Color Glass Condensate and the relation to HERA physics

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Introduction: What is CGC?

- The ultimate form of hadronic matter at high energy
  - “parton saturation” (maximal occupation numbers)

- A firm prediction of first-principle calculations
  - weak coupling (perturbative QCD)
  - strong coupling (AdS/CFT for $\mathcal{N} = 4$ SYM)

- Interesting conceptual aspects
  - high energy limit of scattering amplitudes
  - multiple scattering, saturation, unitarity
  - relation to modern problems in statistical physics

- Interesting consequences for the phenomenology
  - rapid growth of the gluon distribution (HERA)
  - geometric scaling (HERA)
  - particle production in $pA$ and $AA$ collisions (RHIC)

- Decisive tests are coming soon, at LHC!
Motivation: Gluons at HERA

The gluon distribution rises very fast at small $x$! ($\sim 1/x^\lambda$)

$$xG(x, Q^2) \approx \# \text{ of gluons with transverse size } \Delta x_\perp \sim 1/Q \text{ and } k_z = xP$$
Motivation: High density = Weak Coupling

▷ High–energy evolution: An evolution towards increasing density.

▷ High density partonic matter is weakly coupled!
Motivation: High density = Non-linear

A challenging problem though!

High density $\implies$ weak coupling but strong non-linear effects
Gluon evolution at small $x$

- The ‘infrared sensitivity’ of bremsstrahlung favors the emission of ‘soft’ (= small–$x$) gluons

\[ dp \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x} \equiv \alpha_s \, dY \]

\[ Y \equiv \ln \frac{1}{x} \sim \ln s \implies dY = \frac{dx}{x} : \text{“rapidity”} \]

- A probability of $\mathcal{O}(\alpha_s)$ to emit one gluon per unit rapidity.
Gluon evolution at small $x$

- In turn, the emitted gluon can radiate an even softer one

- The ‘price’ of such an additional gluon:

$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x} = \alpha_s Y$$

- Ordering in $x \Rightarrow$ Ordering in (life)time (“glass”):

$$\Delta t \propto \frac{k_z}{k_{\perp}^2} \propto x$$
The blowing–up gluon distribution

\[ xG(x, Q^2) \propto \sum_n \frac{1}{n!} \left( \alpha_s \ln \frac{1}{x} \right)^n \sim e^{\omega \alpha_s Y} \]

\[ Y \equiv \ln \left( \frac{1}{x} \right) \sim \ln s : \text{"rapidity"} \]

"BFKL resummation" (*Balitsky, Fadin, Kuraev, Lipatov, 75–78*)

Conceptual difficulties in the high energy limit
Onset of non–linear dynamics

- The gluon occupation number (or ‘packing factor’):

\[
 n(x, k_\perp, b_\perp) \equiv \frac{dN}{dY d^2k_\perp d^2b_\perp} \sim \frac{1}{Q^2} \times \frac{xG(x, Q^2)}{\pi R^2}
\]

- \( n \sim \langle A_i A^i \rangle \): when \( n \sim 1/\alpha_s \iff A_a^i \sim 1/g \)
The Saturation Momentum

- The gluons must be numerous enough (small $x$) and large enough (low $Q^2$) to strongly overlap with each other.

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x^\lambda}$$

$Q_s^2(x)$ is the saturation momentum, $\alpha_s$ is the strong coupling constant, $G(x, Q_s^2)$ is the gluon distribution, $R$ is the saturation scale, and $\lambda$ is a parameter related to the saturation scale.
At small-\(x\), the struck quark is typically radiated off the gluon distribution in the proton.

- Lorentz boost to the ‘dipole frame’
  \(\gamma^*\) fluctuates into a \(q\bar{q}\) pair which then scatters off the proton.
- The proton still carries most of the total energy!
Dipole factorization for DIS

\[ \sigma_{\gamma^* p}(x, Q^2) = \int_0^1 dz \int d^2 r |\Psi_{\gamma}(z, r; Q^2)|^2 \sigma_{\text{dipole}}(x, r) \]

The dipole–proton scattering amplitude

\[ \sigma_{\text{dipole}}(x, r) = 2 \int d^2 b \ T(x, r, b) \]

- \( T \equiv 1 - S \): The dipole–proton scattering amplitude
- Unitarity bound: \( T \leq 1 \) (\( T = 1 \): ‘black disk limit’)

\[ T = 1 - S \]

BFKL evolution: **Unitarity violation**

- The ‘last’ gluon at small $x$ can be emitted off any of the ‘fast’ gluons with $x' > x$ radiated in the previous steps:

\[
\frac{\partial n}{\partial Y} \simeq \alpha_s n \quad \Rightarrow \quad n(Y) \propto e^{\omega \alpha_s Y}
\]

- Dipole forward scattering amplitude: $T \sim \alpha_s n$
- Unitarity bound ($T \leq 1$) is eventually violated by BFKL!
BFKL evolution: Infrared diffusion

The gluon emission vertex is non–local in transverse space:

\[ \partial_Y n(\rho, Y) = \alpha_s n + \alpha_s \partial_{\rho}^2 n \]

\[ \implies \text{Diffusion in } \rho \equiv \ln k_{\perp}^2 \sim \ln Q^2 \]
Non–linear evolution: **Saturation**

- **High density**: recombination processes leading to saturation

\[
\frac{\partial n}{\partial Y} \simeq \alpha_s \partial_\rho^2 n + \alpha_s n - \alpha_s^2 n^2 = 0 \quad \text{when} \quad n \sim \frac{1}{\alpha_s} \gg 1
\]

- Non–linear equation \(\implies\) stable fixed point at high energy!

- **Unitarity restoration & Hard momentum scale** \(Q_s(Y)\)
Non–linear evolution: **Saturation**

\[ \partial_Y n(\rho, Y) = \alpha_s \partial^2_\rho n + \alpha_s n - \alpha_s^2 n^2 \]

- Cartoon version of BK *(Balitsky–Kovchegov)* equation (99)
- Mean field (large–\(N_c\)) approx. to JIMWLK equation *(CGC)* *(Jalilian-Marian, E.I., McLerran, Weigert, Leonidov, and Kovner, 97–00)*
- Derived to leading–order in perturbative QCD
DIS at strong coupling

(Polchinksi, Strassler, 02; Hatta, E.I., Mueller, 07) see talk by R. Peschanski

- $\lambda \equiv g^2 N_c \gg 1$ with $g^2 \ll 1 \Rightarrow$ AdS/CFT correspondence

- $\mathcal{N} = 4$ SYM $\iff$ classical gravity in the $AdS_5 \times S^5$

- Parton branching at strong coupling:
  No reason to favour special corners of phase–space!

- All partons have branched down to small values of $x$!
Saturation line: weak vs. strong coupling

- No ‘leading–twist’ (no pdf’s !) at $Q^2 > Q_s^2(x)$
  
  all partons lie within the CGC with occupancy $n \sim O(1)$

- Saturation exponent: $Q_s^2(x) \propto 1/x^{\lambda_s} \equiv e^{\lambda_s Y}$
  - weak coupling: $\lambda_s \approx 0.4 g^2 N_c$ (LO BFKL Pomeron)
  - strong coupling: $\lambda_s = 1$ (graviton)
The **Color Glass Condensate**

*(McLerran, Venugopalan, 1994; E.I., Leonidov, McLerran, 2000)*

- **An effective theory** for the evolution towards saturation

- **Small–**$x$** gluons**: Classical color fields radiated by fast color sources ($x' \gg x$) ‘frozen’ in some random configuration $\rho_a$

- $W_Y[\rho]$ : Probability distribution for the color charge density

- Functional evolution equation for $W_Y[\rho]$ : JIMWLK
Deep Inelastic Scattering off the CGC

- $T(r)[\rho]$: scattering off a given configuration $\rho$ of the color sources (multiple scattering in the eikonal approximation)

- Average over $\rho$ with weight function $W_Y[\rho]$ (glass)

$$\langle T(r) \rangle_Y = \int D[\rho] \ W_Y[\rho] \ T(r)[\rho]$$
Saturation momentum

- Saturation front: $T(\rho, Y)$ with $\rho = \ln(1/r^2)$

  $\Rightarrow$ a front interpolating between $T = 0$ and $T = 1$

- The position $\rho_s(Y)$ of the front $\Rightarrow$ saturation momentum

  $\text{BK} \Rightarrow \rho_s(Y) \equiv \ln Q_s^2(Y) \approx \lambda Y$ with $\lambda \approx 4.88\bar{\alpha}_s \sim 1$
Gluon occupation number

- A similar front holds for the ‘unintegrated gluon distribution’

\[ xG(x, Q^2) = \int d^2b \int^Q dk \, k \, n(x, k) \]

- The typical transverse momentum of the gluons is \( \sim Q_s(Y) \)
BK equation: The traveling wave

- The shape of the front is not altered by the evolution

\[ T(\rho, Y) \simeq T(\rho - \rho_s(Y)) \equiv T(r^2 Q_s^2(Y)) \]

- ‘Geometric scaling’
  
  \textit{E.I., Itakura, McLerran (02); Mueller, Triantafyllopoulos (02)}

- Traveling wave picture: Munier, Peschanski (03)

- Relation to Stat Phys: ‘reaction–diffusion’ \( A \rightleftharpoons 2A \)
Geometric scaling

\[ \ln Q^2 = Y \ln \Lambda_{QCD} \]

- \( \rho - \rho_s(Y) = \text{const} \): A line of constant gluon occupancy
  \[ \implies \text{physics must be invariant along any such a line!} \]
- Saturation makes itself felt in the dilute regime \( (Q^2 > Q_s^2) \)
Geometric scaling

- Strictly true only within a finite ‘scaling window’ above $Q_s$,
  which extends with $Y$: $\ln Q_g^2(Y) - \ln Q_s^2(Y) \propto \sqrt{\alpha_s Y}$
Geometric Scaling at HERA

\((Staśto, Golec-Biernat and Kwieciński, 2000)\)

\[\sigma(x, Q^2) \approx \sigma(\tau) \quad \text{with} \quad \tau \equiv \frac{Q^2}{Q_s^2(x)} , \quad Q_s^2(x) = (x_0/x)^\lambda \text{GeV}^2 , \quad \lambda \simeq 0.3\]

\[x \leq 0.01\]
\[Q^2 \leq 450 \text{ GeV}^2\]
\[Q_s^2 \sim 1 \text{ GeV}^2\]
\[\text{for } x \sim 10^{-4}\]
Geometric Scaling at HERA (2)

(Marquet and Schoeffel 2006)

\[ \beta d \sigma / d \beta \]

\[ H1 \text{ data (LRG)} \]
\[ \text{ZEUS data (Mx) } \times 0.85 \]
\[ \text{ZEUS data (LPS) } \times 1.23 \]

\[ 10^{-2} \]
\[ 10^{-1} \]
\[ 1 \]

\[ \tau_d \]
\[ \sigma_{\text{DVCS}} \text{ (nb)} \]

\[ \tau_V \]
\[ \sigma_{\text{MV}} \text{ (nb)} \]

\[ \text{ZEUS data} \]
\[ \text{H1 data} \]

\[ \sqrt{M} = \phi \]

\[ \sqrt{M} = J/\psi \]

\[ \text{ZEUS data} \]
\[ \text{H1 data} \]
Saturation exponent at NLO

D.N. Triantafyllopolous, 2002

\[
\lambda(Y) \equiv \frac{d \ln Q_s^2(Y)}{dY} \approx 0.3
\]

- NLO BFKL + Collinear resummation + Saturation Boundary
The unreasonable effectiveness of the ‘saturation models’

- “Saturation models” \(\equiv\) QCD–inspired models for \(\sigma_{\text{dipole}}\) involving saturation and a reasonable # of free parameters.
- The parameters are fixed by fits to the \(F_2\) data alone!
- Satisfactory description of the ensemble of HERA data at \(x \leq 0.01\).
  All other observables (\(F_2^D, F_L, F_2^c, \rho, J/\psi, \text{DVCS}, \ldots\)) emerge as ‘predictions’.
- Important qualitative predictions of the theory which appear to be consistent with the data.
  - geometric scaling, the transition towards low \(Q^2\) for \(F_2\), a nearly constant \(\sigma_{\text{diff}}/\sigma_{\text{tot}}\) ratio ...
- A similar success for the relevant data at RHIC
  - high–\(p_\perp\) suppression in forward d–Au collisions (‘\(R_{pA}\)’).
Saturation models

- The Golec-Biernat and Wüsthoff model (1999)

\[ \sigma_{\text{dipole}}^{\text{GBW}}(x, r) = 2\pi R^2 \left( 1 - e^{-r^2 Q_s^2(x)} \right), \quad Q_s^2(x) = (x_0/x)^\lambda \text{GeV}^2 \]

- Good fit to the early HERA data with only 3 parameters
- Exact ‘geometric scaling’ built in: \( \sigma^{\text{GBW}}(r^2 Q_s^2(x)) \)

More sophisticated models (pQCD evolution, geometric scaling violations)

- DGLAP–like (also with \( b \) dependence) Bartels, Golec-Biernat, Kowalski (02), Kowalski, Teaney (03), Kowalski, Motyka, Watt (06)
- CGC model (BK eq.) E.I., Itakura, Munier (03) : 3 light quarks
- Improvements of CGC model: heavy quarks, \( b \)–dependence Kowalski, Motyka, Watt (06), Soyez (07)
- FS04 saturation model Forshaw, Shaw (04)
A CGC fit to $F_2$ (G. Soyez, 2007)

$x \leq 10^{-2}$, $Q^2 \leq 150 \text{ GeV}^2$ (281 data points, ZEUS & H1)

4 parameters: $R, x_0 \approx 2 \times 10^{-5}, \gamma \approx 0.26$ and $\lambda \approx 0.22$ ($\chi^2 = 0.90$)
A CGC fit to $F_2$ (G. Soyez, 2007)

- Gluon evolution at small $x$
- AdS/CFT
- CGC & Geometric scaling
- Some consequences for HERA
  - Saturation models
  - GBW
  - CGC fit to $F_2$
    - $F^{2c}$
    - $F^{2D3}$
    - $F^{2D3}$
    - $F^2$ Regge vs Sat
    - DIS Diffraction
    - Soft diffraction (?)
    - Semi-Hard diffraction
A CGC fit to $F_2^c$

Forshaw, Sandapen and Shaw (06)

![Graphs showing the CGC fit to $F_2^c$]
Saturation fits for Diffraction

Forshaw, Sandapen and Shaw (06)  Low $Q^2$

- ZEUS FPC
- FS04 sat $b = 6.8$ GeV$^2$
- FS04 no sat $b = 8$ GeV$^2$
- CGC $b = 6.8$ GeV$^2$
Saturation fits for Diffraction

Forshaw, Sandapen and Shaw (06)  High $Q^2$

- ZEUS FPC
- FS04 sat $b = 6.8 \text{ GeV}^{-2}$
- FS04 no sat $b = 8 \text{ GeV}^{-2}$
- CGC $b = 6.8 \text{ GeV}^{-2}$

$Q^2 = 8 \text{ GeV}^2, 14 \text{ GeV}^2, 27 \text{ GeV}^2, 55 \text{ GeV}^2$
Saturation vs. Regge fits for $F_2$

Forshaw and Shaw (04)  Relatively low $Q^2$

- **Left:** Regge fits (a sum of ‘soft’ + ‘hard’ Pomerons)
- **Right:** 2 saturation fits (FS04 and CGC)
- **Data appear to prefer saturation!**
An ideal laboratory to study saturation/unitarity effects

- sensitive to relatively large dipole sizes
- sensitive to theoretical models (or prejudices)
**DIS Diffraction**

- An ideal laboratory to study saturation/unitarity effects
  - sensitive to relatively large dipole sizes
  - sensitive to theoretical models (or prejudices)

**Original prejudice: “Even for large $Q^2$, diffraction is soft”**

\[ \sigma_{\text{diff}} \propto x^{-2(\alpha_P - 1)} \quad \text{and hence} \quad \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \sim x^{-(\alpha_P - 1)} \quad \text{at small } x \]
Diffractive over inclusive ratio at HERA

Golec-Biernat, Wüsthoff (99); Bartels, Golec-Biernat & Kowalski (02)

\[ \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \]

\[ W(GeV) \]

\[ M_X < 3 \text{ GeV} \]

\[ 3 < M_X < 7.5 \text{ GeV} \]

\[ 7.5 < M_X < 15 \text{ GeV} \]
Diffractive dissociation of the virtual photon

\[
\frac{d\sigma_{\text{diff}}}{d^2b} = \int dz \, d^2r \, |\Psi_{\gamma}(z, r; Q)|^2 \left( T(r, Y) \right)^2
\]

- The photon wavefunction favors small dipoles \((r \sim 1/Q)\)

\[
\frac{d\sigma_{\text{diff}}}{d^2b} \sim \frac{1}{Q^2} \int_{1/Q^2}^{\infty} \frac{dr^2}{r^4} \left( T(r, Y) \right)^2
\]

- The dipole amplitude favors relatively large dipoles:

\[
T(r) \propto r^2 \quad \text{(single scattering)}
\]

- “The integral is dominated by large, non–perturbative, dipoles with size \(r \sim 1/\Lambda_{\text{QCD}}\), hence the soft pomeron!”
Hardening the diffraction

- At sufficiently high energy, gluon saturation cuts off the large dipoles already on the ‘semi–hard’ scale $1/Q_s$ !

$$\frac{d\sigma_{\text{diff}}}{d^2 b} \sim \frac{1}{Q^2} \int_1^{1/Q_s^2} \frac{dr^2}{r^4} \left( r^2 Q_s^2(x) \right)^2 \sim \frac{Q_s^2(x)}{Q^2} \propto x^{-\lambda}$$

- $\sigma_{\text{diff}}$ is dominated by dipole sizes $r \sim 1/Q_s(x)$ !
- $\sigma_{\text{diff}} \propto x^{-\lambda}$: single, hard pomeron increase with $1/x$
  (instead of double soft !)
- $\sigma_{\text{diff}}/\sigma_{\text{tot}} \approx \text{constant} !$ ✓

- ‘Semi–hard diffraction’ ... at intermediate energies !
Stochastic aspects of high–energy QCD

- Classical fields (JIMWLK): no gluon–number fluctuations
- Gluons in the same cascade are correlated with each other
- Saturation & multiple scattering could probe the correlations

\[ \partial_t n(x, t) = \frac{\partial^2}{\partial x} n(x, t) + \alpha n(x, t) - \beta n^2(x, t) \]

diffusion growth recombination

‘Reaction–diffusion’ \( A \rightleftharpoons 2A \quad \text{Munier, Peschanski (03)} \)
DIS with Pomeron loops

- **Statistical physics:** The effects of fluctuations are dramatic! (The front is pulled by the dynamics in its dilute tail)

- **Important consequences for high–energy QCD**
  
  *(Mueller, Shoshi; E.I., Mueller, Munier; E.I., D. Triantafyllopoulos, 2004)*
Front diffusion through fluctuations

- The **stochastic** evolution generates an ensemble of fronts which differ by their saturation momentum \( \rho_s \equiv \ln Q_s^2 \)

\[
\langle \rho_s(Y) \rangle = \lambda Y, \quad \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = DY, \quad D \sim \frac{1}{\ln^3(1/\alpha_s)}
\]

- With increasing energy, the fronts spread from each other \( \implies \text{geometric scaling is progressively washed out!} \)
Dispersion: Fixed coupling

\[ \sigma^2(Y) \sim D\bar{\alpha}_s Y \quad \text{with} \quad D \sim \mathcal{O}(1) \]

\[ \alpha_s = 0.2 \quad \alpha_s = 0.5 \]

Fluctuations effects are clearly important ...

\[ \alpha_s = 0.5 \implies \sigma^2(Y) \approx 10 \quad \text{for} \quad Y = 10 \]

\[ \sigma^2(Y) \gtrsim 1 \implies \text{a totally new picture: ‘diffusive scaling’} \]
Dispersion: Running coupling

- The dispersion keeps rising with $Y$ ...

- ... but now it is tremendously smaller! (by a factor $\sim 100$)

- No physical effect up to unrealistically large $Y$!
Single scattering: 2–gluon exchange

- The dipole scatters off the gluon field in the target

\[ V(r) \sim gt^a r \cdot E_a \implies T(x, r, b) \propto g^2 r^2 \langle E_a \cdot E_a \rangle_x \]

\[ T(x, r, b) \approx \alpha_s r^2 \frac{xG(x, 1/r^2)}{\pi R^2} \equiv \alpha_s n(x, Q^2 \sim 1/r^2) \]

**Weak scattering** \((T \ll 1) \iff \) Low gluon occupation \((n \ll 1/\alpha_s)\)
Multiple scattering: Unitarization

- When decreasing $x$ at fixed $r$: $xG(x, 1/r^2) \sim 1/x^\lambda$
  \[ \implies \text{Unitarity is eventually violated!} \quad (T \gtrsim 1) \]

- Multiple scattering becomes important and restores unitarity

\[ T(x, r) \simeq 1 - \exp \left\{ -\alpha_s r^2 \frac{xG(x, 1/r^2)}{\pi R^2} \right\} \]  
(“Glauber–Mueller”)
No forward jets!

- No large-\(x\) partons \(\implies\) no forward/backward jets in a hadron–hadron collision at strong coupling

\[ |\eta| \lesssim \eta_{\text{max}}(Q) = \ln \frac{\sqrt{s}}{Q} - \ln \frac{1}{x_s(Q)}, \quad x_s(Q) \sim \frac{\Lambda^2}{Q^2 N_c^2} \ll 1 \]