLA
  - MLLA and NMLLA predictions in the "limiting spectrum" ($Q_0 = \Lambda_{QCD}$)
  - and beyond "$Q_0 \neq \Lambda_{QCD}$": prediction for different masses of charged hadrons

- Measured by CDF in $pp \to h + X$ collisions:

$$\underbrace{\left(\frac{d\sigma}{d\ln k_\perp}\right)^h}_{\text{measured}} = \omega \underbrace{\left(\frac{d\sigma}{d\ln k_\perp}\right)_g}_{?} + (1-\omega) \underbrace{\left(\frac{d\sigma}{d\ln k_\perp}\right)_q}_{?}$$



$$\underbrace{\left(\frac{d\sigma}{d\ln k_\perp}\right)_{g\ \ or\ \ q}}_{\text{Measurable!}} = \int dx \left(\frac{d^2\sigma}{dx\, d\ln k_\perp}\right)_{g\ \ or\ \ q}$$

where

$$\left(\frac{d^2\sigma}{dx\, d\ln k_\perp}\right)_{A_0} = \frac{d}{d\ln k_\perp}\sum_A \int_x^1 du D_{A_0}^A\left(u, E\Theta_0, uE\Theta\right) D_A^h\left(\frac{x}{u}, uE\Theta, Q_0\right)$$

$\Rightarrow$ valid $\forall\, x < u$ and dominated by $u \sim 1$!!

Small x $(x \ll 1)$ approximation $\Rightarrow$ analytical results

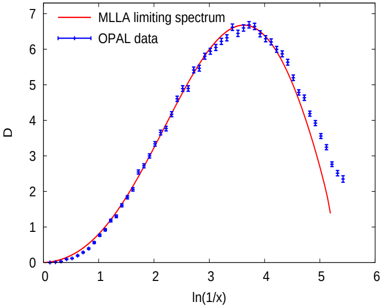
- $D_{A_0}^{A}(u = \mathcal{O}(1)\ldots)$: DGLAP; $\quad D_A^h(\frac{x}{u} \ll 1\ldots)$: (N)MLLA
- Perturbative Parameter of the expansion, kinematics and variables:
  - $\gamma_0^2 \simeq \alpha_s(\ell + y)$; $\quad \ell = \ln(1/x)$, $\quad y = \ln(k_\perp/\Lambda)$, $\quad k_\perp \approx xE\Theta$
- Double differential inclusive cross section:

$$\left(\frac{d^2\sigma}{d\ell dy}\right)_{A_0} \stackrel{x\ll 1}{\approx} \frac{d}{dy}\left[< C >_{A_0}(\ell, y)\tilde{D}_g^h(\ell, y)\right]$$

  - $< C >_{A_0}(\ell, y) \simeq c_0(\ell, y) \times 1 + c_1(\ell, y) \times \sqrt{\alpha_s} + c_2(\ell, y) \times \alpha_s + \mathcal{O}(\alpha_s^{3/2})$: color current (varying color factor): evolution of the jet between Θ_0 and Θ.
  - $\tilde{D}_g^h(\ell, y)$: ((N)MLLA evolution equations) describes the hump-backed plateau in the limit $Q \gg Q_0 \sim \Lambda_{QCD}$ (limiting spectrum)

Reminder

Hump-backed plateau $\tilde{D}^h \equiv \mathcal{K}^{ch} \times \frac{1}{\sigma}\frac{d\sigma}{d\ln(1/x)}$ at Z^0 peak: $Q = 91.2$ GeV



$* \; Q \gg Q_0 \sim \Lambda_{QCD}, \; \gamma_0 \approx 0.5$

“Next-to-MLLA” ev. eq. for the single inclusive spectrum

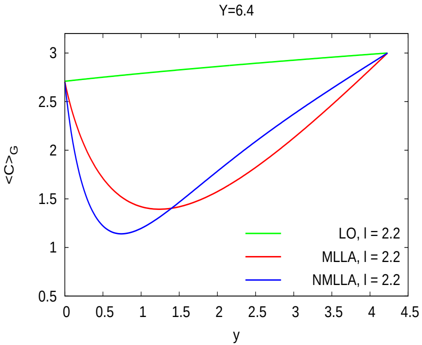
$$\underbrace{\tilde{D}_g^h(\ell, y)}_{gluon\ jet} = \delta(\ell) + \int_0^{\ell} d\ell' \int_0^{y} dy' \gamma_0^2(\ell'+y')\Big[\underbrace{1}_{DLA} - (a_1 + a_2\,\psi_\ell(\ell', y'))\,\delta(\ell'-\ell)\Big] \times \tilde{D}_g^h(\ell', y')$$

$$\underbrace{\tilde{D}_q^h(\ell, y)}_{quark\ jet} = \delta(\ell) + \frac{C_F}{N_c} \dots \tilde{D}_g^h(\ell', y') \dots \quad \underbrace{\ell = \ln(1/x)}_{infrared}, \ \underbrace{y = \ln(k_\perp/\Lambda)}_{collinear}$$

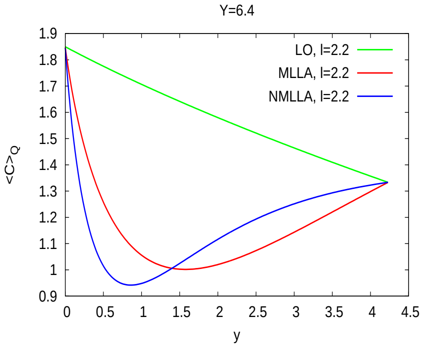
- DLA term: $\propto \mathcal{O}(1)$
- hard single logs: $\propto a_1 \sim \mathcal{O}(\sqrt{\alpha_s})$ & $\propto a_2(\psi_\ell = \frac{\partial D^h}{\partial \ell}) \sim \mathcal{O}(\alpha_s)$
- $\tilde{D}_g^h(\ell, y) = (\ell + y) \iint \frac{d\omega d\nu}{(2\pi i)^2} e^{\omega\ell} e^{\nu y} \int_0^\infty \frac{\mathrm{d}s}{\nu+s} \left(\frac{\omega(\nu+s)}{(\omega+s)\nu}\right)^{\sigma_0} \left(\frac{\nu}{\nu+s}\right)^{\sigma_1+\sigma_2} e^{-\sigma_3\, s}$
  - $\sigma_0 = \frac{1}{\beta_0(\omega-\nu)}, \qquad \sigma_1 = \frac{a_1}{\beta_0}, \qquad \sigma_2 = -\frac{a_2}{\beta_0}(\omega - \nu), \qquad \sigma_3 = -\frac{a_2}{\beta_0} + \lambda$
- $\tilde{D}_q^h \simeq \frac{C_F}{N_c}\left(1 + r_1\sqrt{\alpha_s} + r_2\alpha_s\right)\tilde{D}_g^h$

Color current $< \mathbf{C} >_{A_0} (\ell, \mathbf{y})$ at $\mathbf{Q} = \mathbf{155}$ GeV (CDF)

Gluon



Quark



- NMLLA sources: $< C >_{A_0} = \mathcal{O}(\alpha_s)$ and $\frac{d}{dy} < C >_{A_0} = \mathcal{O}(\alpha_s)$
- $\frac{d\sigma}{dy}(y) = \int d\ell \left(\frac{d^2\sigma}{d\ell dy}\right) \equiv \int d\ell \frac{d}{dy}\left[< C >_{A_0} (\ell, y)\tilde{D}_g^h(\ell, y)\right]$ comes now!

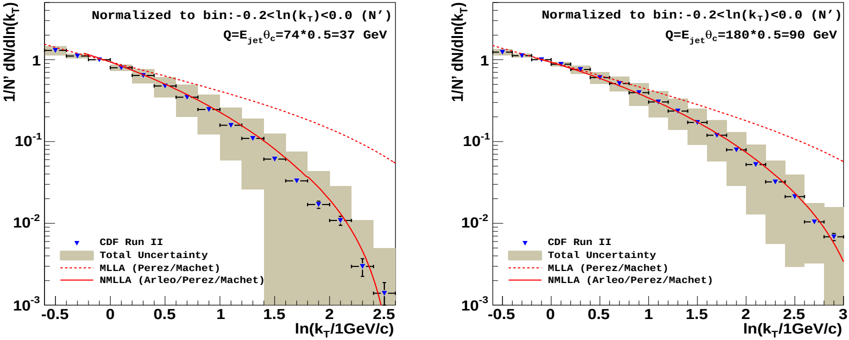
Limitations of the method⇒ranges of validity in MLLA and NMLLA

- 2 contradictory conditions should be simultaneously satisfied: $x \ll 1$ and $\alpha_s(k_\perp \approx xE\Theta) \ll 1$
- At too large $k_\perp$, the small x approximation fails ($k_\perp \approx xE\Theta$) because of positivity problems $\Rightarrow \exists k_{\perp_{max}} = f(Q)$ (NMLLA>MLLA).
- When $k_\perp \to \Lambda_{QCD}$, the perturbative expansion fails ($\alpha_s(k_\perp) \to \infty$)
  - $k_\perp > k_{\perp\, min} \Rightarrow y = \ln\left(\frac{k_\perp}{\Lambda}\right) > 1.4 \Leftrightarrow k_\perp > 1$ GeV

$\Rightarrow \exists$ range of validity: $\boxed{k_{\perp\, min} \leq k_\perp \leq k_{\perp\, max}}$ (NMLLA>MLLA) which increases when energy scale increases.

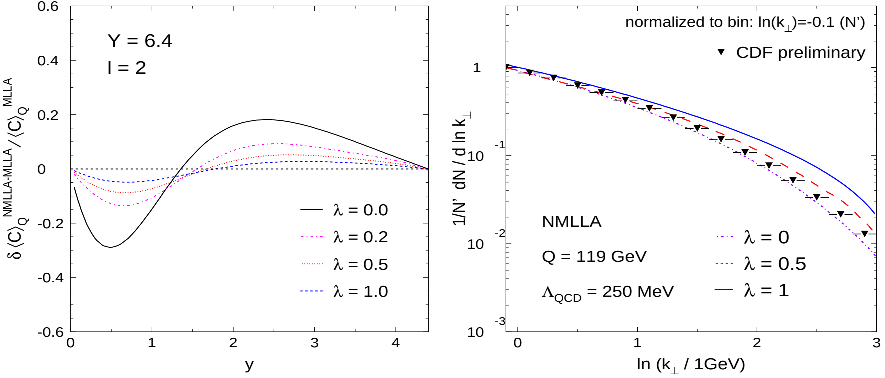
$\rightarrow$ LHC will supply tests of pQCD in a much larger domain of $k_\perp$ than the Tevatron.

Phys.Rev.Lett. **100** (2008) 052002



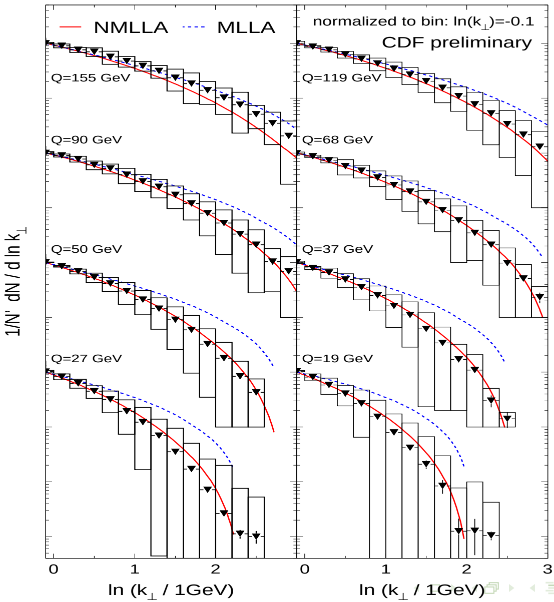
- preliminary results from CDF in very good agreement with NMLLA expectations
- range of validity enlarged in NMLLA

Beyond the limiting spectrum

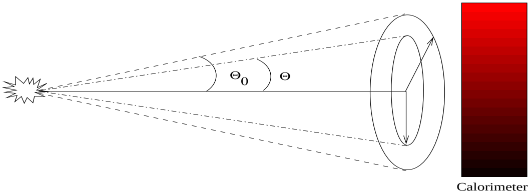


- $\lambda = \ln(Q_0/\Lambda_{QCD}) = 0$: limiting spectrum
- $\lambda = \ln(Q_0/\Lambda_{QCD}) \neq 0$: beyond

Phys.Rev.Lett. **100** (2008) 052002



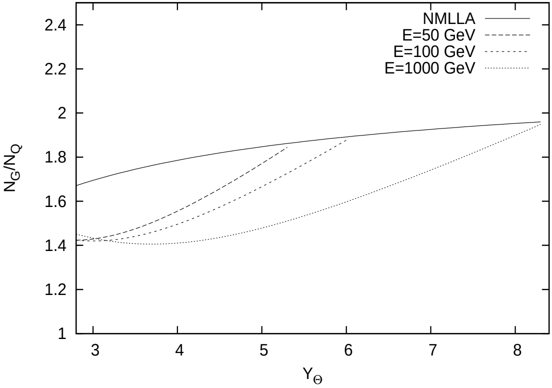
Multiplicities in sub-jets with jet axis from color current



- Average multiplicity ($\simeq$ number of particles) inside the sub-jet of half-opening angle Θ:
  - $\hat{N}^h_{A_0}(\Theta_0,\Theta) \approx \frac{1}{N_c}\langle C\rangle_{A_0}(\Theta_0,\Theta) N^h_G(\Theta)$; $N^h_G(\Theta) \simeq \exp\left(\int \gamma(E\Theta)d\Theta\right)$
  - $\gamma \simeq \sqrt{\alpha_s}(1 - a_1\sqrt{\alpha_s} - a_2\alpha_s)$ from Next-to-MLLA ev. eq.
  - $A_0 = Q$, $\langle C\rangle_Q(\Theta \equiv \Theta_0) = C_F$; $A_0 = G$, $\langle C\rangle_G(\Theta \equiv \Theta_0) = N_c$
- Ratio: $r(\Theta_0,\Theta) = \frac{N_G}{N_Q}(\Theta_0,\Theta) = \frac{\langle C\rangle_G}{\langle C\rangle_Q}(\Theta_0,\Theta)$
  - $r(\Theta \equiv \Theta_0) \simeq \frac{N_c}{C_F}(1 - r_1\sqrt{\alpha}_s - r_2\alpha_s)$ from Next-to-MLLA ev. eq.

Multiplicity ratio for jet $\Theta_0 \sim 1$ and sub-jet $\Theta \leq \Theta_0$

Ochs & Pérez-Ramos, Phys. Rev. D **78** (2008) 034010



- $\boxed{Y_\Theta = \ln\left(\frac{E\Theta}{Q_0}\right)}$: unreliability for $Y_\Theta \lessapprox 3$ or $E\Theta \lessapprox 5$ GeV

Conclusions

- Very good agreement between NMLLA predictions and the CDF data
  - in a broader $k_\perp$ range than in MLLA
  - NMLLA$\rightarrow$MLLA asymptotically
- Further test of LPHD hypothesis (partons roughly behave as hadrons)
  - pQCD successfully predicts the shape of $\frac{1}{\sigma}\frac{d\sigma}{d\ln k_\perp}$
    - also confirmed for multiplicities, multiplicity correlators (KNO problems where an analogous set of NMLLA corrections was included!), hump-backed plateau
- Limiting spectrum proves once again to be the most successful to describing the data
- The gluon to quark average multiplicity ratio inside a sub-jet follows the same trend as that inside a jet (full event) but suffers a variation of $\approx$20% from the color current (not measured yet)