Dynamical parton distribution functions *

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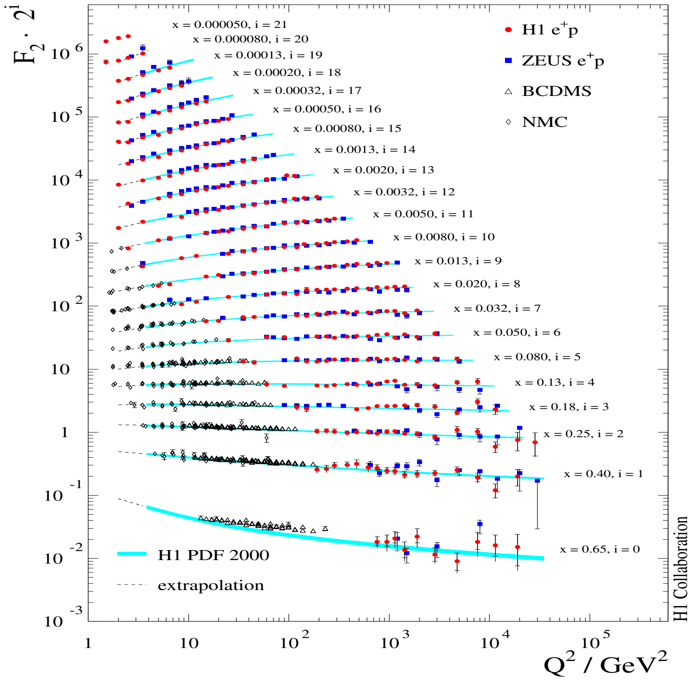
Outline

- The dynamical parton model
    - definition
    - comparison with the “standard” approach
    - determination of the parton distribution functions from experiments
- The perturbative stability of the longitudinal structure function F_L at small–x
- Predictions for F_L
    - dynamical parton model
    - “standard” approach
- Summary and conclusions

Scaling violations of $F_2(x,Q^2)$ and QCD

- At fixed x and $Q^2 \gtrsim 1$ GeV^2 , the structure function of the proton F_2 appears to depend logarithmically on Q^2



- This behaviour arises from perturbative QCD (pQCD), which dictates the Q^2–evolution of the underlying parton distributions $f(x,Q^2),\ f = q,\ \bar{q},\ g$

- The parton distributions are fixed at a specific input scale $Q^2 = Q_0^2$, mainly by experiment, only their evolution to any $Q^2 > Q_0^2$ being predicted by pQCD

The dynamical parton model

Pdf's are extracted from DIS data by two essentially different approaches, based on a different choice of the input distributions at some low scale Q_0:

- Standard: $Q_0 > 1$ GeV fixed, input distributions unrestricted; e.g. MRST, CTEQ

- Dynamical: pdf's at $Q > 1$ GeV are QCD radiatively generated from *valence*–like (positive) input distributions at a $Q_0 \equiv \mu < 1$ GeV, like GRV

$$xf(x, Q_0^2) \sim x^{a_f}(1-x)^{b_f} \quad \text{with} \quad a_f > 0$$

  - Positive definite parton distributions
  - More restrictive ansatz: less uncertainties, slightly higher χ^2
  - QCD predictions for $x < 10^{-2}$, subsequently confirmed by experiments
  - Useful for connecting nonperturbative models with the actually measured distributions at $Q > 1$ GeV

  - Can they improve the perturbative stability of the structure function F_L?

F_L *in perturbative QCD*

- In the $\overline{\mathrm{MS}}$ factorization scheme, with fixed number of flavors $n_f = 3$:

$$\frac{1}{x}\,F_L = C_{L,ns} \otimes q_{ns} + \frac{2}{9}\,\left(C_{L,q} \otimes q_s + C_{L,g} \otimes g\right) + \frac{1}{x}\,F_L^c$$

- The heavy flavor (dominantly charm) contribution F_L^c to F_L is taken as given by fixed–order NLO perturbation theory

  E. Laenen, S. Riemersma, J. Smith, W.L. van Neerven, NPB 392, 162 (1993)

  S. Riemersma, J. Smith, W.L. van Neerven, PLB 347, 143 (1995)

$$q_s = \sum_{q=u,d,s} (q+\bar{q}), \quad q^{+}_{ns,3} = u + \bar{u} - (d + \bar{d}), \quad q^{+}_{ns,8} = u + \bar{u} + d + \bar{d} - 2(s + \bar{s})$$

- Perturbative expansion of the coefficient functions:

$$C_{L,i}(\alpha_s, x) = \sum_{n=1} \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^n c^{(n)}_{L,i}(x)$$

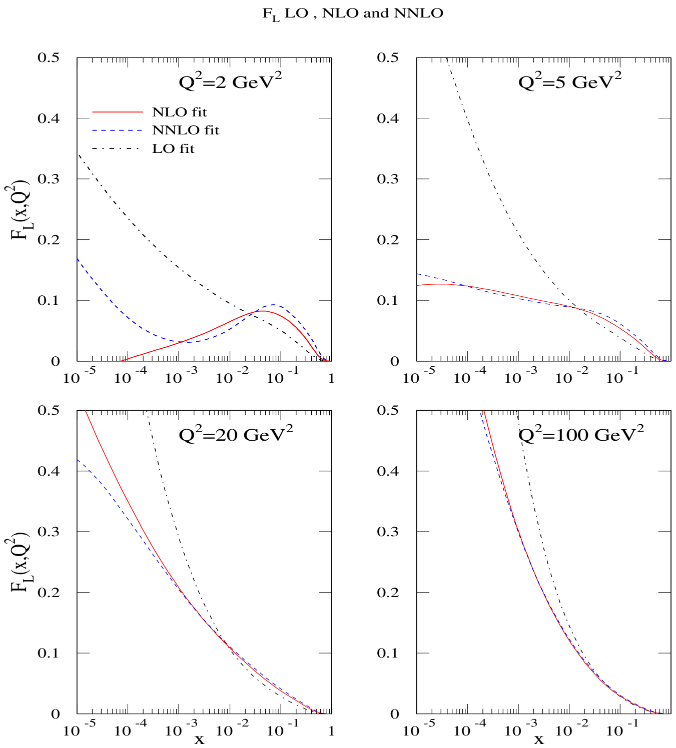
- In LO: $c^{(1)}_{L,ns} = \frac{16}{3}x$, $c^{(1)}_{L,ps} = 0$, $c^{(1)}_{L,g} = 24x(1-x)$

$$c^{(n)}_{L,q} = c^{(n)}_{L,ns} + c^{(n)}_{L,ps}$$

- At small x: $xc^{(2)}_{L,i}$(negative) constant, overwhelmed by the $xc^{(3)}_{L,i} \sim -\ln x$ terms

The problem of the perturbative stability of F_L

- Sensitive test of the reliability of perturbative QCD (pQCD) : study of the perturbative stability of $F_L(x,Q^2)$ at $x \lesssim 10^{-3}$ and $Q^2 \gtrsim \mathcal{O}(2-3$ GeV$^2)$



A.D. Martin et al., PLB 531, 216 (2002);
PLB 635, 305 (2006)

Fixed order pQCD instability related to:

- NNLO α_s^3 contribution to the coefficient function $xc_L^{(3)} \sim -\ln x$ at small x, while $xc_L^{(1)}$ and $xc_L^{(2)}$ are small and constant
- Behaviour of the parton distributions (pdf's) at small–x: NNLO effects are reduced when pdf's are steep

S. Moch, J.A.M. Vermaseren, A. Vogt, PLB 606, 123 (2005)

F_L *in the dynamical parton model approach*

NLO dynamical pdf's are quite steep in the very small–x region already at rather low Q^2, and in fact steeper than their common standard counterparts

M. Glück, P. Jimenez-Delgado, E. Reya, EPJC 53, 355 (2008)

Reliable predictions of F_L:

NLO and NNLO dynamical parton model analysis of recent F_2 data (no F_L) at $x \lesssim 10^{-3}$

The extracted partons are used to predict F_L

A.D. Martin et al., PLB 635, 305 (2006)

- For comparison, standard pdf set with $Q_0^2 = 1.5$ GeV^2 taken from

M. Glück, C. P., E. Reya, EPJC 50, 29 (2007)

Determination of the parton distributions

- The valence $q_v = u_v$, d_v and sea $w = \bar{q}$, g distributions are parametrized at the input scale $Q_0^2 = 0.5$ GeV^2 as follows

$$
\begin{aligned}
x\, q_v(x, Q_0^2) &= N_{q_v} x^{a_{q_v}} (1-x)^{b_{q_v}} (1 + c_{q_v}\sqrt{x} + d_{q_v} x + e_{q_v} x^{1.5}) \\
x\, w(x, Q_0^2) &= N_w x^{a_w} (1-x)^{b_w} (1 + c_w\sqrt{x} + d_w x)
\end{aligned}
$$

- Sea breaking effects are not considered: $\bar{q} \equiv \bar{u} = \bar{d}$ and $s = \bar{s}$

- The normalizations N_{u_v}, N_{d_v} and N_g are fixed by $(\Sigma(x, Q^2) \equiv \Sigma_{q=u,d,s}(q + \bar{q}))$:

$$\int_0^1 u_v dx = 2, \qquad \int_0^1 d_v dx = 1, \qquad \int_0^1 x(\Sigma + g) dx = 1$$

- All Q^2-evolutions are performed in Mellin n-moment space, the program QCD-PEGASUS has been used for the NNLO evolutions

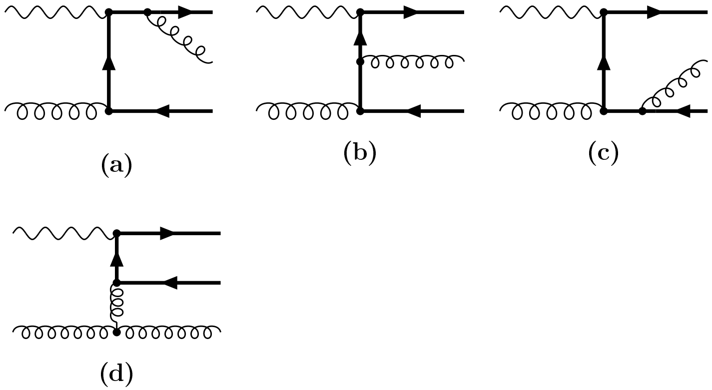
A. Vogt, Comput. Phys. Commun. 170, 65 (2005)

Heavy flavor contribution to $F_{2,L}(x, Q^2)$

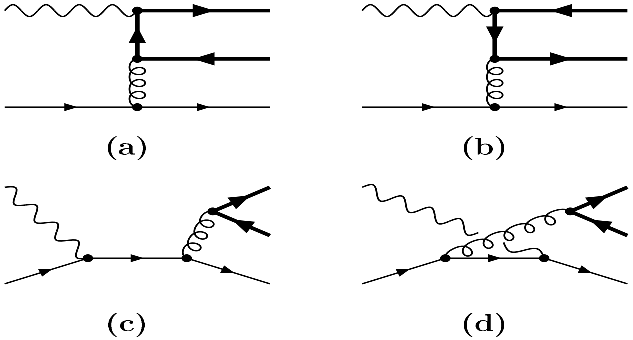
- The heavy flavor (charm) contribution F_2^c is taken as given by the fixed-order NLO perturbation theory

  Laenen, Riemersma, Smith, van Neerven, NP B392, 162 (1993)
  Riemersma, Smith, van Neerven, PL B347, 143 (1995)

- It is due to the NLO gluon bremsstrahlung process:



  to the Bethe-Heitler (a-b) and Compton processes (c-d):



  and to virtual corrections

The Fixed flavor scheme (FFS)

- In the FFS heavy quarks ($h = c$, b, t) are considered as external particles, not included among the partons in the colorless hadrons:
  h participate to DIS only via subprocesses like $\gamma^* g \to h\bar{h}$ rather than $\gamma^* h \to h$

- $\gamma^* h \to h$ is relevant in the variable flavor scheme (VFS), besides the light u, d, s quarks, the heavy quarks are also considered to form an intrinsic part of hadrons: considering both subprocesses would amount to double counting

We work in the FFS with $n_f = 3$: no resummations or “intrinsic” heavy quark distributions are needed, even at $Q^2 \gg m_h^2$

M. Glück, E. Reya, M. Stratmann, NPB 422, 37 (1994);
M. Glück, P. Jimenez-Delgado, E. Reya, EPJC 53, 355 (2008)

- A NNLO calculation of heavy quark production is not yet available in the FFS

- The small bottom contribution turns out to be negligible for our purposes

Parameter values

- The following data sets from DIS processes have been used:
  small-x and large-x H1 F_2^p data
  fixed target BCDMS data for F_2^p and F_2^n
  proton and deuteron NMC data
- Total of 740 data points; degrees of freedom dof = 720, χ^2 evaluated by adding in quadrature statistical and systematic errors

	NNLO				NLO			
	u_v	d_v	$\bar{q}$	g	u_v	d_v	$\bar{q}$	g
N	0.621	0.191	0.439	20.28	0.531	0.306	0.481	20.65
a	0.333	0.868	0.074	0.974	0.316	0.869	0.051	1.394
b	2.725	4.786	12.62	6.519	2.821	4.691	14.58	11.88
c	-9.059	65.36	2.212	—	-8.682	44.83	-2.262	15.88
d	53.55	1.622	7.745	—	54.99	-5.365	21.65	—
e	-36.98	-41.12	—	—	-40.09	-21.84	—	—
χ^2/dof	1.037				1.073			
$\alpha_s(M_Z^2)$	0.112				0.113			

- Standard fit to the same data: similar values of χ^2 and $\alpha_s(M_Z^2)$

M. Glück, C. P., E. Reya, EPJC 50, 29 (2007)

Update of the GRV98 pdf's

- The resulting NLO sea and gluon distributions are in agreement with the GJR ones, obtained from a global analysis

  M. Glück, P. Jimenez-Delgado, E. Reya, EPJC 53, 355 (2008)

- GJR: update of LO and NLO GRV98, including
  - New/improved data: improved HERA data
    new Tevatron Drell-Yan data
    high-E$_T$ jet data
  - Uncertainty estimates

- LO , NLO($\overline{\mathrm{MS}}$) and NLO(DIS) GJR dynamical pdf's are available at

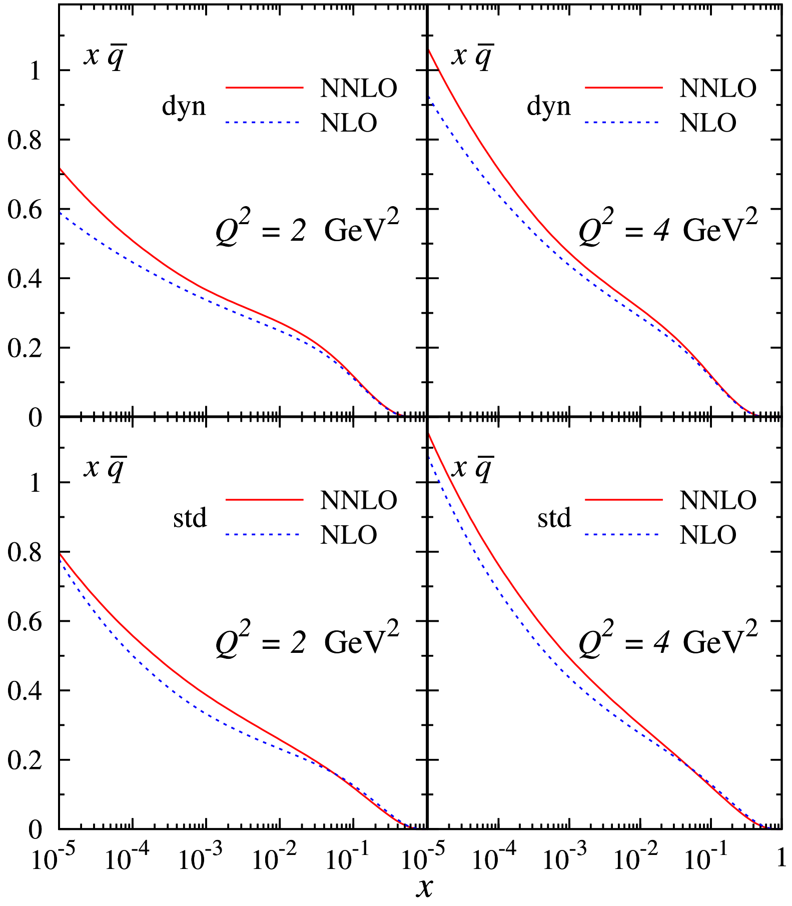
  http://doom.physik.uni-dortmund.de/pdfserver

- Compatible with GRV98

- NNLO in progress

Resulting sea distributions

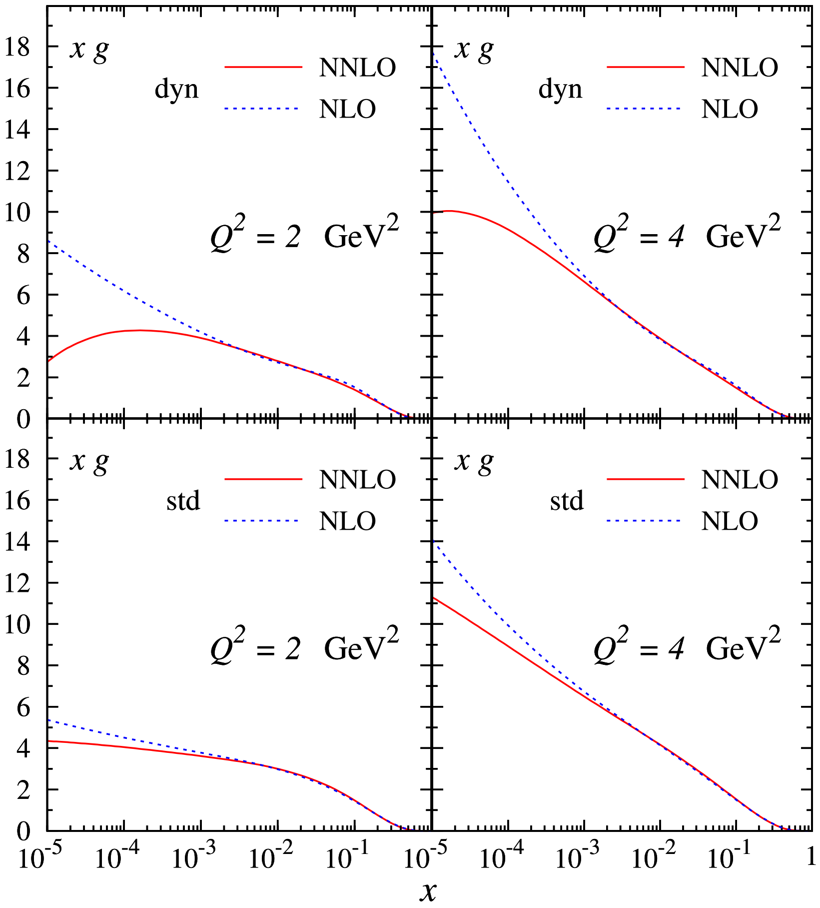
- The dynamical (dyn) NLO sea distribution has a similar small–x dependence as the standard (std) one: the valence–like sea input vanishes very slowly as $x \to 0$ (corresponding to a small value of $a_{\bar{q}}$, $a_{\bar{q}} \simeq 0.05$)



- The NNLO sea distribution $x\bar{q}$ is larger (steeper) than the NLO one

Resulting gluon distributions

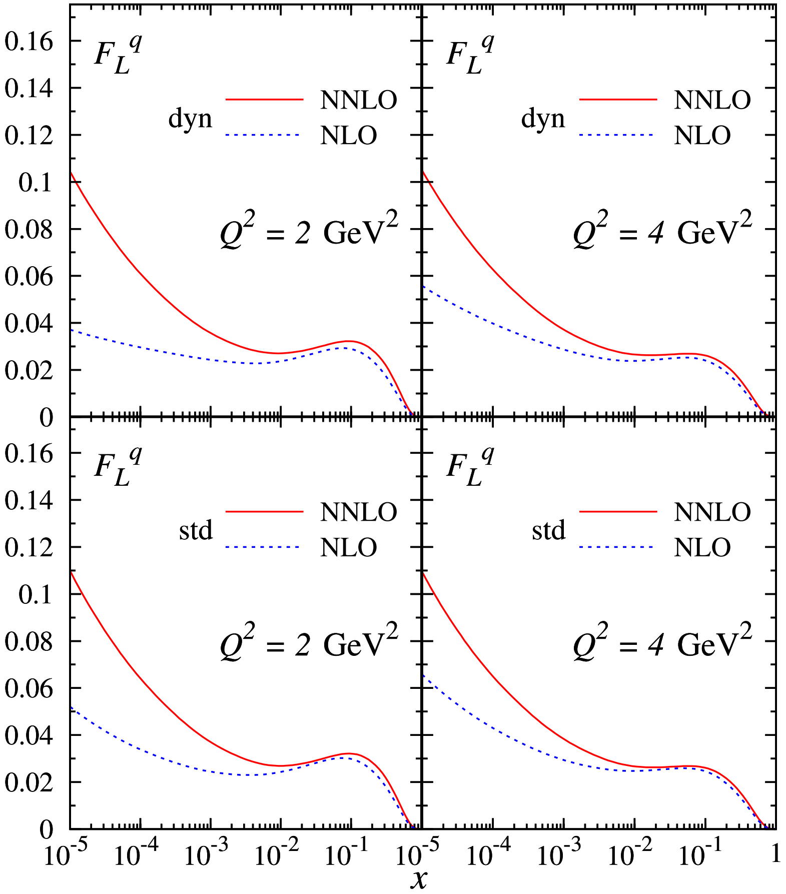
- The dynamically generated (dyn) NLO gluon is steeper as $x \longrightarrow 0$ than the gluon distribution obtained from conventional ‘standard’ (std) fits



- At NNLO the gluon distribution xg is flatter as x decreases and, in general, falls below the NLO one in the small–x region

Light quark contribution F_L^q to F_L

- The dynamical NLO and NNLO sea distributions have a rather similar small–x dependence as the standard ones

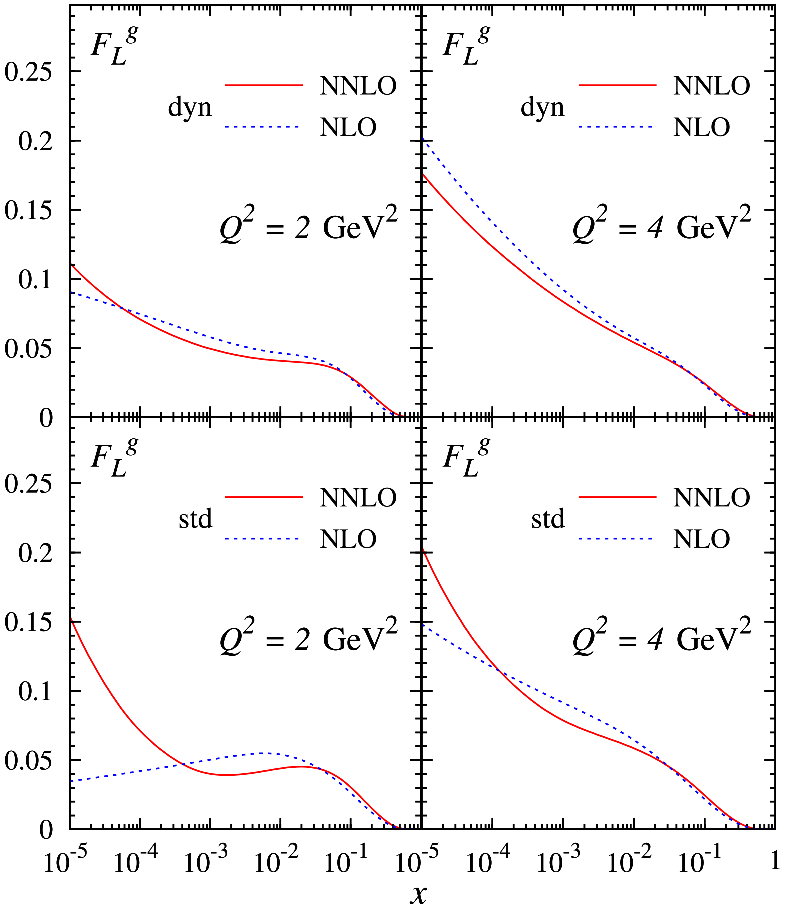


$$F_L^q + F_L^g = F_L - F_L^c$$

- The perturbative instability of the subdominant quark contribution F_L^q as obtained in a standard fit does not improve for the dynamical (sea) quark distributions

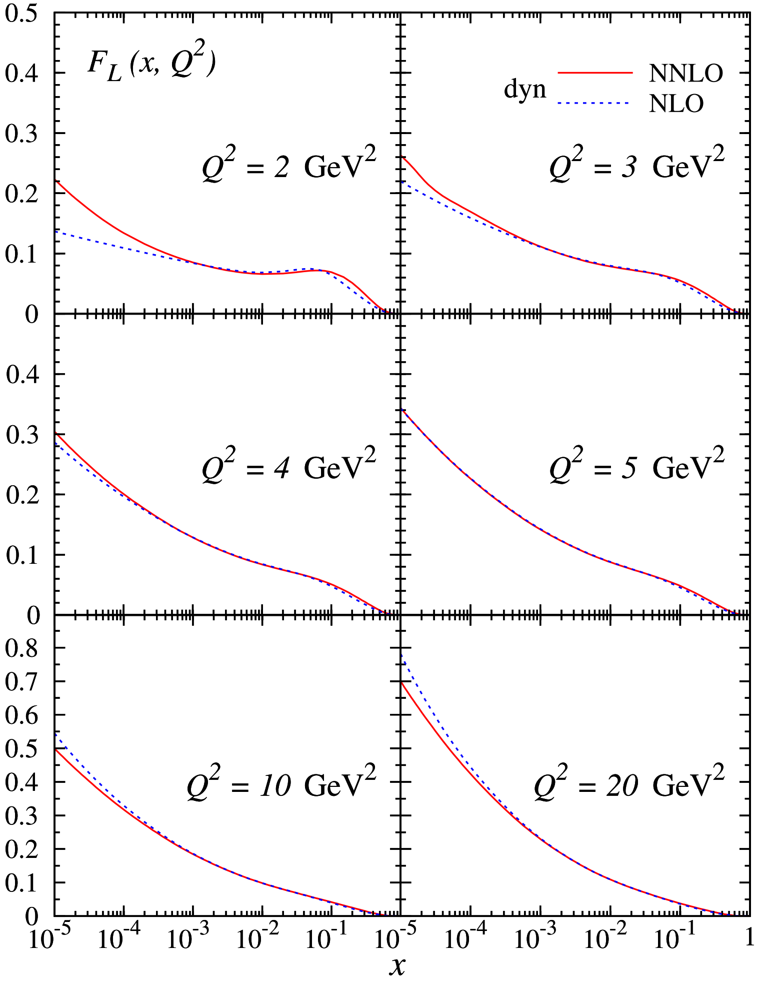
Gluon contribution F_L^g to F_L

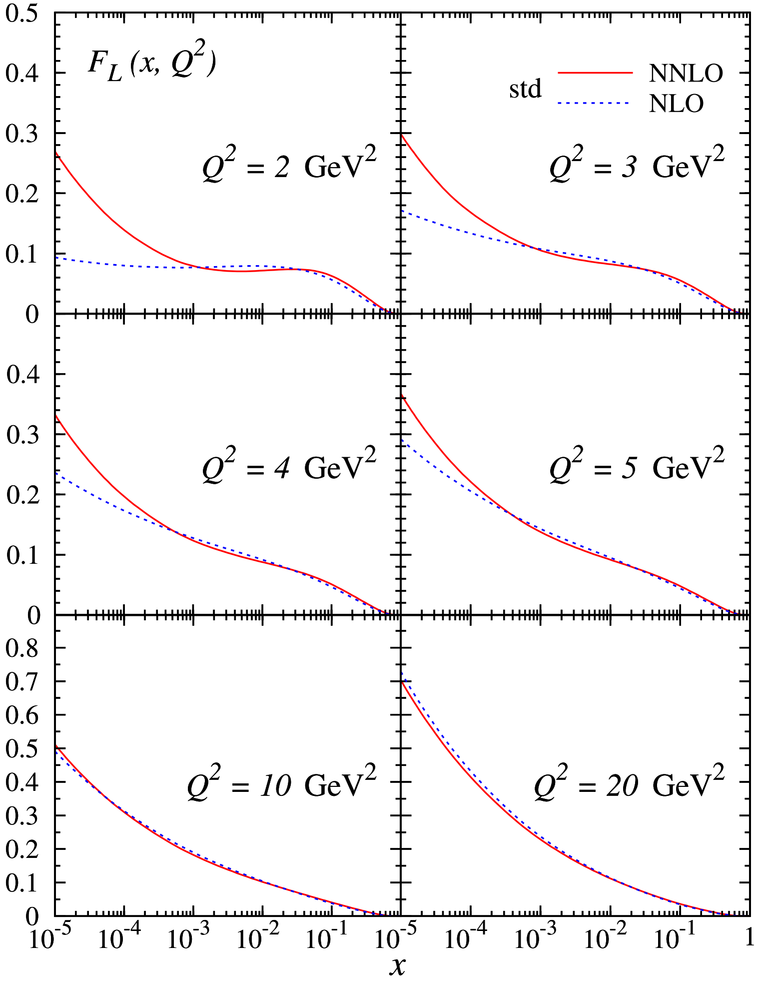
- The instability disappears almost entirely for the dominant dynamical gluon contribution already at $Q^2 \simeq 2$ GeV^2



$$F_L^g = \frac{2}{9} x\, C_{L,g} \otimes g$$

Dynamical predictions for the total F_L





- The dynamical predictions become perturbatively stable already at the relevant low values of $Q^2 \gtrsim \mathcal{O}(2-3 \text{ GeV}^2)$
- Standard results: stability has not been fully reached even at $Q^2 = 5$ GeV^2

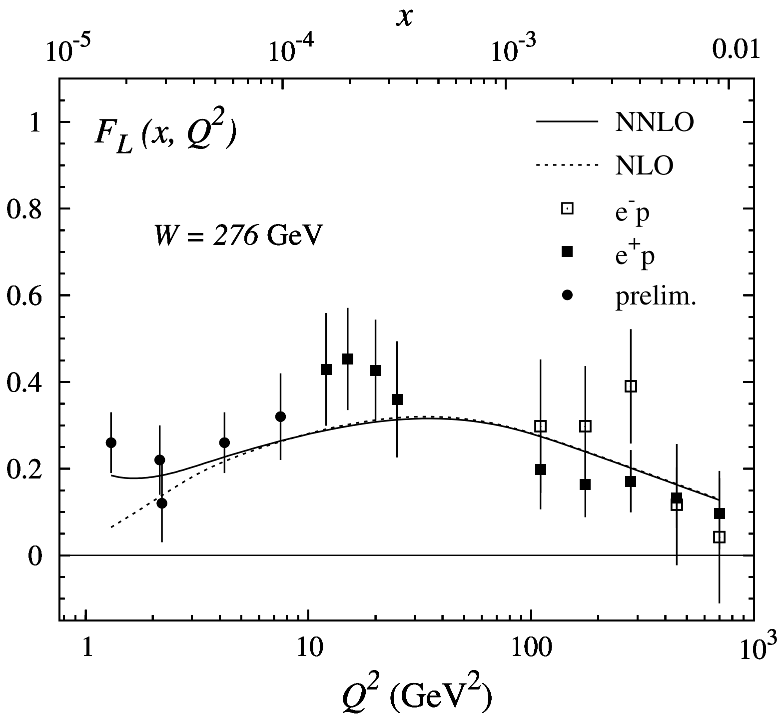
Comments on the results

- For $Q^2 \lesssim 2$ GeV^2, nonperturbative (higher twist) contributions to F_L and F_2 become relevant. The dynamical NLO twist–2 fit slightly undershoots the HERA data for F_2 at $Q^2 \simeq 2$ GeV^2 in the small–x region

- The NLO/NNLO instabilities implied by the standard fit results by Martin et al., are even more violent. This is mainly due to their negative gluon (negative $F_L(x, Q^2)$)

- The perturbative stability in any scenario becomes in general better the larger Q^2, typically beyond 5 GeV^2: Q^2–evolutions eventually force any parton distribution to become sufficiently steep in x

Dynamical predictions for F_L vs HERA-H1 data



- Dynamical, leading twist, results are in full agreement with present measurements
- Data in contrast to expectations based on negative parton distributions and structure functions at small values of x
- F_L: positive defined at $Q^2 \geq \mu^2 = 0.5$ GeV^2, although leading twist– 2 predictions need not necessarily be confronted with data below $\simeq$ 2 GeV^2

Summary and conclusions

- NNLO and NLO dynamical parton distributions are determined from an analysis of recent DIS data
- The extreme perturbative NNLO/NLO instability of F_L at low Q^2, noted by Martin et al., is not an indication of a genuine problem of pQCD
- It is an artefact of the commonly utilized standard gluon distributions
- It is reduced considerably already at $Q^2 = 2 - 3$ GeV^2 when utilizing dynamical pdf's at NLO and NNLO
- Stability of F_L: an advantage of the dynamical parton model approach to pQCD!