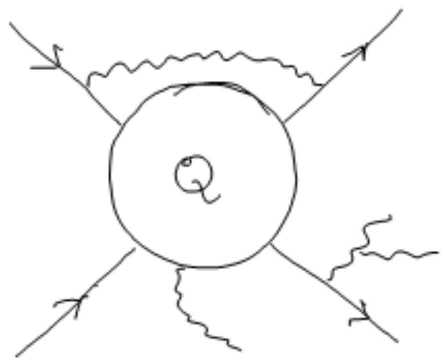
Superleading logarithms in QCD

- Soft gluons in QCD: an introduction.
- Gaps between jets I: the old way (< 2001).
- A second example: Higgs plus two jets.
- Gaps between jets II: the new way (< 2006).
- Superleading logarithms: the newer way?

JF, A. Kyrieleis, M. Seymour: JHEP 0608:059, 2006.
JF, M. Sjödahl: JHEP 0709:119, 2007.
JF, A. Kyrieleis, M. Seymour: JHEP 0809:128, 2008.

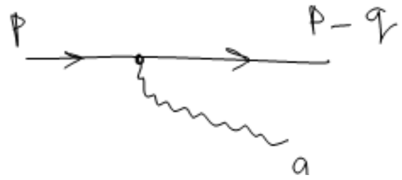
Given a particular hard scattering process we can ask how it will be dressed with additional radiation (perturbatively calculable):



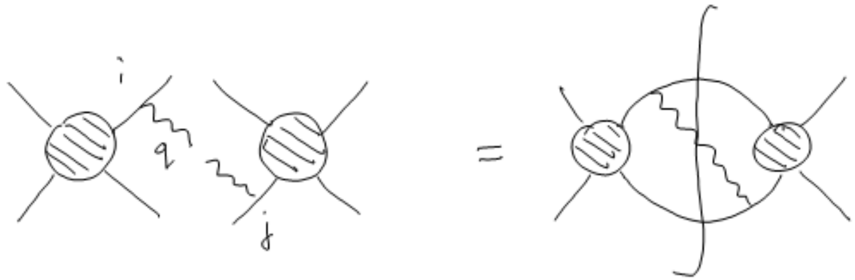
This question may not be interesting a priori because hadronization could wreck any underlying partonic correlations. However experiment reveals that the hadronization process is 'gentle'.

The most important emissions are those involving either <u>collinear</u> quarks/gluons or <u>soft</u> gluons. By important we mean that the usual suppression in the strong coupling is compensated by a large logarithm.

SOFT GLUONS:



$$= 2g p_\mu \delta_{\lambda\lambda'} T^a_{ij}$$

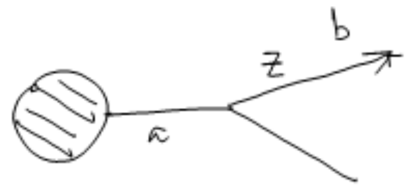


$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi} \frac{dE}{E} \frac{d\Omega}{2\pi} \sum_{ij} C_{ij} E^2 \, \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q}$$

- Only have to consider soft gluons off the external legs of a hard subprocess since internal hard propagators cannot be put on shell.
- Virtual corrections are included analogously….of which more later….
- Only need to consider gluons.
- Colour factor is the “problem”.

COLLINEAR EMISSIONS:

Colour structure is easier. It is as if emission is off the parton to which it is collinear ~ "classical branching".



$$d\sigma_{n+1} = d\sigma_n \, \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} dz \; P_{ba}(z)$$

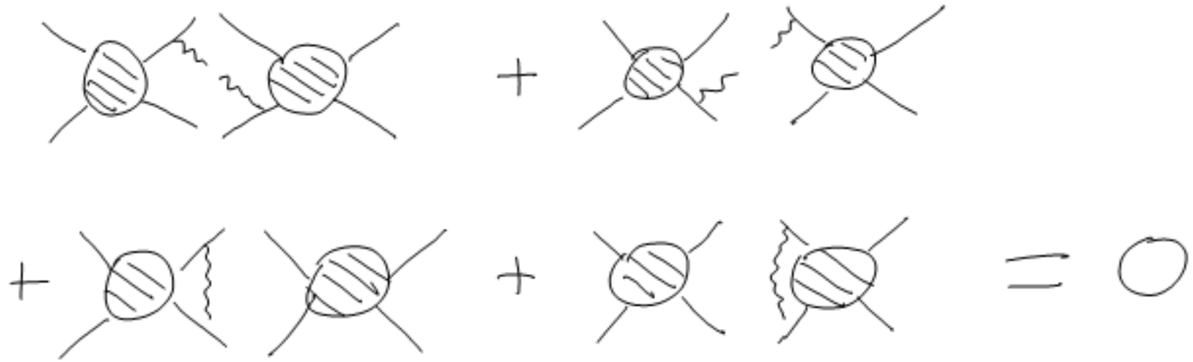
In the Monte Carlos: soft and/or collinear evolution is handled simultaneously using "angular ordered parton evolution".

Folklore: OK only in the large N_c approximation where colour simplifies hugely. Also assumes azimuthal averaging.

Not all observables are affected by soft and/or collinear enhancements

Intuitive: imagine the e^+e^- total cross-section. It cannot care that the outgoing quarks may subsequently radiate additional soft and/or collinear particles (causality and unitarity).

Bloch-Nordsieck: *soft gluon corrections cancel in "sufficiently inclusive" observables.*

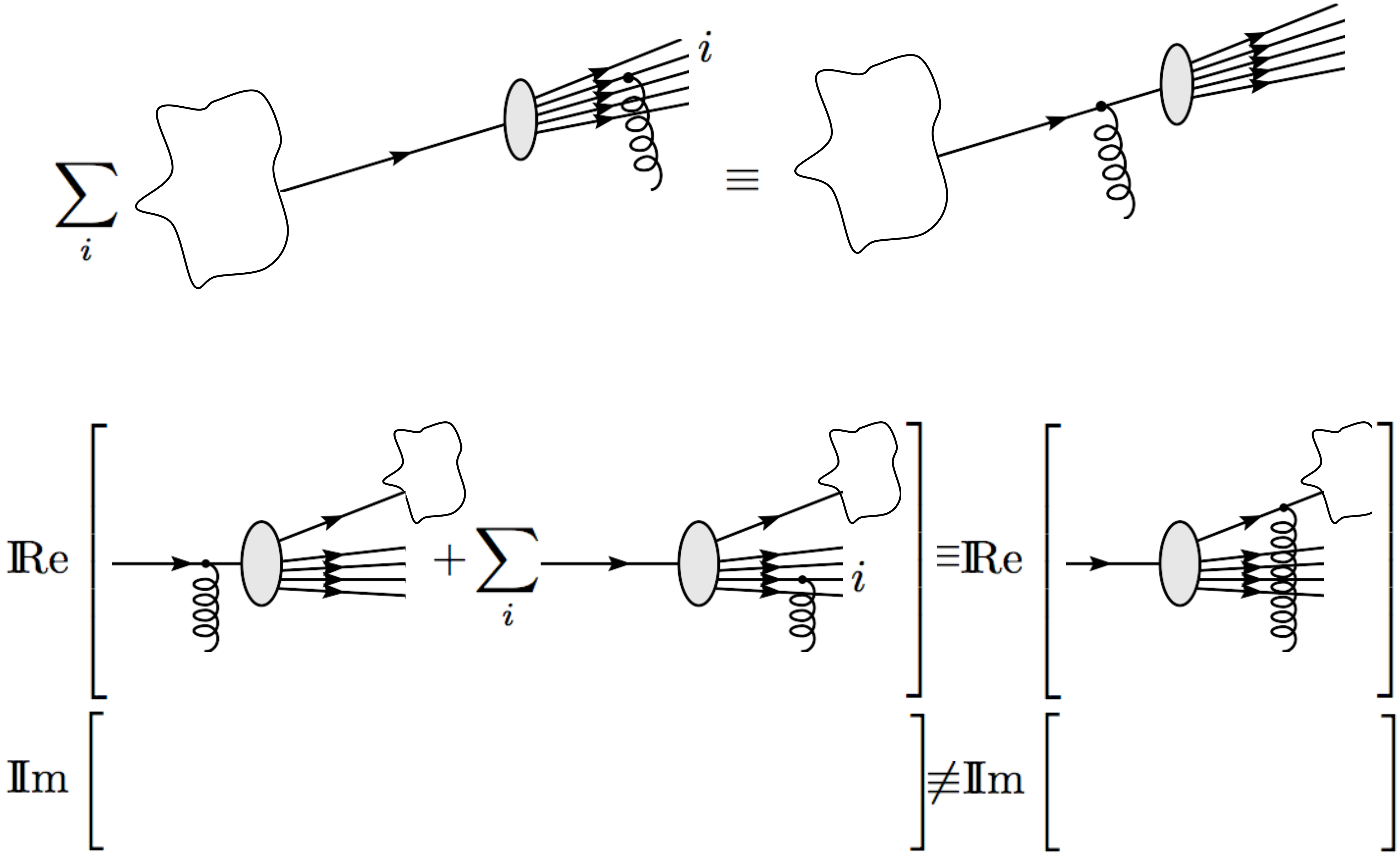


Miscancellation can be induced by restricting the real emissions in some way.

All observables are "sufficiently inclusive" to guarantee that the would-be soft divergence cancels (no detector can detect zero energy particles). But <u>the miscancellation may leave behind a logarithm</u>, e.g. if real emissions are forbidden above μ then virtual corrections give

$$\alpha_s \int_\mu^Q \frac{dE}{E} = \alpha_s \ln \frac{Q}{\mu}$$

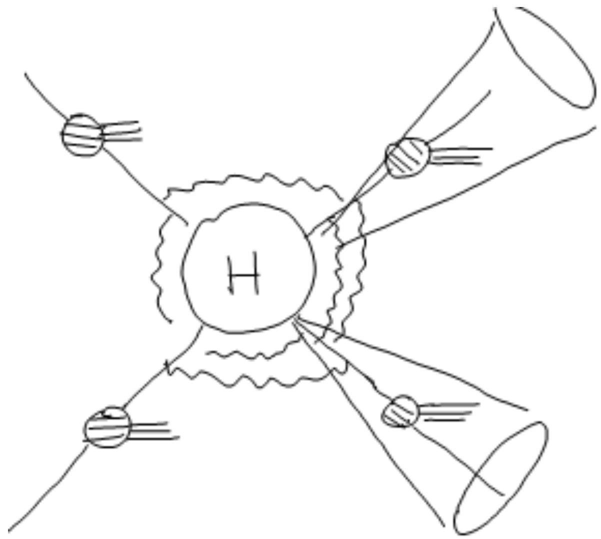
COHERENCE:



$\sum_i$
i
$\equiv$
$\mathbb{R}\mathrm{e}$
$+\sum_i$
$\equiv \mathbb{R}\mathrm{e}$
$\mathbb{I}\mathrm{m}$
$\neq \mathbb{I}\mathrm{m}$

COHERENCE:

It is exploited to factorize collinear emissions from soft, wide angle, gluon emissions.



The failure of the “coherence identity” for the imaginary part will be significant later.

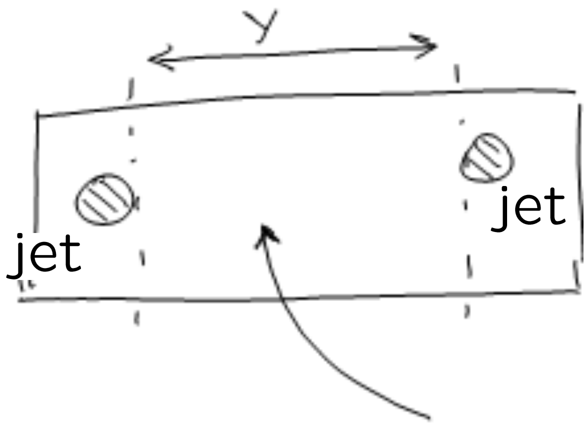
Soft gluon corrections will be important for observables that insist on only *small deviations from lowest order kinematics*.

In such cases real radiation is constrained to a small corner of phase space and BN miscancellation induces large logarithms.

If V measures 'distance' from the lowest order kinematics:

Event shapes such as thrust ($V = 1 - T$)
Production near threshold (top, W/Z) ($V = 1 - M^2/\hat{s}$)
Drell-Yan at low p_T (W/Z or Higgs) ($V = p_T^2/\hat{s}$)
Deep-inelastic scattering at large x ($V = 1 - x$)
Gaps between jets....

GAPS BETWEEN JETS:



Jets produced with $p_T = Q \gg Q_0$

No radiation in between jets with $k_T > Q_0$

Observable restricts emission in the gap region therefore expect

$$\alpha_s^n \ln^n(Q/Q_0)$$

i.e. do not expect collinear enhancement since we sum inclusively over the collinear regions of the incoming and outgoing partons.

We start with the original calculation of Oderda & Sterman…and work only with quark-quark scattering.

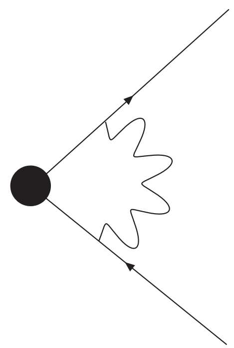
Real emissions are forbidden in the phase-space region

$$-Y/2 < y < Y/2$$
$$k_T > Q_0$$

"*By Bloch-Nordsieck, all other real emissions cancel and we therefore only need to compute the* <u>*virtual*</u> *soft gluon corrections to the primary hard scattering*."

$e^+e^- \to q\bar{q}$ case is very simple:

$$\sigma_{\text{gap}} = \sigma_0 \ \exp\left(-C_F \frac{\alpha_s}{\pi} Y \ln\left(\frac{Q}{Q_0}\right)\right)$$



The virtual gluon is integrated over "in gap" momenta, i.e. the region where real emissions are forbidden.

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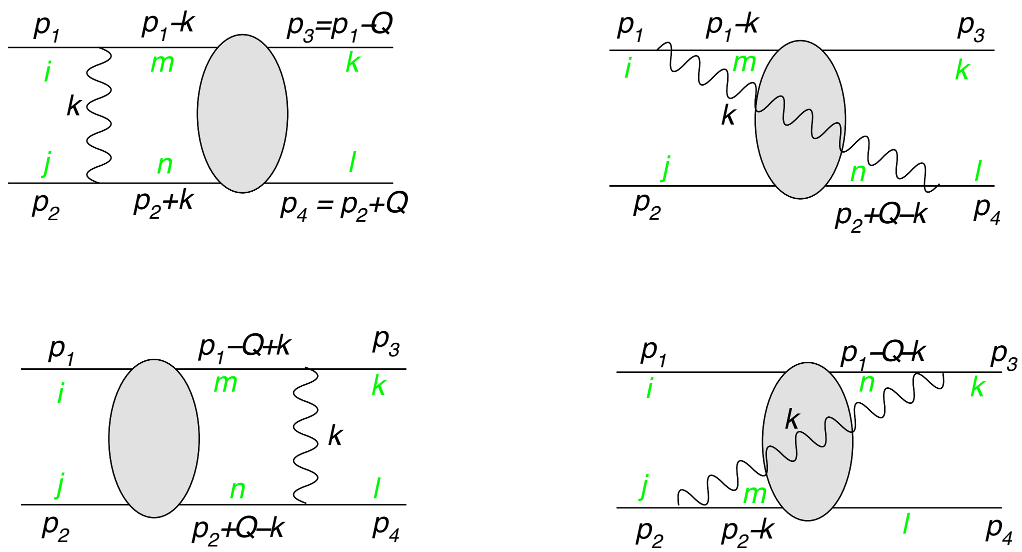


The virtual gluon is integrated over "in gap" momenta, i.e. the region where real emissions are forbidden.

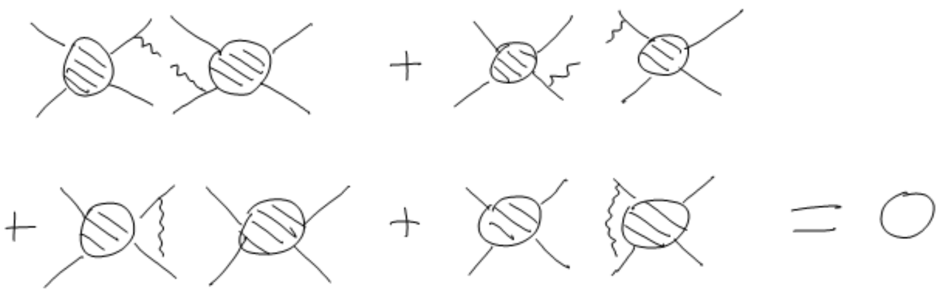
(plus two others)

Coulomb gluons

- I have skipped over a subtle issue…..the real-virtual cancellation of soft gluons occurs point-by-point in (y, k_T) only between the *real parts* of the virtual correction and the real emission.

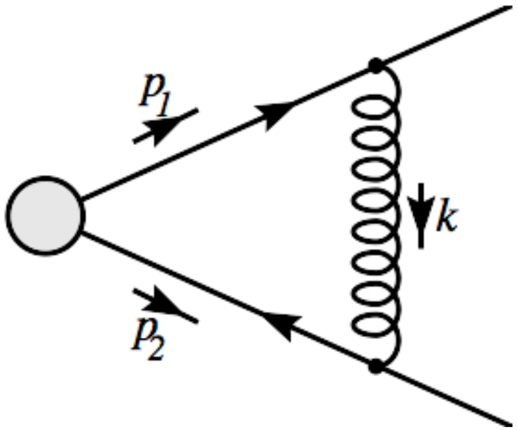
- The imaginary part obviously cancels if the soft gluon is closest to the cut….but what about subsequent evolution? Might this spoil the real-virtual cancellation below Q_0?

- No, it does not. The "non-cancelled" iπ terms exponentiate to produce a pure phase in the amplitude → no physical effect, i.e. it is "as if they cancelled".



e^+e^- revisited:

The colour structure is simple enough that the Coulomb gluons lead only to a phase even above Q_0.



$$\mathcal{A}_1 = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F\left(Y - i\pi\right)\mathcal{A}_0$$

eikonal $k^2=0$

Coulomb $p_1{}^2=p_2{}^2=0$

$$\mathcal{A} = e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0$$

$$\sigma = \mathcal{A}^\star\mathcal{A} = \mathcal{A}_0^\star e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y+i\pi)}e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0$$

i π terms cancel

Back to gaps between jets…

The amplitude can be projected onto a colour basis:

$$(M)_{ij}^{kl} = M^{(1)} C_{ijkl}^{(1)} + M^{(8)} C_{ijkl}^{(8)}$$

$$\begin{aligned} C_{ijkl}^{(8)} &= (T^a)_{ik}(T^a)_{jl} \\ C_{ijkl}^{(1)} &= \delta_{ik}\delta_{jl}. \end{aligned}$$

i.e. $\mathbf{M} = \begin{pmatrix} M^{(1)} \\ M^{(8)} \end{pmatrix}$ and $\sigma = \mathbf{M}^\dagger \mathbf{S}_V \mathbf{M}$

$$\mathbf{S}_V = \begin{pmatrix} N^2 & 0 \\ 0 & \frac{N^2-1}{4} \end{pmatrix}$$

Iterating the insertion of soft virtual gluons builds up the N^{th} order amplitude:

$$\mathbf{M} = \exp\left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_T}{k_T}\, \mathbf{\Gamma} \right) \mathbf{M}_0$$

The factorial needed for exponentiation arises as a result of ordering the transverse momenta of successive soft gluons, i.e.

$$Q_0 \ll k_{T1} \ldots \ll k_{TN} \ll Q$$

where the evolution matrix is

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{N^2-1}{4N}\rho(Y,\Delta y) & \frac{N^2-1}{4N^2} i\pi \\ i\pi & -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N}\rho(Y,\Delta y) \end{pmatrix}$$

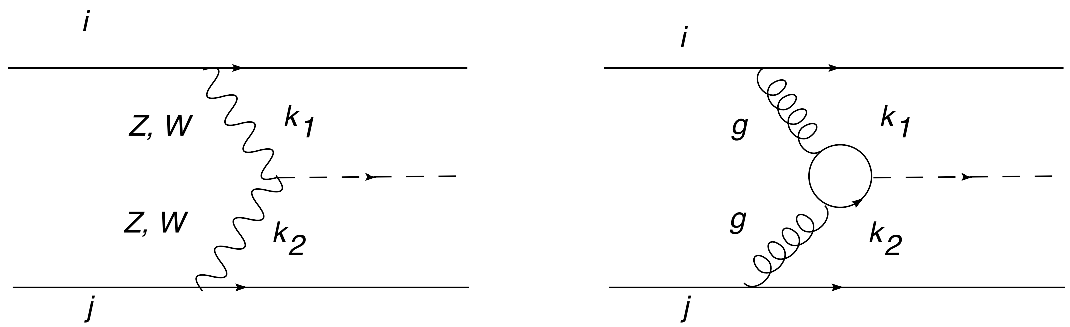
$\Delta y =$ distance between jet centres $\qquad Y =$ size of gap

In qq → qq the colour structure is more complicated than e^+e^- and the Coulomb gluons no longer exponentiate into a phase above Q_0 (due to the presence of the real parts of the virtual corrections).

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{N^2-1}{4N}\rho(Y,\Delta y) & \frac{N^2-1}{4N^2} i\pi \\ i\pi & -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N}\rho(Y,\Delta y) \end{pmatrix}$$

Coulomb gluons are relevant

An example: Higgs plus two jets



- To reduce backgrounds and to focus on the VBF channel, experimenters will make a veto on additional radiation between the tag jets, i.e. no additional jets with

$$k_T \geq Q_0$$

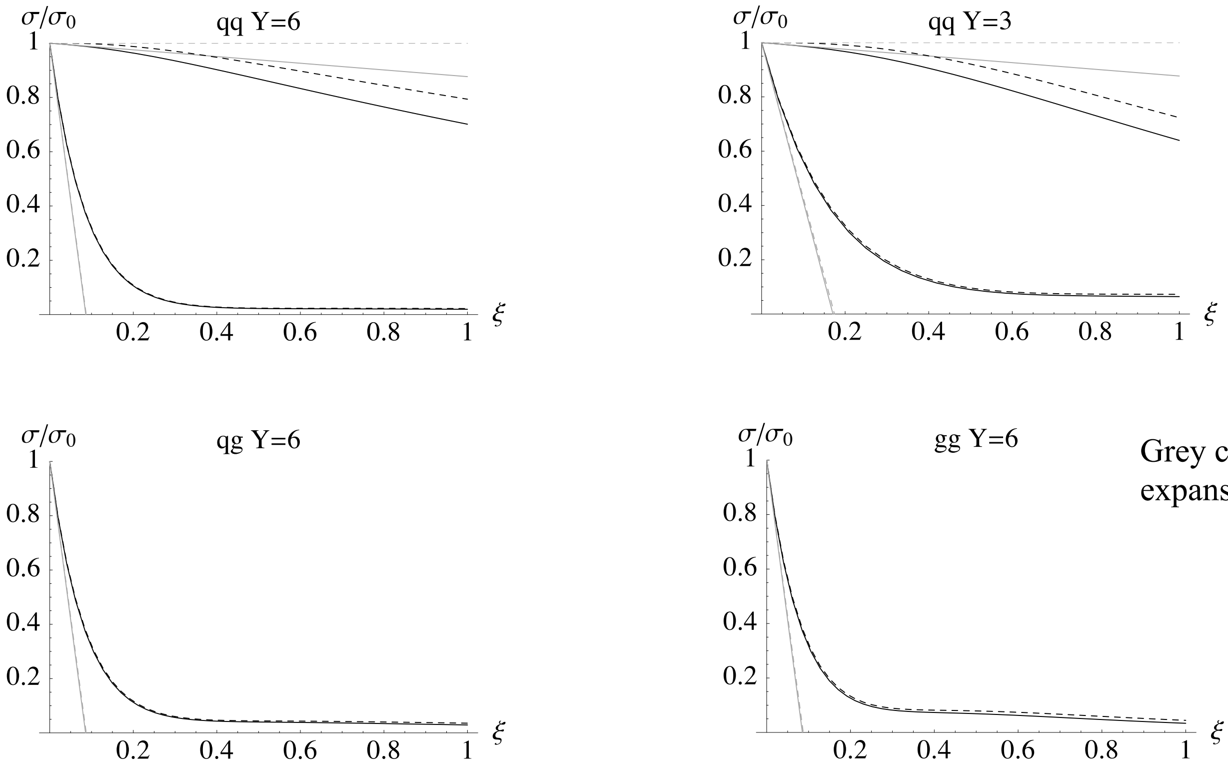
- Soft gluon effects will induce logarithms:

$$\alpha_s^n \ln^n(Q/Q_0)$$

$Q = \text{transverse momentum of tag jets}$

Resummation proceeds almost exactly as for “gaps between jets”

$$\mathbf{M} = \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\,\mathbf{\Gamma}\right)\mathbf{M}_0 \qquad \rho(Y,\Delta y) \to \frac{1}{2}(\rho(Y,2|y_3|)+\rho(Y,2|y_4|))$$



Grey curves = lowest order expansion of black curves.

Only the colour of the exchange matters.

$$\xi = \int_{Q_0}^{Q}\frac{dq}{q}\alpha_s(q) \approx 0.2$$ for 100 GeV jets and a 20 GeV veto, i.e. resummation is important at LHC

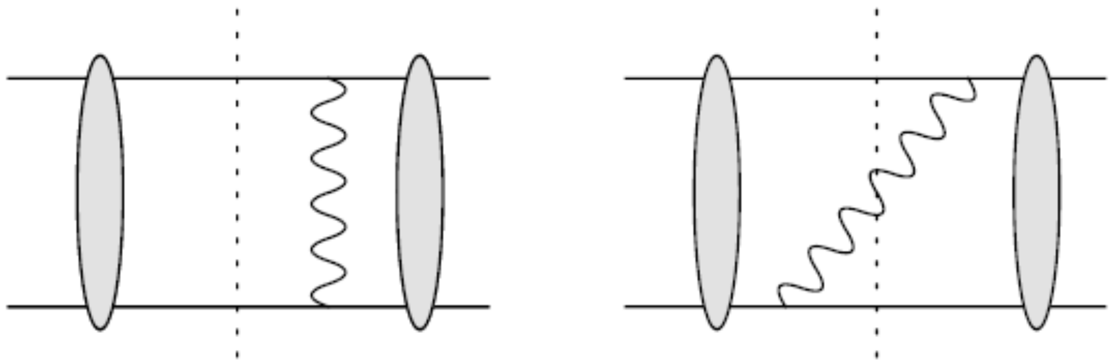
JF & Malin Sjödahl (2007)

- Fixed order calculations cannot account adequately for the effect of a veto.

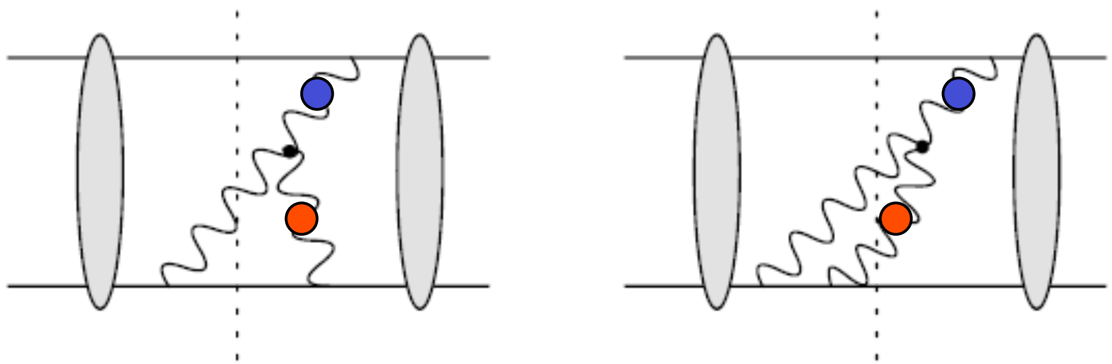
- How much is this physics already present in parton shower Monte Carlos?

- gg-VBF interference is present but is negligibly small ($< 1\%$).

But there is a big fly in the ointment: these observables are *non-global*



Such real & virtual corrections cancel.



But these do not if the gluon marked with a red blob is in the forbidden region: the 2nd cut is not allowed.

So the cancellation does not hold…..

real and virtual

It fails only once we start to evolve emissions (such as those denoted by the blue blob in the above) which lie *outside of the gap* region and which have $k_T > Q_0$

$|y| > Y/2$

If $k_T < Q_0$ then subsequent evolution also has $k_T < Q_0$ and cancellation works.

- The miscancellation is telling us that this observable is sensitive to soft gluon emissions outside of the gap, even though the observable sums inclusively over that region.

- Not a surprise once we realise that emissions outside of the gap can subsequently radiate back into the gap.

- We must therefore include any number of emissions outside of the gap and their subsequent evolution.

- Colour structure makes this impossible using current technology.

- We could aim to compute the all orders non-global corrections in the leading N_c approximation. Dasgupta, Salam, Appleby, Seymour, Delenda, Banfi

- Instead we choose to compute the "<u>one hard emission out of the gap</u>" contribution without any approximation on the colour.

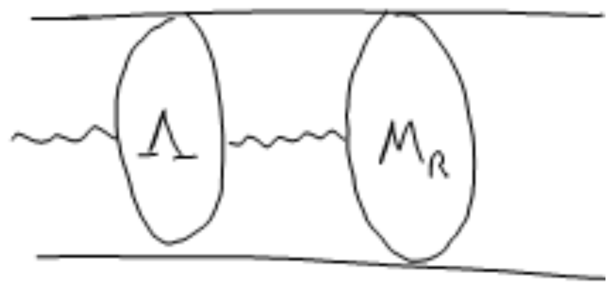
Two new ingredients still sticking to quark-quark scattering

1) How to add a real gluon to the four-quark amplitude

$$\mathbf{M}_R = \boxed{\mathbf{D}} \cdot \mathbf{M}$$

2) How to evolve the five-parton amplitude

$$\mathbf{M}_R(Q_0) = \exp\left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \boxed{\mathbf{\Lambda}} \right) \mathbf{M}_R(k_T)$$



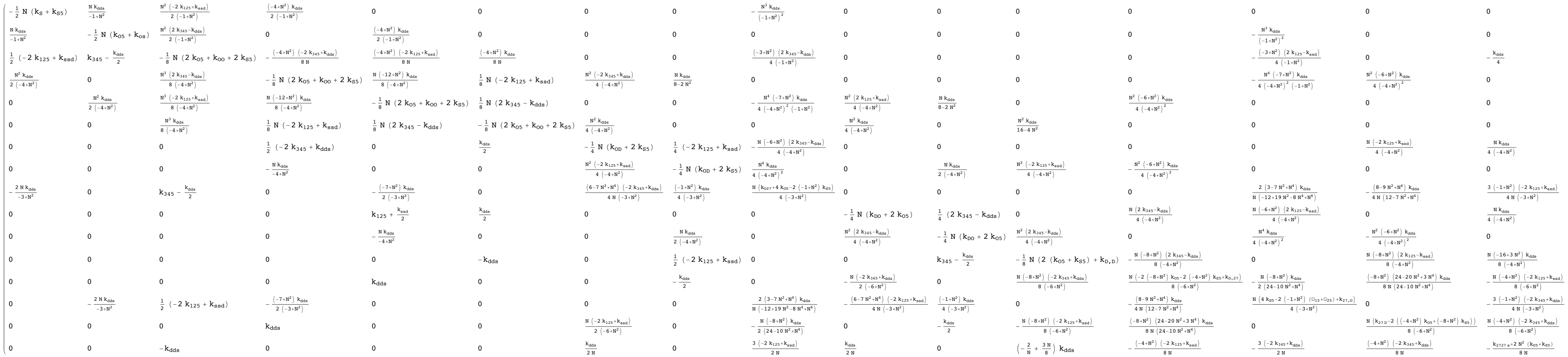
Kyrieleis & Seymour

$$\mathbf{D}^{\mu} = \begin{pmatrix} \frac{1}{2}(-h_1^{\mu} - h_2^{\mu} + h_3^{\mu} + h_4^{\mu}) & \frac{1}{4N}(-h_1^{\mu} - h_2^{\mu} + h_3^{\mu} + h_4^{\mu}) \\ 0 & \frac{1}{2}(-h_1^{\mu} - h_2^{\mu} + h_3^{\mu} + h_4^{\mu}) \\ \frac{1}{2}(-h_1^{\mu} + h_2^{\mu} + h_3^{\mu} - h_4^{\mu}) & \frac{1}{4N}(h_1^{\mu} - h_2^{\mu} - h_3^{\mu} + h_4^{\mu}) \\ 0 & \frac{1}{2}(-h_1^{\mu} + h_2^{\mu} - h_3^{\mu} + h_4^{\mu}) \end{pmatrix} \qquad h_i^{\mu} = \frac{1}{2} k_T \frac{p_i^{\mu}}{p_i \cdot k}$$

$$\begin{aligned}
\mathbf{\Lambda} = & \begin{pmatrix} \frac{N}{4}(Y - i\pi) + \frac{1}{2N} i\pi & \left(\frac{1}{4} - \frac{1}{N^2}\right) i\pi & -\frac{N}{4} s_y Y & 0 \\ i\pi & \frac{N}{4}\left(2Y - i\pi\right) - \frac{3}{2N} i\pi & 0 & 0 \\ -\frac{N}{4} s_y Y & 0 & \frac{N}{4}(Y - i\pi) - \frac{1}{2N} i\pi & -\frac{1}{4} i\pi \\ 0 & 0 & -i\pi & \frac{N}{4}\left(2Y - i\pi\right) - \frac{1}{2N} i\pi \end{pmatrix} \\
& + \begin{pmatrix} N & 0 & 0 & 0 \\ 0 & N & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 0 & N \end{pmatrix} \frac{1}{4} \rho(Y, 2\left|y\right|) \\
& + \begin{pmatrix} C_F & 0 & 0 & 0 \\ 0 & C_F & 0 & 0 \\ 0 & 0 & C_F & 0 \\ 0 & 0 & 0 & C_F \end{pmatrix} \frac{1}{2} \rho(Y, \Delta y) \\
& + \begin{pmatrix} \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2} s_y \lambda\right) & \frac{1}{4}\left(\frac{1}{2} s_y \lambda\right) \\ 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(\frac{1}{2} s_y \lambda\right) \\ \frac{N}{4}\left(-\frac{1}{2} s_y \lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & \frac{1}{4}\left(-\frac{1}{2}\lambda\right) \\ \frac{1}{2} s_y \lambda & \left(\frac{N}{4} - \frac{1}{N}\right)\left(\frac{1}{2} s_y \lambda\right) & -\frac{1}{2}\lambda & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) \end{pmatrix}
\end{aligned}$$

Recently extended to all five parton amplitudes:

e.g. gg → ggg



Malin Sjödahl (2008)

…and most recently to arbitrary n-parton amplitudes:

$$\mathbf{\Gamma} = \frac{1}{2} Y \mathbf{T}_t^2 + \boxed{i\pi \mathbf{T}_1 \cdot \mathbf{T}_2} + \frac{1}{4} \sum_{i \in F} \rho(Y; 2|y_i|) \mathbf{T}_i^2$$
$$+ \frac{1}{2} \sum_{(i<j) \in L} \lambda(Y; |y_i| + |y_j|, |\phi_i - \phi_j|) \mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{2} \sum_{(i<j) \in R} \lambda(Y; |y_i| + |y_j|, |\phi_i - \phi_j|) \tag{3.12}$$

Easy to see it is final state collinear safe but not initial state collinear safe.

i.e. $\mathbf{\Gamma} \sim \mathbf{T}_i + \mathbf{T}_j$ only for *i* and *j* collinear *and* in final state

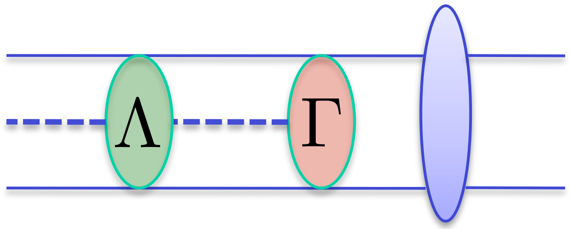
JF, Kyrieleis & Seymour (2008)

The complete cross-section for one real emission outside of the gap is thus

$$\sigma_R = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\mathrm{out}}\frac{dy\,d\phi}{2\pi}$$

$$\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}^\dagger\right)\mathbf{D}_\mu^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Lambda}^\dagger\right)\mathbf{S}_R$$

$$\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Lambda}\right)\mathbf{D}^\mu \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\mathbf{M}_0$$



And the corresponding contribution when the out-of-gap gluon is virtual is

$$\sigma_V = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\text{out}}\frac{dy\,d\phi}{2\pi}$$
$$\left[\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}^\dagger\right)\mathbf{S}_V\right.$$
$$\left.\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\gamma\exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\mathbf{M}_0 + \text{c.c.}\right]$$

Adds one "out of the gap" virtual gluon

Conventional wisdom: when the out of gap gluon becomes collinear with either incoming quark or either outgoing quark the real and virtual contributions should cancel.

This cancellation operates for **final state collinear emission**:

$$\mathbf{D}^{\mu\dagger}(\mathbf{\Lambda}^{\dagger})^{n-m}\mathbf{S}_R\mathbf{\Lambda}^m\mathbf{D}_\mu + (\mathbf{\Gamma}^{\dagger})^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m\gamma + \gamma^{\dagger}(\mathbf{\Gamma}^{\dagger})^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m = \mathbf{0}$$

But it fails for **initial state collinear emission**:

The problem is entirely due to the emission of Coulomb gluons.

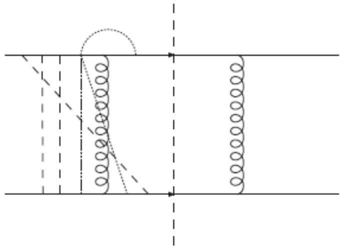
Cancellation *does* occur for n = 1, 2 and 3 gluons relative to lowest order but not for larger n. This is the lowest order where the Coulomb gluons do not trivially cancel.

The non-cancelling diagrams…..

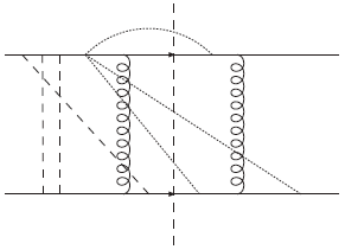
Dotted line is the out-of-gap gluon.

Dashed lines are in-gap & Coulomb gluons.

Springs are hard scatter gluons.



(a)



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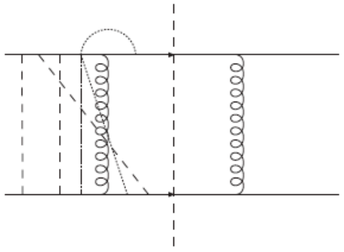
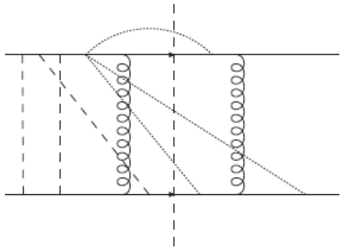
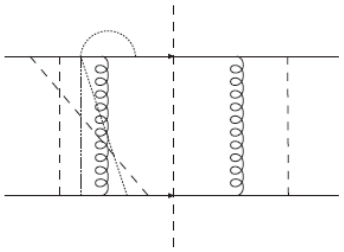


(c)



(d)



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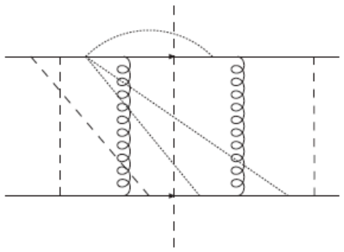

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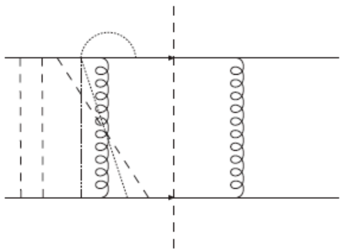
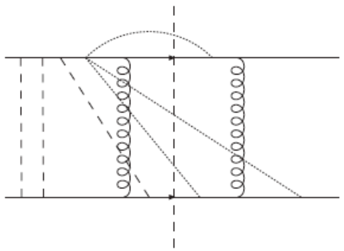
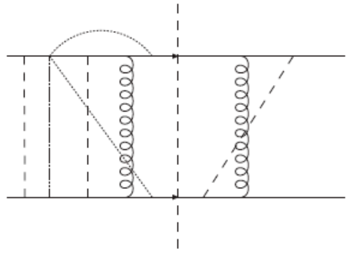


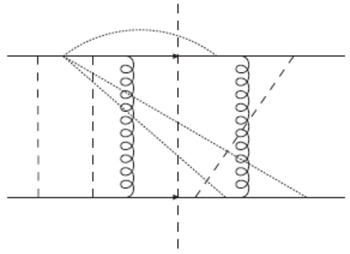
(g)



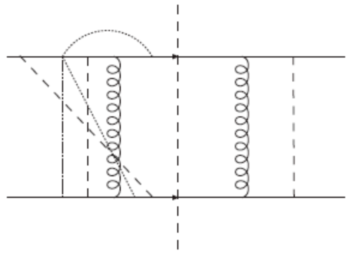
(h)



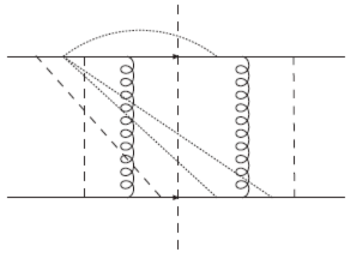
(a)



(b)



(c)



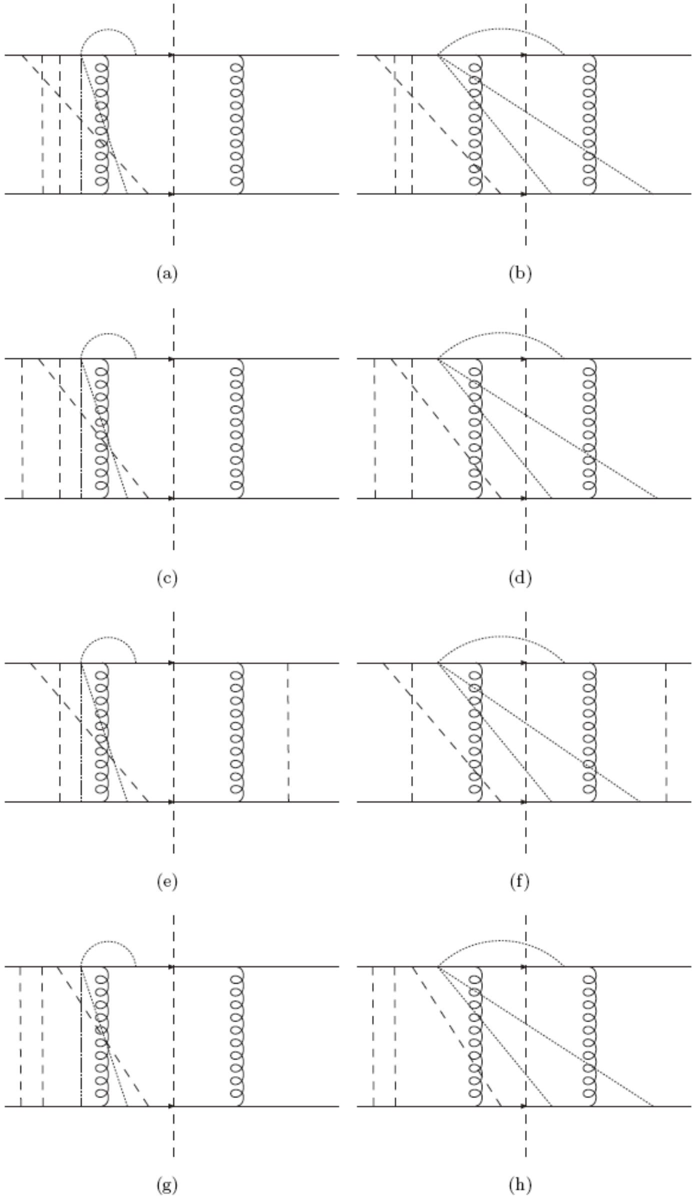
(d)

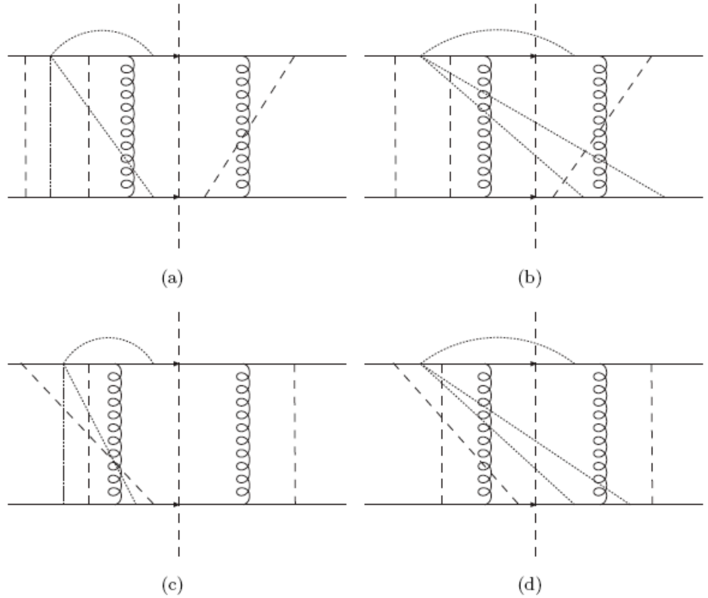
The non-cancelling diagrams…..

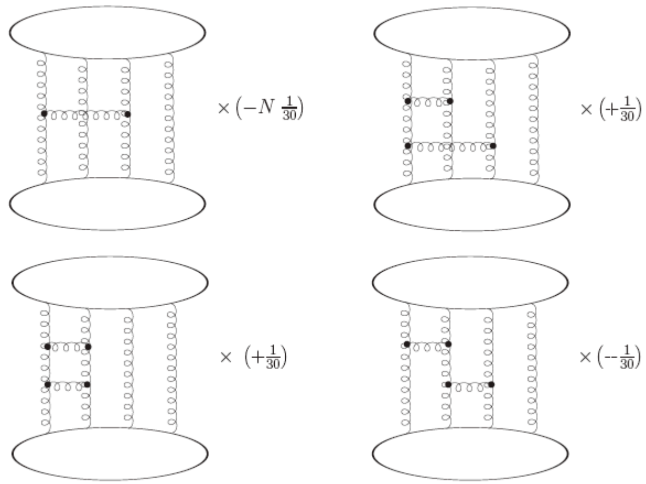
Dotted line is the out-of-gap gluon.

Dashed lines are in-gap & Coulomb gluons.

Springs are hard scatter gluons.







Colour traces ~ small-x physics?

What are we to make of a non-cancelling collinear divergence?

$$\sigma \sim \sigma_0 \; \alpha^4 L^4 \pi^2 Y \int_{\mathrm{out}} dy$$

Cannot actually have infinite rapidity with $k_T > Q_0$

Need to go beyond soft gluon approximation in collinear limit:

$$\int d^2k_T \int\limits_{\mathrm{out}} dy \quad \left.\frac{d\sigma}{dyd^2k_T}\right|_{\mathrm{soft}} \to \int d^2k_T \left[\int\limits^{y_{\max}} dy \quad \left.\frac{d\sigma}{dyd^2k_T}\right|_{\mathrm{soft}} + \int\limits_{y_{\max}}^{\infty} dy \quad \left.\frac{d\sigma}{dyd^2k_T}\right|_{\mathrm{collinear}} \right]$$

$$\int\limits_{y_{\max}}^{\infty} dy \quad \left.\frac{d\sigma}{dyd^2k_T}\right|_{\mathrm{collinear}} = \int\limits_{y_{\max}}^{\infty} dy \; \left(\left.\frac{d\sigma_{\mathrm{R}}}{dyd^2k_T}\right|_{\mathrm{collinear}} + \left.\frac{d\sigma_{\mathrm{V}}}{dyd^2k_T}\right|_{\mathrm{collinear}} \right)$$

Soft approximation:

$$\int dz\, \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) \to \int dy$$

Real collinear emission:

$$\int_{y_{\max}}^{\infty} dy \left.\frac{d\sigma_{\mathrm{R}}}{dy\,d^2k_T}\right|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)\frac{q(x/z,\mu^2)}{q(x,\mu^2)} A_{\mathrm{R}}$$

$$= \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)\left(\frac{q(x/z,\mu^2)}{q(x,\mu^2)} - 1\right) A_{\mathrm{R}} + \int_0^{1-\delta} dz \frac{1}{2}\frac{1+z^2}{1-z} A_{\mathrm{R}}$$

Virtual collinear emission:

$$y > y_{\max}$$

implies

$$\delta \approx \frac{k_T}{Q}\exp\left(y_{\max} - \frac{\Delta y}{2}\right)$$

$$\int_{y_{\max}}^{\infty} dy \left.\frac{d\sigma_{\mathrm{V}}}{dy\,d^2k_T}\right|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) A_{\mathrm{V}}$$

If $A_{\mathrm{R}} + A_{\mathrm{V}} = 0$
then the divergence would cancel leaving behind a regularized splitting which would correspond to the DGLAP evolution of the incoming quark pdf. These purely collinear logs could then be resummed by selecting the scale of the pdf to be the jet scale Q.

But as we have seen, the Coulomb gluons spoil this cancellation. Instead we have

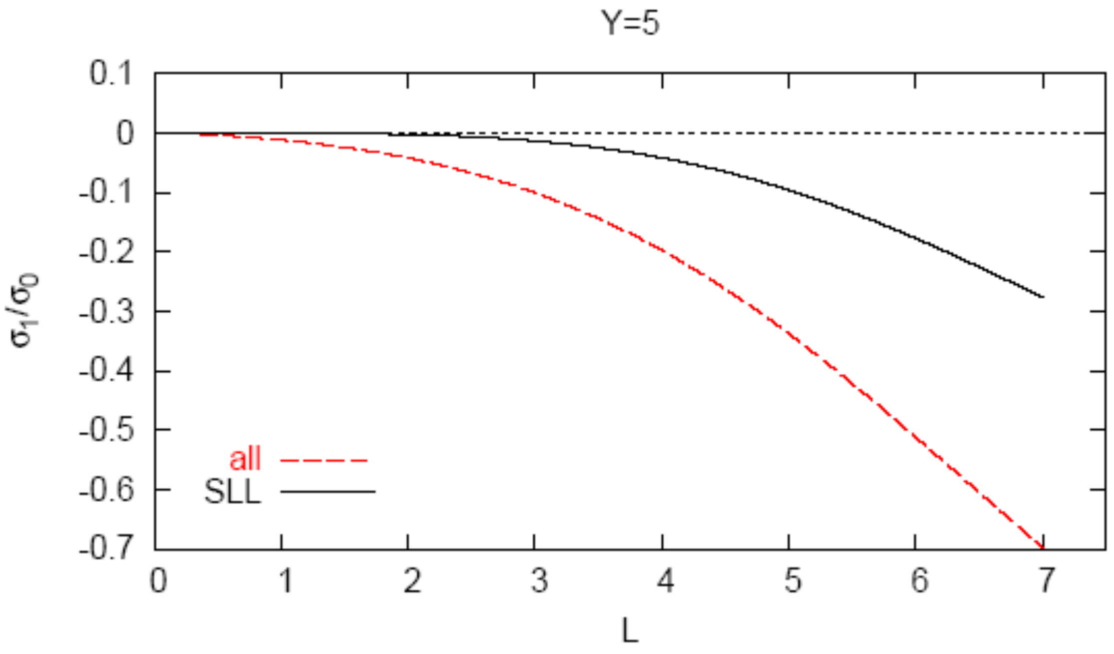
$$\int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)(A_{\mathrm{R}}+A_{\mathrm{V}}) = \ln\left(\frac{1}{\delta}\right)(A_{\mathrm{R}}+A_{\mathrm{V}}) + \text{subleading}$$
$$\approx \left(-y_{\max} + \frac{\Delta y}{2} + \ln\left(\frac{Q}{k_T}\right)\right)(A_{\mathrm{R}}+A_{\mathrm{V}})$$

Hence

$$\int_{Q_0}^{Q} \frac{dk_T}{k_T} \int_{\mathrm{out}} dy \to \int_{Q_0}^{Q} \frac{dk_T}{k_T} \left(\int^{y_{\max}} dy + \left(-y_{\max} + \ln\frac{Q}{k_T}\right)\right) = \frac{1}{2}\ln^2\frac{Q}{Q_0}$$

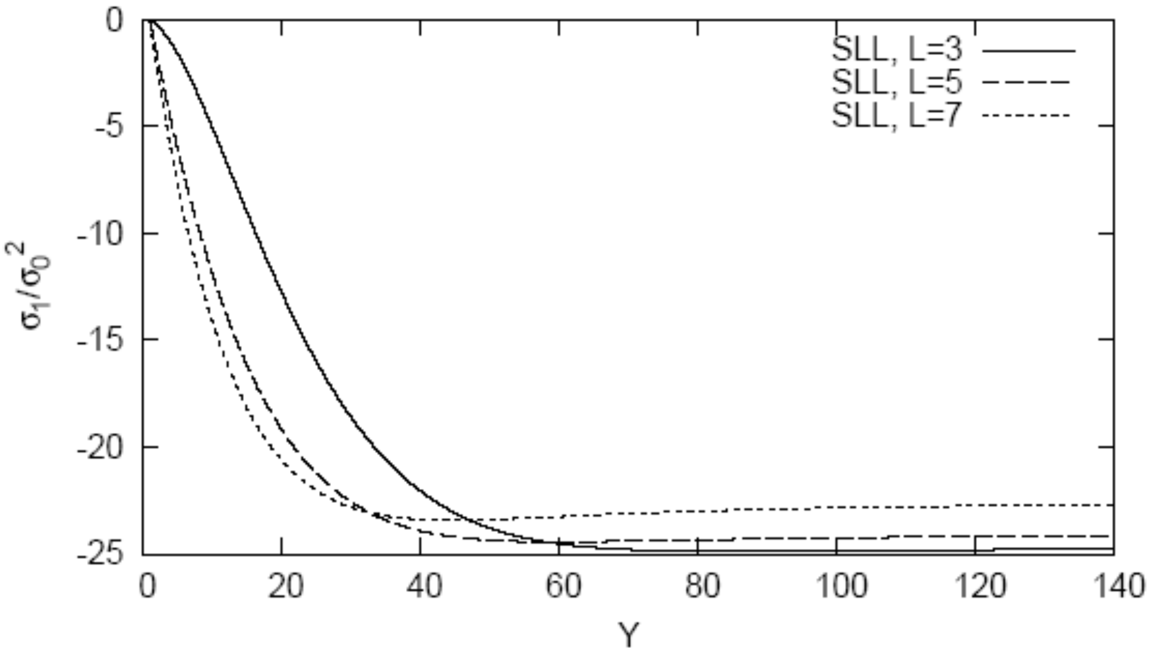
The final result for the “one emission out-of-gap” cross-section is

$$\sigma_{1,\mathrm{SLL}} = -\sigma_0 \left(\frac{2\alpha_s}{\pi}\right)^4 \ln^5\left(\frac{Q}{Q_0}\right) \pi^2 Y \frac{(3N^2-4)}{480}$$



Modest but potentially not a negligible phenomenological impact.

We already knew single non-global logs are potentially important (but can be reduced by taking a small cone radius).
Appleby & Seymour

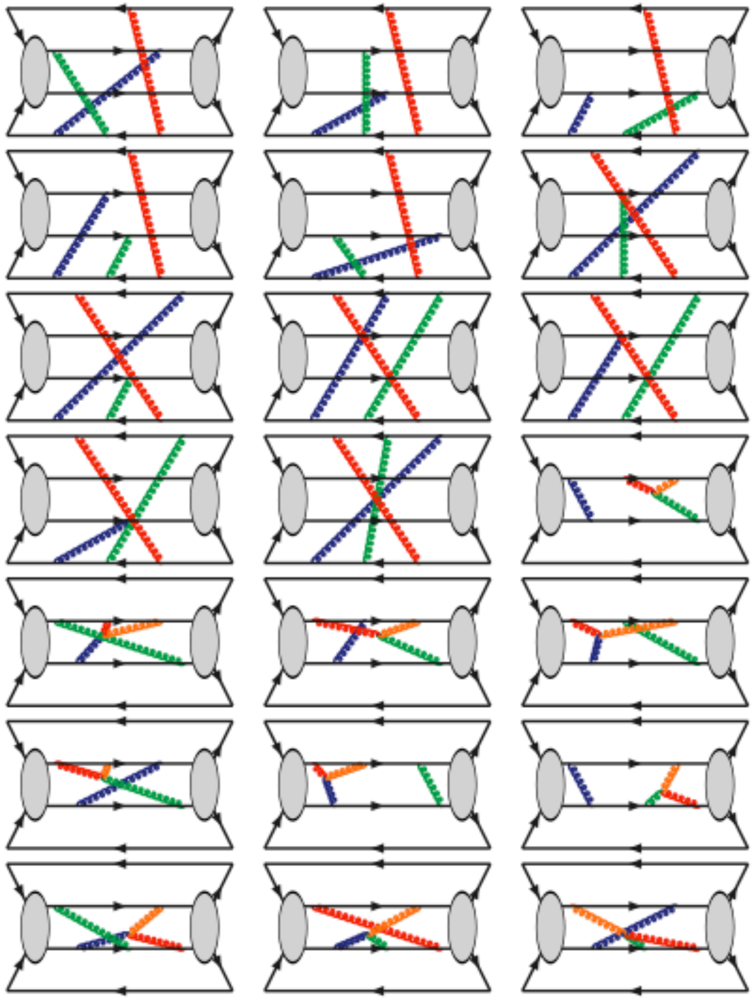


Intriguing link to non-linear effects in small-x physics.

Marchesini & Mueller.
Banfi, Marchesini & Smye.

A fixed order cross check:

- Gluons are added in all possible ways to trace diagrams and colour factors calculated using COLOUR.
- Diagrams are then cut in all ways consistent with strong ordering.
- At fourth order there are 10,529 diagrams and 1,746,272 after cutting.
- Super-leading terms are seen at fourth order, confirms our calculation.
- Can go to higher orders (more gluons out of gap) and also check k_T ordering assumption.



James Keates

Concluding comments on super-leading logs:

- Need to add the contribution from $n > 1$ out-of-gap gluons.

- The $\alpha_s^4 L^5$ term we just computed *cannot* be cancelled by an $n > 1$ contribution.

- To get the "leading" logs correct requires a "next-to-leading" calculation of the evolution matrices etc. (Dixon, Mert Aybat, Sterman)

- Shocking: large collinear enhancements in an observable that sums inclusively over the collinear region.
  Conventional wisdom says expect soft enhancement but not soft-collinear, i.e. constitutes a breakdown of collinear factorization ("plus prescription" fails) and of coherence.

- Implications for other observables?

- Recently extended to all partonic sub-processes. (JF, Kyrieleis, Seymour)

Conclusions

- Are the super-leading logarithms really there? Implications?
  [coherent states/remnants? k_T ordering?]

- Soft re-summation may be important for Higgs-plus-two-jet production.

- “Standard” non-global effects have not yet been included in Higgs-plus-two-jet production.

- Pressing need to establish how reliable existing resummations, based on parton shower Monte Carlos, actually are.