Heavy Quarks in MRST/MSTW Global Fits

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Will discuss Charm $\sim 1.5\mathrm{GeV}$, bottom $\sim 4.3\mathrm{GeV}$, and at end strange $\sim 0.3\mathrm{GeV}$ as heavy flavours.

Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x, Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Note that n_f is effective number of light quarks. Can be $3, 4$ or 5.

Does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in perturbative expansion. Usually achieved by definition of heavy flavour parton distributions and solution of evolution equations.

Additional problem FFNS known up to NLO (Laenen et al), but are not defined at NNLO – $\alpha_S^3 C_{2,Hg}^{FF,3}$ unknown.

Variable Flavour

High scales $Q^2 \gg m_H^2$ massless partons. Behave like up, down (strange always in this regime. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZM,n_f} \otimes f_j^{n_f}(Q^2).$$

Partons in different number regions related to each other perturbatively.

$$f_j^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),$$

Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ (Buza et al) containing $\ln(Q^2/m_H^2)$ terms relate $f_i^{n_f}(Q^2)$ and $f_i^{n_f+1}(Q^2)$ $\rightarrow$ correct evolution for both.

Want a **General-Mass Variable Flavour Number Scheme** (VFNS) taking one from the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

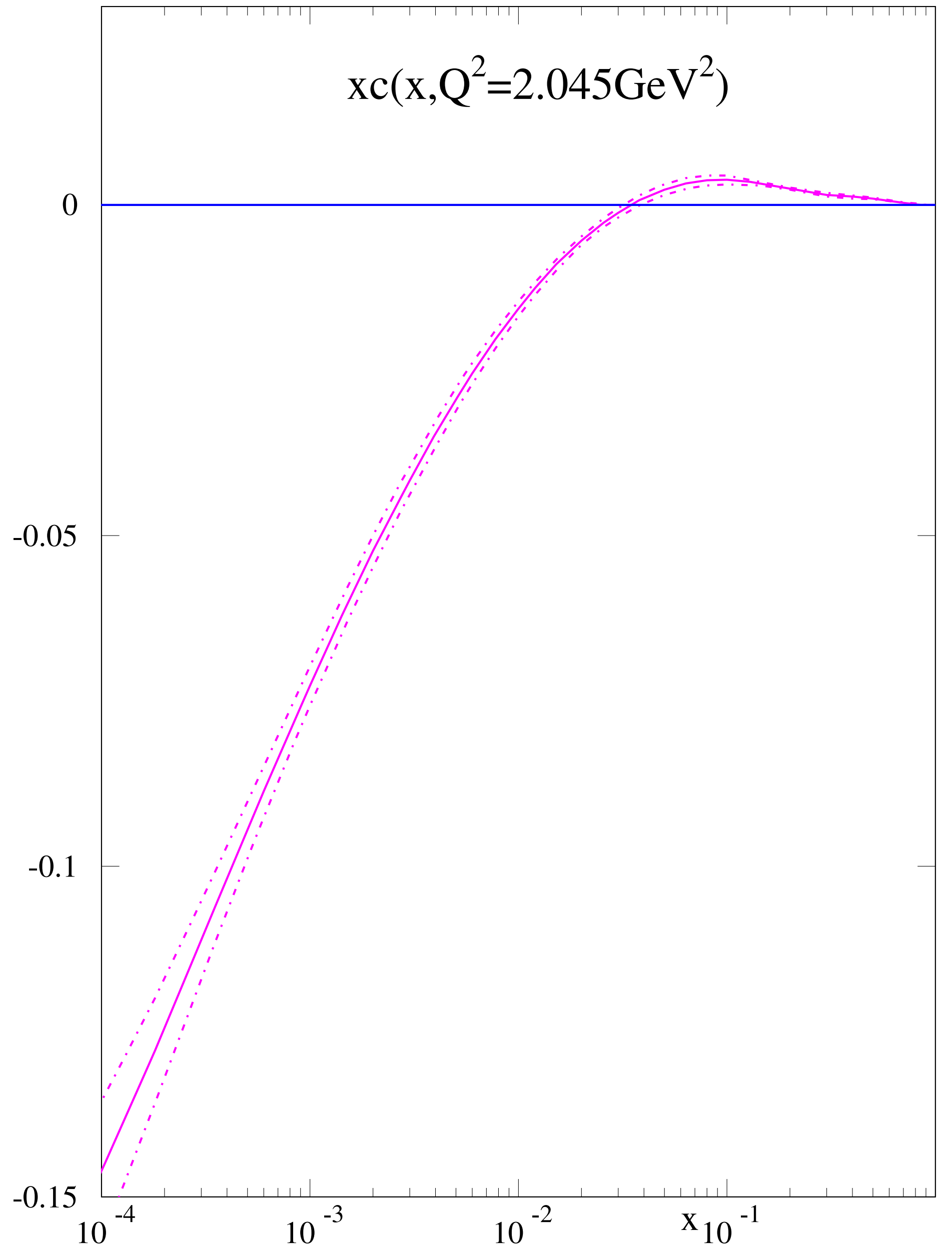
At NLO the partons remain continuous if transition point is taken as $Q^2 = m_H^2$. ZM-VFNS possible, if inaccurate.

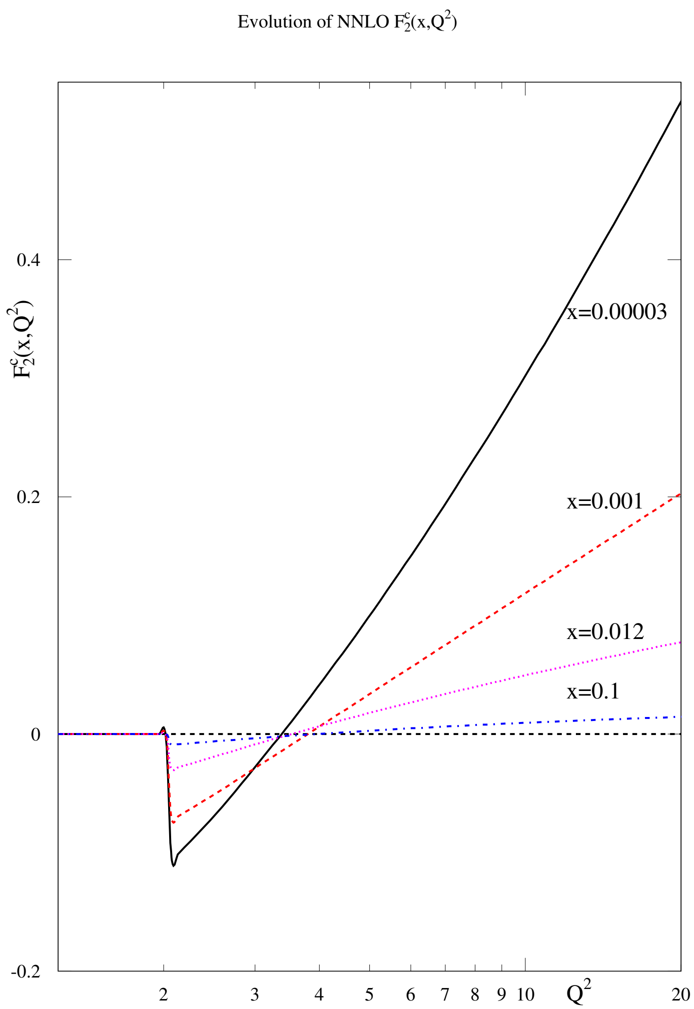
At NNLO lead to discontinuities in partons.

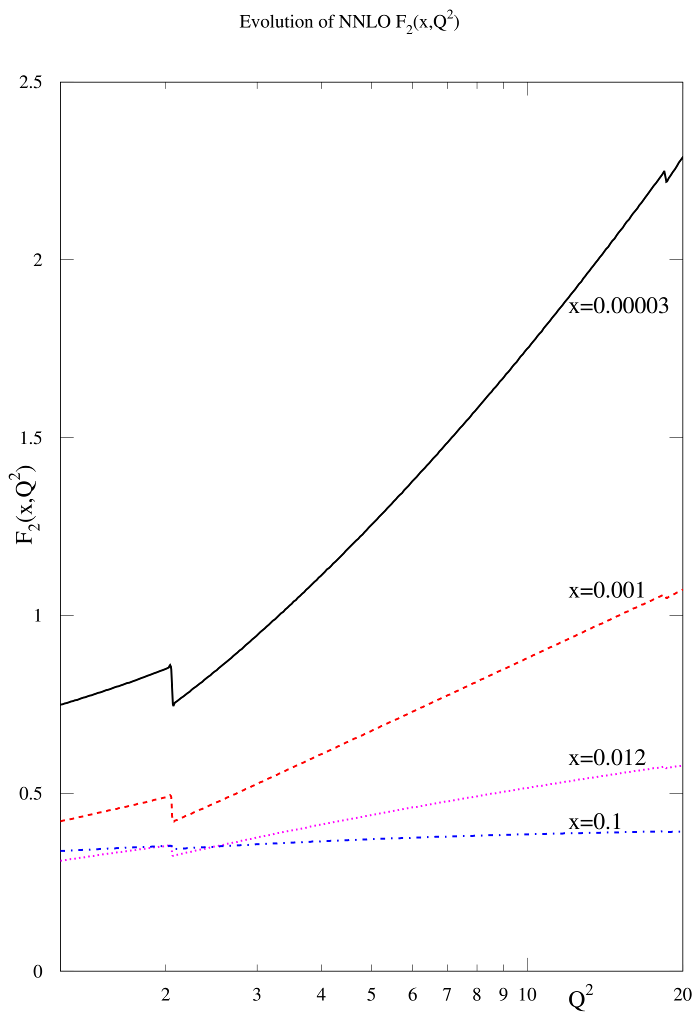
Heavy flavour no longer turns on from zero at $\mu^2 = m_c^2$

$$(c + \bar{c})(x, m_c^2) = A_{Hg}^2(m_c^2) \otimes g(m_c^2)$$

In practice turns on from negative value, (for general gluon).







Leads to huge discontinuity in $F_2^c(x,Q^2)$. Still significant in $F_2^{Tot}(x,Q^2)$.

ZM-VFNS not really feasible at NNLO. Want $\rightarrow$ Need.

The GM-VFNS can be defined by demanding equivalence of the n_f light flavour and n_f+1 light flavour descriptions at all orders – above transition point $n_f \to n_f+1$

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2)$$
$$\equiv C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF,n_f}(Q^2/m_H^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2),$$

which at $\mathcal{O}(\alpha_S)$ gives

$$C_{2,Hg}^{FF,n_f,(1)}(\frac{Q^2}{m_H^2}) = C_{2,HH}^{VF,n_f+1,(0)}(\frac{Q^2}{m_H^2}) \otimes P_{qg}^0 \ln(Q^2/m_H^2) + C_{2,Hg}^{VF,n_f+1,(1)}(\frac{Q^2}{m_H^2}),$$

The VFNS coefficient functions tend to the massless limits as $Q^2/m_H^2 \to \infty$.

However, $C_j^{VF}(Q^2/m_H^2)$ only uniquely defined in this limit.

Can swap $\mathcal{O}(m_H^2/Q^2)$ terms between $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and $C_{2,g}^{VF,1}(Q^2/m_H^2)$.

Original ACOT prescription violated threshold $W^2 > 4m_H^2$ since only needed one quark in final state rather than quark-antiquark pair.

(TR-VFNS) recognised ambiguity in definition of $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ for first time and removed it by making $(d\,F_2/d\ln Q^2)$ continuous at transition (in gluon sector).

Smoothness guaranteed at $Q^2 = m_H^2$ – but complicated.

Various other alternatives. Most recently Tung, Kretzer, Schmidt have come up with the ACOT(χ) prescription which I interpret as

$$C_{2,HH}^{VF,0}(Q^2/m_H^2, z) = \delta(z - Q^2/(Q^2 + 4m_H^2)).$$

$$\to F_2^{H,0}(x, Q^2) = (h + \bar{h})(x/x_{max}, Q^2), \qquad x_{max} = Q^2/(Q^2 + 4m_H^2)$$

$\to C_{2,HH}^{ZM,0}(z) = \delta(1-z)$ for $Q^2/m_H^2 \to \infty$. Also $W^2 = Q^2(1-x)/x \geq 4m_H^2$.

For VFNS to remain simple (and physical) at all orders is necessary to choose

$$C_{2,HH}^{VF,n}(Q^2/m_H^2, z) = C_{2,HH}^{ZM,n}(z/x_{max}).$$

Have adopted this.

One more problem in defining VFNS. Ordering for $F_2^H(x,Q^2)$ different above and below transition point.

	Below	**Above**
LO	$\frac{\alpha_S}{4\pi} C_{2,Hg}^{FF,n_f,(1)} \otimes g^{n_f}$	$C_{2,HH}^{VF,n_f+1,(0)} \otimes (h+\bar{h})$
NLO	$\left(\frac{\alpha_S}{4\pi}\right)^2 (C_{2,Hg}^{FF,n_f,(2)} \otimes g^{n_f} + C_{2,Hq}^{FF,n_f,(2)} \otimes \Sigma^{n_f})$	$\frac{\alpha_S}{4\pi} (C_{2,HH}^{VF,n_f+1,(1)} \otimes h^+ + C_{2,Hg}^{VF,n_f+1,(1)} \otimes g^{n_f+1})$
NNLO	$\left(\frac{\alpha_S}{4\pi}\right)^3 \sum_i C_{2,Hi}^{FF,n_f,(3)} \otimes f_i^{n_f}$	$\left(\frac{\alpha_S}{4\pi}\right)^2 \sum_j C_{2,Hj}^{VF,n_f+1,(2)} \otimes f_j^{n_f+1}$.

Switching direct from fixed order to same order when going from n_f to n_f+1 flavours $\longrightarrow$ discontinuity.

Must make some decision how to deal with this.

ACOT type schemes have used e.g.

NLO $$\frac{\alpha_S}{4\pi}C^{FF,n_f,(1)}_{2,Hg} \otimes g^{n_f} \to \frac{\alpha_S}{4\pi}(C^{VF,n_f+1,(1)}_{2,HH} \otimes (h+\bar{h}) + C^{VF,n_f+1,(1)}_{2,Hg} \otimes g^{n_f+1}),$$

i.e., same order of α_S above and below.

But LO evolution below and NLO evolution above. Slope discontinuous.

TR have used e.g.

LO $$\frac{\alpha_S(Q^2)}{4\pi}C^{FF,n_f,(1)}_{2,Hg}(Q^2/m_H^2) \otimes g^{n_f}(Q^2) \to \frac{\alpha_S(M^2)}{4\pi}C^{FF,n_f,(1)}_{2,Hg}(1) \otimes g^{n_f}(M^2)$$
$$+C^{VF,n_f+1,(0)}_{2,HH}(Q^2/m_H^2) \otimes (h+\bar{h})(Q^2),$$

i.e. freeze higher order α_S term when going upwards through $Q^2 = m_H^2$.

This difference in choice can be phenomenologically important.

In order to define our VFNS at NNLO, need $\mathcal{O}(\alpha_S^3)$ heavy flavour coefficient functions for $Q^2 \leq m_H^2$ and to be frozen for $Q^2 > m_H^2$. However, not calculated.

Model using known leading threshold logarithms (Laenen and Moch) and leading $\ln(1/x)$ term from k_T-dependent impact factors Catani, Ciafaloni and Hautmann.

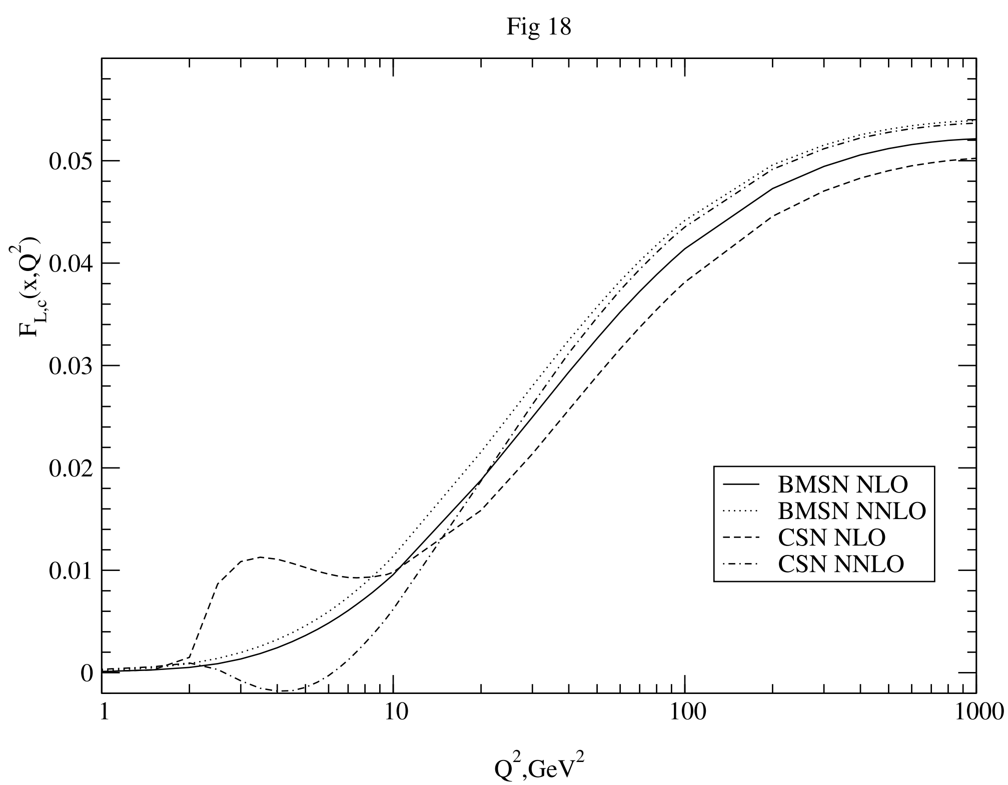
Have to also consider ambiguities in definition of scheme for $F_L(x,Q^2)$. If charm explicitly in proton, zeroth order contribution to $C^c_L(x,Q^2) = \frac{4m_c^2}{Q^2}\delta(1 - z/(x(1+\frac{m_c^2}{Q^2}))$, which disappears at high Q^2.

Cancels between orders in properly defined GM -VFNS. However, large near m_c^2 while other terms suppressed by v^3 (v is the velocity of the heavy quark in the centre-of-mass frame). If implemented can lead to peculiar behaviour for Q^2 slightly above m_c^2 – particularly at NNLO where charm distributions start off negative. Chosen to be absent in our scheme.

Example of competing definitions of schemes for $F^c_L(x,Q^2)$ (Chuvakin, Smith, van Neerven).

One choice gives positive "bump" at NLO and negative "bump" at NNLO. Other is smooth.

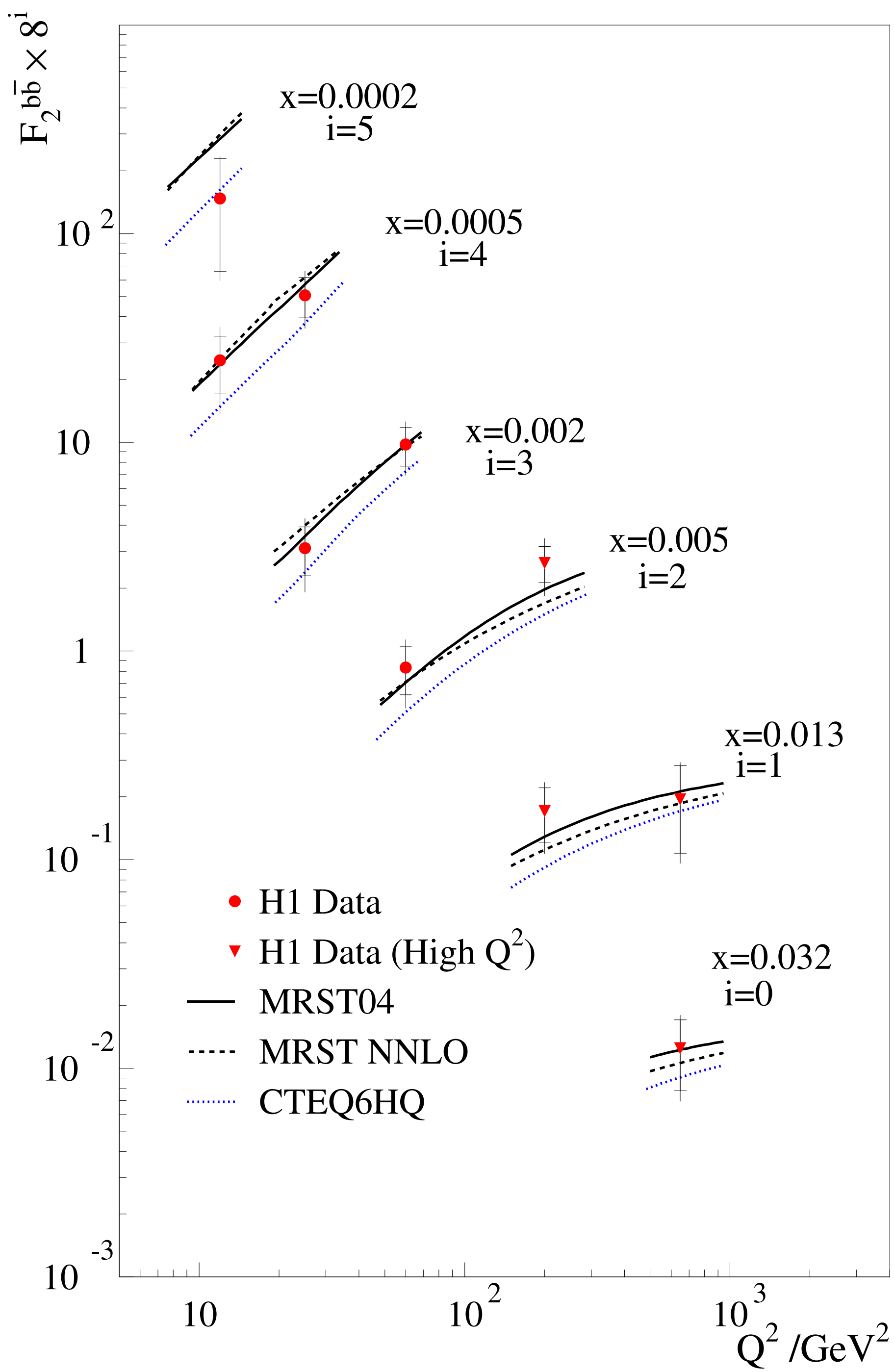
All TR-VFNS schemes and SACOT(χ) schemes have no zeroth order contribution.



Ordering is the main difference in the NLO predictions from MRST and CTEQ in the comparison to (published) H1 data on $F_2^b(x,Q^2)$. (Preliminary data lower.)

$\mathcal{O}(\alpha_S^2)$ part is dominant at for $Q^2 \leq m_c^2$. "Frozen" part remains significant. Clearly improves match to data.

Now raised $m_b = 4.3\text{GeV}$ to $m_b = 4.75\text{GeV}$ $\rightarrow$ slight relative suppression at low scales compared to plots.

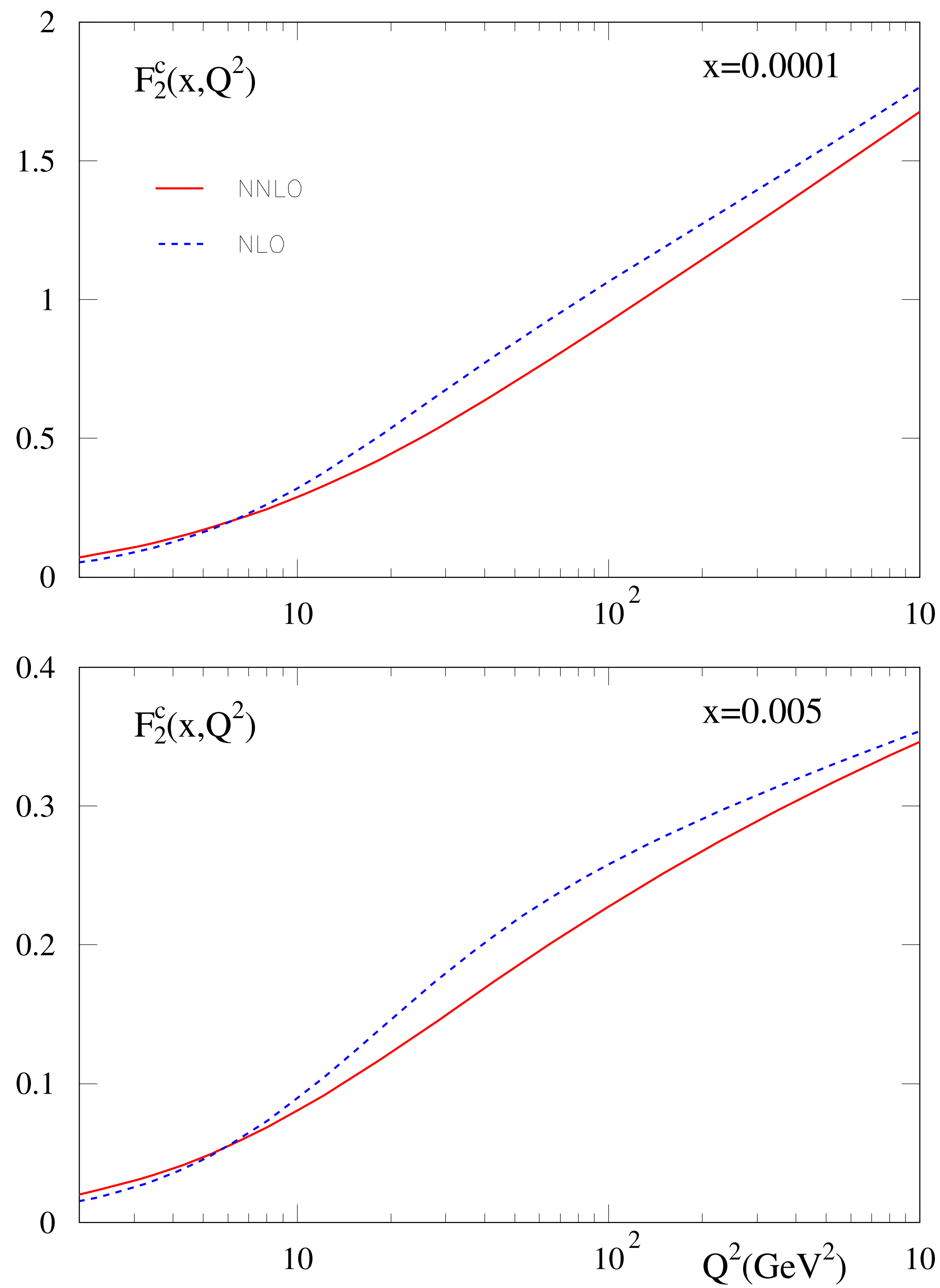


NNLO consequences.

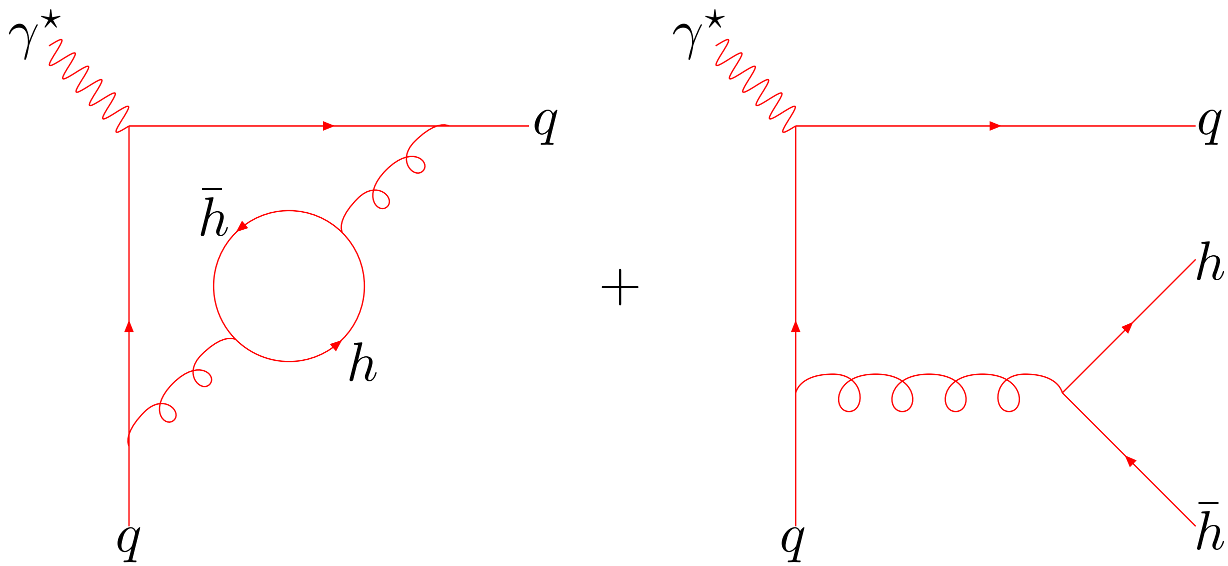
NNLO $F_2^c(x,Q^2)$ starts from higher value at low Q^2.

At high Q^2 dominated by $(c+\bar{c})(x,Q^2)$. This has started evolving from negative value at $Q^2 = m_c^2$. Remains lower than at NLO for similar evolution.

General trend – $F_2^c(x,Q^2)$ flatter in Q^2 at NNLO than at NLO. Important effect on gluon distribution going from one to other.



Remember caveat at NNLO. At NNLO also get contribution due to heavy flavours away from photon vertex.



Strictly, left-hand type diagram and soft parts of right-hand type diagram should be light flavour structure function, and hard part of right-hand type diagram contributes to $F_2^H(x, Q^2)$ (Chuvakin, Smith, van Neerven).

Soft part of right cancels $\ln^3(Q^2/m_H^2)$ divergences in virtual corrections (left).

Can be implemented (depends on separation parameter), but each contribution tiny. At moment all in light flavours.

Importance of treating heavy flavour correctly illustrated at NNLO with MRST2006 partons.

Previous approximate NNLO sets used (declared) approximate VFNS at flavour thresholds.

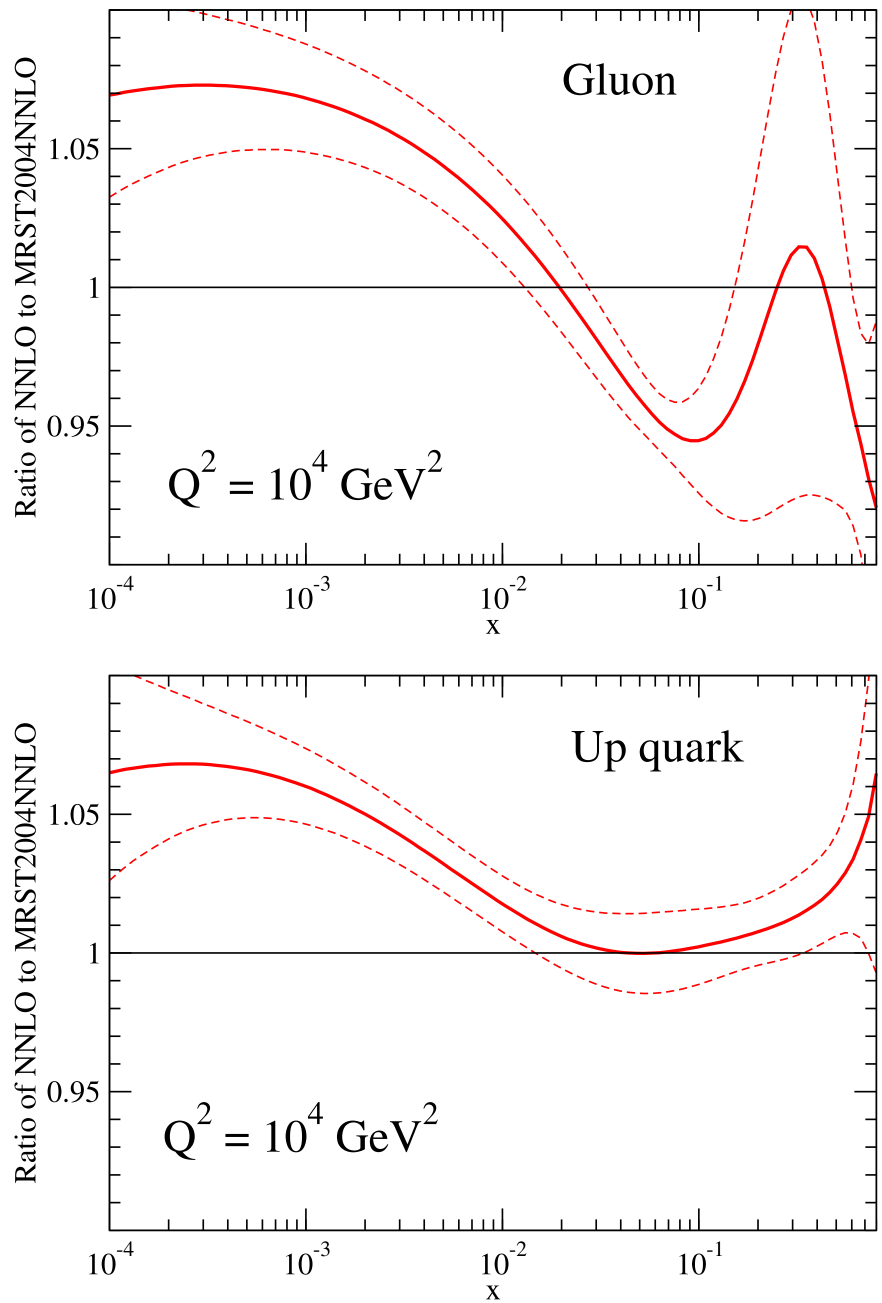
Full VFNS $\rightarrow$ flatter evolution of charm

$\rightarrow$ bigger gluon and more evolution of light sea and bigger α_S.

$\rightarrow$ 6% increase in σ_W and σ_Z at the LHC.

This is a correction not uncertainty.

Very important changes nonetheless.



With hindsight check effect of change in flavour prescription for NLO.

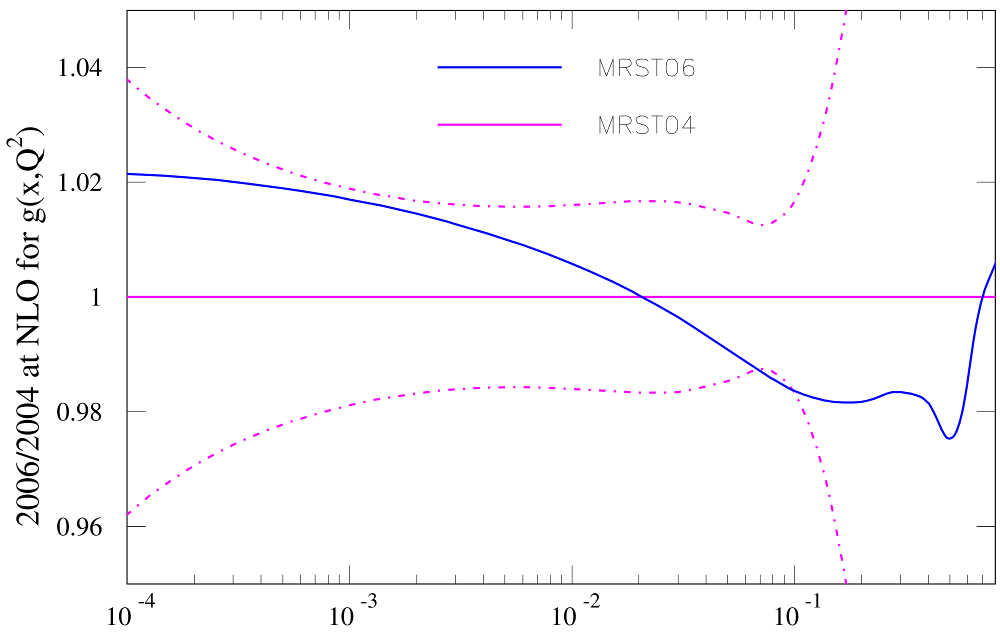
Compare MRST2004 (with 2001 uncertainties) to unofficial "MRST2006 NLO".

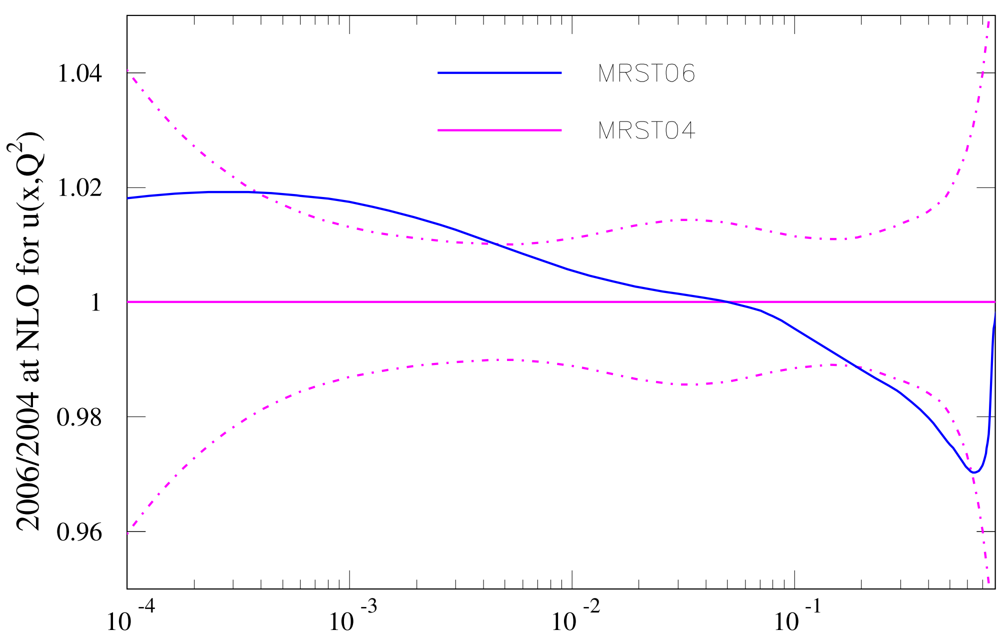
Fit to same data as 2006 NNLO set.

Same trend for partons and α_S as at NNLO, but a lot smaller. Max of 2% changes.

$\rightarrow$ 2% increase in σ_W and σ_Z at the LHC.

Can be same size as quoted uncertainties. This is a genuine theory uncertainty due to competing but equally valid choices. Ambiguity decreases at higher orders.





Dependence on m_c at NLO in 2008 fits (very prelim).

Vary m_c in steps of $0.1\mathrm{GeV}$.

m_c (GeV)	χ^2_{global} 2699 pts	$\chi^2_{F_2^c}$ 83 pts	$\alpha_s(M_Z^2)$
1.1	2734	262	0.1184
1.2	2631	187	0.1189
1.3	2570	134	0.1196
1.392	2550	108	0.1202
1.4	2550	107	0.1203
1.5	2552	97	0.1210
1.6	2581	104	0.1217
1.7	2635	129	0.1223

Clear correlation between m_c and $\alpha_S(M_Z^2)$.

For low m_c overshoot low Q^2 medium x data badly.

Preference for $m_c = 1.4\mathrm{GeV}$. Towards lower end of pole mass determinations. Uncertainty from fit $\sim 0.1 - 0.15\mathrm{GeV}$.

Dependence on m_c at NNLO in 2008 fits (prelim).

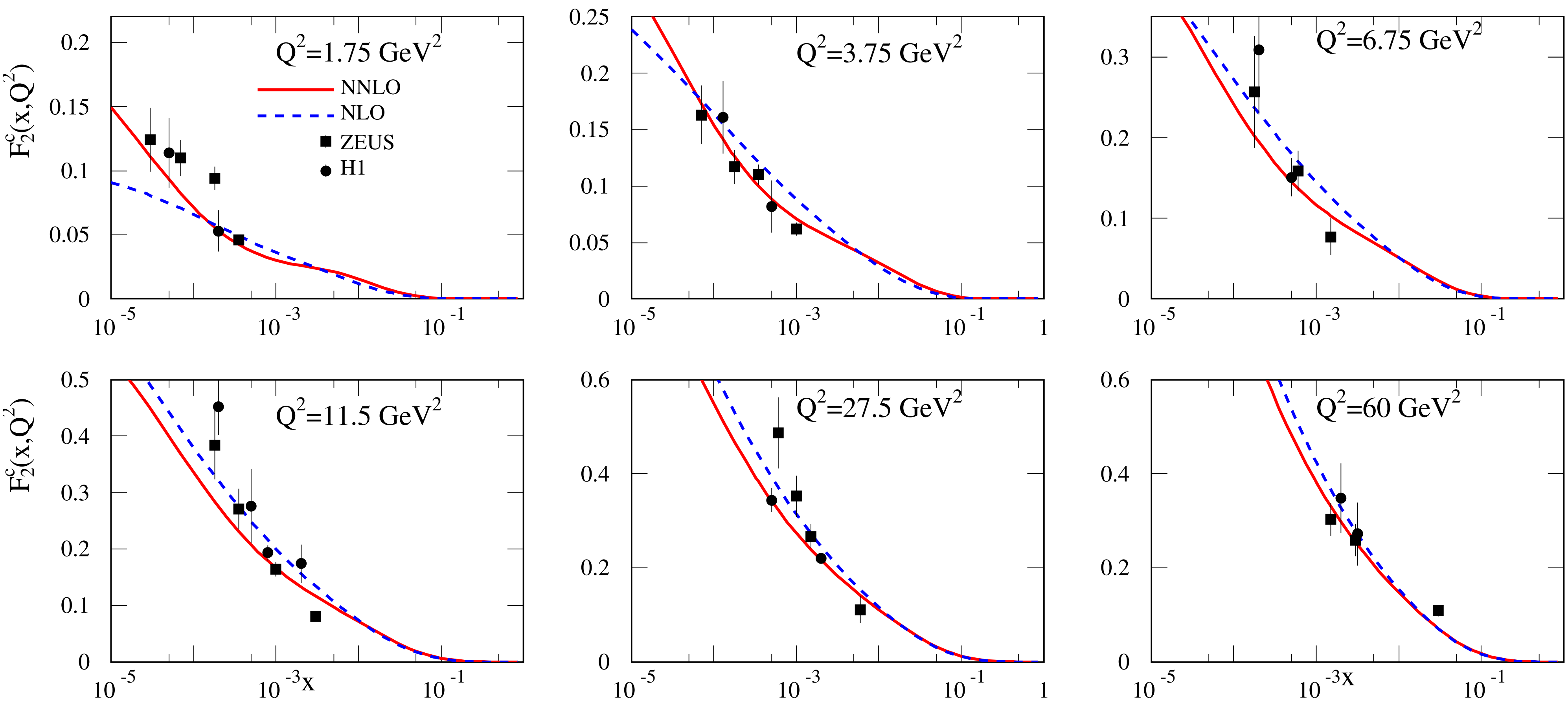
m_c (GeV)	χ^2_{global} 2615 pts	$\chi^2_{F_2^c}$ 83 pts	$\alpha_s(M_Z^2)$
1.1	2522	127	0.1157
1.2	2477	97	0.1161
1.3	2462	83	0.1165
1.304	2462	83	0.1165
1.4	2474	87	0.1168
1.5	2521	112	0.1173
1.6	2574	147	0.1177
1.7	2640	191	0.1181

Again clear correlation between m_c and $\alpha_S(M_Z^2)$, but less variation in latter.

Preference for $m_c = 1.3\mathrm{GeV}$. Very much lower end of pole mass determinations. Uncertainty from fit $\sim 0.1 - 0.15\mathrm{GeV}$.

More consistency between best global fit and fit to charm data.

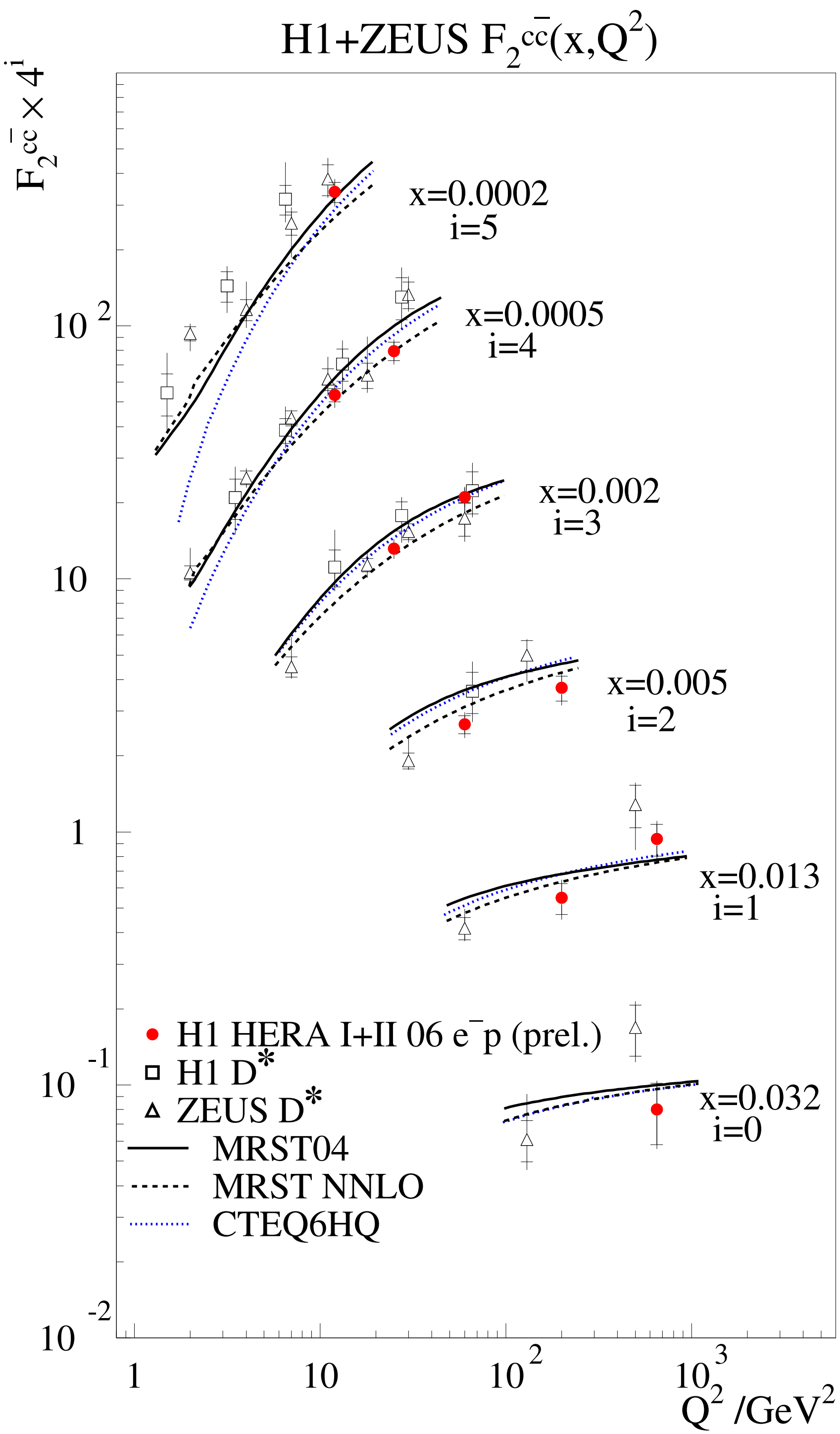
NNLO and NLO fits to charm data (prelim)



Clearly NNLO improves match to lowest Q^2 data, where NLO always too low.

NNLO generally better shape, but undershoots some $Q^2 \sim 10 - 20\mathrm{GeV}^2$ data.

NNLO shape seems to best match high statistics preliminary H1 charm data as well.



Extension to charged currents.

General procedure works on same principles - can now produce single charm quark from strange (and some down) so threshold at $x_{max} = 1/(1 + m_c^2/Q^2)$. Fairly straightforward to NLO.

However, massive FFNS coefficient functions not known at $\mathcal{O}(\alpha_S^2)$ (only asymptotic limits Buza et al).

Needed in our GM-VFNS at low Q^2 at NLO, and at all Q^2 at NNLO - though in later case subtracted so tend to known massless limit for large Q^2/m_c^2.

Initial proposal – assume mass-dependence the same as for neutral current functions but with threshold in $4m_c^2$ replaced by threshold in m_c^2.

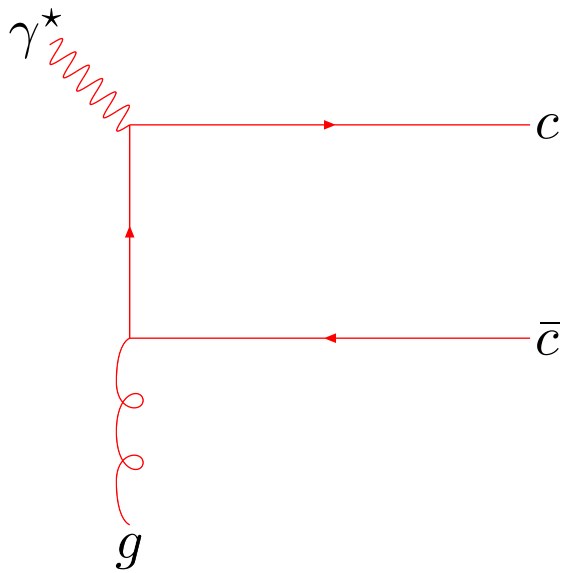
Various complications. In particular important for processes such as dimuon production at CCFR, NuTeV. Cross-section given by

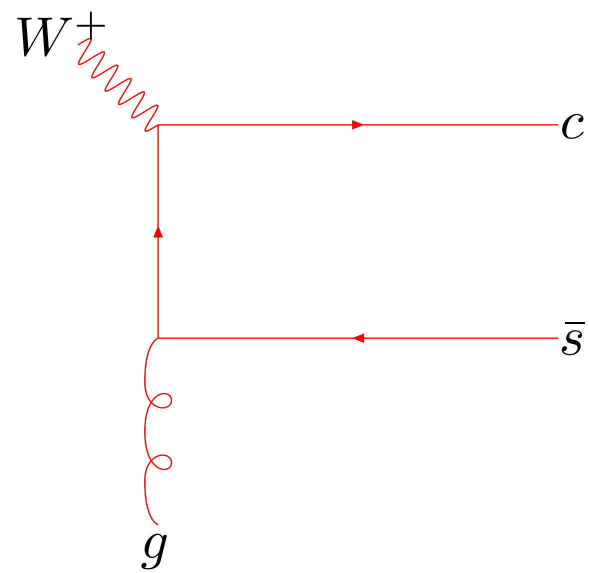
$$d\sigma \propto \left(1 - y + \frac{y^2}{2} - \frac{M_N x y}{2E_\nu}\right) F_2(x) - \frac{y^2}{2} F_L(x) \pm y\left(1 - \frac{y}{2}\right) x F_3,$$

where $y \sim 0.3 - 0.8$, so all terms are important.

$\mathcal{O}(\alpha_S^2)$ dominated by gluon. Simple prescription gives large corrections to $F_2(x, Q^2)$. Smaller threshold $\longrightarrow$ longer convolution length than neutral current.

But consider comparison at $\mathcal{O}(\alpha_S)$.





For NC coefficient is finite and positive. For CC co-linear divergence from *light* quark. After subtraction approx factor $(1 + 2\log(1-z))$ at low Q^2. Finite parts of NC and CC (after $\log(Q^2/m_c^2)$ subtraction) converge to each other high at Q^2 .

NC positive at $\mathcal{O}(\alpha_S^2)$ with threshold $\log$ enhancements. Obtain CC by change in kinematics and $(1 + 2m_c^2/Q^2 \log(1-z))$ factor. $\rightarrow$ correct large Q^2/m_c^2 limit.

$\mathcal{O}(\alpha_S^2)$ contribution to $C_{3g}^{CC}(x, m_c^2, Q^2)$ non-zero for finite Q^2/m_c^2. However, vanishes at both $Q^2/m_c^2 \to \infty$ and $W^2/m_c^2 \to \infty$. Must be implemented else potentially important at low Q^2 and x.

Complication in ordering for longitudinal CC charm production.

In massless limit lowest order contribution

$$F_{L,c}^{CC}(x, Q^2) = \alpha_S(C_{L,g}(x) \otimes g(x, Q^2) + C_{L,q}(x) \otimes s(x, Q^2)).$$

At low Q^2 zeroth order contribution ($\xi = x(1 + m_c^2/Q^2)$)

$$F_{L,c}^{CC}(x, Q^2) = \frac{m_c^2}{m_c^2 + Q^2} s(\xi, Q^2).$$

Difference in orders below and above (opposite to NC $F_2(x, Q^2)$). Choose to obtain correct limits in both regimes with continuity, i.e. above LO for $Q^2 < m_c^2$ and for $Q^2 > m_c^2$

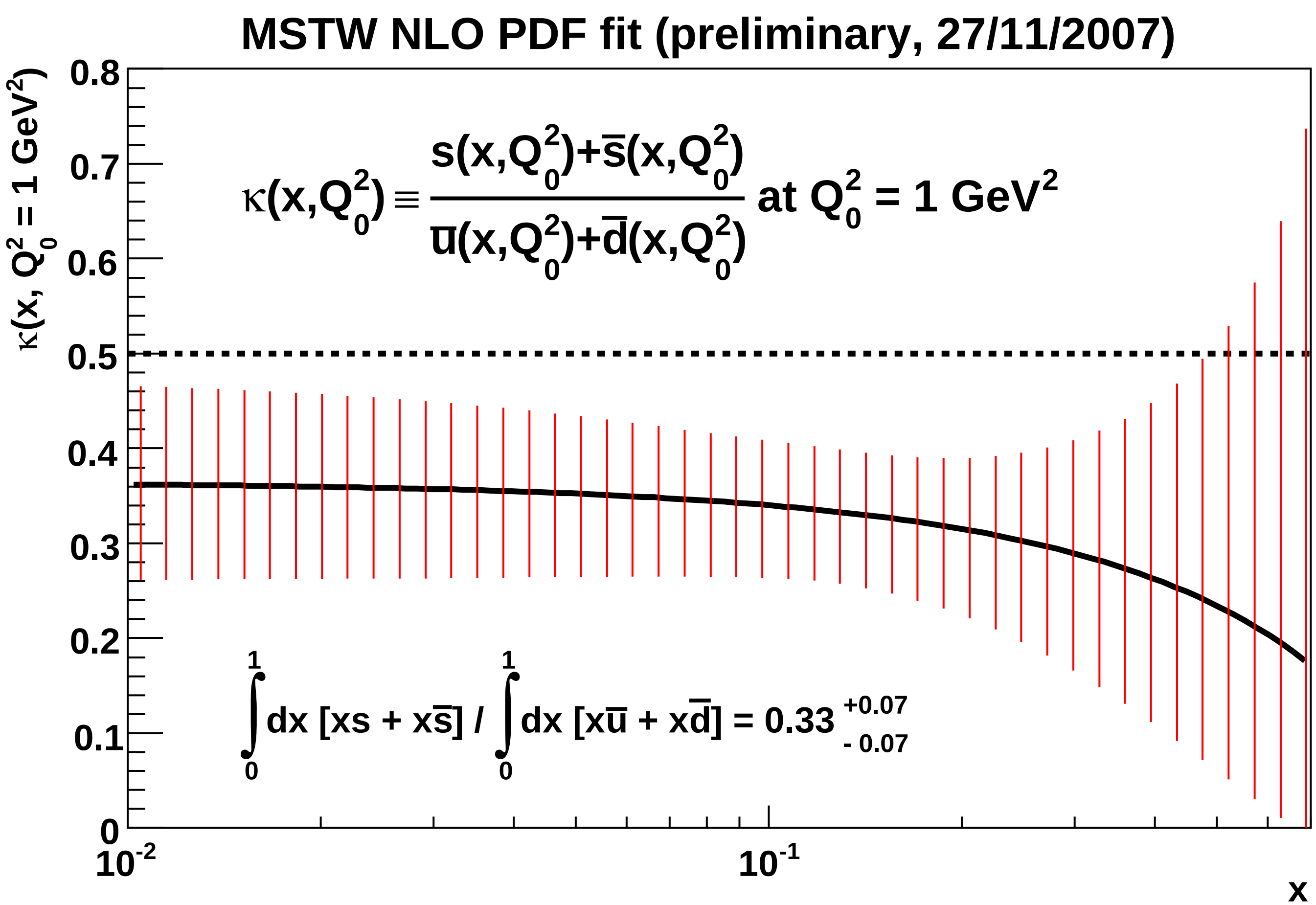
$$F_{L,c}^{CC}(x, Q^2) = \frac{m_c^2}{m_c^2 + Q^2} s(\xi, Q^2) + \left(1 - \frac{m_c^2}{Q^2}\right) \alpha_S(C_{L,g}(x) \otimes g(x, Q^2) + C_{L,q}(x) \otimes s(x, Q^2)),$$

where first term dies away leading to normal massless limit.

Generalise to higher orders.

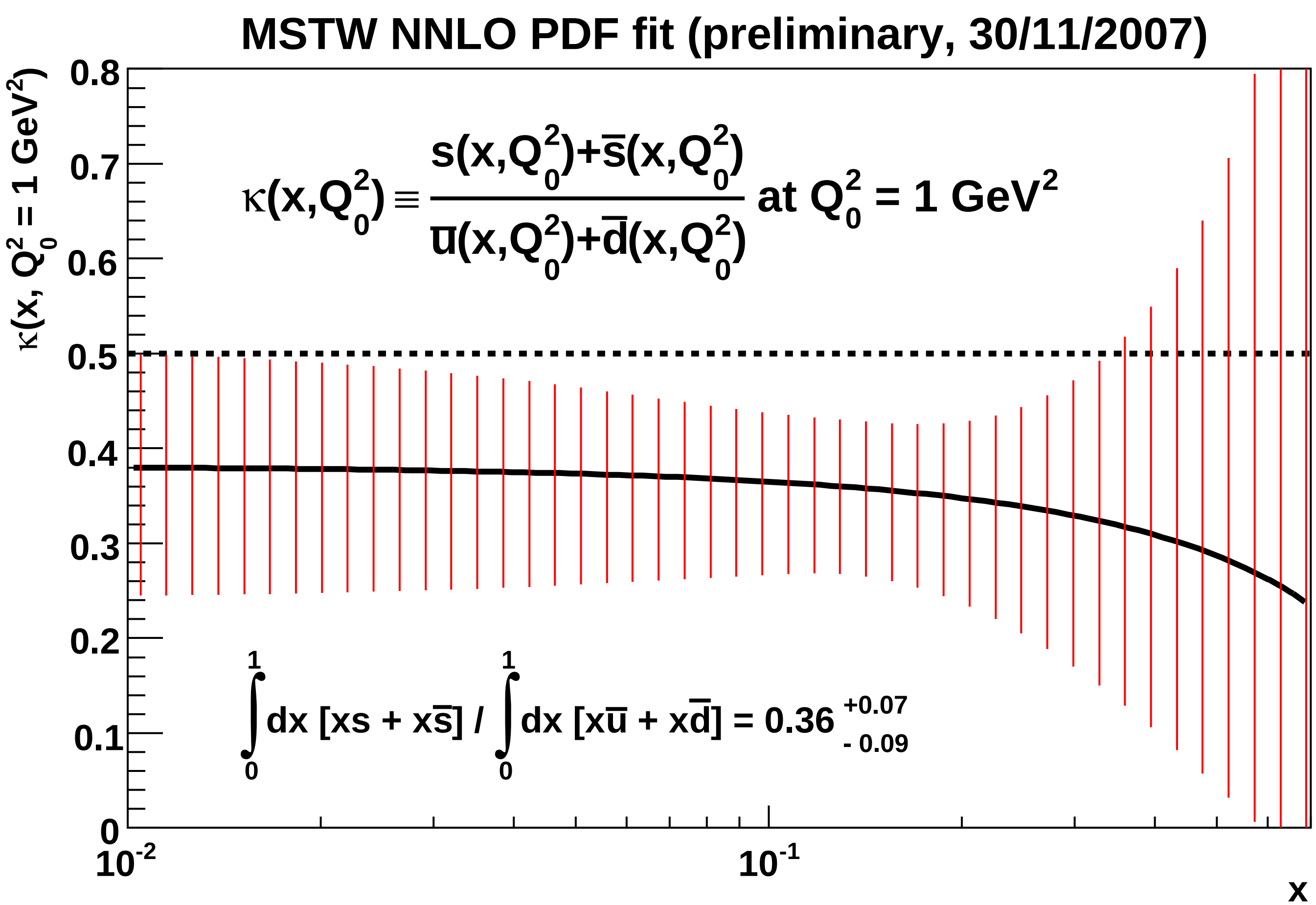
Extraction of strange sensitive, to varying degrees, to these details.

Find slightly reduced ratio of strange to non-strange sea compared to previous default single factor $\kappa = 0.5$.



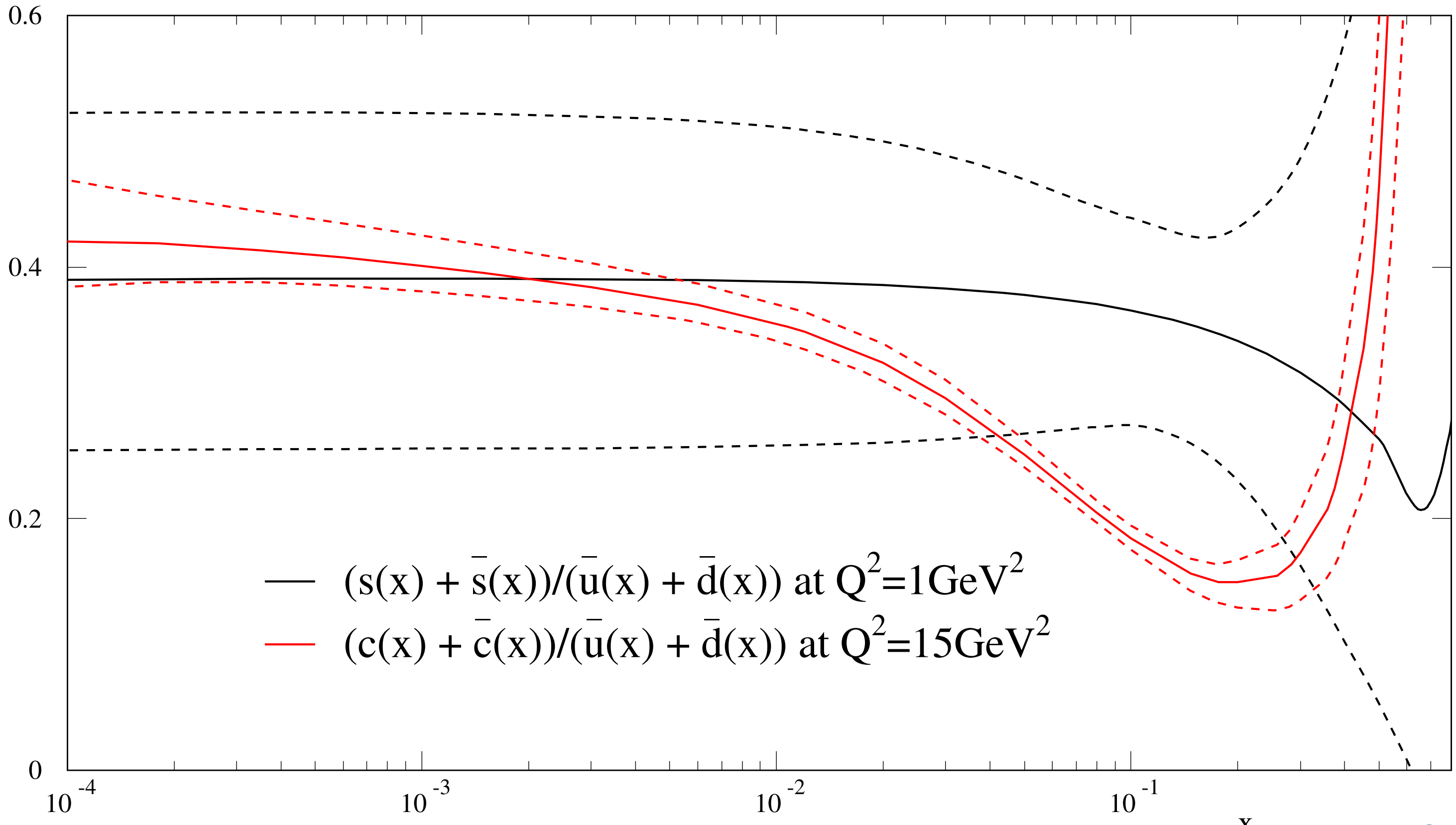
With chosen prescription for charged current heavy flavour, results rather stable from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

NNLO similar overall to NLO. Not completely obvious *a priori* that this would be so.



Strange itself has some non-insignificant mass, and this should qualitatively lead to suppression compared to light sea quarks up and down.

When c and b turn on they evolve like massless quarks, but always lag behind. $\rightarrow$ some suppression at all x for finite Q^2.



$c + \bar{c}$ evolved through $\sim 7-8$ times input scale similar to $s + \bar{s}$ at $Q^2 = 1\mathrm{GeV}^2$. Do not expect exact correspondence, but very good except $c + \bar{c}$ more suppressed at $x \sim 0.1$. (Implication for $s + \bar{s}$ from recent HERMES $K^{\pm}$ data).

Note $c + \bar{c}$ is harder as $x \to 1$ than $\bar{u} + \bar{d}$.

This is more the case at NNLO compared to NLO.

Largely driven by "input" contribution

$$(c + \bar{c})(x, m_c^2) = A_{Hg}(z, Q^2 = m_c^2) \otimes g(x/z, Q^2),$$

where as $z \to 1$

$$A_{Hg}(z, Q^2 = m_c^2) \approx -\frac{10}{9} \frac{\alpha_S^2}{(4\pi)^2} \ln^3(1 - z).$$

Very small absolute effect, but sign of high-x intrinsic charm appearing in perturbative series? (Renormalons - higher twist related to perturbative series)

$\to$ question of intrinsic charm. Contribution of $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$.

In VFNS coefficient functions ambiguous at $\mathcal{O}(m_c^2/Q^2)$. Cancels exactly in perturbative contributions between orders.

For intrinsic charm ambiguity of order $\Lambda_{QCD}^2/m_c^2 * m_c^2/Q^2 = \mathcal{O}(\Lambda_{QCD}^2/Q^2)$, i.e. genuine higher twist.

CTEQ examine possibilities. What about MSTW?

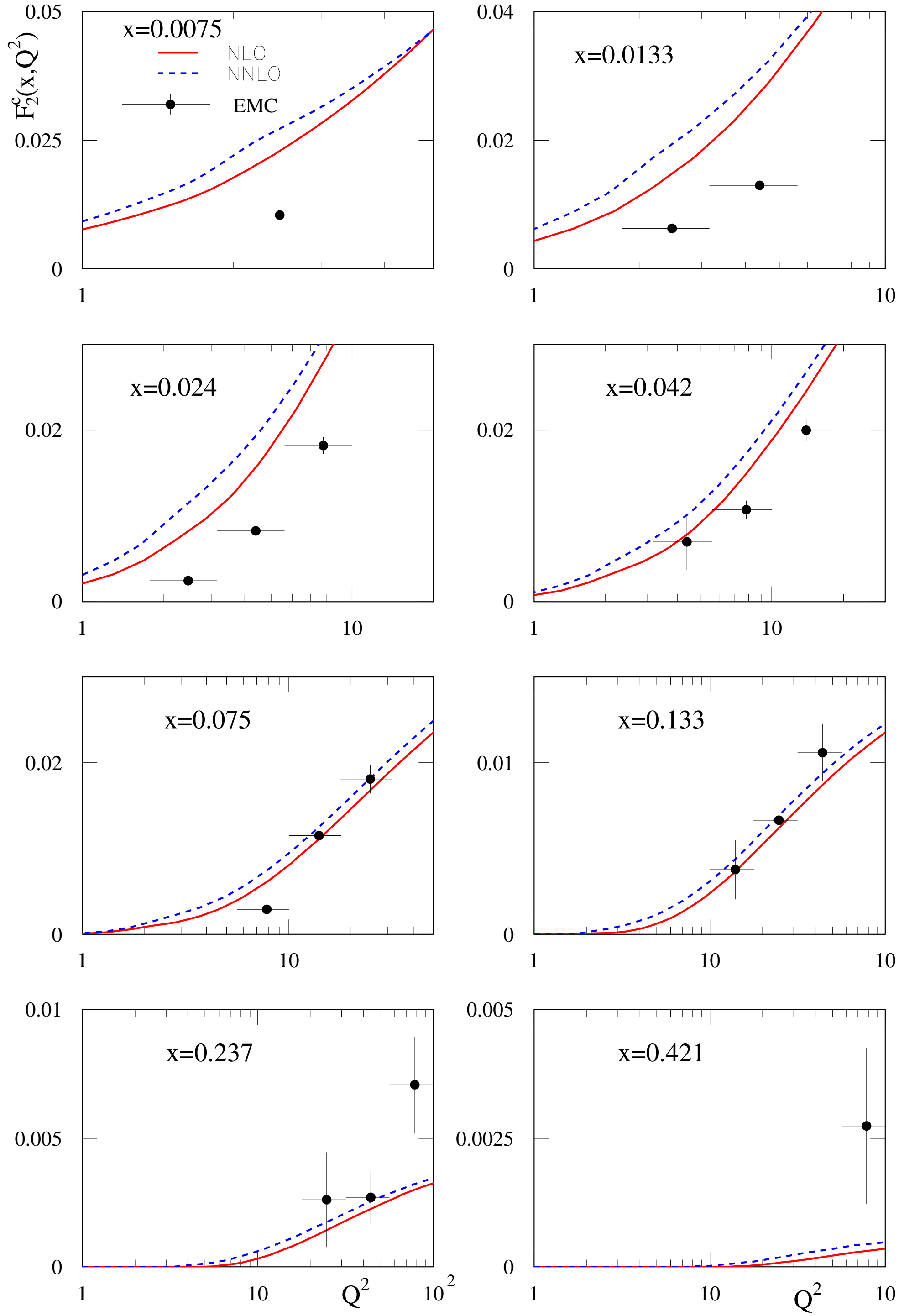
Intrinsic Charm?

First compare to EMC data.

NNLO threshold corrections cause increase.

At fixed NLO and NNLO with $m_c = 1.4\text{GeV}$ some room at $x > 0.2$, but more tendency to overshoot.

Comparison not great overall.



Always have problem with heavy flavours that use $W^2 = 4m_c^2$ as threshold, whereas in reality need to produce mesons, i.e. W^2 a bit greater.

In practice higher twist effect, but EMC data not so far from threshold.

Implement $m_c^2 \rightarrow m_c^2(1+\Lambda^2/m_c^2)$ in threshold dependent parts of coefficient functions, where Λ^2 is a binding energy, e.g. $\Lambda = 0.2\mathrm{GeV}$.

No change in off-threshold parts of coefficient functions, and certainly not in PDF definitions.

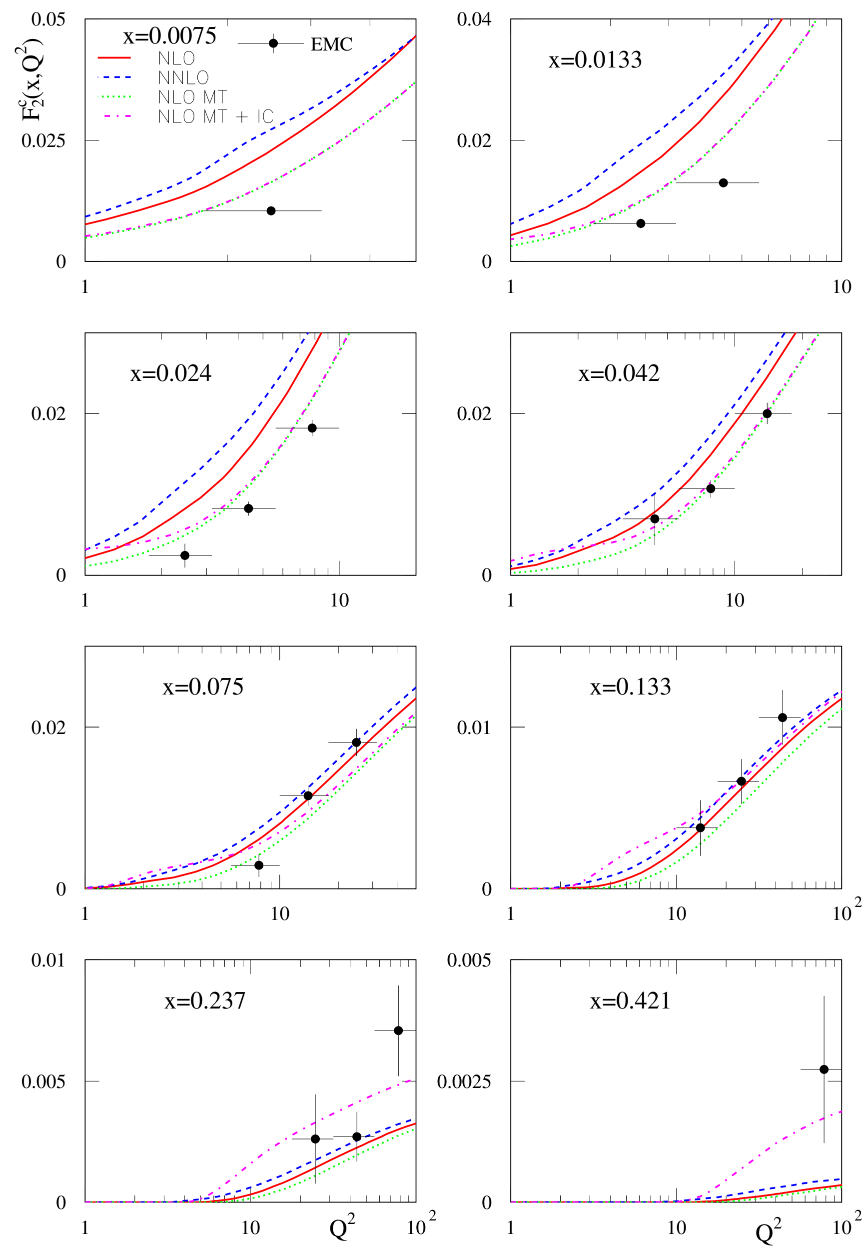
Model nonperturbative corrections for EMC data – much smaller proportional effect for HERA data (but not completely negligible).

Green curve has nonperturbative threshold corrections. Clearly matches lower Q^2 EMC data better. Slightly smaller at higher x.

Pink curve now includes 0.3% contribution of Brodsky, et al model for intrinsic charm (with correct threshold). $\approx 1/20$ of CTEQ upper limit.

Seems about maximum that can be included even with nonperturbative suppression of other contribution.

Note Hoffmann and Moore claimed 0.3% as best fit in 1983 – with very old PDFs and only LO perturbative contribution.



Conclusions

Use a GM-VFNS as default for heavy flavours, as done since 1998. Coefficient functions for heavy quarks now based on those in ACOT(χ) – leads to physically sensible and simple VFNS. However, ordering different in two competing approaches for a variety of quantities. Higher order effect. Sometimes large.

Full NNLO VFNS, has small amount of necessary modelling. Improves fit to lowest x and Q^2 data. Important impact on gluon. (Caveat on uniqueness of $F_2^c(x, Q^2)$ beyond NLO- Chuvakin et al).

Recent changes in T-Roberts prescription lead to differences in partons of at most 2% at NLO. Inherent theoretical uncertainty. Decreases with perturbative order.

Implemented detailed prescription for charged currents. Needs more modelling at low Q^2 but contains physics constraints and tends to correct asymptotic limit.

Order-by-order stability in strange extraction from dimuon data. Strange behaves roughly like mass-suppressed quark with evolution starting from scale $\sim 0.1GeV^2$.

Look for evidence of intrinsic charm. EMC data require nonperturbative corrections for good comparison. If data at all sensible imply some high-x intrinsic charm, but cannot have much more than 0.3% contribution.