Heavy Quarks

... lessons we've learned

& future applications

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Conspirators:
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J.-Y. Yu, J. Owens, J. Morfin, W. Tung, C. Keppel, ...

Ringberg Workshop
6 October 2008

m = 0: Massless case.

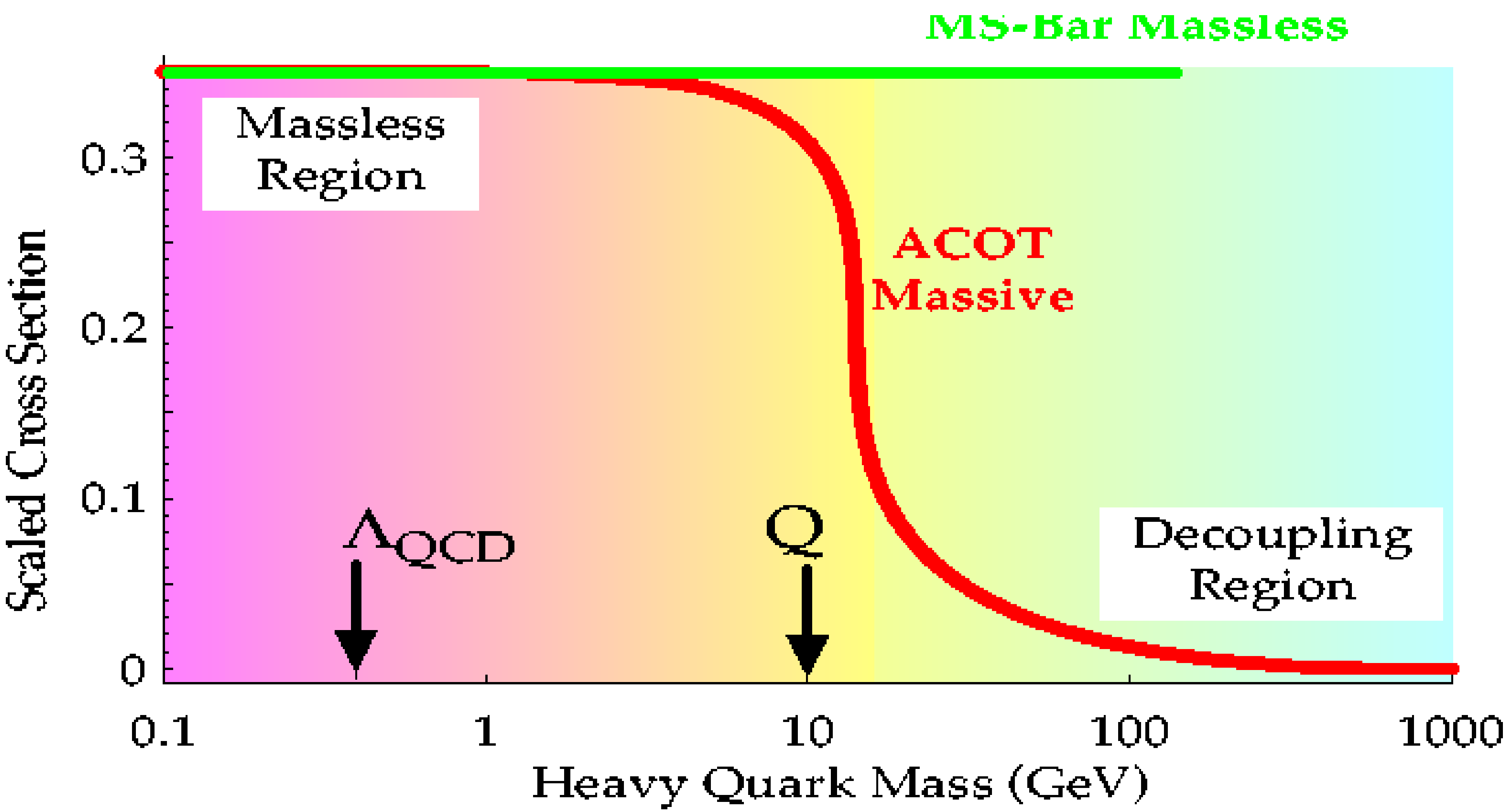
Mass plays no dynamic role

Well understood.

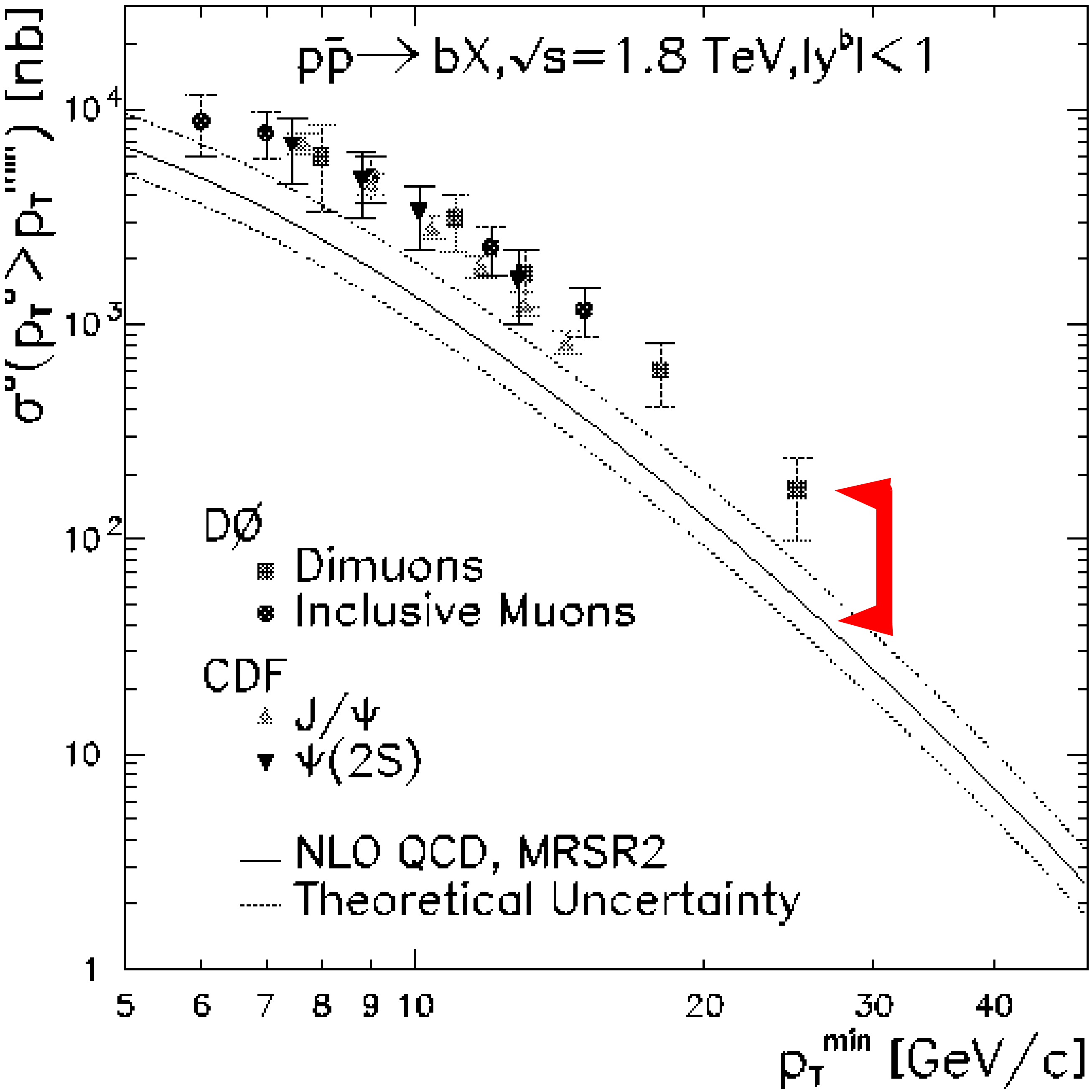
m = ∞: Infinite case.

Mass Decouples.

We can forget about this object



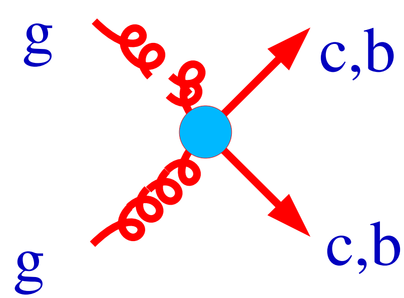
HISTORY

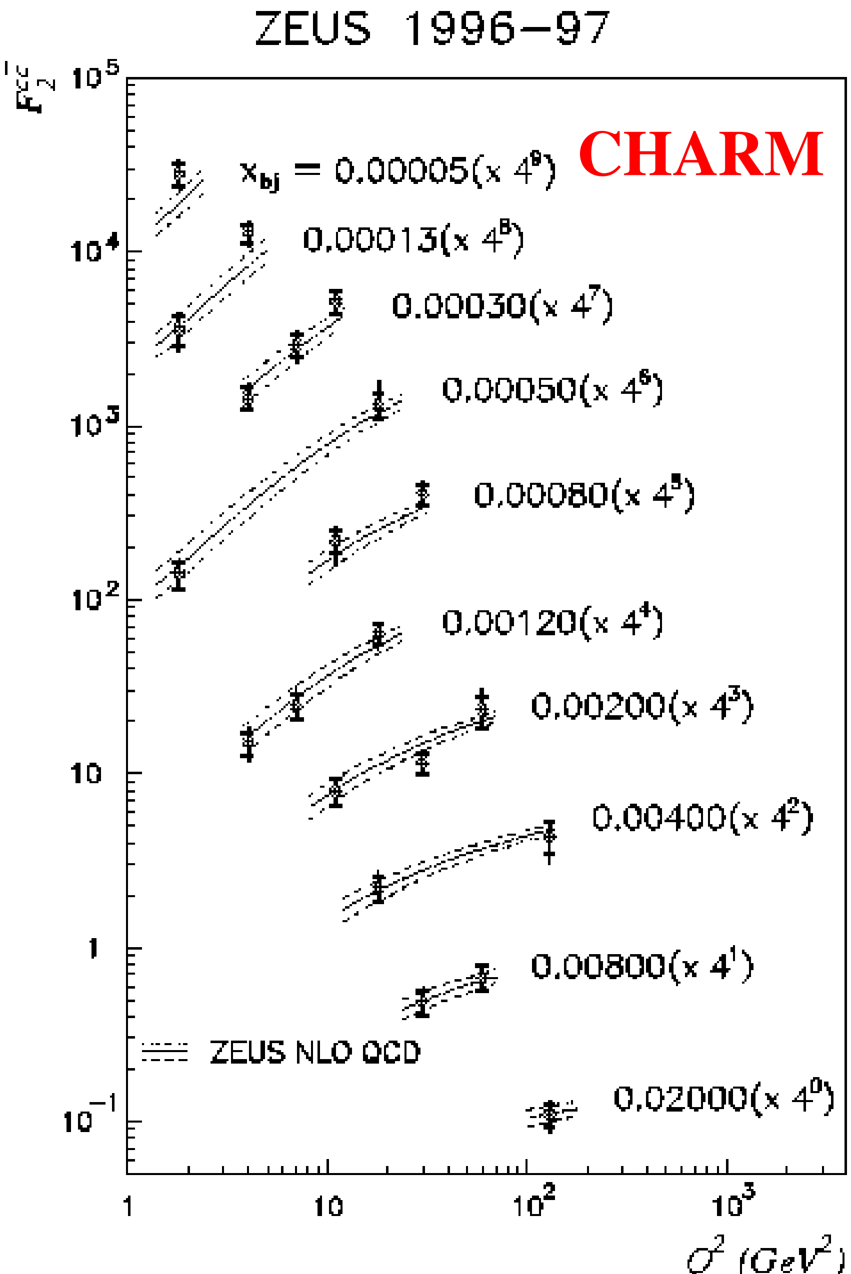


Comparison of Run I data with NLO Theory

Data is high by factor of 2x or 3x

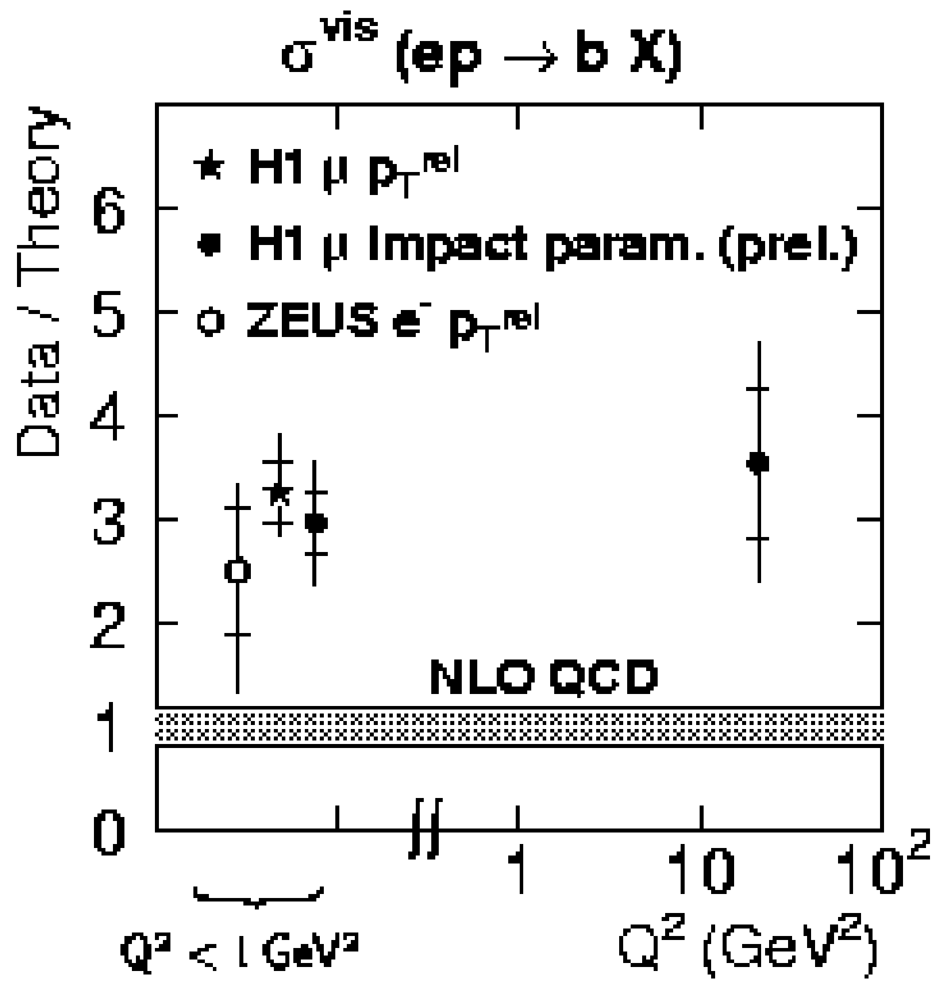
... even given μ variation



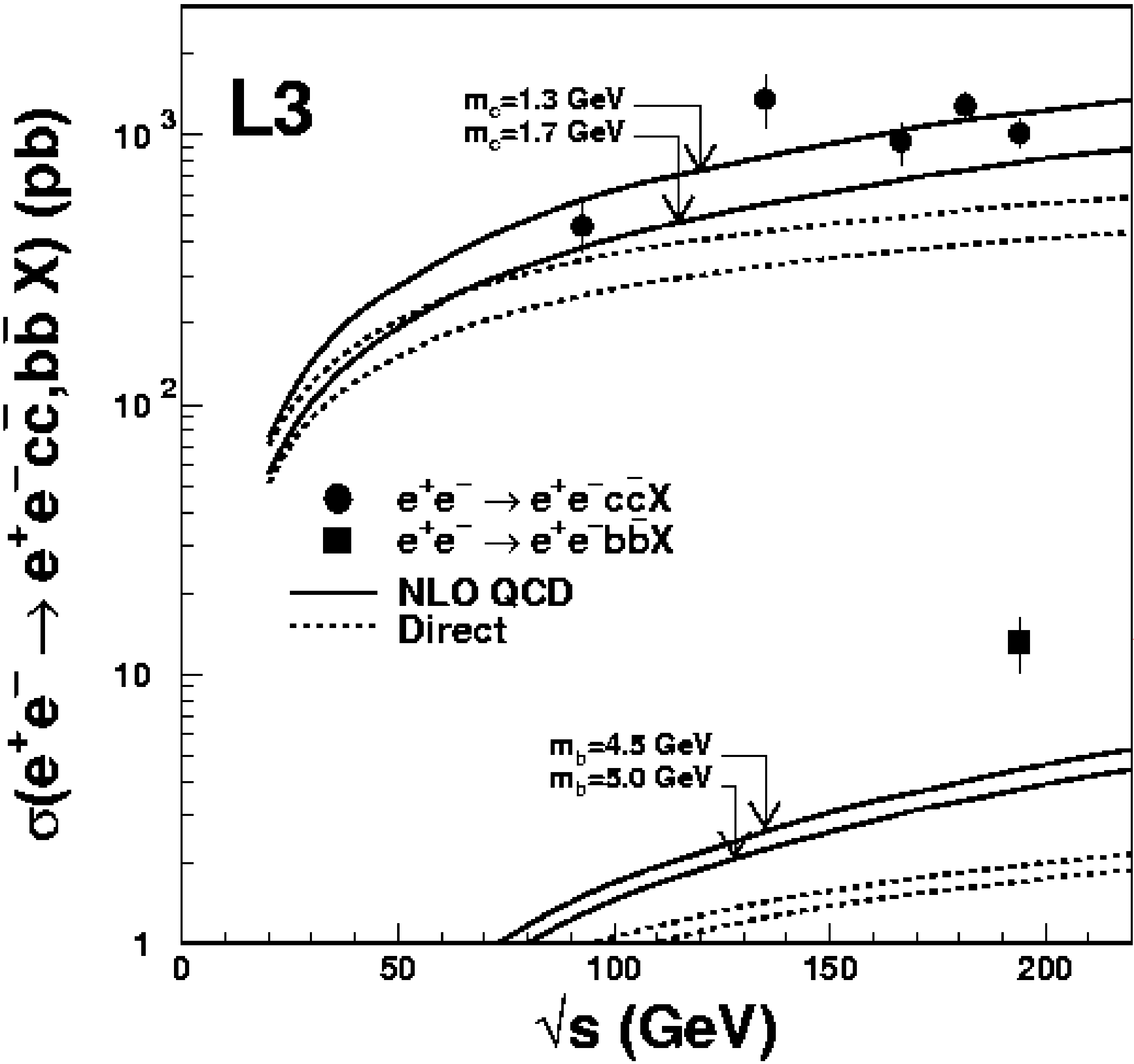


Charm production matches
well with NLO calculation

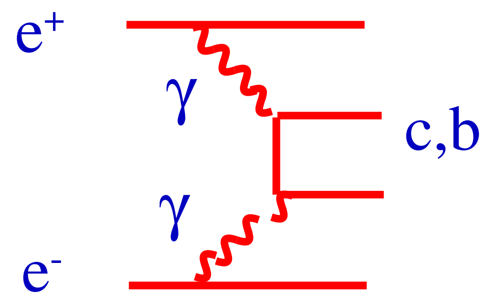
BEAUTY



Data is high by factor
of 2x or 3x



Comparison of LEP data with NLO Theory



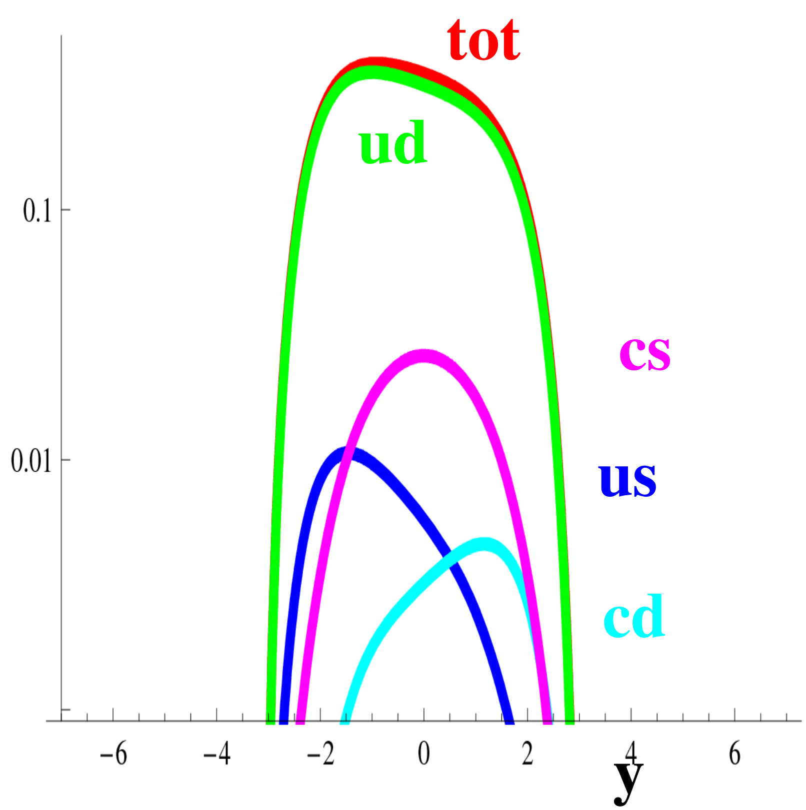
Charm is reasonable

Bottom data is high by factor of 2x or 3x

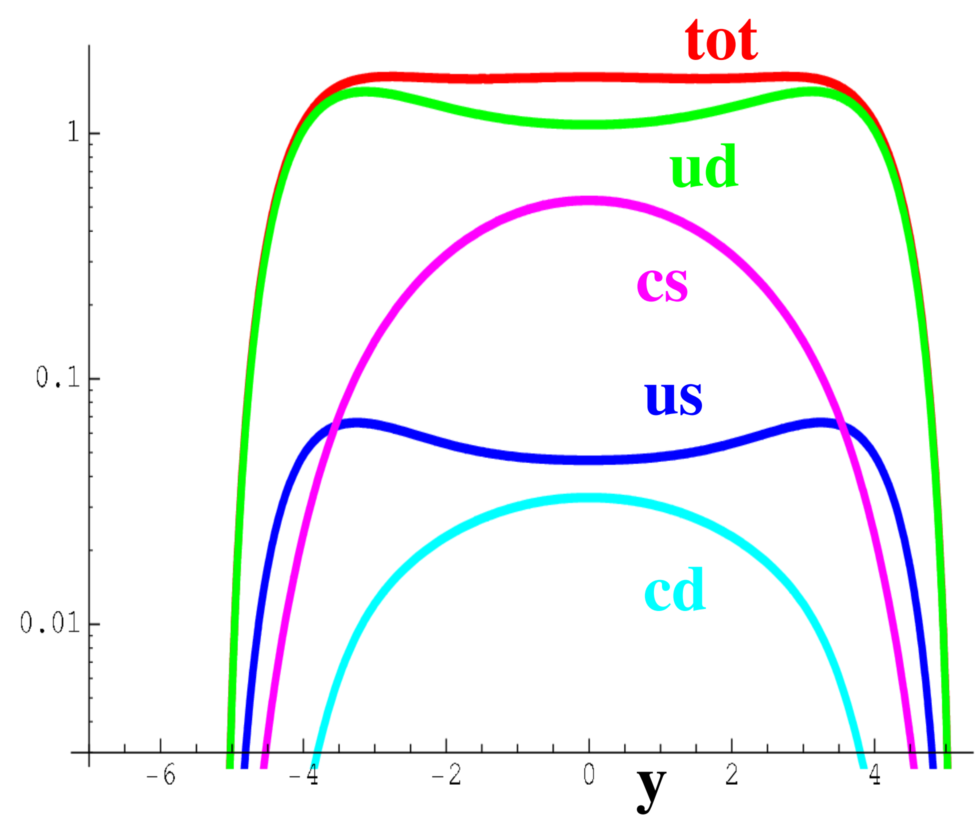
FUTURE

W & Z at LHC: *"Heavy Quarks" play important role*

dσ/dy(W^+) at Tevatron



dσ/dy(W^+) at LHC



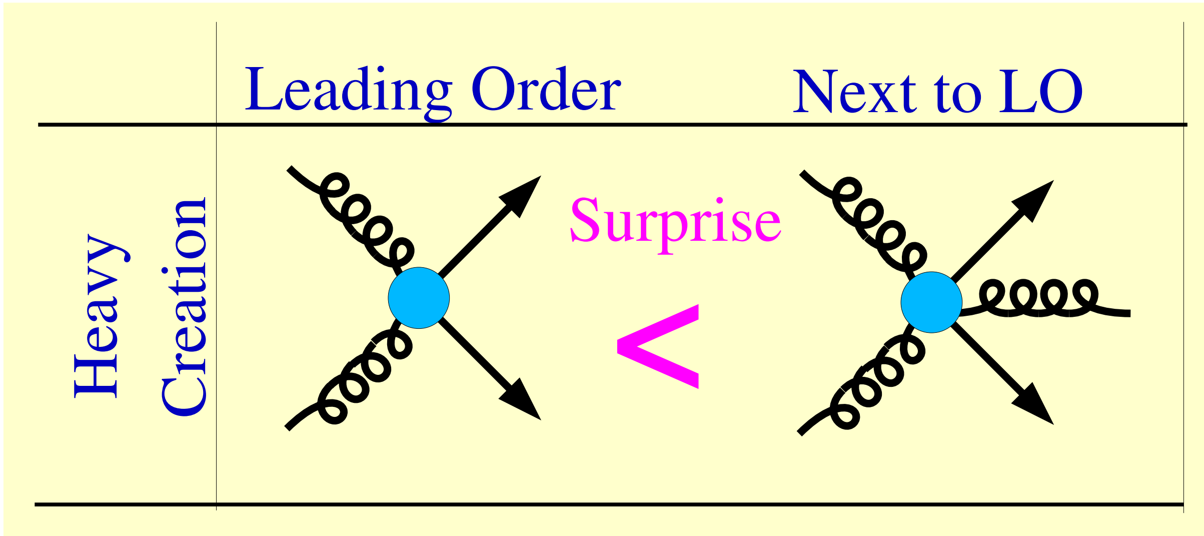
- Larger fraction of heavy quarks
- W/Z are "Benchmark" Cross Sections

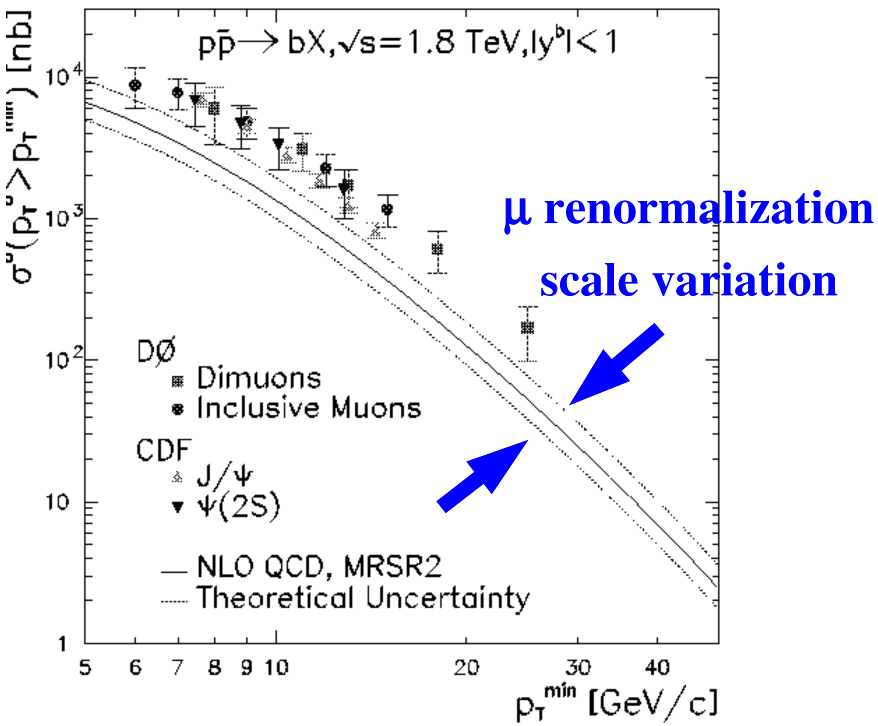
... will be measured in early run

HEAVY is a relative term

Calculating Heavy Quarks

Part I





Surprise:

NLO / LO Big

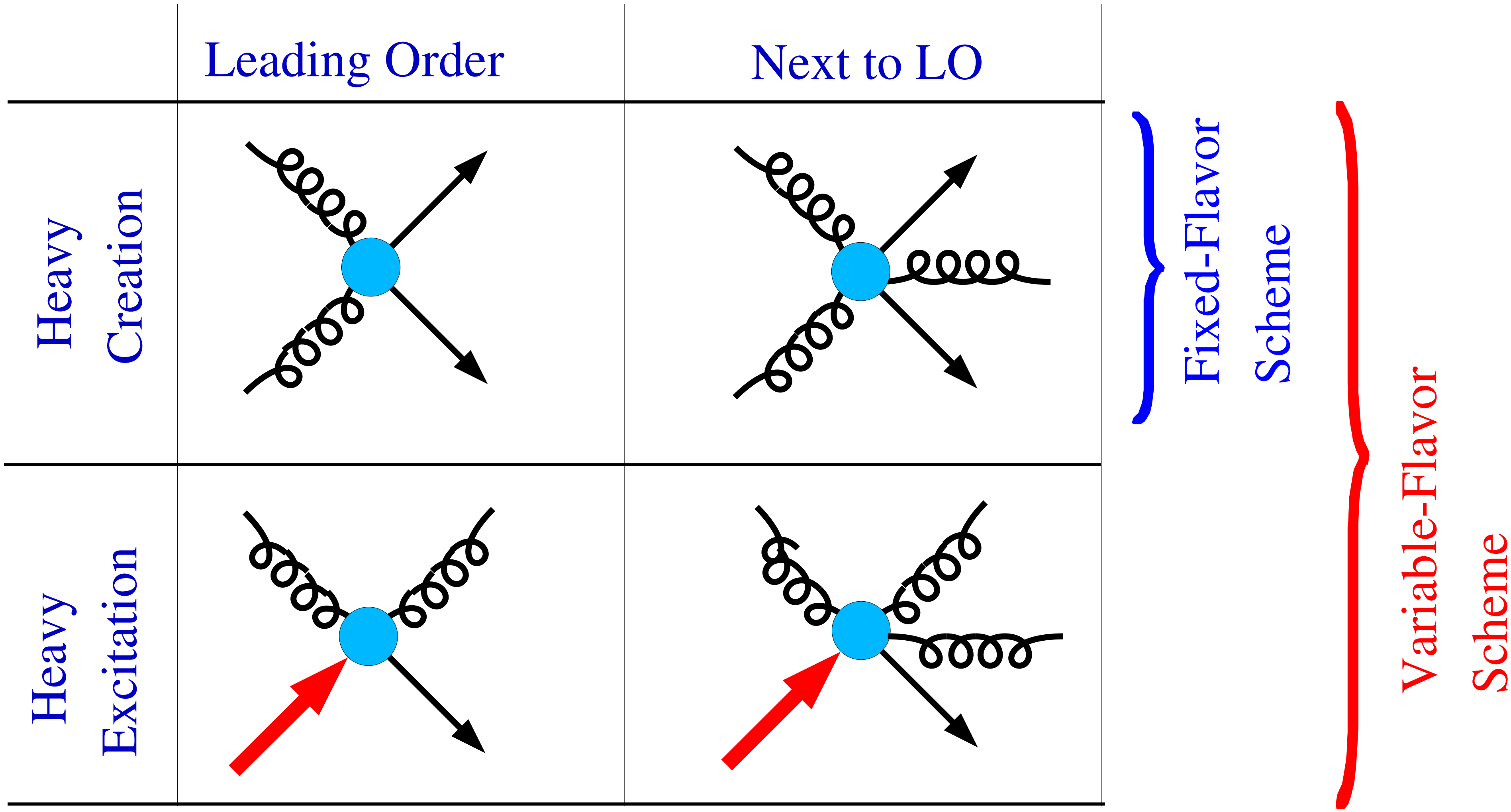
But, ...

theory still

below data

Nason, Dawson, Ellis

Beenakker, Kuijf, Van Neerven, Smith



heavy quark is integral to the proton

Calculating Heavy Quarks

Part II

“state of the art”

VFN: Variable Flavor Number: Introduce new partonic components when the scale μ exceeds the heavy quark mass.

E.g., charm and bottom are included in the proton at high enough scales

Massless MS-bar Evolution:

Evolution kernels in DGLAP are Mass-Independent: $\partial f \sim P \otimes f$.

ACOT *(Aivazis, Collins, Olness, Tung)* A general framework for including the heavy quark components. *Phys.Rev.D50:3102-3118,1994.*

S-ACOT *(Simplified-ACOT)* ACOT with the simplification that initial-state heavy quark masses can be set to zero. *Phys.Rev.D62:096007,2000.*

χ-Prescription: ACOT-χ & S-ACOT-χ:

As above with a generalized slow-rescaling $x \to x(1+(m_1+m_2)^2/Q^2)$

... *see:* R.S. Thorne, W.K. Tung, arXiv:0809.0714 [hep-ph]

Problem: Heavy Quark introduces new scale: $\log(\frac{Q^2}{\mu^2})$ $\log(\frac{M_H^2}{\mu^2})$

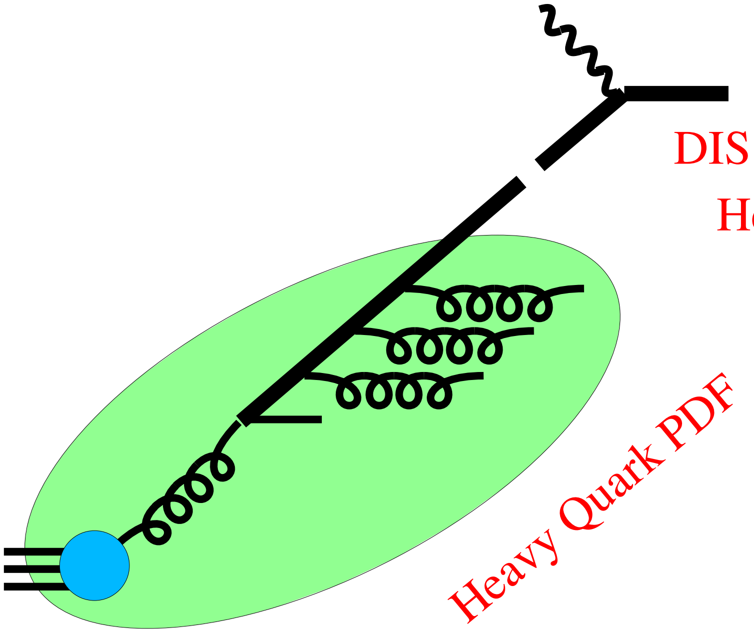
... life gets interesting.

Solution: Resum $Log(M_H)$ in the Heavy Quark PDF's:

... i.e., as in the ACOT renormalization scheme

ACOT, PRD 50, 3102

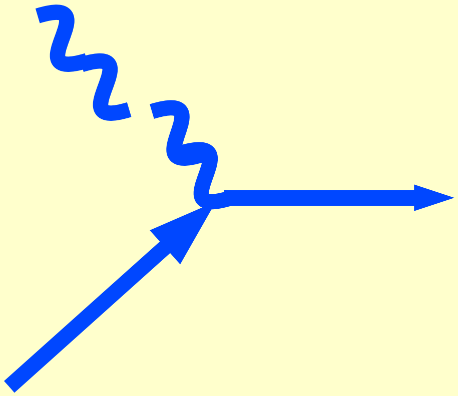
DGLAP equation
Resums iterative splittings
inside the proton



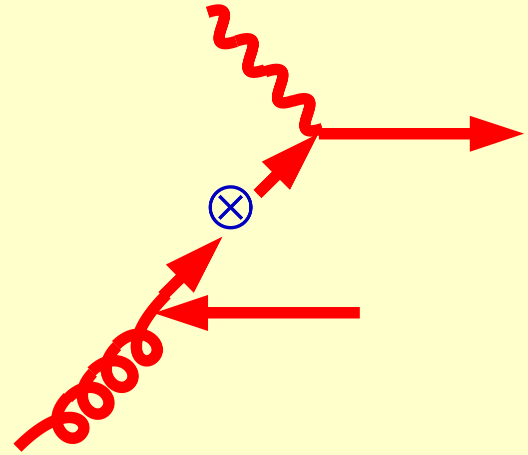
Result: We can describe the full kinematic range from low to high

implemented in the CTEQ6HQ PDF's with finite M_Q

Massless MS-Bar Evolution: Why it is valid



$$\int f(P\to a)\otimes\sigma(a\to c)$$



$$\int f(P\to g)\otimes{}^{1}P(g\to a)\otimes\sigma(a\to c)$$

near threshold (M_H~Q) $\approx\ f(P\to g)\otimes{}^{1}P(g\to a)$

${}^{1}P$ splittings must match

Near threshold(M_H~Q), mass effects cancel between HE and SUB

Above threshold(M_H<<Q), mass effects can be ignored

Choice of DGLAP Kernels is a Scheme Choice!!!

This is NOT an approximation

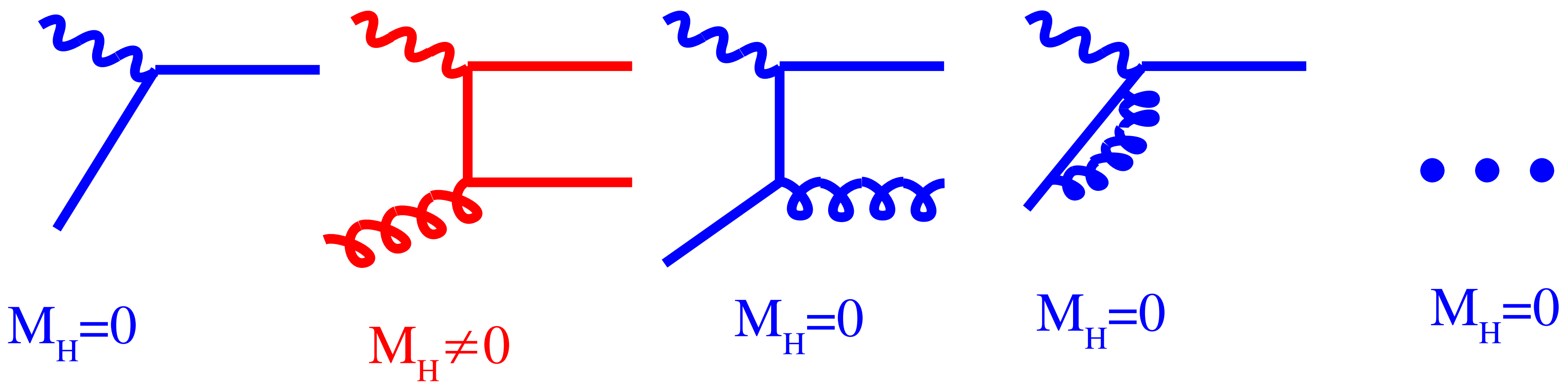
Simplified-ACOT Scheme: S-ACOT

Development: Factorization proof extended to Heavy Quark case.

Collins (1998)

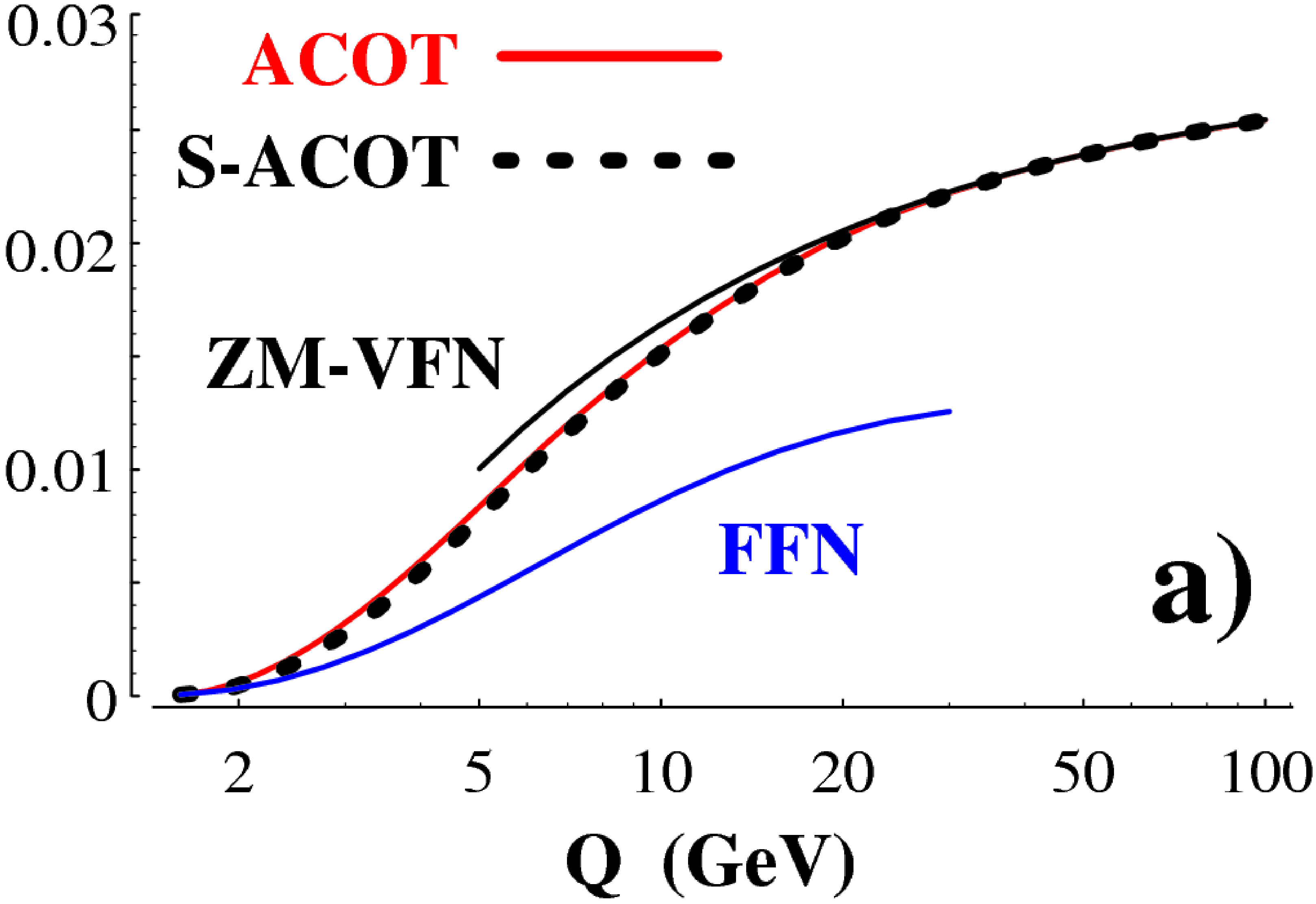
Observation: Simplified-ACOT Scheme: Set M_H=0 on incoming HQ lines

Kramer, Olness, Soper (2000)



Result: 1) Comparable Numerics & 2) Simpler Calculations

Simplified result numerically comparable to full result



... what about the analytic result???

Which would you prefer to calculate?

ACOT

$$
\begin{aligned}
f_1^Q(\hat{s}_1) &= \frac{8}{\Delta'^2}\Big\{ -\Delta^2(S_+\Sigma_{++} - 2m_1m_2S_-)I_{\xi'} + 2m_1m_2S_-\Big(\frac{1}{\hat{s}_1}[\Delta'^2 + 4m_2^2\Sigma_{+-}] \\
&+ 2\Sigma_{+-} - \Sigma_{-+} + \frac{\Sigma_{++}+\hat{s}_1}{2} + \frac{\hat{s}_1+m_2^2}{\Delta'\hat{s}_1}\left[\Delta'^2 + 2\Sigma_{+-}\Sigma_{++} + \left(m_2^2+Q^2\right)\hat{s}_1\right] L_{\xi'}\Big) \\
&+ S_+\Big(\frac{-m_2^2\Sigma_{++}}{(\hat{s}_1+m_2^2)\hat{s}_1}(\Delta^2 + 4m_2^2\Sigma_{+-}) - \frac{1}{4(\hat{s}_1+m_2^2)}\Big[3\Sigma_{++}^2\Sigma_{-+} + 4m_2^2(10\Sigma_{++}\Sigma_{+-} \\
&- \Sigma_{+-}\Sigma_{-+} - m_1^2\Sigma_{++}) + \hat{s}_1[-7\Sigma_{++}\Sigma_{-+} + 18\Delta^2 - 4m_1^2(7Q^2 - 4m_2^2 + 7m_1^2)] \\
&+ 3\hat{s}_1^2[\Sigma_{+-} - 2m_1^2] - \hat{s}_1^3\Big] + \frac{\hat{s}_1+m_2^2}{2\Delta'}\Big[\frac{-2}{\hat{s}_1}\Sigma_{++}\left(\Delta^2 + 2\Sigma_{+-}\Sigma_{++}\right) \\
&+ \left(4m_1^2m_2^2 - 7\Sigma_{+-}\Sigma_{++}\right) - 4\Sigma_{+-}\hat{s}_1 - \hat{s}_1^2\Big] L_{\xi'}\Big)\Big\} \\
f_2^Q(\hat{s}_1) &= \frac{16}{\Delta'^4}\Big\{ -2\Delta^4 S_+ I_{\xi'} + 2m_1m_2S_-\Big(\frac{\hat{s}_1+m_2^2}{\Delta'}\left(\Delta'^2 - 6m_1^2Q^2\right) L_{\xi'} \\
&- \frac{\Delta'^2(\hat{s}_1+\Sigma_{++})}{2(\hat{s}_1+m_2^2)} + \left(2\Delta'^2 - 3Q^2\left(\hat{s}_1+\Sigma_{++}\right)\right)\Big) + S_+\Big(-2(\Delta^2 - 6m_1^2Q^2)(\hat{s}_1+m_2^2) \\
&- 2\left(m_1^2+m_2^2\right)\hat{s}_1^2 - 9m_2^2\Sigma_{+-}^2 + \Delta^2\left(2\Sigma_{++} - m_2^2\right) + 2\hat{s}_1\left(2\Delta^2 + \left(m_1^2 - 5m_2^2\right)\Sigma_{+-}\right) \\
&+ \frac{\left(\Delta'^2 - 6Q^2\left(m_2^2+\hat{s}_1\right)\right)\Sigma_{++}\left(\hat{s}_1+\Sigma_{++}\right)}{2(\hat{s}_1+m_2^2)} - \frac{2\Delta^2}{\hat{s}_1}\left(\Delta^2 + 2(2m_2^2+\hat{s}_1)\Sigma_{+-}\right) \\
&+ \frac{(\hat{s}_1+m_2^2)}{\Delta'}\Big[\frac{-2}{\hat{s}_1}\Delta^2(\Delta^2 + 2\Sigma_{+-}\Sigma_{++}) - 2\hat{s}_1(\Delta^2 - 6m_1^2Q^2) \\
&- (\Delta'^2 - 18m_1^2Q^2)\Sigma_{++} - 2\Delta^2(\Sigma_{++} + 2\Sigma_{+-})\Big] L_{\xi'}\Big)\Big\} \\
f_3^Q(\hat{s}_1) &= \frac{16}{\Delta'^2}\Big\{ -2\Delta^2 R_+ I_{\xi'} + 2m_1m_2R_-\Big(1 - \frac{\Sigma_{-+}}{\hat{s}_1} + \frac{(\hat{s}_1+m_2^2)(\hat{s}_1+\Sigma_{+-})}{\Delta'\hat{s}_1} L_{\xi'}\Big) \\
&+ R_+\Big(\Sigma_{-+} - 3\Sigma_{+-} - \frac{2}{\hat{s}_1}\left(\Delta^2 + 2m_2^2\Sigma_{+-}\right) - \frac{(\hat{s}_1 - \Sigma_{-+})(\hat{s}_1+\Sigma_{++})}{2(\hat{s}_1+m_2^2)} \\
&+ \frac{\hat{s}_1+m_2^2}{\Delta'\hat{s}_1}\left[-\hat{s}_1^2 + 4\left(m_1^2\Sigma_{-+} - \Delta^2\right) - 3\hat{s}_1\Sigma_{+-}\right] L_{\xi'}\Big)\Big\}
\end{aligned}
$$

with

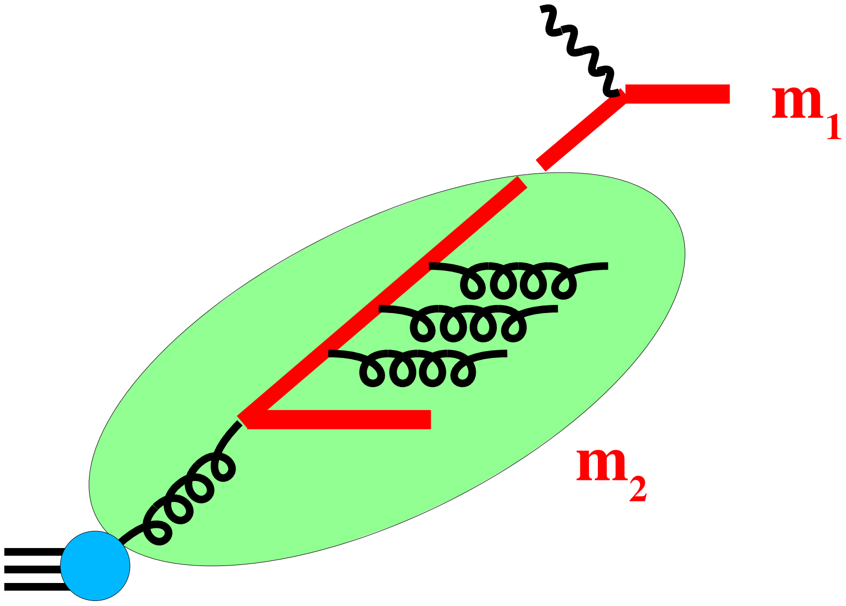
$$L_{\xi'} \equiv \ln\left(\frac{\Sigma_{++} + \hat{s}_1 - \Delta'}{\Sigma_{++} + \hat{s}_1 + \Delta'}\right)$$

and

$$I_{\xi'} = \left(\frac{\hat{s}_1 + 2m_2^2}{\hat{s}_1^2} + \frac{\hat{s}_1 + m_2^2}{\Delta'\hat{s}_1^2}\Sigma_{++} L_{\xi'}\right) .$$

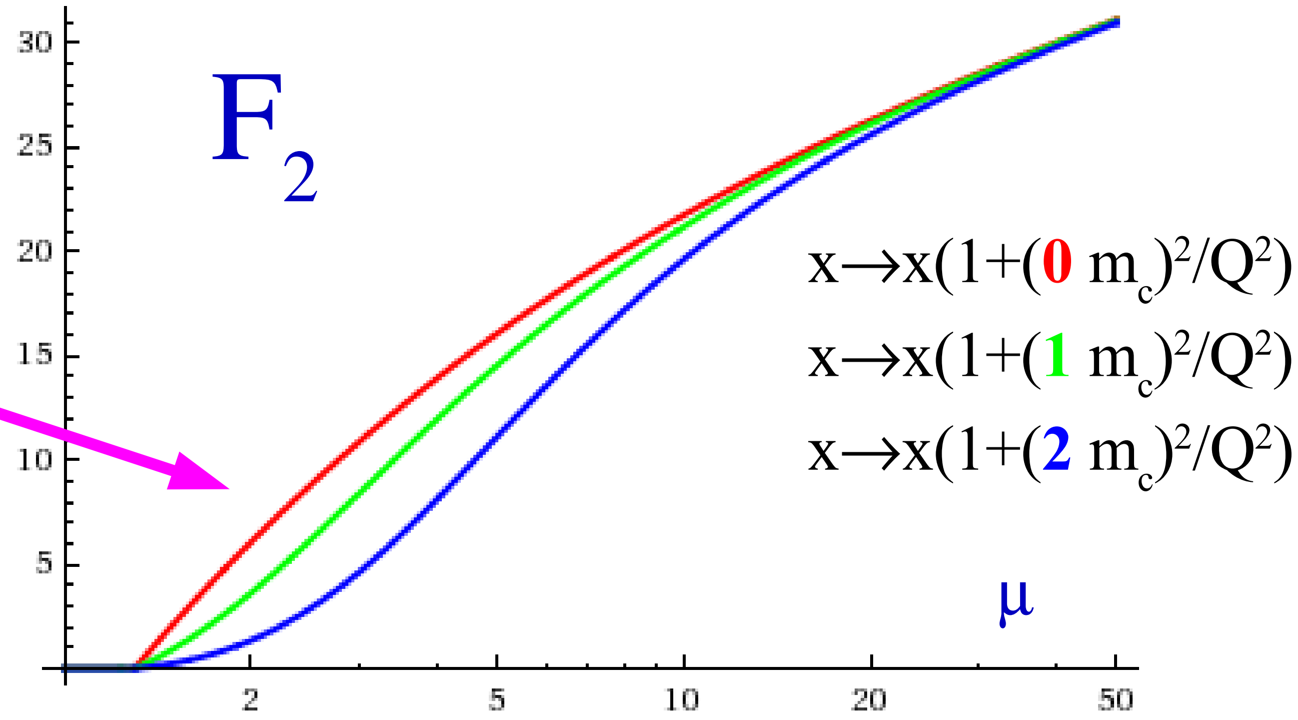
S-ACOT

$$
\begin{aligned}
C_2^{(Vq)(1)} &= C_2(F)\frac{x}{2}\left[\frac{1+x^2}{1-x}\left(\ln\frac{(1-x)}{x} - \frac{3}{4}\right) + \frac{1}{4}(9+5x)\right]_+ , \\
C_1^{(Vq)(1)} &= \frac{1}{2x}C_2^{(Vq)(1)} - C_2(F)\frac{1}{2}x , \\
C_3^{(Vq)(1)} &= \frac{1}{x}C_2^{(Vq)(1)} - C_2(F)(1+x) ,
\end{aligned}
$$

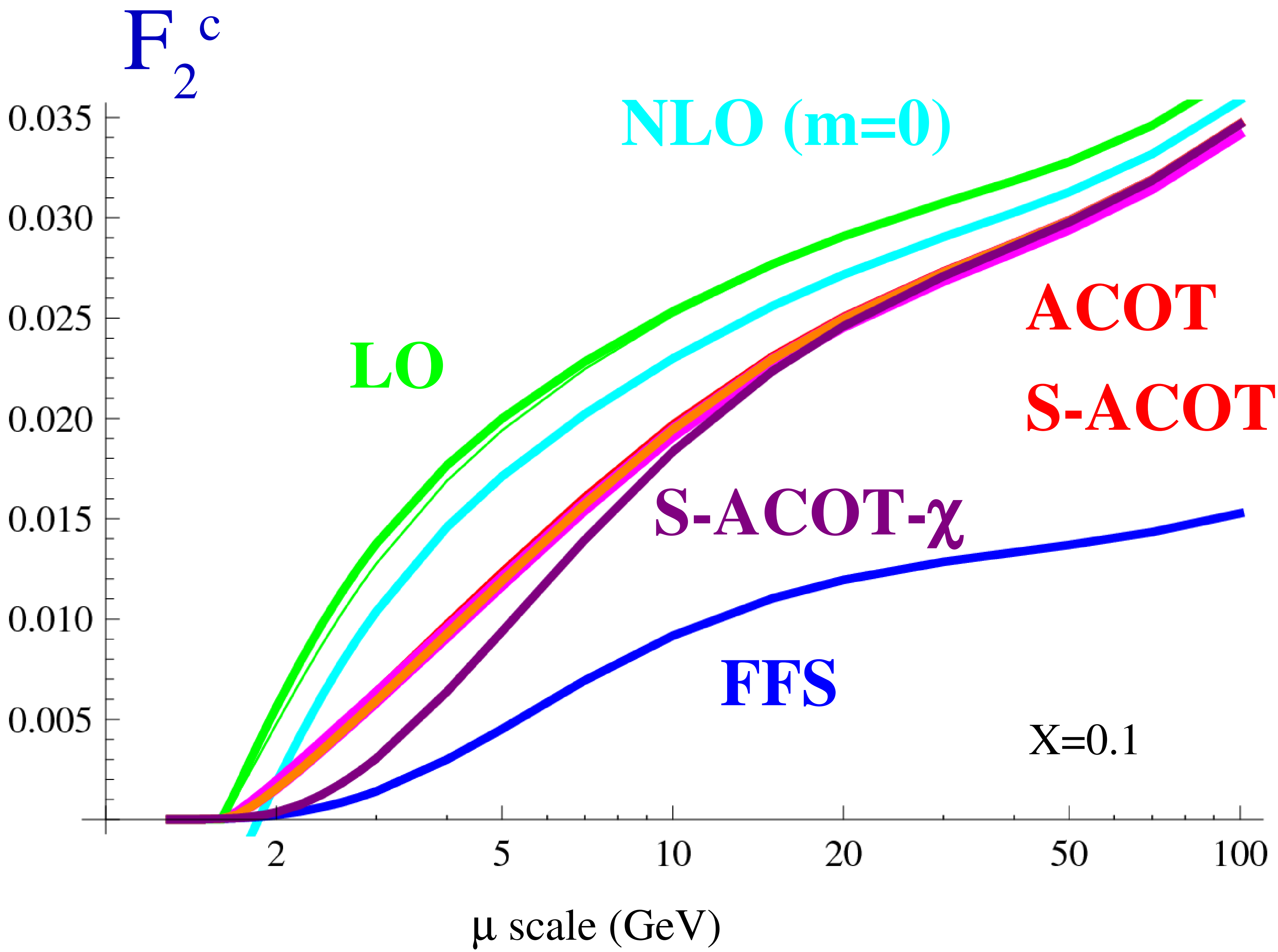


$$\chi = x\left[1+\frac{(m_1+m_2)^2}{Q^2}\right]$$

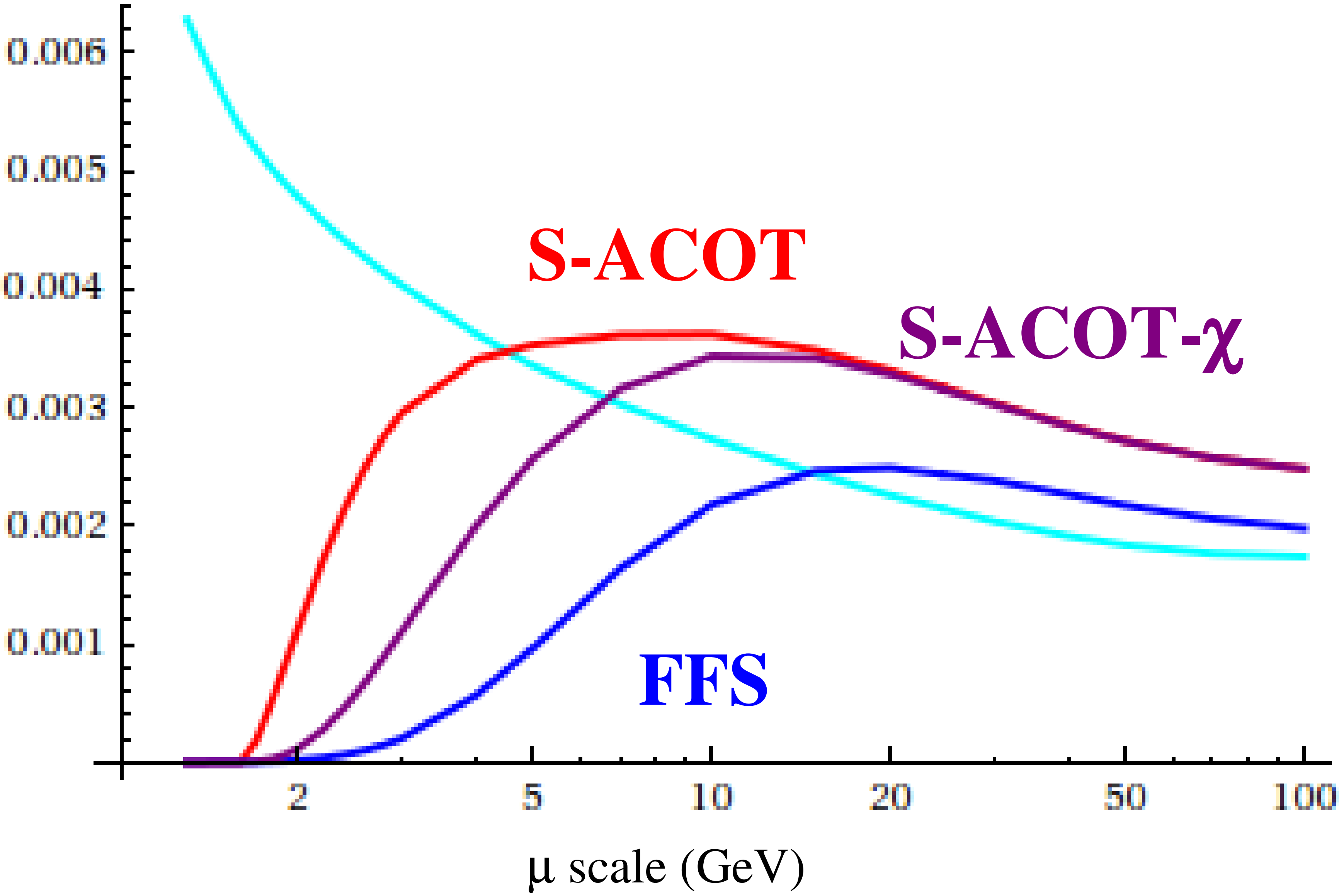
Kinematic suppression at threshold



Results

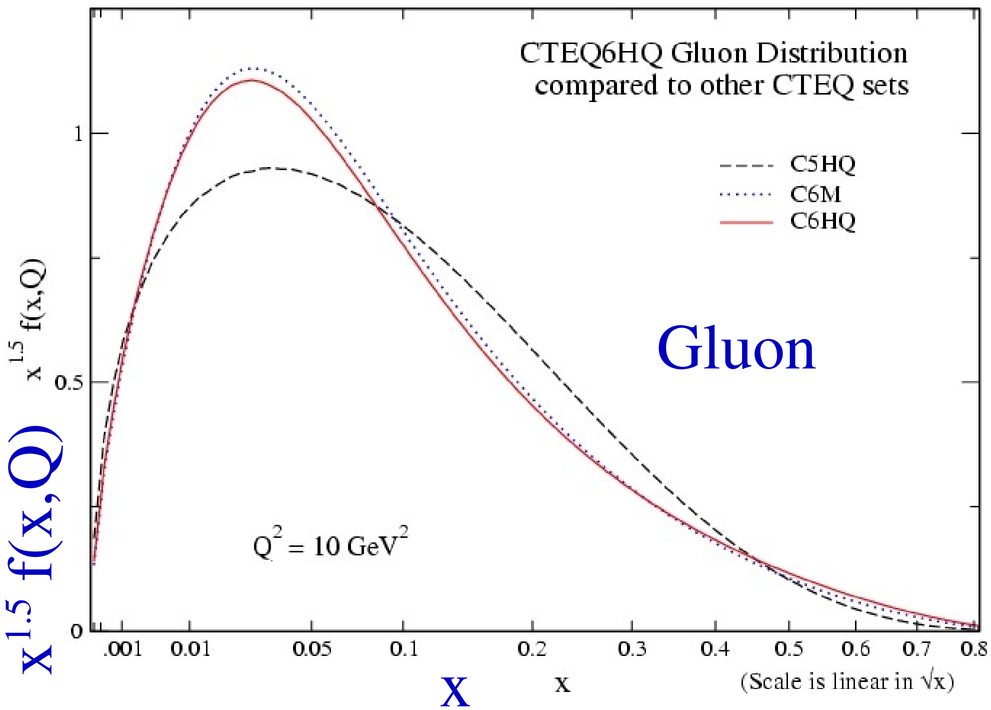


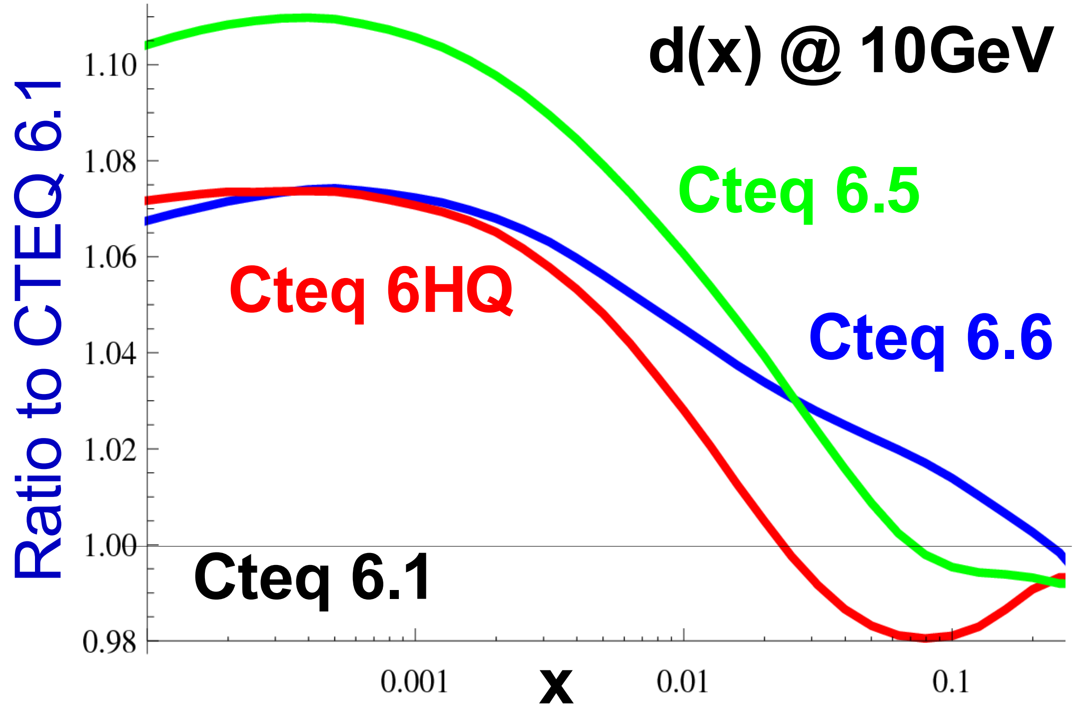
F_2^c
0.035
0.030
0.025
0.020
0.015
0.010
0.005
NLO (m=0)
LO
ACOT
S-ACOT
S-ACOT-χ
FFS
X=0.1
2
5
10
20
50
100
μ scale (GeV)


0.006
0.005
0.004
0.003
0.002
0.001
S-ACOT
S-ACOT-χ
FFS
2
5
10
20
50
100
μ scale (GeV)

Impact

Heavy Quark Schemes: CTEQ6M, CTEQ6HQ, CTEQ5M





- Large shift C5M⇒C6M
  - *(New DIS & Jet data)*
- Charm PDF tied to gluon *(g →cc)*
- Small visual difference but ...

Shift due to both scheme and uncertainty

χ^2/DOF

Set	# pts	6HQ	6M	6M⊗GM	6HQ⊗ZM
ZEUS	104	0.91	0.98	2.84	3.72
H1	484	1.02	1.04	1.50	1.22
TOTAL	1925	1.04	1.06	1.26	1.30

Mixed Schemes

S. Kretzer, H.L. Lai, F. I. Olness and W.K. Tung. Phys.Rev.D69:114005,2004

Where does it make a difference??? *... only HERA is sensitive*

Data set	# pts	CTEQ6HQ	CTEQ6M	C6M⊗GM	C6HQ⊗ZM
Bcdms_p	339	370 (1.09)	370 (1.09)	370 (1.11)	373 (1.10)
Bcdms_d	251	269 (1.07)	279 (1.11)	274 (1.07)	281 (1.12)
Zeus	104	94 (0.91)	102 (0.98)	258 (2.84)	387 (3.72)
H1a	126	124 (0.99)	130 (1.03)	135 (1.11)	123 (0.98)
H1b	129	103 (0.80)	111 (0.86)	119 (0.84)	104 (0.80)
H1c	229	266 (1.16)	261 (1.14)	474 (2.11)	364 (1.59)
Nmc_p	201	304 (1.51)	299 (1.49)	273 (1.35)	366 (1.82)
Nmc_d/p	123	112 (0.91)	111 (0.91)	111 (0.90)	114 (0.92)
Ccfr_F2	69	90 (1.30)	120 (1.74)	116 (1.82)	107 (1.55)
Ccfr_F3	86	35 (0.41)	37 (0.43)	36 (0.40)	36 (0.42)
E605	119	102 (0.86)	103 (0.86)	101 (0.86)	102 (0.86)
Cdf_wasy	11	9 (0.78)	9 (0.83)	9 (0.83)	9 (0.78)
E866	15	5 (0.34)	6 (0.43)	6 (0.43)	5 (0.34)
D0_jet	90	71 (0.79)	49 (0.55)	49 (0.55)	71 (0.79)
Cdf_jet	33	55 (1.66)	50 (1.51)	50 (1.51)	55 (1.66)
All	1925	2008 (1.04)	2037 (1.06)	2431 (1.26)	2496 (1.30)

CTEQ6HQ CTEQ6M MIXED MIXED

A 3 σ effect

HERA experiments sensitive to Mixed scheme

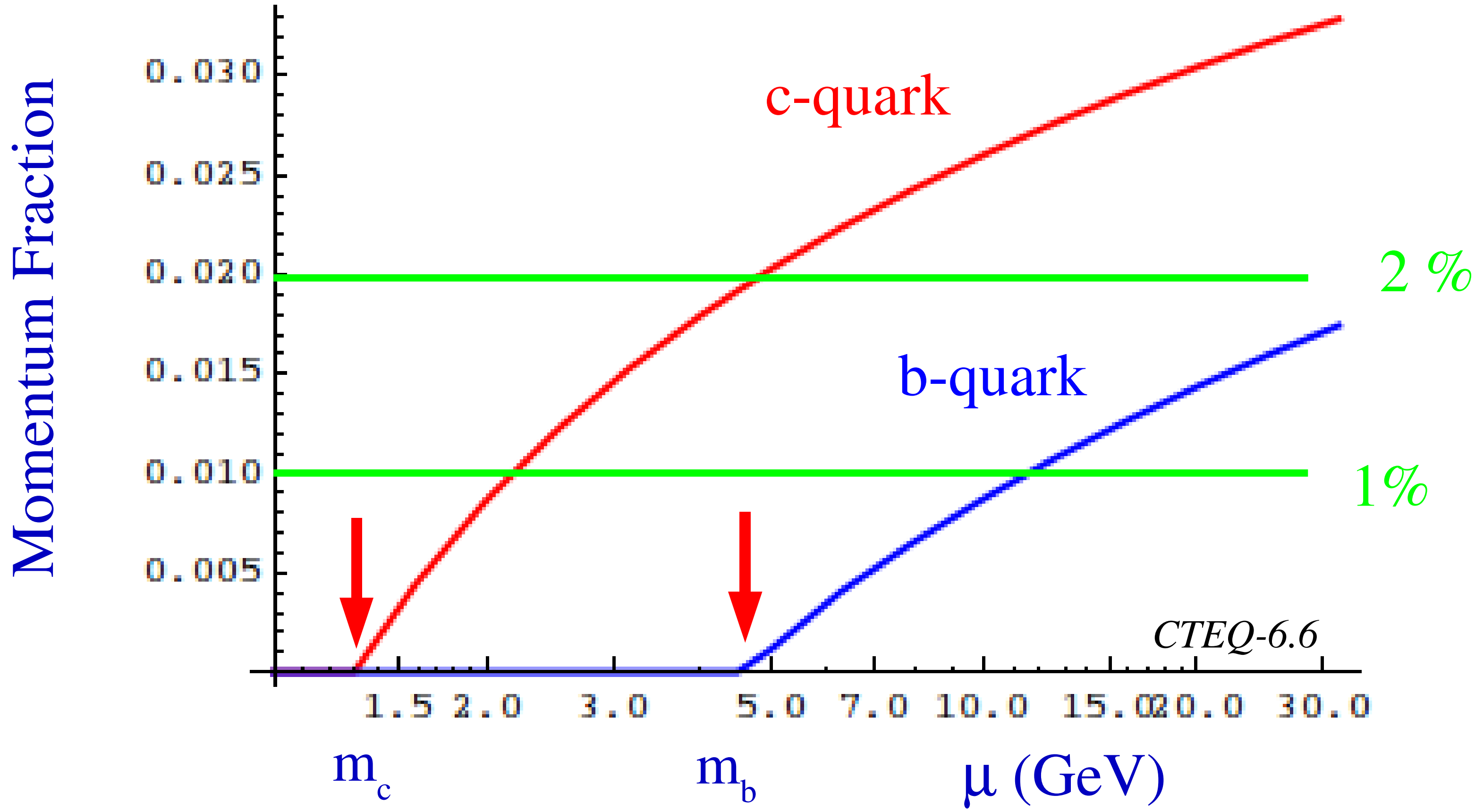
Encouraging that C6M and C6HQ are comparable

Encouraging that Mixed schemes yield large χ^2

Will affect PDFs in region of low x and low Q

Intrinsic Heavy Quarks

Are there Intrinsic Heavy Quarks???

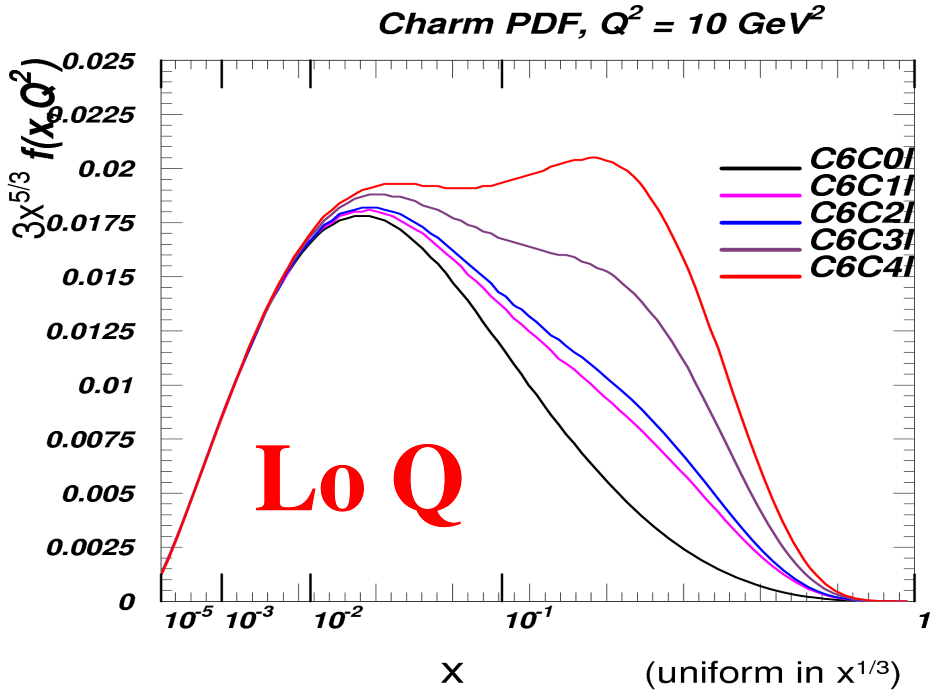


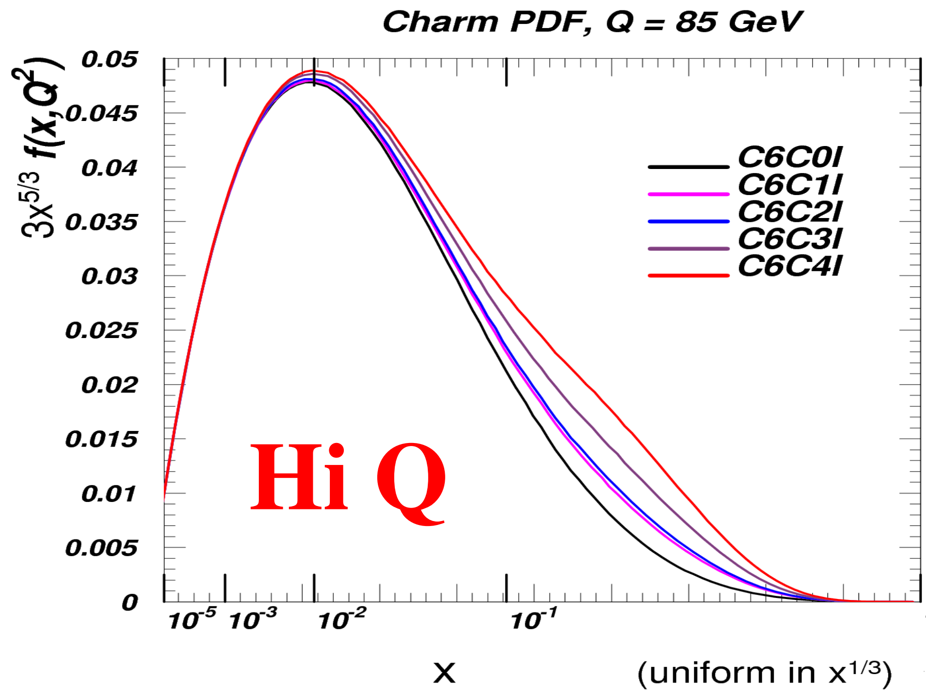
* Most sensitive near threshold
* What happens if we allow the evolution to determine charm?

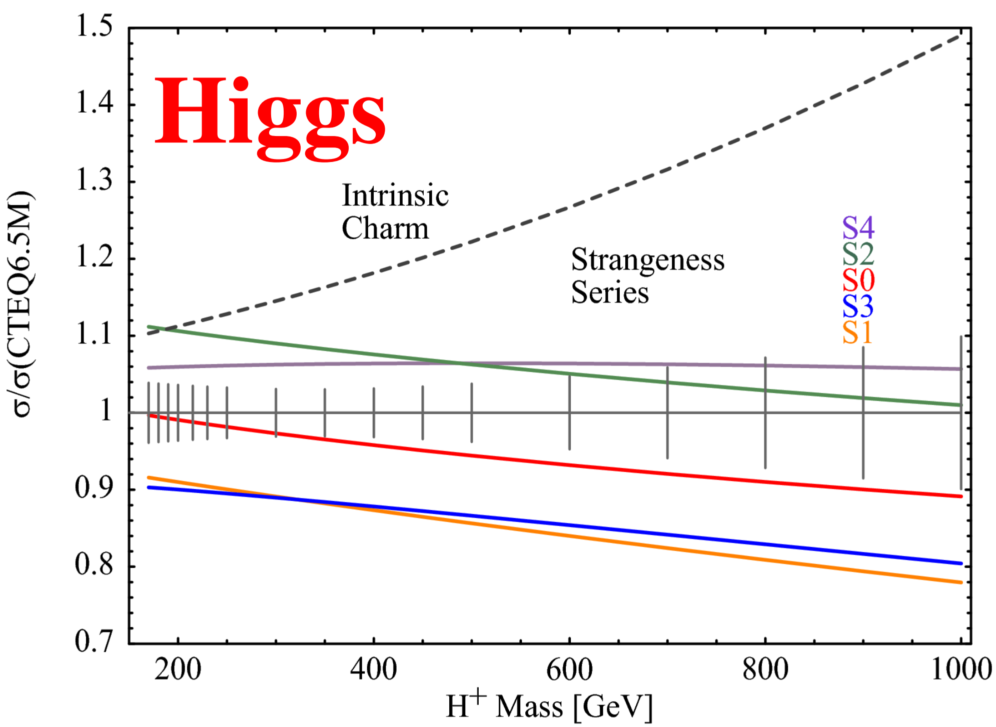
Zero:	No intrinsic charm
Positive:	Intrinsic charm
Negative:	Inconsistent

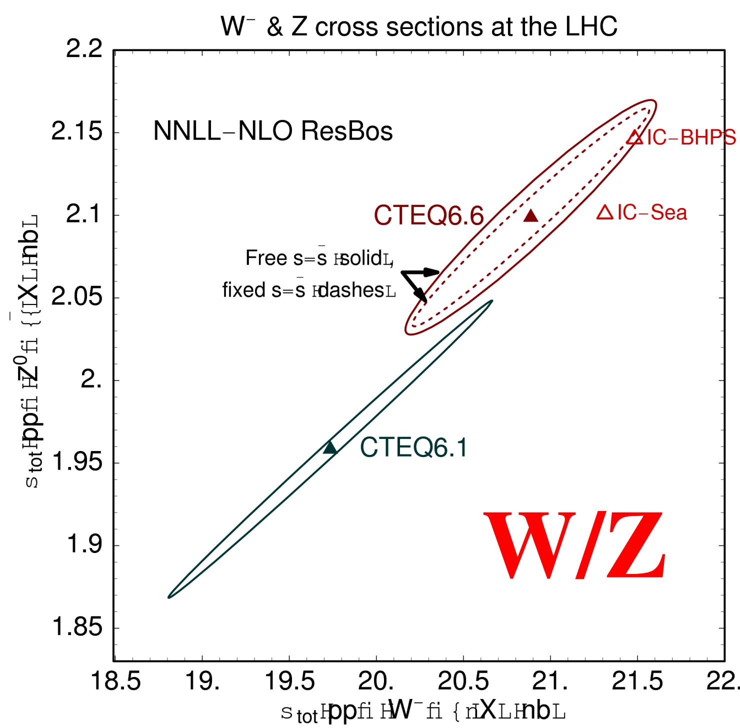
J. Pumplin, Phys.Rev.D75:054029,2007.

Impact of Intrinsic Heavy Quarks









J. Pumplin, Phys.Rev.D75:054029,2007.

Nadolsky, Phys.Rev.D78:013004,2008.

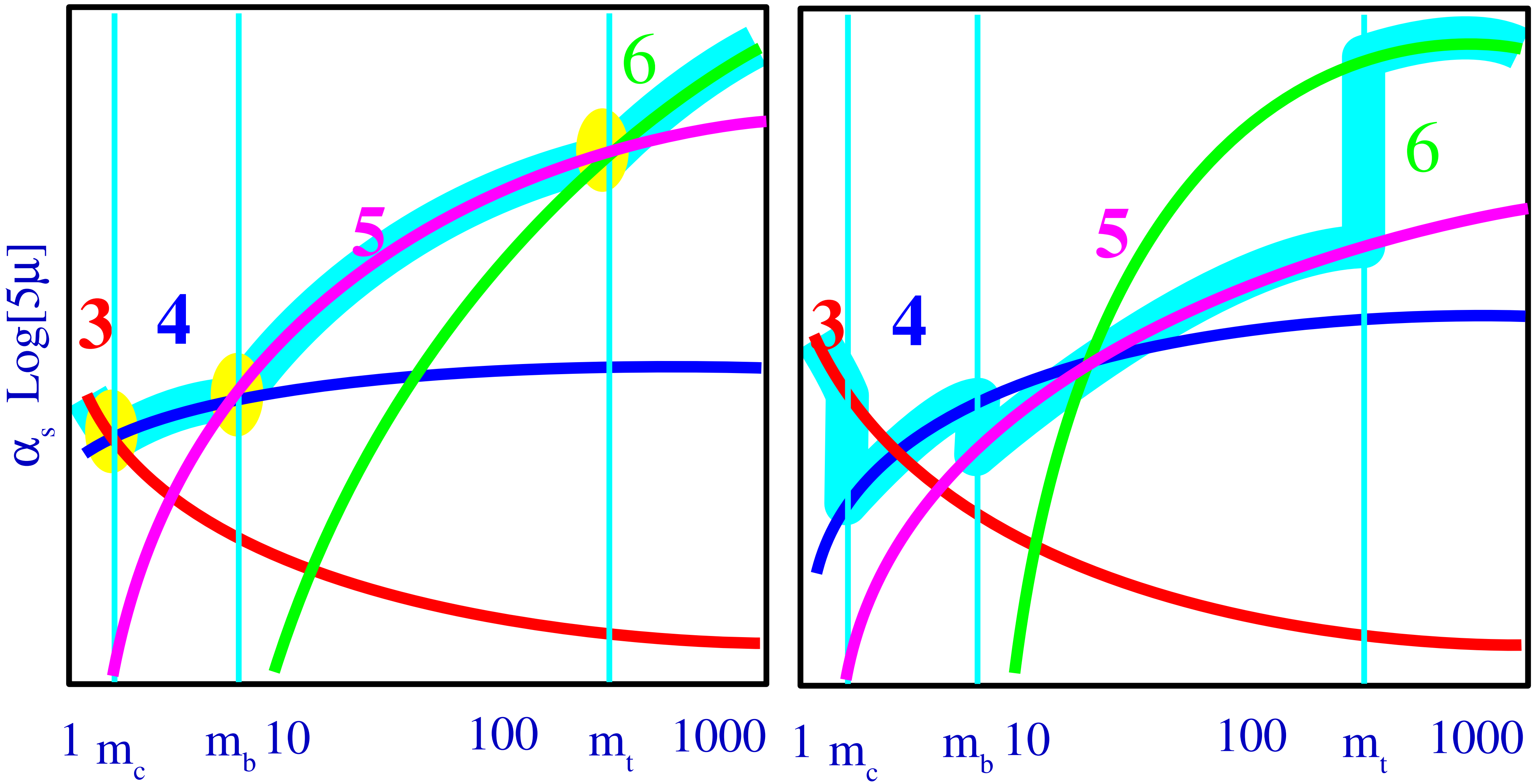
NNLO

A proposal for NNLO PDF implementation

α_s as a function of μ for various flavor numbers

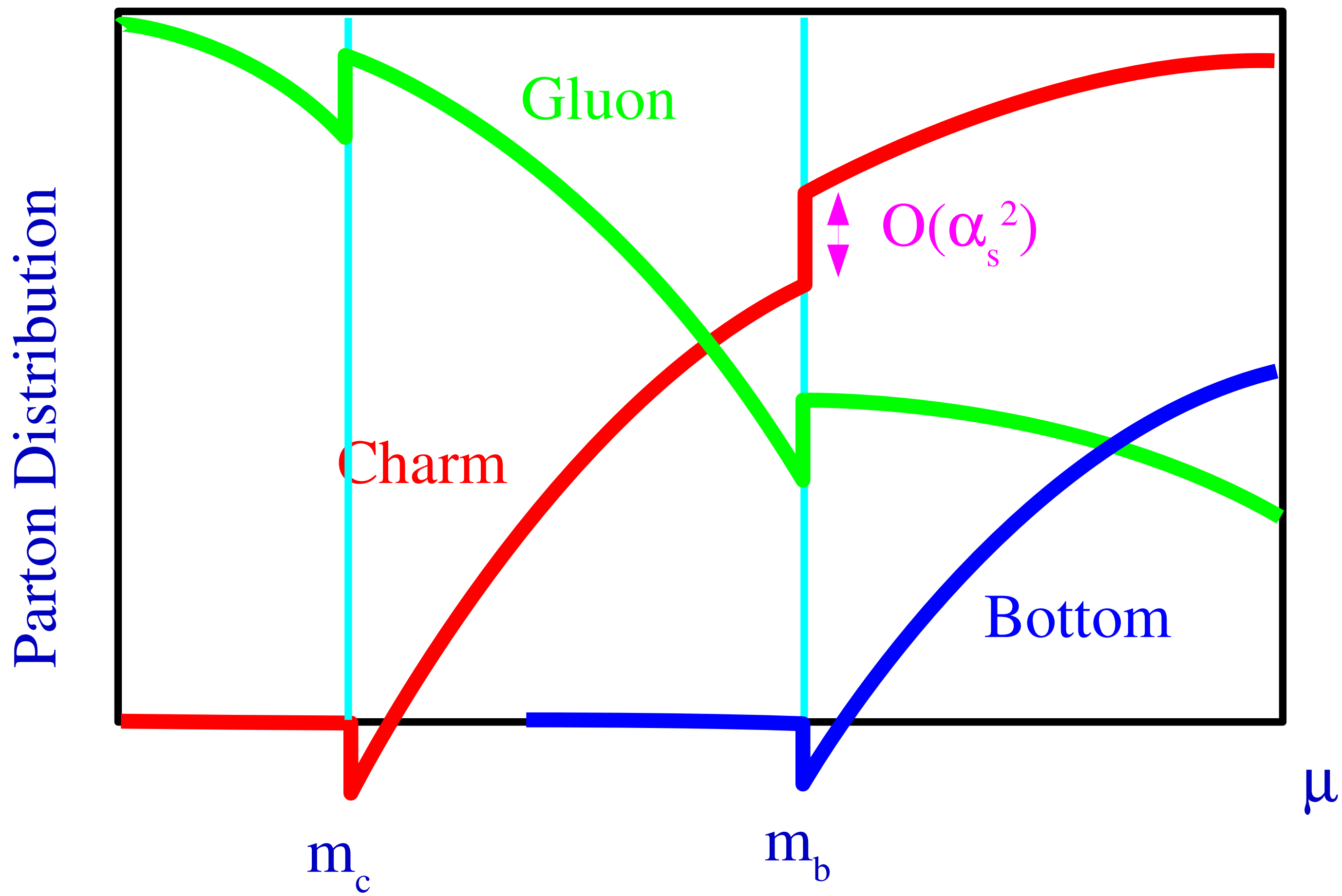
At 1-loop and 2-loops, continuous at thresholds

At $O(\alpha_s^3)$, not even continuous at thresholds



$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2}\alpha^3_{(n_f-1)}(M) + O\left(\alpha^4_{(n_f-1)}\right)$$

f(x,μ) as a function of μ for various flavor numbers



Not continuous at $O(\alpha_s^2)$

$$f_k^{n_f+1}(\mu^2, m_H^2) = A_{kj}(\mu^2/m_H^2) \otimes f_j^{n_f}(\mu^2),$$

relate N and N+1 PDF's

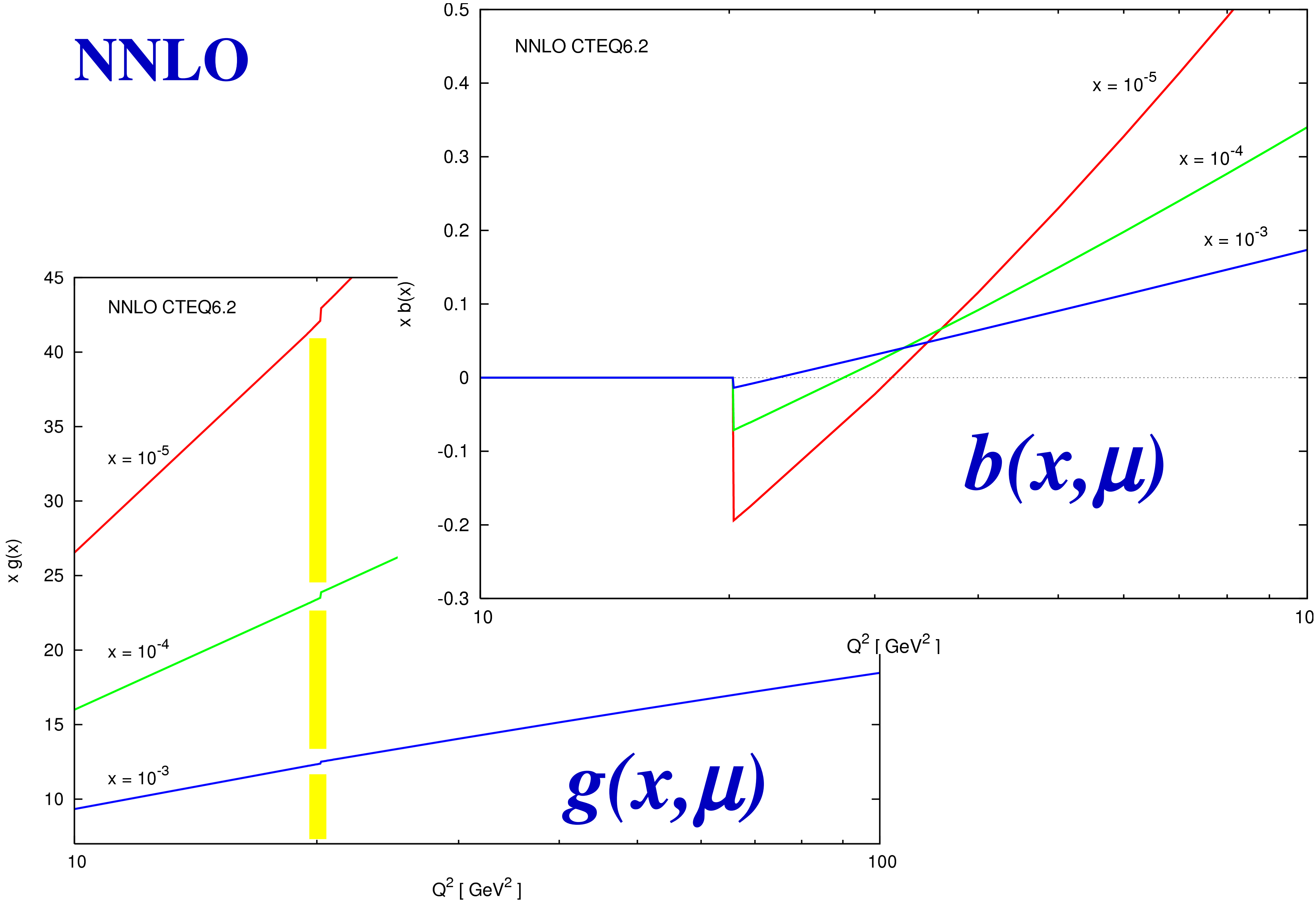
implied relation of C's

$$\begin{aligned} F(x,Q^2) &= C_k^{FFNS}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) \\ &= C_j^{VFNS}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2) \equiv C_j^{VFNS}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) \end{aligned}$$

Note:
FFNS ~ N
VFNS ~ N+1

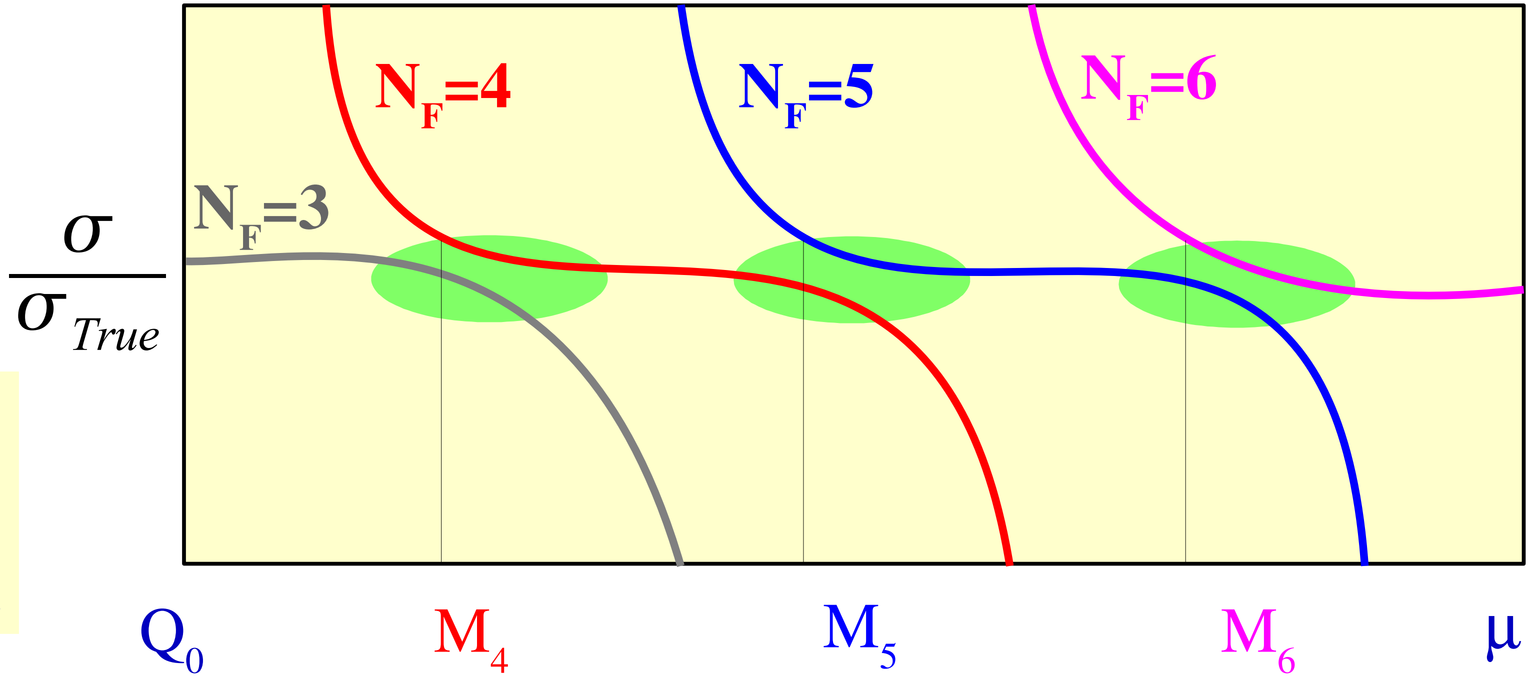
f(x,μ) as a function of μ for various flavor numbers

NNLO



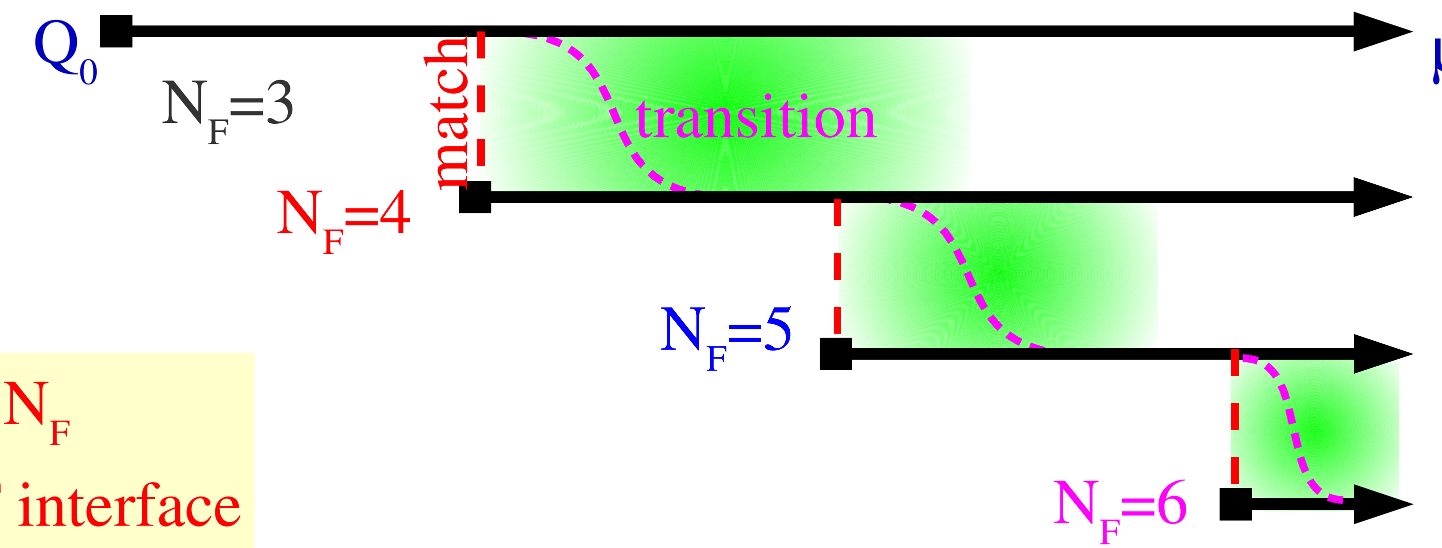
Match at μ=m
Transition at ...

For μ~m,
N and N+1
Schemes Co-exist



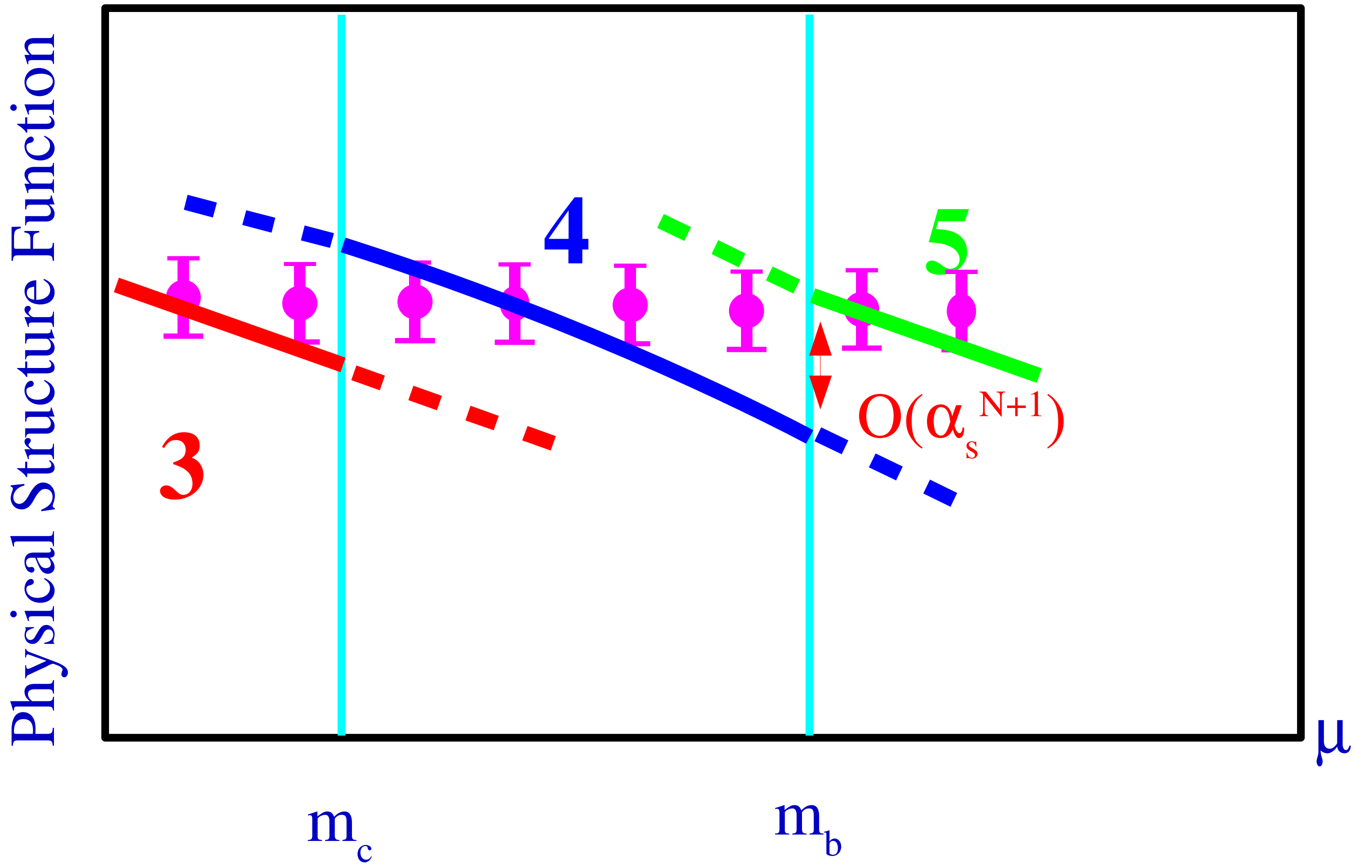
$f_{a/p}(x, Q, N_F)$

new



- Freedom to specify N_F
- Requires N_F in PDF interface
- Simplified Numerics

A multi-flavor scheme is truly a patchwork



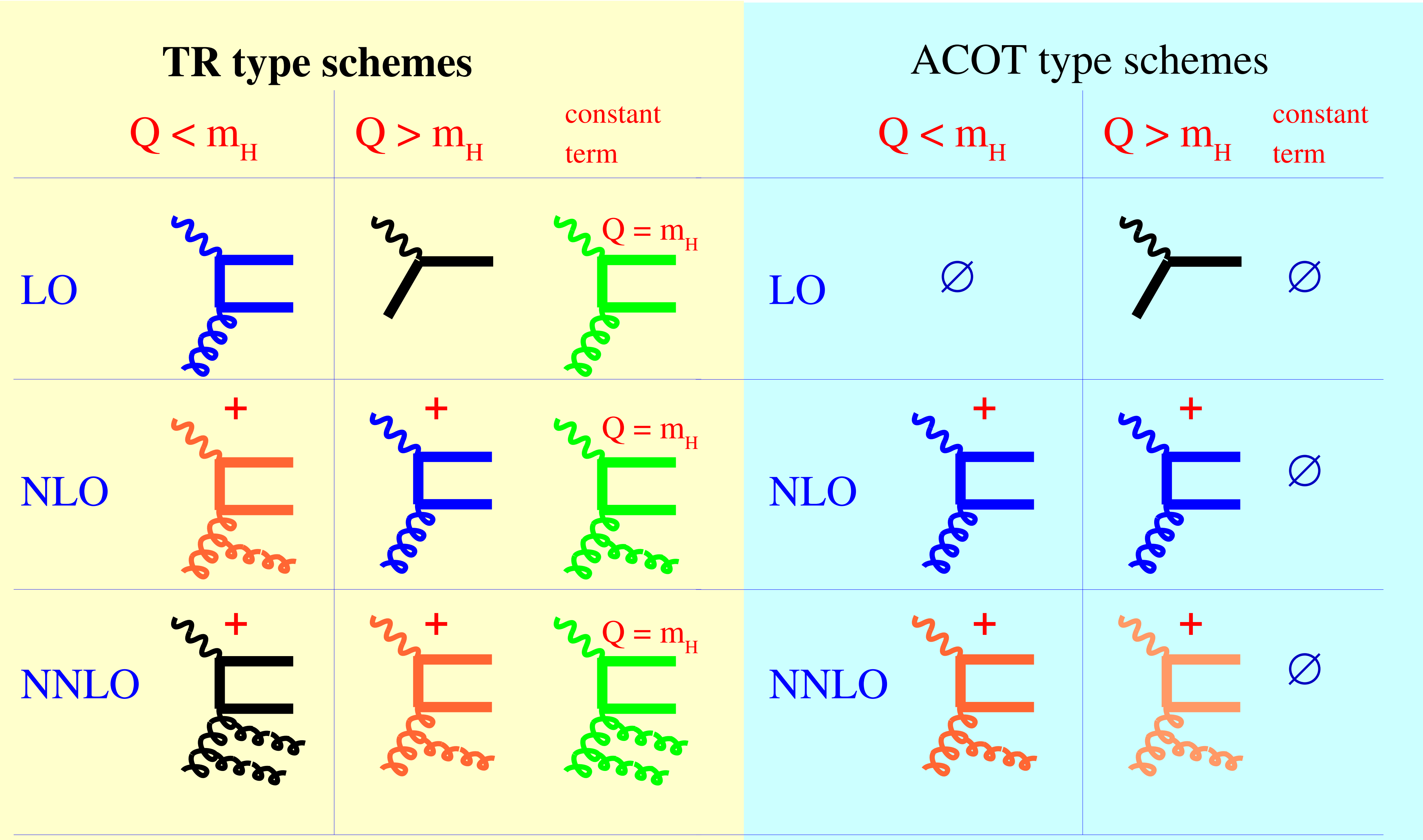
* Difference represents the theoretical uncertainty
* Gaps will decrease with higher orders *(they must as physical quantities)*

(note: gaps of PDF's and α_s do not--these are unphysical quantities)

* If data prefers one scheme ⇒ optimal perturbative organization
* Gaps between schemes reflects limit of theory uncertainty

Scheme choices and global fits

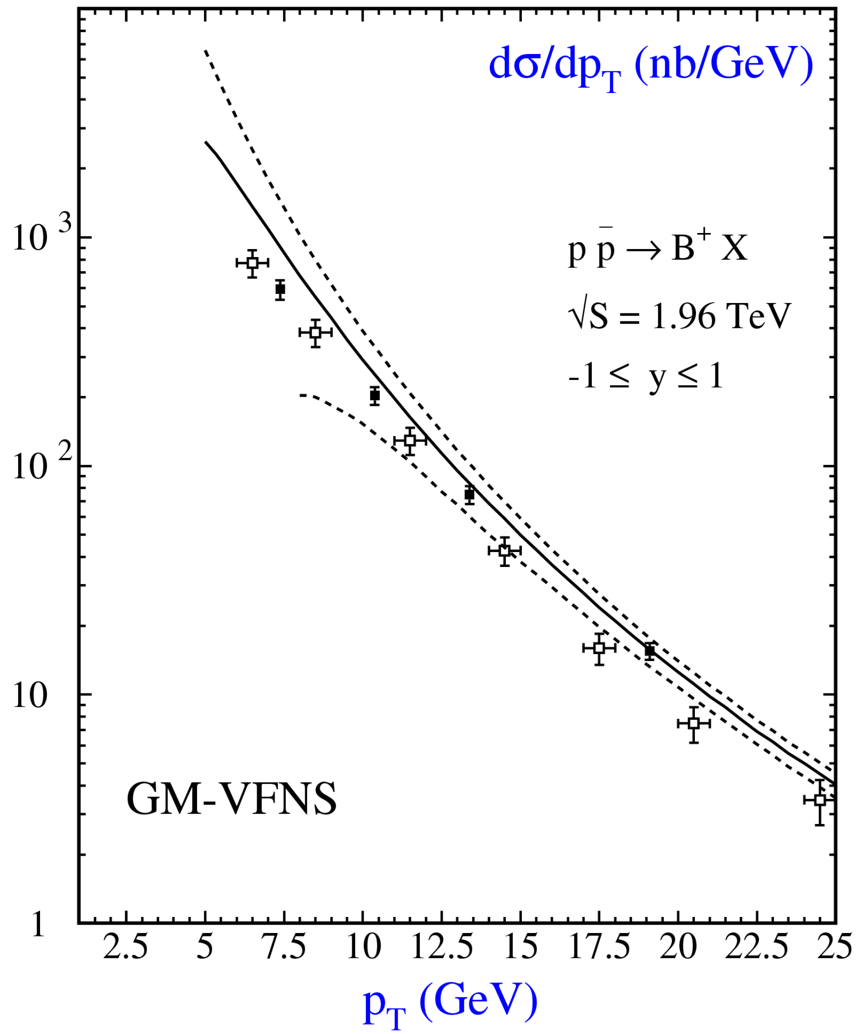
Schematic Summary of Schemes

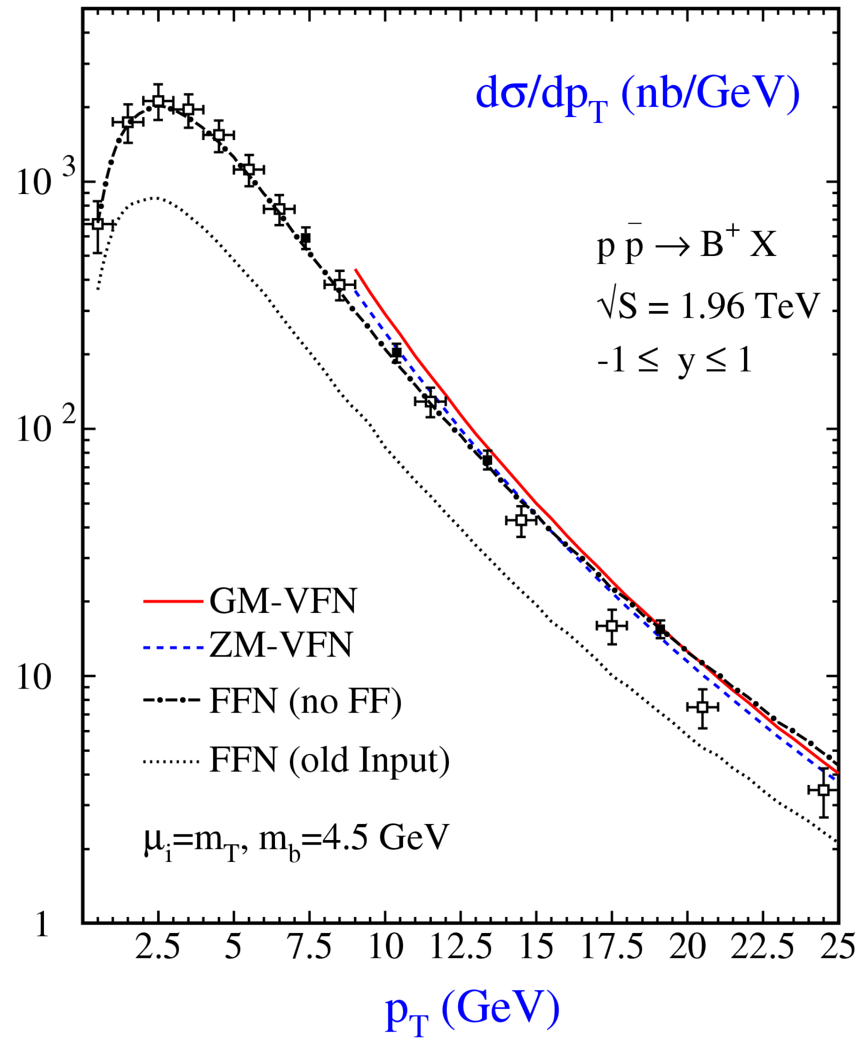


Are we ready for the LHC

... yes ... no

Inclusive B-Meson Hadroproduction in a GM-VFN

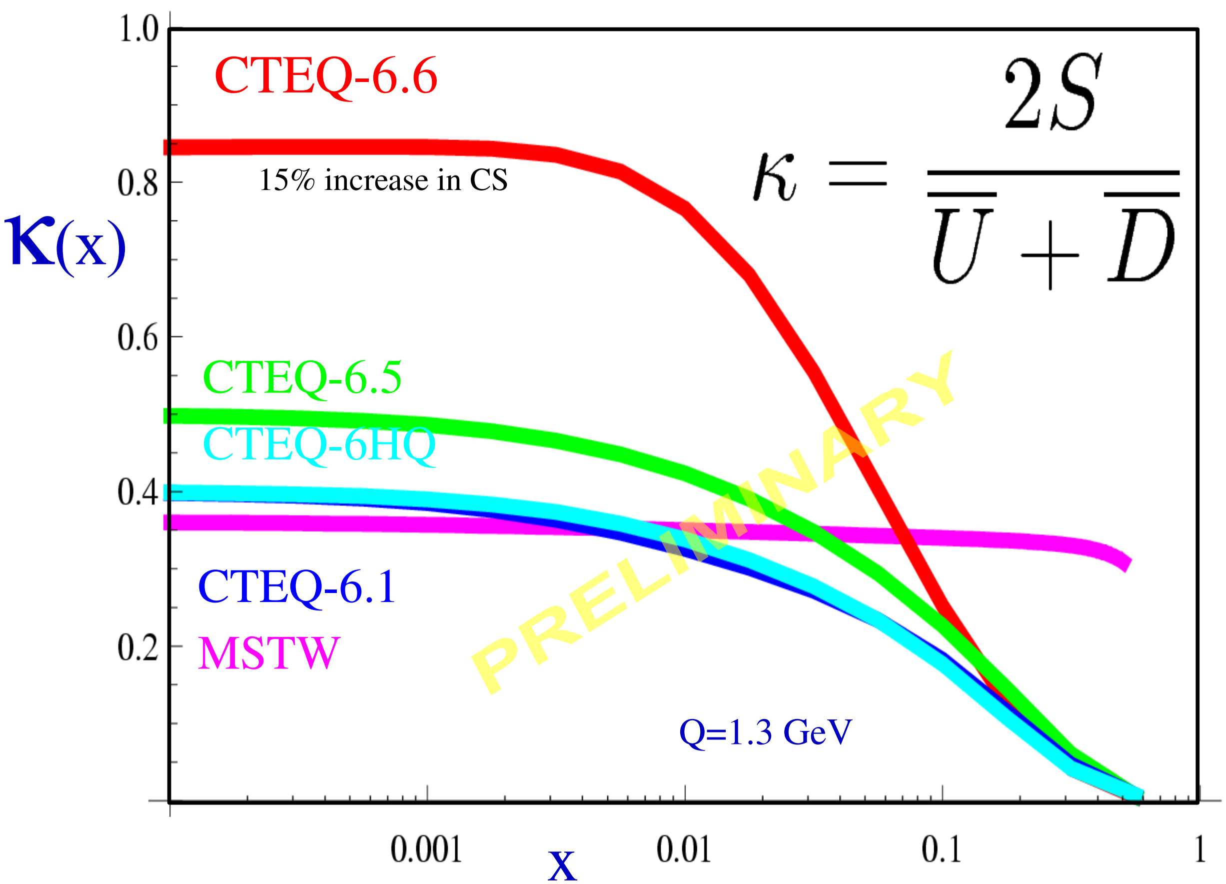


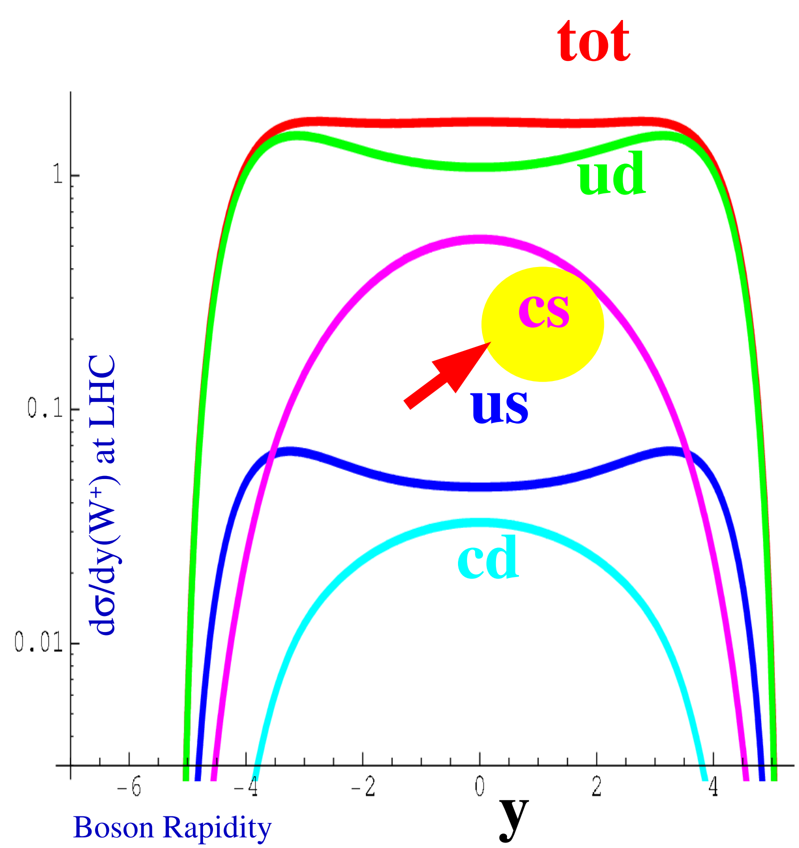


- GM-VFN prediction yields agreement w/ Run II data
- Contrast FFN w/ old input

Kniehl, Kramer, Schienbein, Spiesberger, Phys.Rev.D77:014011,2008.

Impact of HQ PDFs on W/Z at LHC





⇒ Different Y Distribution

... but HQ Uncertainties will feed into LHC "Benchmark" processes

Conclusions

Conclusions

* Historically: Calculation of Heavy Quarks challenging

* Significant Progress:

New theoretical tools enable reliable calculations

Heavy Quarks are included in DGLAP evolution

Heavy Quarks masses are included in Global Analysis

* Current Challenges

Complex at NNLO

Heavy Quarks play a more prevalent role at the LHC

Need to accommodate these effects and uncertainties
to make best use of HERA & Tevatron data.
Also, be prepared to cross-check using early LHC data

Thanks to: I. Schienbein, J.-Y. Yu, S. Berge, P. Nadolsky, J. Owens, W. Tung, S. Kretzer, S. Kuhlmann, J. Pumplin, H. Lai, T.Bolton, P. Spentzouris, D. Mason, M. Shaevitz, K. McFarland, U.K. Yang,