

# $D^*$ MESONS IN PHOTOPRODUCTION AT HERA

H. Spiesberger  
Univ. Mainz

based on work in collaboration with  
B. Kniehl, G. Kramer, I. Schienbein

- Theoretical framework: heavy quark production  
the General-Mass Variable-Flavor-Number Scheme  
for 1-particle inclusive heavy-meson production  
cf. talks by F. Olness, R. Thorne (structure functions)
- Numerical results for  $D^*$  meson production  
in photoproduction at HERA:  $\gamma^* p \rightarrow D^* X$   
cf. talk by K. Lipka (experimental results)

## Massive or Massless Heavy Quarks?

$m \neq 0 \longrightarrow$

- correct threshold behavior  
no collinear divergences from  $c \rightarrow c + g$   
but terms  $\propto \log(\mu/m)$  with  $\mu = Q, p_T, \dots$
- large corrections at large  $\mu$

$m = 0 \longrightarrow$

- not reliable at heavy quark threshold
- QCD prediction: DGLAP (RG) evolution resums  
large logarithms  $\log(\mu/m)$
- more reliable at large  $\mu$

**Goal:** combine massive (low scale) and massless (high scale) calculations

- exploit freedom to choose an appropriate factorization scheme

## GENERAL-MASS VARIABLE FLAVOR NUMBER SCHEME

- The problem:

Conventionally, PDFs and FFs are defined in the  $\overline{\text{MS}}$  scheme

$\overline{\text{MS}}$  scheme is based on a massless calculation

Massless and massive calculations contain different singularities

Can not use  $\overline{\text{MS}}$  PDFs and FFs in a massive calculation?

...details →

## MASS SINGULARITIES

### (1) massive calculation $m \neq 0$

example: gluon radiation:  $\gamma\gamma \rightarrow c\bar{c}g$

$$\int \frac{d^3 k}{2k_0} \frac{1}{(p+k)^2 - m^2} = \int \frac{d^3 k}{2k_0} \frac{1}{2pk} \longrightarrow$$

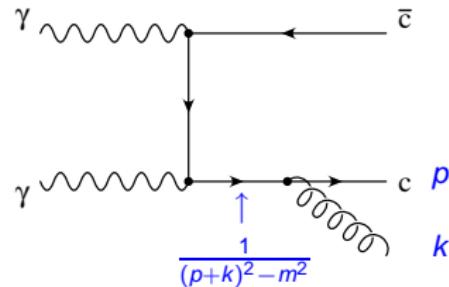
$$\int \frac{d \cos \theta}{E - p \cos \theta} = \frac{1}{p} \log \left( \frac{E+p}{E-p} \right) \rightarrow \frac{1}{p} \log \left( \frac{E^2}{m^2} \right)$$

$$\text{using } E = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2E}$$

NLO corrections contain **logarithms**  $\sim \log \frac{p_T^2}{m^2}$

( $\rightarrow$  mass singularities, collinear log; associated with initial or final state partons)

Numerically well-defined, but potentially large



### (2) calculation with $m = 0$

dimensional regularization ( $D = 4 - 2\epsilon$ )

$\log(m^2)$ -terms correspond to  $\frac{1}{\epsilon}$ -poles

in dim.reg: coupling  $g^2 \rightarrow g^2 \mu^{2\epsilon}$  and phase space contains  $S^{-\epsilon} \rightarrow$  terms

$$\frac{1}{\epsilon} \mu^{2\epsilon} S^{-\epsilon} = \frac{1}{\epsilon} \exp \left( \epsilon (\ln \mu^2 - \ln S) \right) \rightarrow \frac{1}{\epsilon} \left( 1 + \epsilon \log \frac{\mu^2}{S} \right)$$

## GENERAL-MASS VARIABLE FLAVOR NUMBER SCHEME

- The problem:

Conventionally, PDFs and FFs are defined in the  $\overline{\text{MS}}$  scheme

$\overline{\text{MS}}$  scheme is based on a massless calculation

Massless and massive calculations contain different singularities

Can not use  $\overline{\text{MS}}$  PDFs and FFs in a massive calculation?

---

- The solution:

Match massless and massive calculations:

$$d\sigma_{\text{sub}} = \lim_{m \rightarrow 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\text{MS}}}$$

The subtracted cross section (in a massive calculation)

$$d\hat{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma_{\text{sub}}$$

can be used with  $\overline{\text{MS}}$  parton distribution and fragmentation functions

→ The **GM-VFNS** (general-mass variable flavor number scheme)

## FACTORIZATION FORMULAE FOR $A + B \rightarrow H + X$

$$A + B \rightarrow H + X: \quad d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\sigma(ij \rightarrow kX) \otimes D_k^H(z)$$

sum over all possible subprocesses  $i + j \rightarrow k + X$

Parton distribution functions:

$$f_i^A(x_1, \mu_F), f_j^B(x_2, \mu_F)$$

**non-perturbative** input

long distance

universal

Hard scattering

cross section:

$$d\sigma(\mu_F, \mu'_F, \alpha_s(\mu_R), [\frac{m_h}{p_T}])$$

**perturbatively** computable

short distance

(coefficient functions)

Fragmentation functions:

$$D_k^H(z, [\mu'_F])$$

**non-perturbative** input

long distance

universal

Accuracy:

light hadrons:  $\mathcal{O}((\Lambda/p_T)^p)$  with  $p_T$  hard scale,  $\Lambda$  hadronic scale,  $p = 1, 2$

heavy hadrons:  $\mathcal{O}((m_h/p_T)^p)$  if  $m_h$  neglected in  $d\sigma$

Details (subprocesses, PDFs, FFs; mass terms) depend on  
the **Heavy Flavour Scheme**

- collinear logs:  $\log(p_T^2/m^2) = \log(p_T^2/\mu^2) + \log(\mu^2/m^2)$ , terms with  $\log(\mu^2/m^2)$ : subtracted from hard part and absorbed in parton distribution and fragmentation functions resummed by DGLAP evolution equations
- Parton distribution functions for  $g, u, d, s$ , and  $c$ , charm is a parton:  $f_c \neq 0$
- VFNS:  $f_c = 0$  below,  $f_c \neq 0$  above threshold;  $\longrightarrow$  GM-VFNS with  $m \neq 0$
- Fragmentation functions, e.g. for  $c \rightarrow D^*$ :  $D_c^{D^*}(z, \mu_F^2)$  with non-perturbative input and perturbative RG evolution

- collinear logs:  $\log(p_T^2/m^2) = \log(p_T^2/\mu^2) + \log(\mu^2/m^2)$ , terms with  $\log(\mu^2/m^2)$ : subtracted from hard part and absorbed in parton distribution and fragmentation functions resummed by DGLAP evolution equations
- Parton distribution functions for  $g, u, d, s$ , and  $c$ , charm is a parton:  $f_c \neq 0$
- VFNS:  $f_c = 0$  below,  $f_c \neq 0$  above threshold;  $\rightarrow$  GM-VFNS with  $m \neq 0$
- Fragmentation functions, e.g. for  $c \rightarrow D^*$ :  $D_c^{D^*}(z, \mu_F^2)$  with non-perturbative input and perturbative RG evolution

- technically involved:
    - calculation with  $m \neq 0$
    - mass factorization with massive regularization
  - + large collinear logarithms  $\ln \frac{\mu^2}{m^2}$  resummed in evolved  $f_c(x, \mu^2)$  and  $D_c^{D^*}(x, \mu^2)$
  - +  $(\frac{m}{p_T})^n$  included
- $\Rightarrow$  good for smaller  $p_T$ :  $0 < p_T^2 \lesssim m^2$  and  $p_T^2 \gg m^2$

Mass terms contained in the hard scattering coefficients:

$$d\hat{\sigma}(\mu_F, \mu_{F'}, \alpha_s(\mu_R), \frac{m}{p_T})$$

Two ways to derive them:

- (1) Compare **massless limit** of a massive fixed-order calculation with a massless  $\overline{\text{MS}}$  calculation to determine subtraction terms

OR

- (2) Perform **mass factorization** using partonic PDFs and FFs

## (1) SUBTRACTION TERMS FOR THE GM-VFNS FROM MASSLESS LIMIT

- Compare limit  $m \rightarrow 0$  of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...) with massless  $\overline{\text{MS}}$  calculation (Aurenche et al., Aversa et al., ...)

$$\lim_{m \rightarrow 0} d\tilde{\sigma}(m) = d\hat{\sigma}_{\overline{\text{MS}}} + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{sub}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\text{MS}}}$$

## (1) SUBTRACTION TERMS FOR THE GM-VFNS FROM MASSLESS LIMIT

- Compare limit  $m \rightarrow 0$  of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...) with massless  $\overline{\text{MS}}$  calculation (Aurenche et al., Aversa et al., ...)

$$\lim_{m \rightarrow 0} d\tilde{\sigma}(m) = d\hat{\sigma}_{\overline{\text{MS}}} + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{sub}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\text{MS}}}$$

- Subtract  $d\sigma_{\text{sub}}$  from **massive** partonic cross section while **keeping mass terms**

$$d\hat{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma_{\text{sub}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$  dependence

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization  $\otimes$  **massive** short distance cross sections

## (1) SUBTRACTION TERMS FOR THE GM-VFNS FROM MASSLESS LIMIT

- Compare limit  $m \rightarrow 0$  of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...) with massless  $\overline{\text{MS}}$  calculation (Aurenche et al., Aversa et al., ...)

$$\lim_{m \rightarrow 0} d\tilde{\sigma}(m) = d\hat{\sigma}_{\overline{\text{MS}}} + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{sub}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\text{MS}}}$$

- Subtract  $d\sigma_{\text{sub}}$  from **massive** partonic cross section while **keeping mass terms**

$$d\hat{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma_{\text{sub}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$  dependence

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization  $\otimes$  **massive** short distance cross sections

- Treat contributions with charm in the initial state with  $m = 0$
- Massless limit: technically non-trivial, map from phase-space slicing to subtraction method

## Mass factorization

Subtraction terms are associated to mass singularities:

can be described by

**partonic PDFs and FFs** for collinear splittings  $a \rightarrow b + X$

- initial state:

$$f_{g \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(x) \ln \frac{\mu^2}{m^2}$$

$$f_{Q \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[ \frac{1+z^2}{1-z} \left( \ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+$$

$$f_{g \rightarrow g}^{(1)}(x, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \frac{1}{3} \ln \frac{\mu^2}{m^2} \delta(1-x)$$

- final state:

$$d_{g \rightarrow Q}^{(1)}(z, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(z) \ln \frac{\mu^2}{m^2}$$

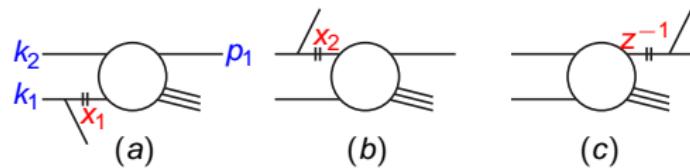
$$d_{Q \rightarrow Q}^{(1)}(z, \mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[ \frac{1+z^2}{1-z} \left( \ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+$$

- Other partonic distribution functions are zero to this order in  $\alpha_s$

Mele, Nason; Kretzer, Schienbein; Melnikov, Mitov

## (2) SUBTRACTION TERMS FOR THE GM-VFNS VIA MASS FACTORIZATION

**g** and light  $q(\bar{q})$   
collinear emission  
from initial or final state



(a) direct  $\gamma$  parton  $a$  in  $p$

$$\begin{aligned} d\sigma_{\text{sub}}(\gamma a \rightarrow QX) &= \int_0^1 dx_1 f_{a \rightarrow i}^{(1)}(x_1, \mu_F^2) d\hat{\sigma}^{(0)}(\gamma i \rightarrow QX)[x_1 k_1, k_2, p_1] \\ &\equiv f_{a \rightarrow i}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(\gamma i \rightarrow QX) \end{aligned}$$

(b) resolved  $\gamma$

$$\begin{aligned} d\sigma_{\text{sub}}(ab \rightarrow QX) &= \int_0^1 dx_2 f_{b \rightarrow j}^{(1)}(x_2, \mu_F^2) d\hat{\sigma}^{(0)}(aj \rightarrow QX)[k_1, x_2 k_2, p_1] \\ &\equiv f_{b \rightarrow j}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(aj \rightarrow QX) \end{aligned}$$

(c) final state

$$\begin{aligned} d\sigma_{\text{sub}}(ab \rightarrow QX) &= \int_0^1 dz d\hat{\sigma}^{(0)}(ab \rightarrow kX)[k_1, k_2, z^{-1} p_1] d_{k \rightarrow Q}^{(1)}(z, \mu_F'^2) \\ &\equiv d\hat{\sigma}^{(0)}(ab \rightarrow kX) \otimes d_{k \rightarrow Q}^{(1)}(z) \end{aligned}$$

## Subprocesses for photoproduction

- direct photon:
  - dominated by  $\gamma + g \rightarrow c + \bar{c}$  (LO)
  - at NLO: 1-loop diagrams,  
gluon bremsstrahlung  $\gamma + g \rightarrow c + \bar{c} + g$
  - also  $\gamma + q \rightarrow c + \bar{c} + q$  and
  - charm-initiated:  $\gamma + c \rightarrow g + c$
- resolved photon:
  - gluons, light quarks, and charm in the proton
  - gluons, light quarks, and charm in the photon
- every parton can fragment to the heavy meson:  
fragmentation functions for  $c \rightarrow D^*$ ,  $g \rightarrow D^*$ ,  $q \rightarrow D^*$

# GM-VFNS SUBPROCESSES FOR HADRONIC PROCESSES

Light partons

- 1  $gg \rightarrow qX$
- 2  $gg \rightarrow gX$
- 3  $qg \rightarrow gX$
- 4  $qg \rightarrow qX$
- 5  $q\bar{q} \rightarrow gX$
- 6  $q\bar{q} \rightarrow qX$
- 7  $qg \rightarrow \bar{q}X$
- 8  $qg \rightarrow \bar{q}'X$
- 9  $qg \rightarrow q'X$
- 10  $qq \rightarrow gX$
- 11  $qq \rightarrow qX$
- 12  $q\bar{q} \rightarrow q'X$
- 13  $q\bar{q}' \rightarrow gX$
- 14  $q\bar{q}' \rightarrow qX$
- 15  $qq' \rightarrow gX$
- 16  $qq' \rightarrow qX$

⊕ charge conjugated processes

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

Heavy quark initiated ( $m = 0$ )

- 1 -
- 2 -
- 3  $Qg \rightarrow gX$
- 4  $Qg \rightarrow QX$
- 5  $Q\bar{Q} \rightarrow gX$
- 6  $Q\bar{Q} \rightarrow QX$
- 7  $Qg \rightarrow \bar{Q}X$
- 8  $Qg \rightarrow \bar{q}X$
- 9  $Qg \rightarrow qX$
- 10  $QQ \rightarrow gX$
- 11  $QQ \rightarrow QX$
- 12  $Q\bar{Q} \rightarrow qX$
- 13  $Q\bar{q} \rightarrow gX, q\bar{Q} \rightarrow gX$
- 14  $Q\bar{q} \rightarrow QX, q\bar{Q} \rightarrow qX$
- 15  $Qq \rightarrow gX, qQ \rightarrow gX$
- 16  $Qq \rightarrow QX, qQ \rightarrow qX$

Mass effects:  $m \neq 0$

- 1  $gg \rightarrow QX$
- 2 -
- 3 -
- 4 -
- 5 -
- 6 -
- 7 -
- 8  $qg \rightarrow \bar{Q}X$
- 9  $qg \rightarrow QX$
- 10 -
- 11 -
- 12  $q\bar{q} \rightarrow QX$
- 13 -
- 14 -
- 15 -
- 16 -

Applications available for

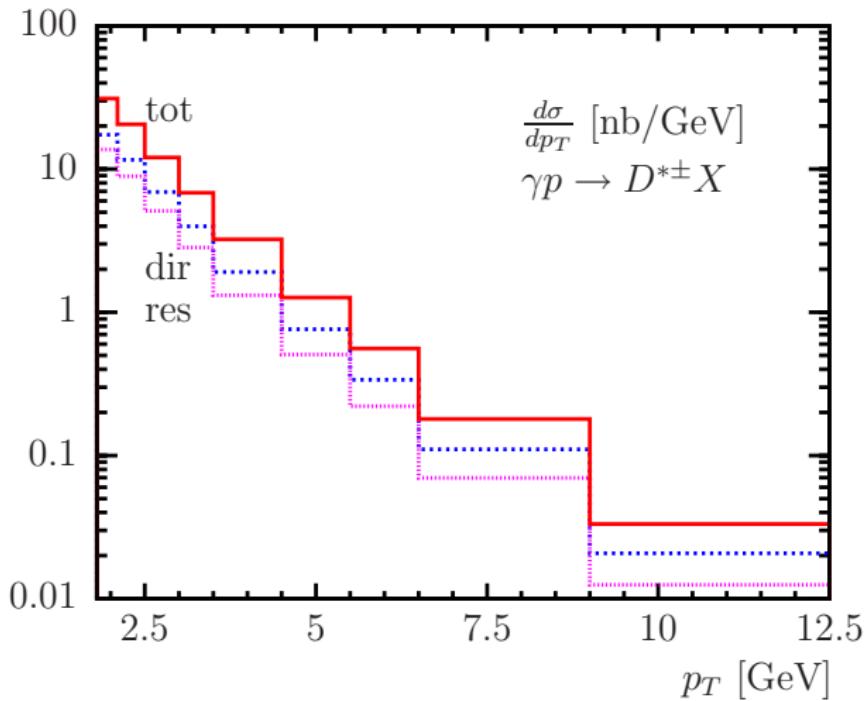
- $\gamma + \gamma \rightarrow D^{*\pm} + X$   
direct and resolved contributions
- $\gamma^* + p \rightarrow D^{*\pm} + X$   
photoproduction, this talk
- $p + \bar{p} \rightarrow (D^0, D^{*\pm}, D^\pm, D_s^\pm, \Lambda_c^\pm) + X$   
good description of Tevatron data
- $p + \bar{p} \rightarrow B + X$   
works for Tevatron data at large  $p_T$
- work in progress for  $e + p \rightarrow D + X$

# NUMERICAL RESULTS

Exemplify results for

- $p_T, \eta$  distributions
  - $1.5 \text{ GeV} \leq p_T \leq 12.5 \text{ GeV}, |\eta| \leq 1.5$
  - photoproduction:  $Q^2 \leq 2 \text{ GeV}^2$
  - $100 \text{ GeV} \leq W_{\gamma p} \leq 285 \text{ GeV}$
- compare with H1 preliminary data
- charm mass:  $m = 1.5 \text{ GeV}$
  - $\alpha_s$  at NLO with  $\Lambda_{N_f=4}^{\overline{\text{MS}}} = 0.328 \text{ GeV}$ , i.e.  $\alpha_s(M_Z^2) = 0.1180$
  - independent choice of renormalization and factorization scales:  
 $\mu_i = \xi_i \sqrt{p_T^2 + m^2}, i = R, F, F'$ , default:  $\xi_i = 1$
  - PDFs: proton: CTEQ6.5, photon: GRV
  - fragmentation functions: KKKS 2008

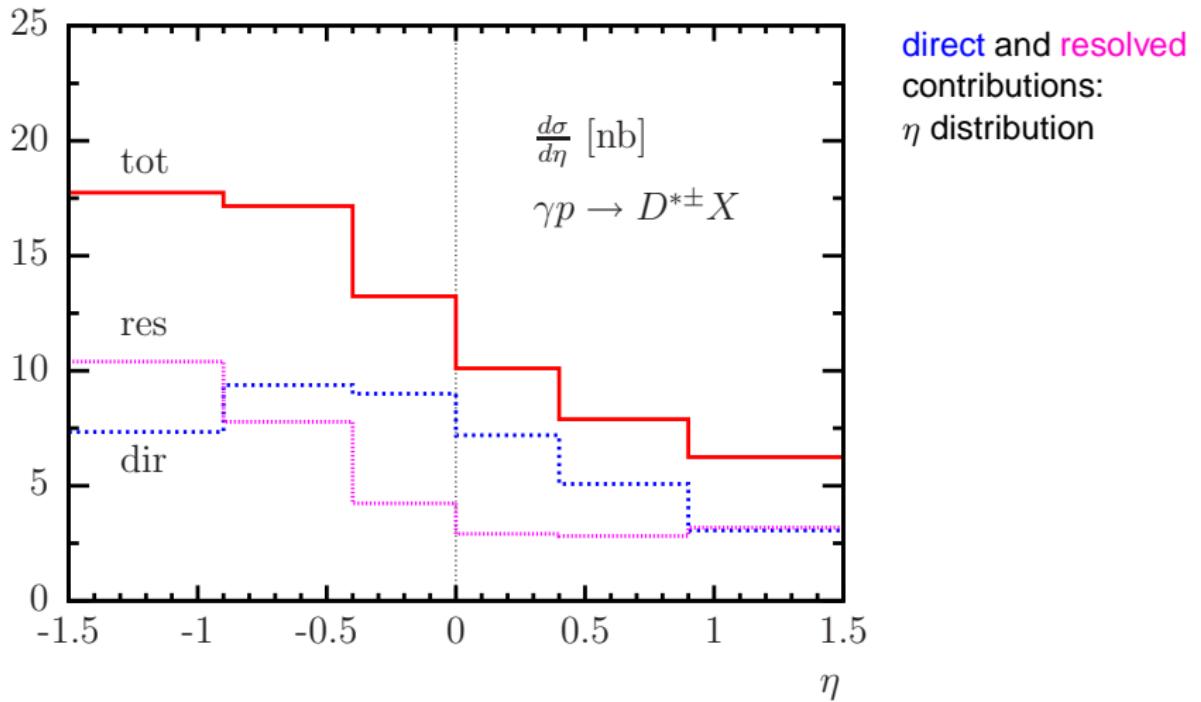
## DIRECT AND RESOLVED PARTS



direct and resolved contributions:  
 $p_T$  distribution

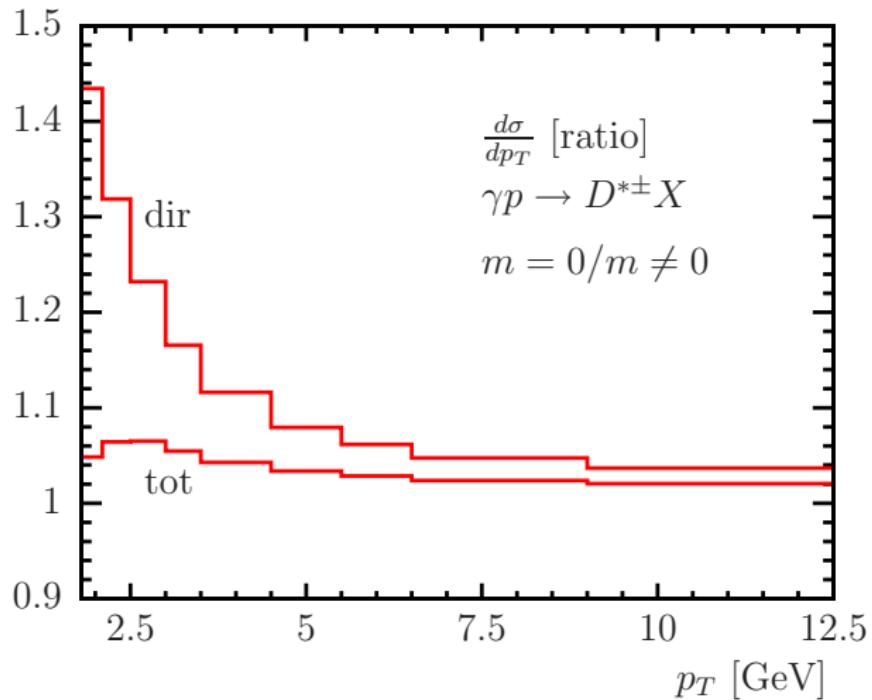
resolved part  
dominated by  
charm PDF

## DIRECT AND RESOLVED PARTS

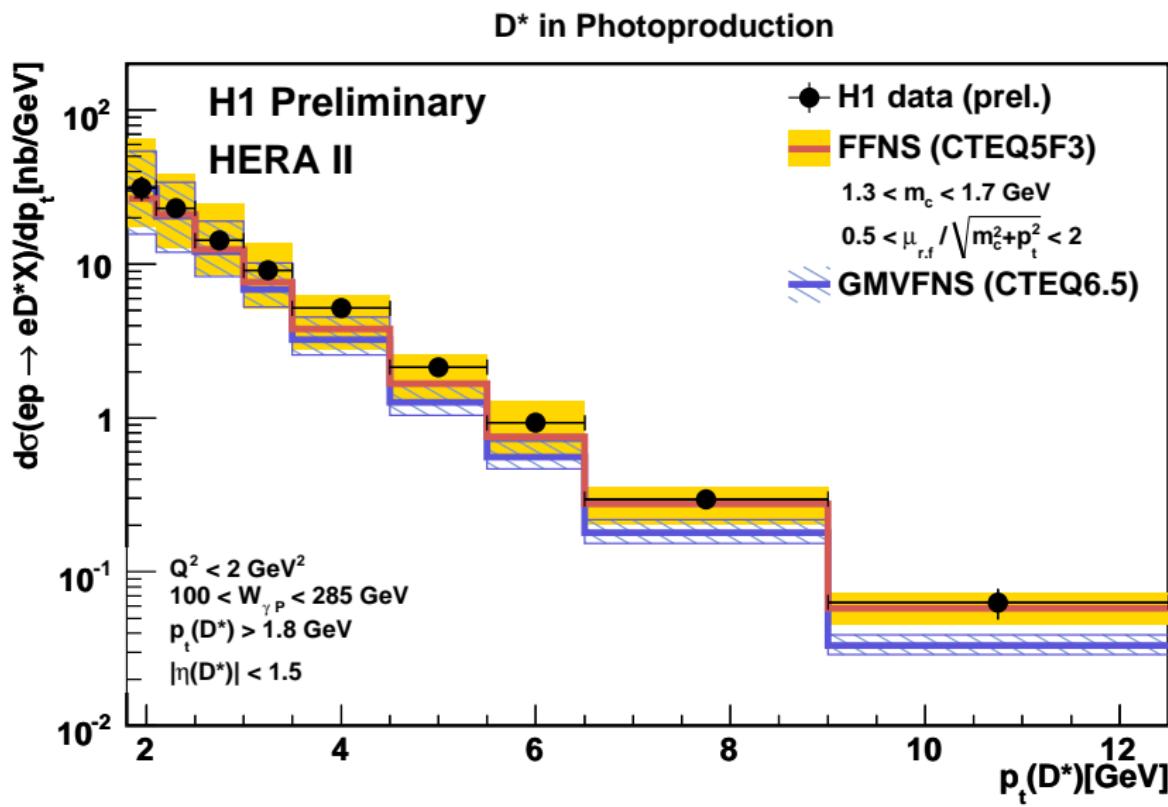


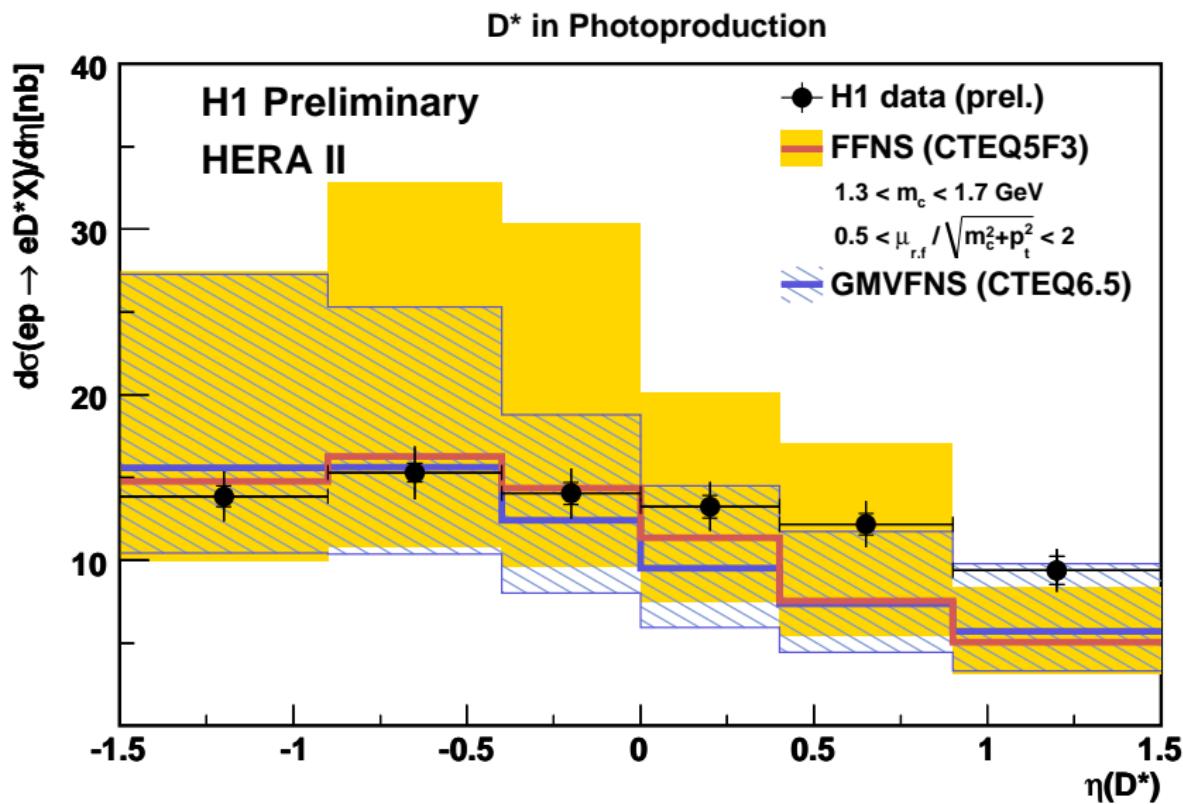
direct and resolved  
contributions:  
 $\eta$  distribution

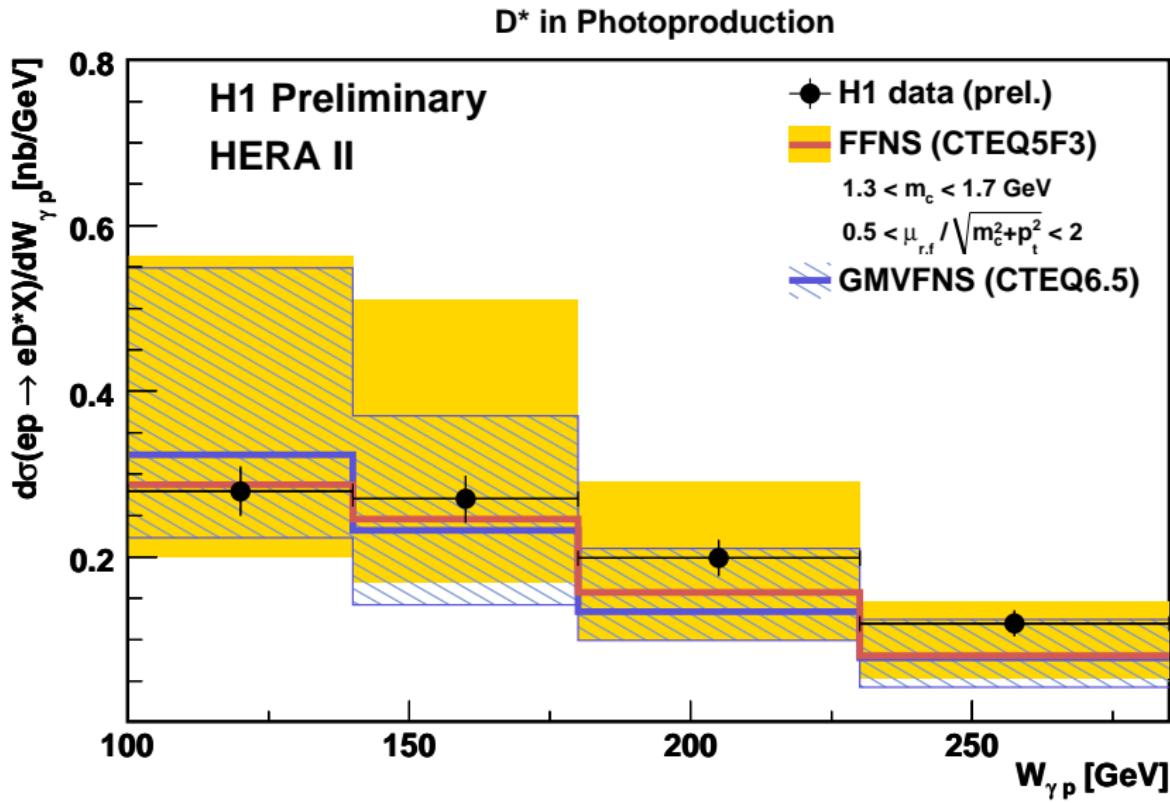
## MASS EFFECTS



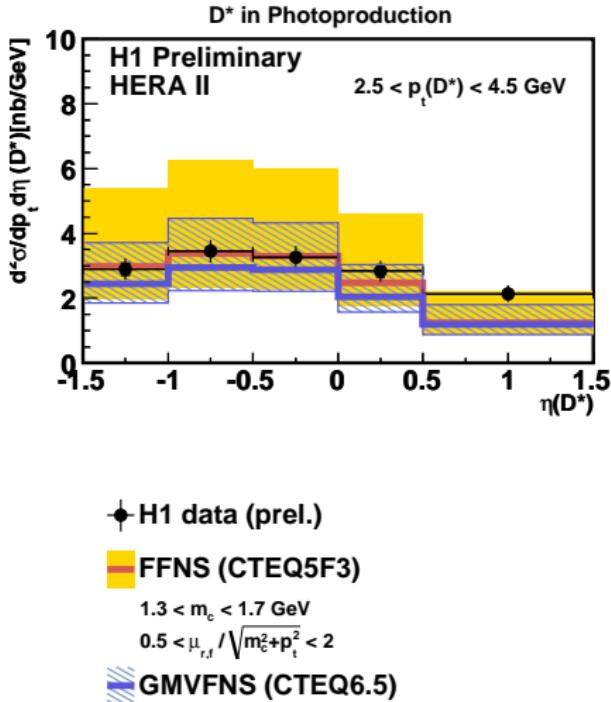
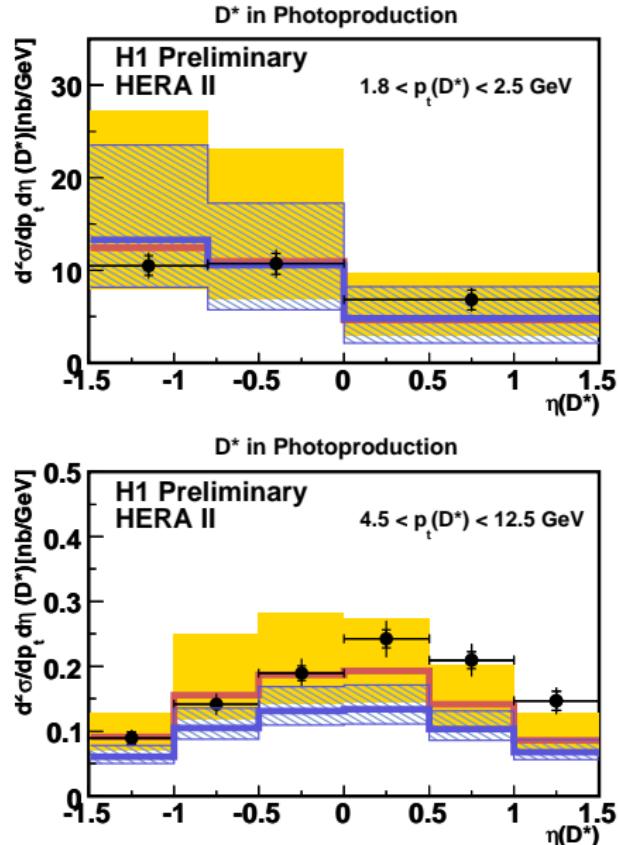
mass effects  
suppressed in  $\sigma_{\text{tot}}$



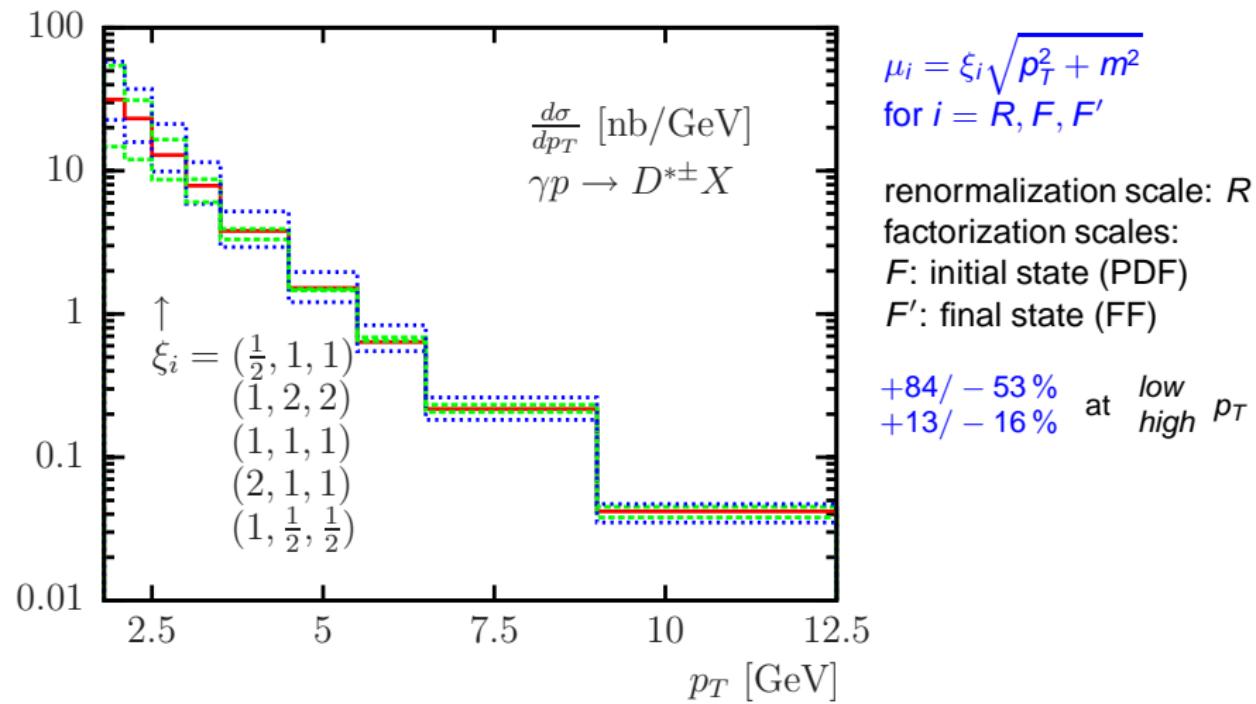




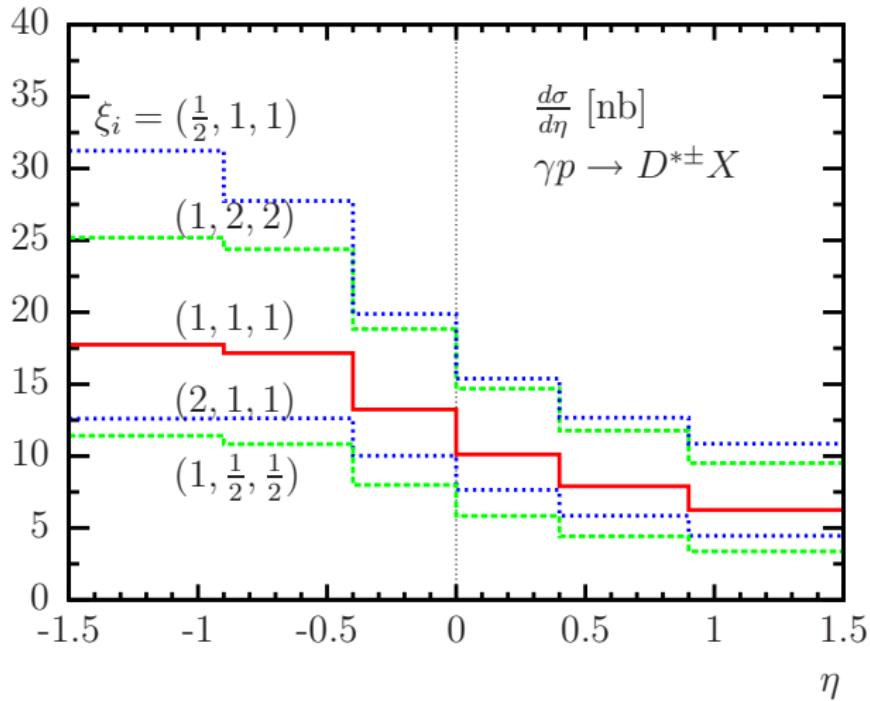
# COMPARISON WITH H1 PRELIMINARY DATA



## SCALE DEPENDENCE



## SCALE DEPENDENCE

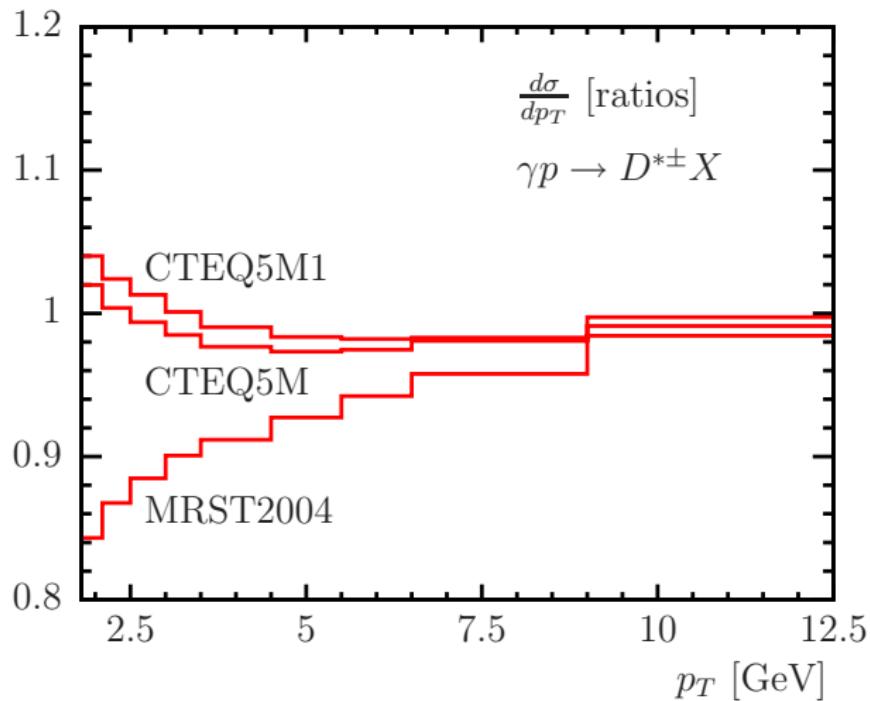


scale dependence:  
rapidity distribution

Large scale uncertainties at small  $p_T \rightarrow$   
need further improvements:

- matching of  $3 \leftrightarrow 4$  flavor schemes
- threshold behaviour of charm-initiated subprocesses

... work in progress

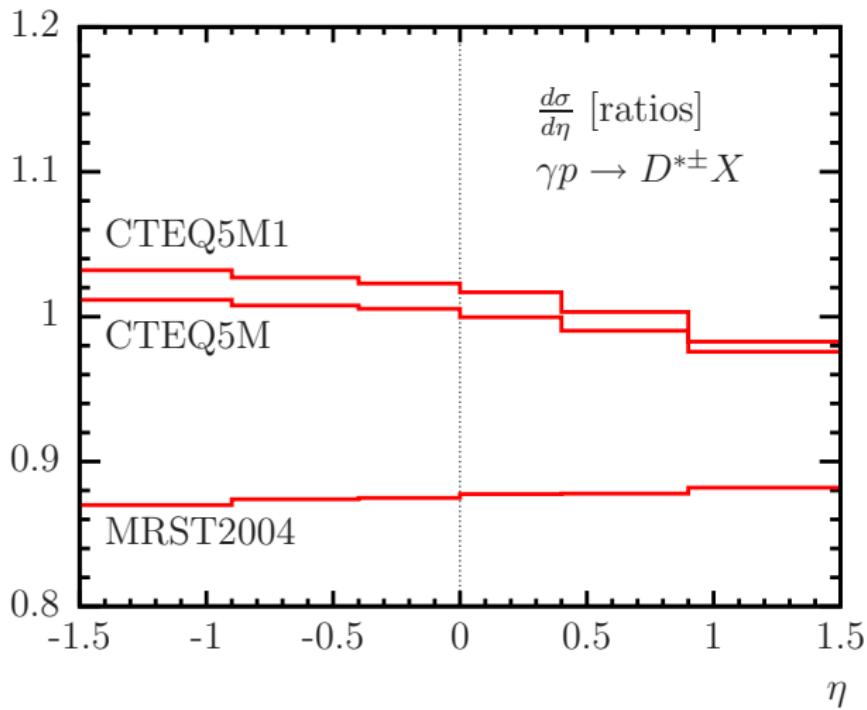


ratio of cross sections  
normalized to  
CTEQ6.5

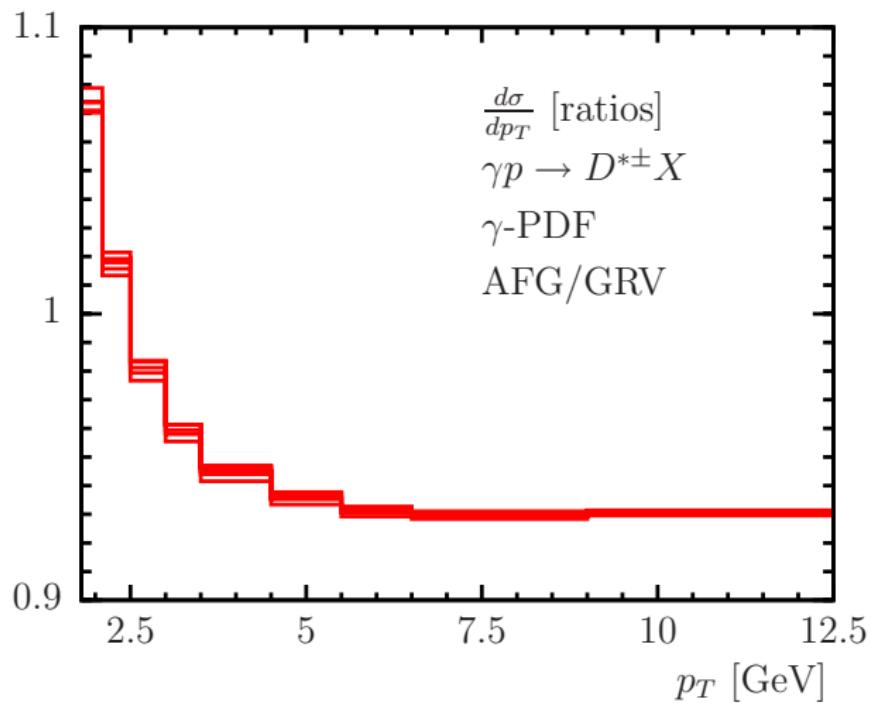
largest influence  
from varying PDF input  
at small  $p_T$

but small compared to  
scale uncertainty

## PDF INPUT



PDF input:  
rapidity distribution

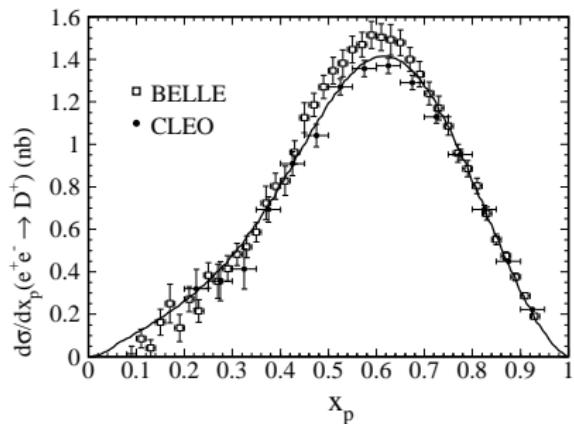


ratio of cross sections

uncertainties  
from  $\gamma$  PDF input  
slightly smaller

default: **GRV**  
compared with **AFG**:  
Aurenche, Fontannaz,  
Guillet, EPJC44 (2005)  
5 sets (low/high  $\mu_0^2$ , soft/hard  
non-perturbative gluon)

## FRAGMENTATION FUNCTIONS



FF for  $c \rightarrow D^*$   
from fitting to  $e^+e^-$  data

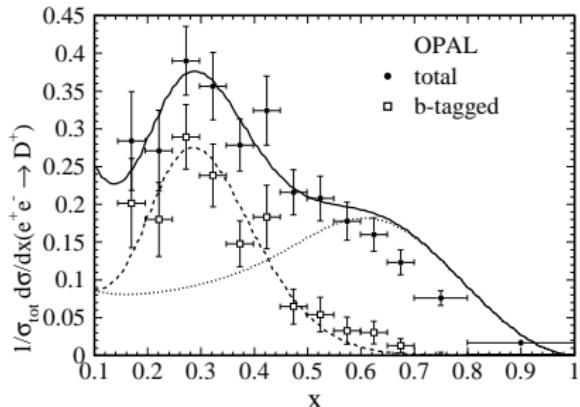
2008 analysis based on GM-VFNS  
 $\mu_0 = m$

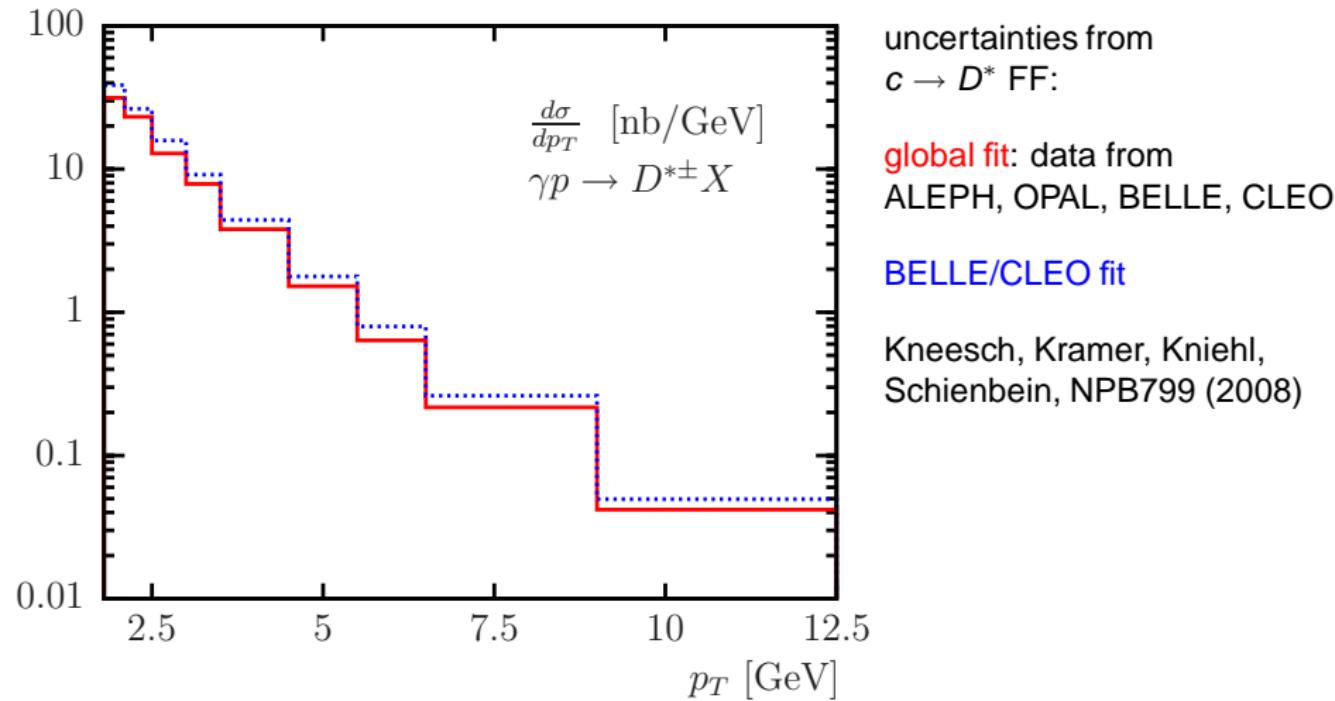
global fit: data from  
ALEPH, OPAL, BELLE, CLEO

BELLE/CLEO fit

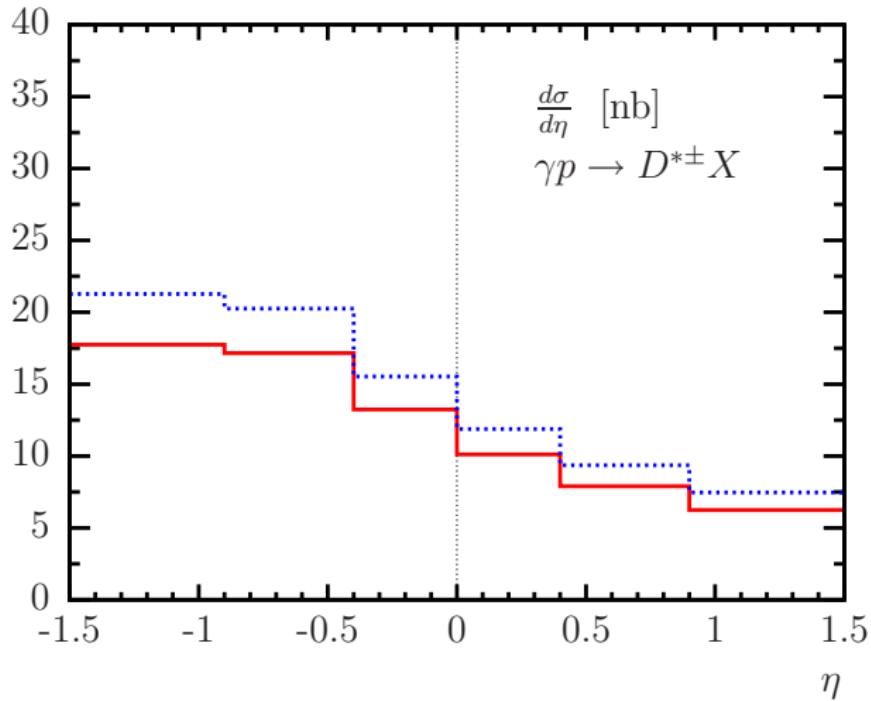
KKKS: Kneesch, Kramer, Kniehl,  
Schienbein, NPB799 (2008)

tension between low and high energy  
data sets → speculations about non-  
perturbative (power-suppressed) terms



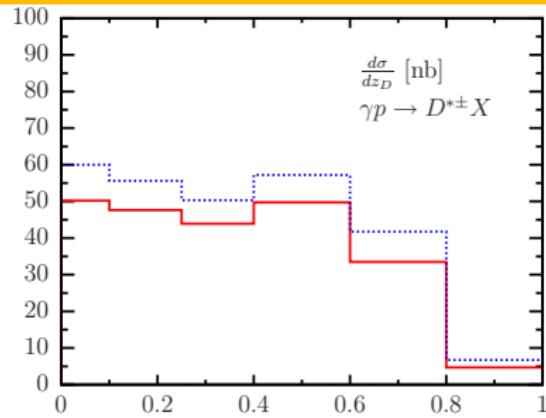


## FF INPUT

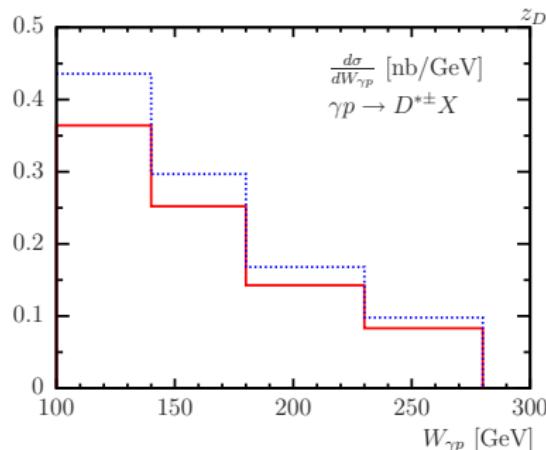


FF input:  
rapidity distribution

## FF INPUT



inelasticity  
 $z_D = E(D^*)/E(\gamma^*)$   
(in  $p$  rest frame)



$\gamma^* p$  cms energy:  $W_{\gamma p}$

## Summary

- The General-Mass Variable-Flavor-Number Scheme:  
a theoretical framework for one-particle inclusive heavy quark production  
  
resummed large logarithms in universal PDFs and FFs  
mass terms fully kept at  $O(\alpha_s)$
- Numerical results for  $D^*$  meson production in photoproduction at HERA  
  
more results available for  $\gamma\gamma$ ,  $p\bar{p}$ ,  $pp$  collisions, more to come: DIS