Color Glass Condensate and the relation to HERA physics

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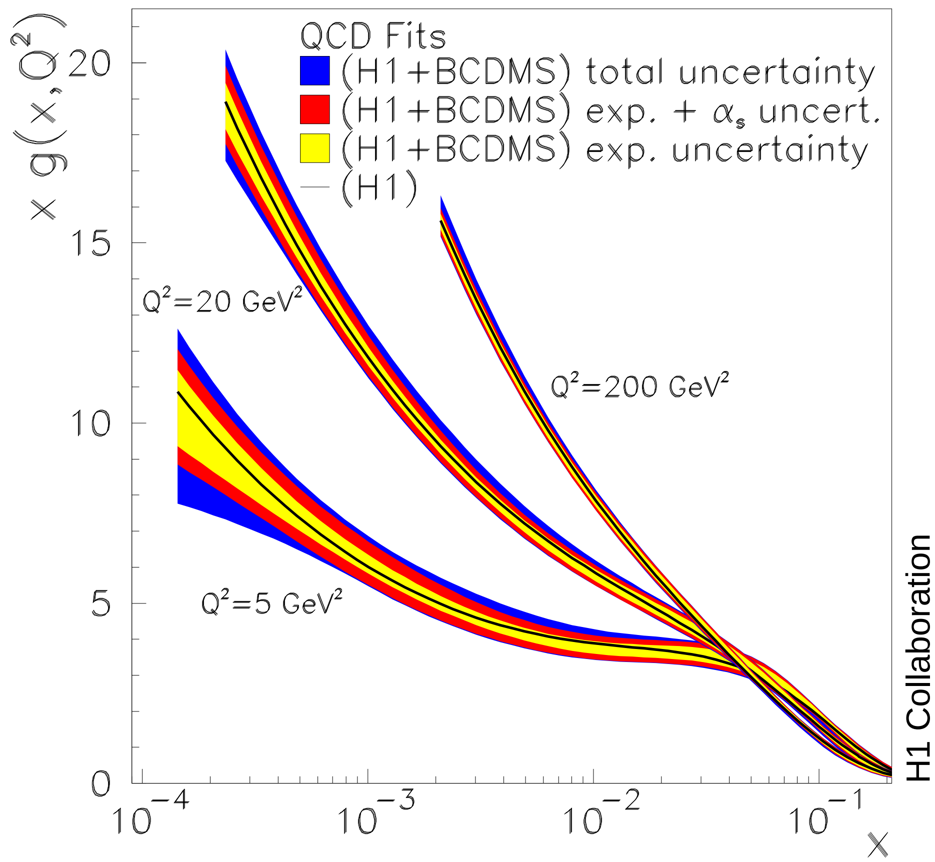
Introduction: What is CGC ?

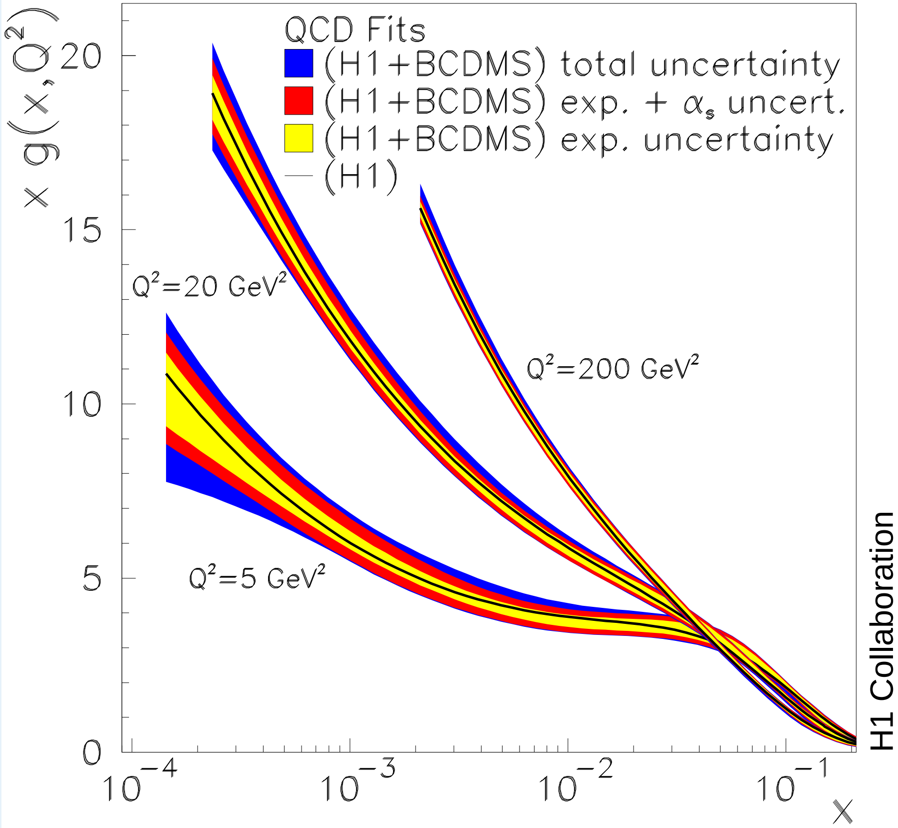
- The ultimate form of hadronic matter at high energy
  - "parton saturation" (maximal occupation numbers)
- A firm prediction of first–principle calculations
  - weak coupling (perturbative QCD)
  - strong coupling (AdS/CFT for $\mathcal{N} = 4$ SYM)
- Interesting conceptual aspects
  - high energy limit of scattering amplitudes
  - multiple scattering, saturation, unitarity
  - relation to modern problems in statistical physics
- Interesting consequences for the phenomenology
  - rapid growth of the gluon distribution (HERA)
  - geometric scaling (HERA)
  - particle production in pA and AA collisions (RHIC)
- Decisive tests are coming soon, at LHC !

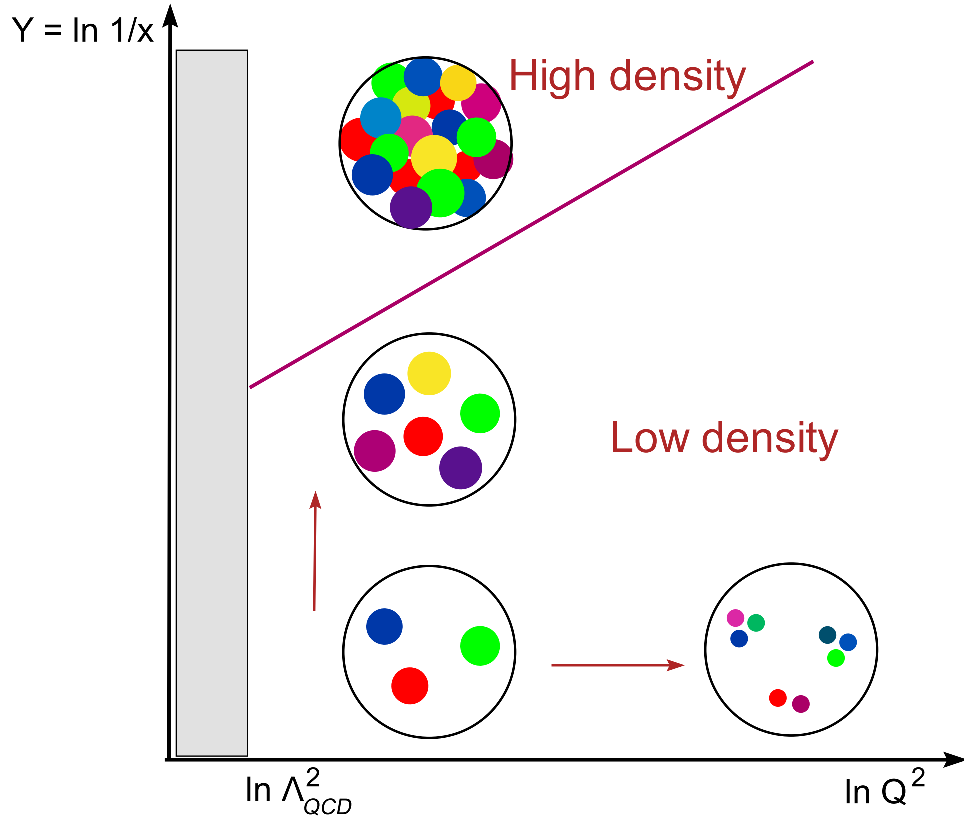
Motivation: Gluons at HERA

▷ The gluon distribution rises very fast at small x ! $(\sim 1/x^{\lambda})$



$xG(x,Q^2) \approx$ # of gluons with transverse size $\Delta x_\perp \sim 1/Q$ and $k_z = xP$

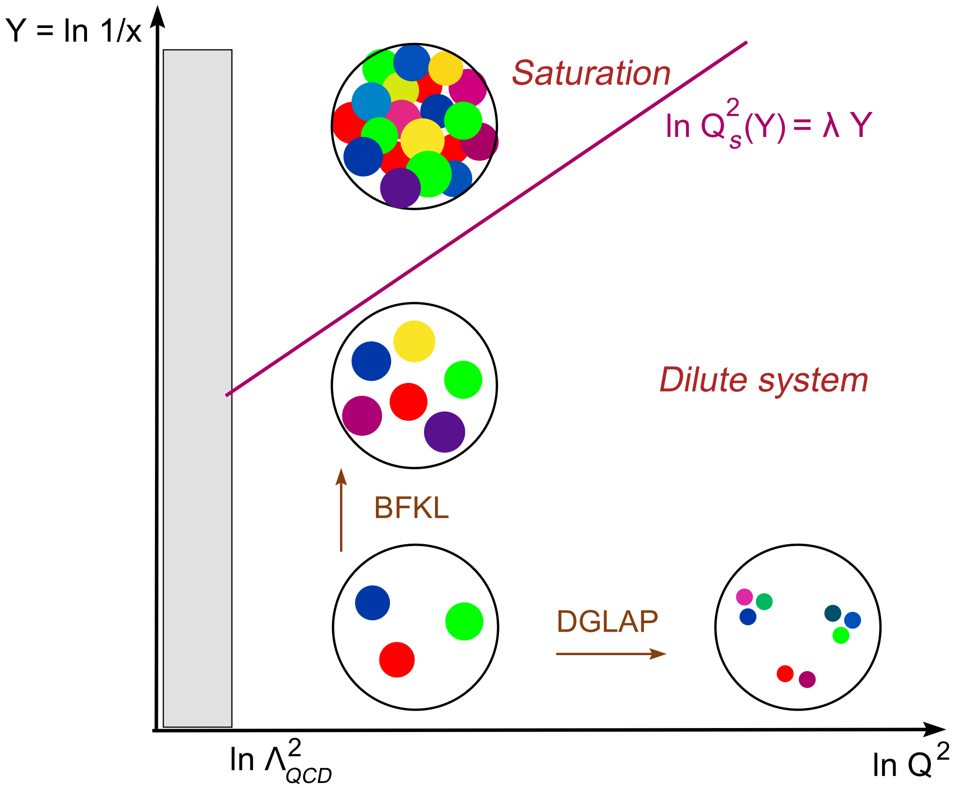




▷ High–energy evolution : An evolution towards increasing density.

▷ High density partonic matter is weakly coupled !

Motivation: High density = Non-linear

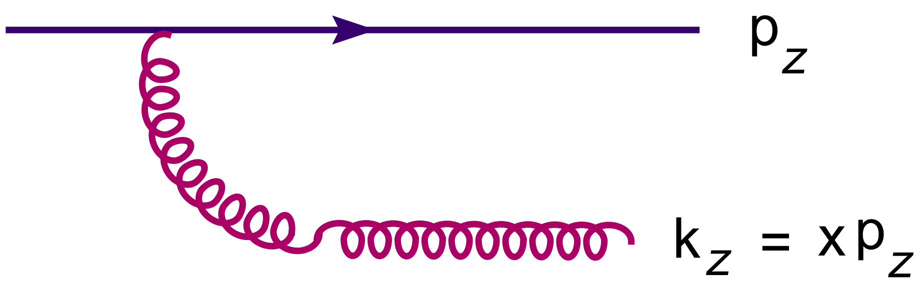


- ▷ A challenging problem though !

  High density $\Longrightarrow$ weak coupling but strong non–linear effects

Gluon evolution at small x

- The ‘infrared sensitivity’ of bremsstrahlung favors the emission of ‘soft’ (= small–x) gluons



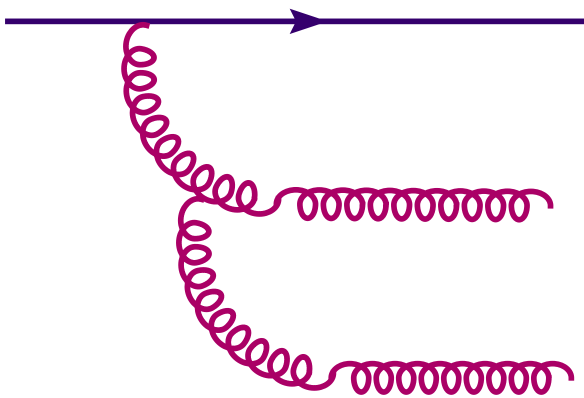
$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x} \equiv \alpha_s \, dY$$

$$Y \equiv \ln \frac{1}{x} \sim \ln s \implies dY = \frac{dx}{x} : \quad \text{“rapidity”}$$

- A probability of $\mathcal{O}(\alpha_s)$ to emit one gluon per unit rapidity.

Gluon evolution at small x

- In turn, the emitted gluon can radiate an even softer one



- The 'price' of such an additional gluon:

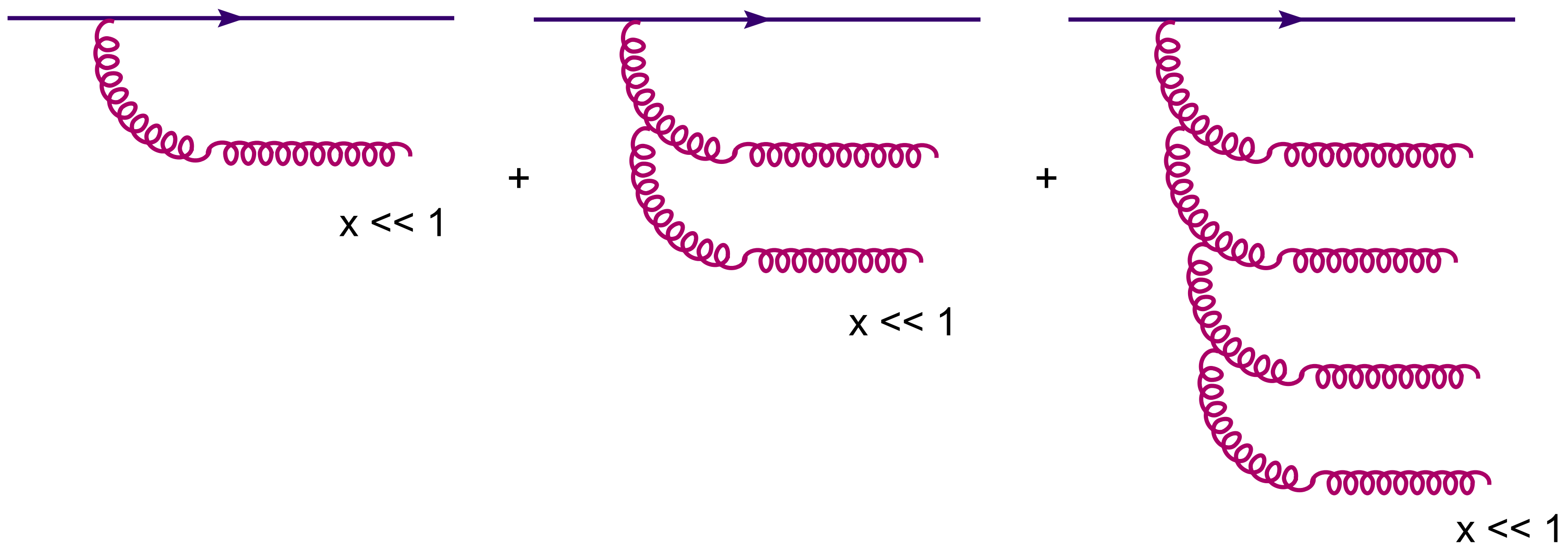
$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{\mathrm{d}x_1}{x_1} = \alpha_s \ln \frac{1}{x} = \alpha_s Y$$

- Ordering in $x \Longrightarrow$ Ordering in (life)time ("glass") :

$$\Delta t \propto \frac{k_z}{k_\perp^2} \propto x$$

BFKL evolution

- The blowing–up gluon distribution



$$xG(x,Q^2) \;\propto\; \sum_n \frac{1}{n!}\left(\alpha_s \ln\frac{1}{x}\right)^n \;\sim\; \mathrm{e}^{\omega\alpha_s Y}$$

$$Y \equiv \ln(1/x) \sim \ln s \;:\quad \text{“rapidity”}$$

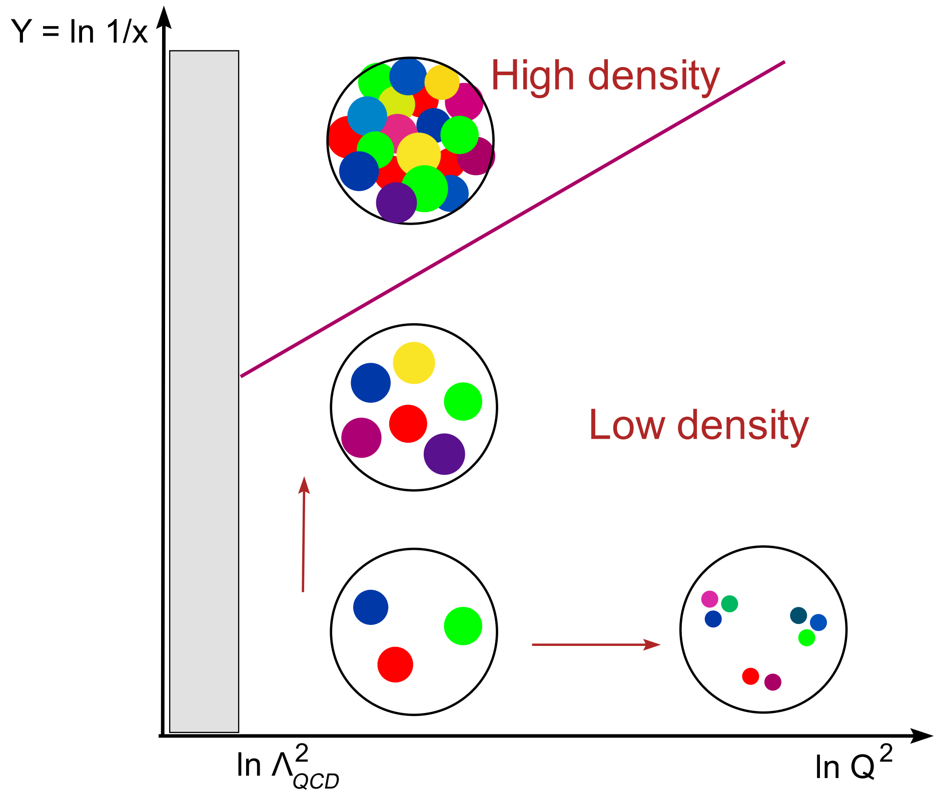
- “BFKL resummation” *(Balitsky, Fadin, Kuraev, Lipatov, 75–78)*
- Conceptual difficulties in the high energy limit

Onset of non–linear dynamics

- The gluon occupation number (or ‘packing factor’) :

$$n(x, k_\perp, b_\perp) \equiv \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2k_\perp\,\mathrm{d}^2b_\perp} \sim \frac{1}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2}$$

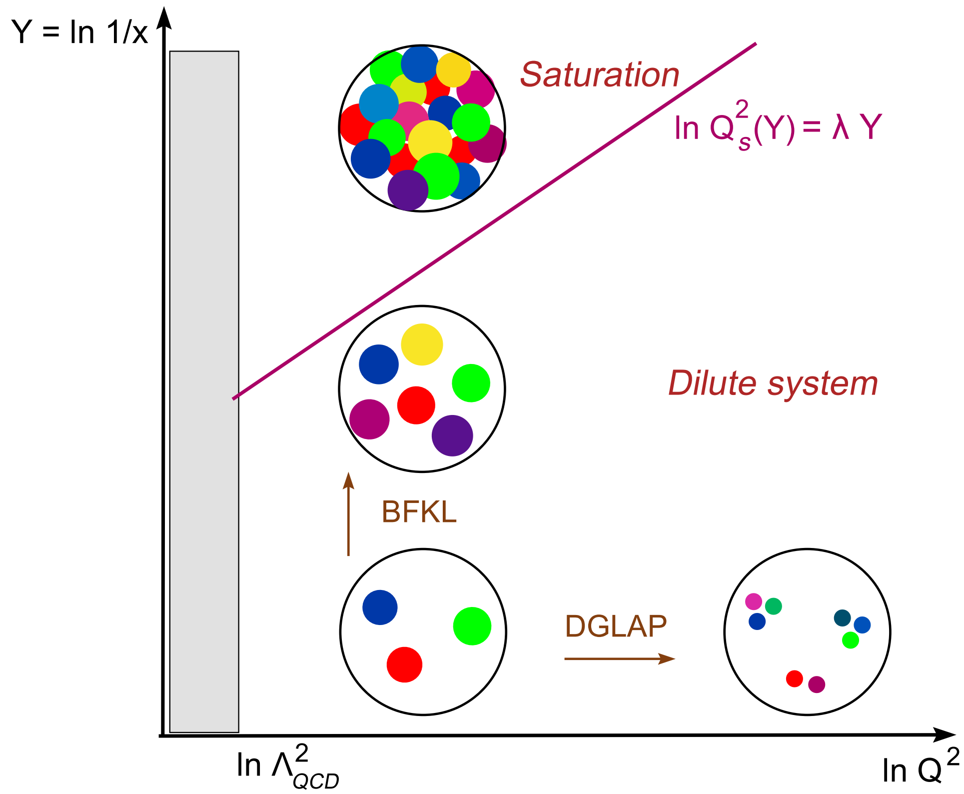
- $n \sim \langle A^i A^i \rangle$: when $n \sim 1/\alpha_s \iff A^i_a \sim 1/g$



The Saturation Momentum

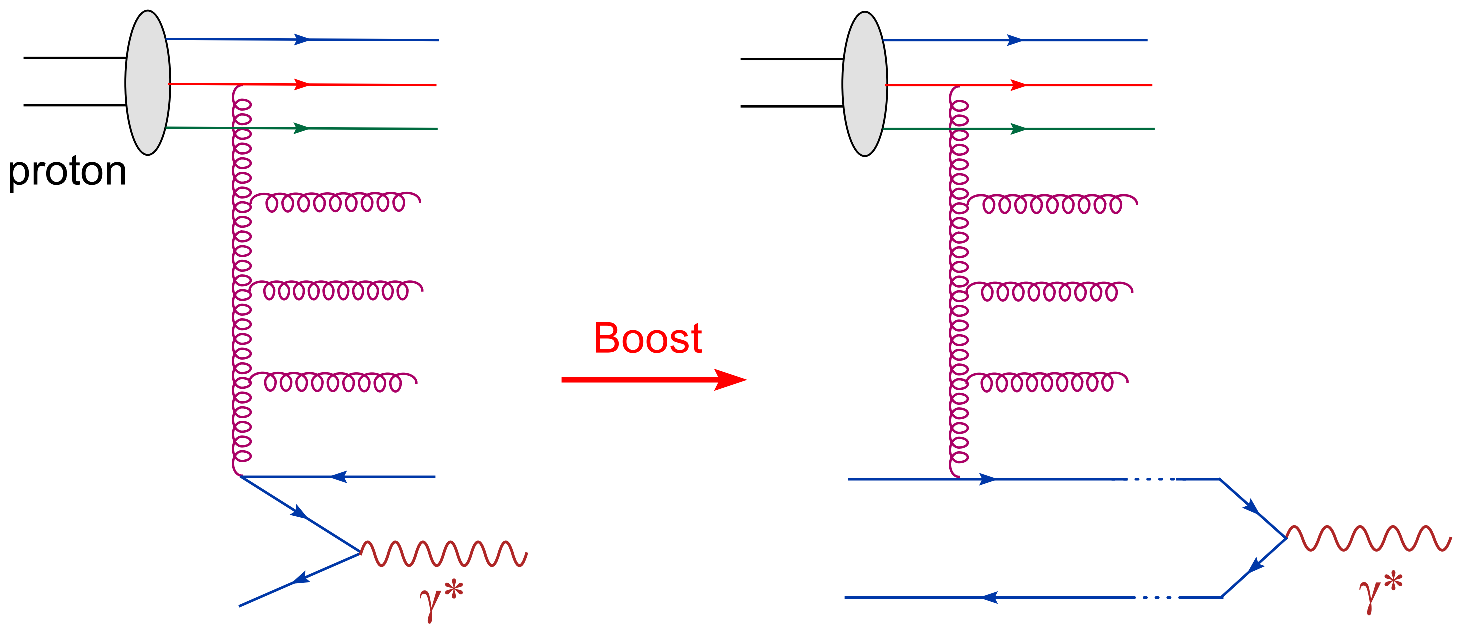
- The gluons must be numerous enough (small x) and large enough (low Q^2) to strongly overlap with each other.

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x^\lambda}$$



Dipole factorization for DIS

- At small–x, the struck quark is typically radiated off the gluon distribution in the proton



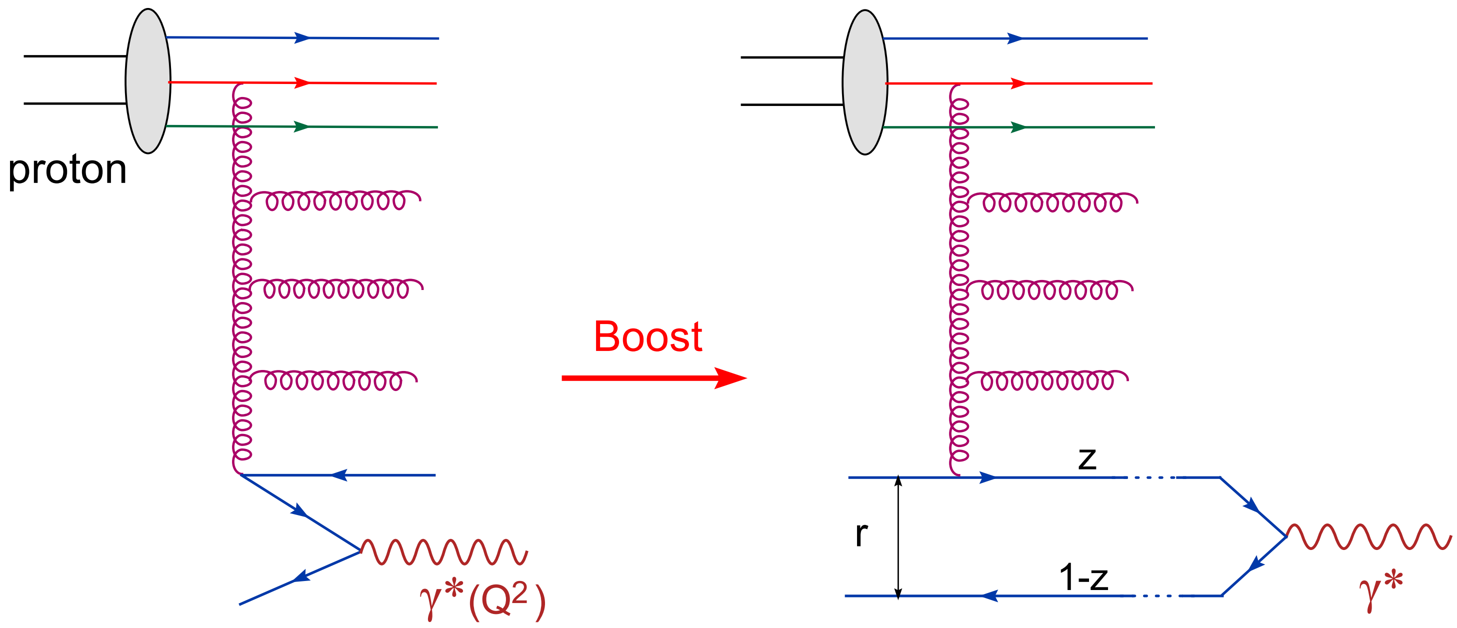
- Lorentz boost to the 'dipole frame'
  γ^* fluctuates into a $q\bar{q}$ pair which then scatters off the proton.
- The proton still carries most of the total energy !

Dipole factorization for DIS

Introduction
Motivation
Gluon evolution at small x
- Small-x evolution
- BFKL
- Saturation momentum
- Dipole frame
- BFKL equation
- Non–linear evolution
- Non–linear evolution

AdS/CFT
CGC & Geometric scaling
Some consequences for HERA
Backup

$$\sigma_{\gamma^* p}(x, Q^2) = \int_0^1 \mathrm{d}z \int \mathrm{d}^2 r \, |\Psi_\gamma(z, r; Q^2)|^2 \, \sigma_{\mathrm{dipole}}(x, r)$$



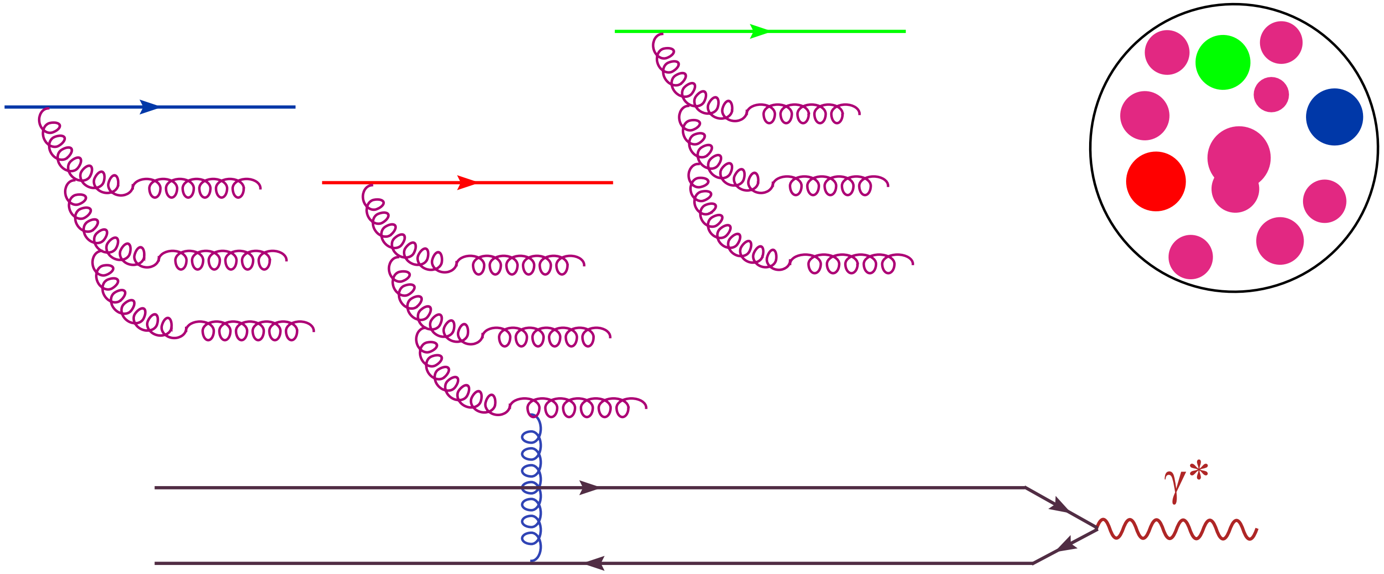
$$\sigma_{\mathrm{dipole}}(x, r) = 2 \int \mathrm{d}^2 b \;\; T(x, r, b)$$

- $T \equiv 1 - S$: The dipole–proton scattering amplitude
- Unitarity bound: $T \leq 1$ ($T = 1$: ‘black disk limit’)

BFKL evolution: Unitarity violation

- The 'last' gluon at small x can be emitted off any of the 'fast' gluons with $x' > x$ radiated in the previous steps :

$$\frac{\partial n}{\partial Y} \simeq \alpha_s n \qquad \Longrightarrow \qquad n(Y) \propto \mathrm{e}^{\omega \alpha_s Y}$$



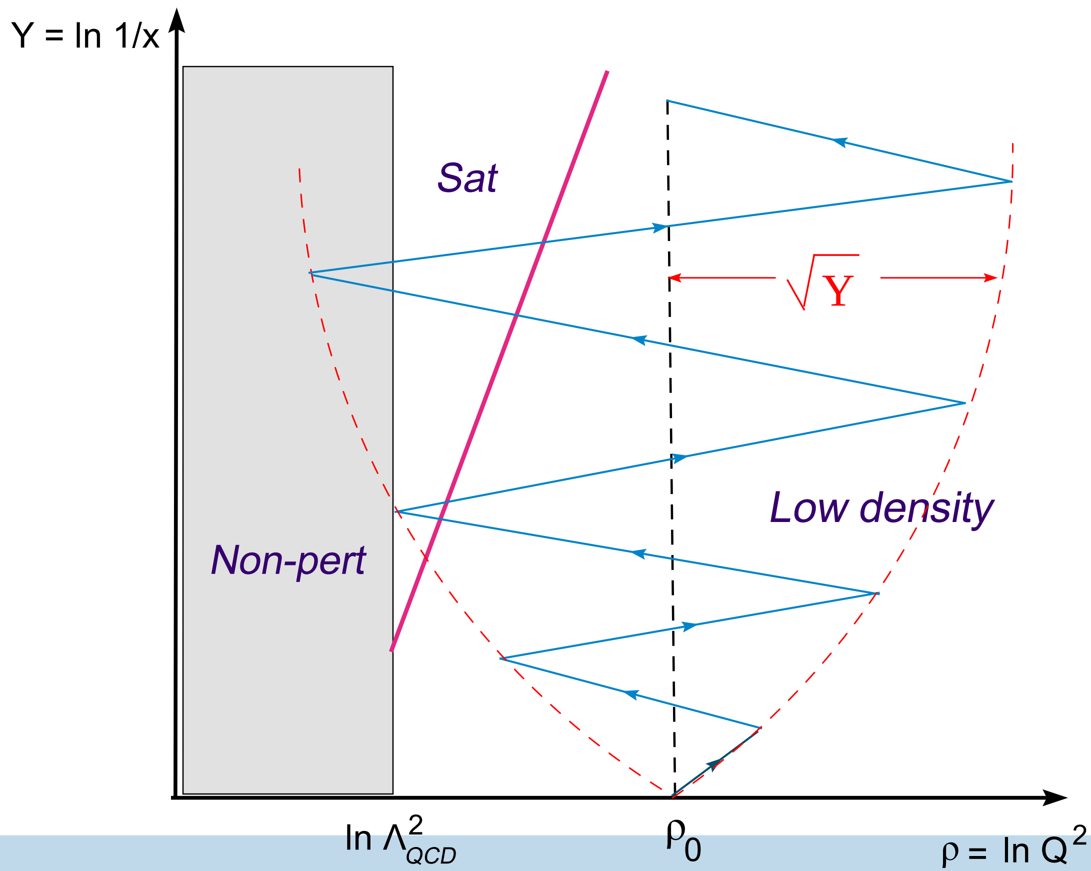
- Dipole forward scattering amplitude: $T \sim \alpha_s n$
- Unitarity bound $(T \leq 1)$ is eventually violated by BFKL !

BFKL evolution: Infrared diffusion

- The gluon emission vertex is non–local in transverse space:

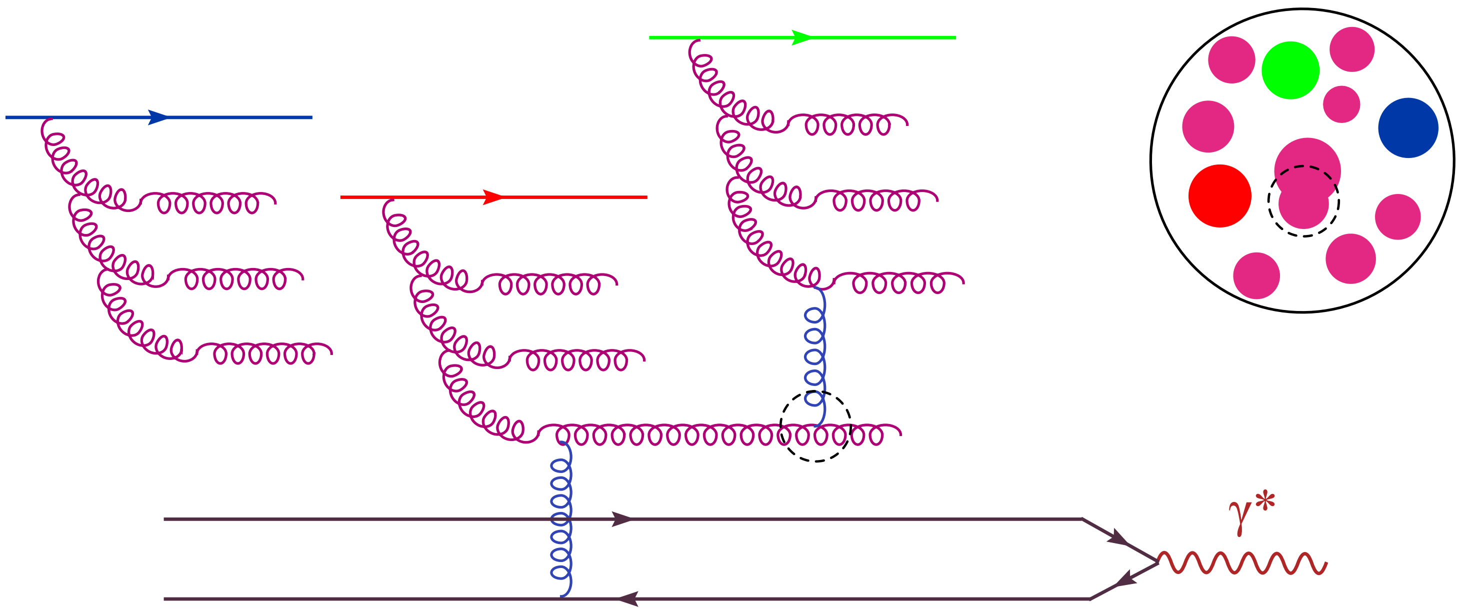
$$\partial_Y n(\rho, Y) = \alpha_s n + \alpha_s \partial_\rho^2 n$$

$\Longrightarrow$ Diffusion in $\rho \equiv \ln k_\perp^2 \sim \ln Q^2$



Non–linear evolution: Saturation

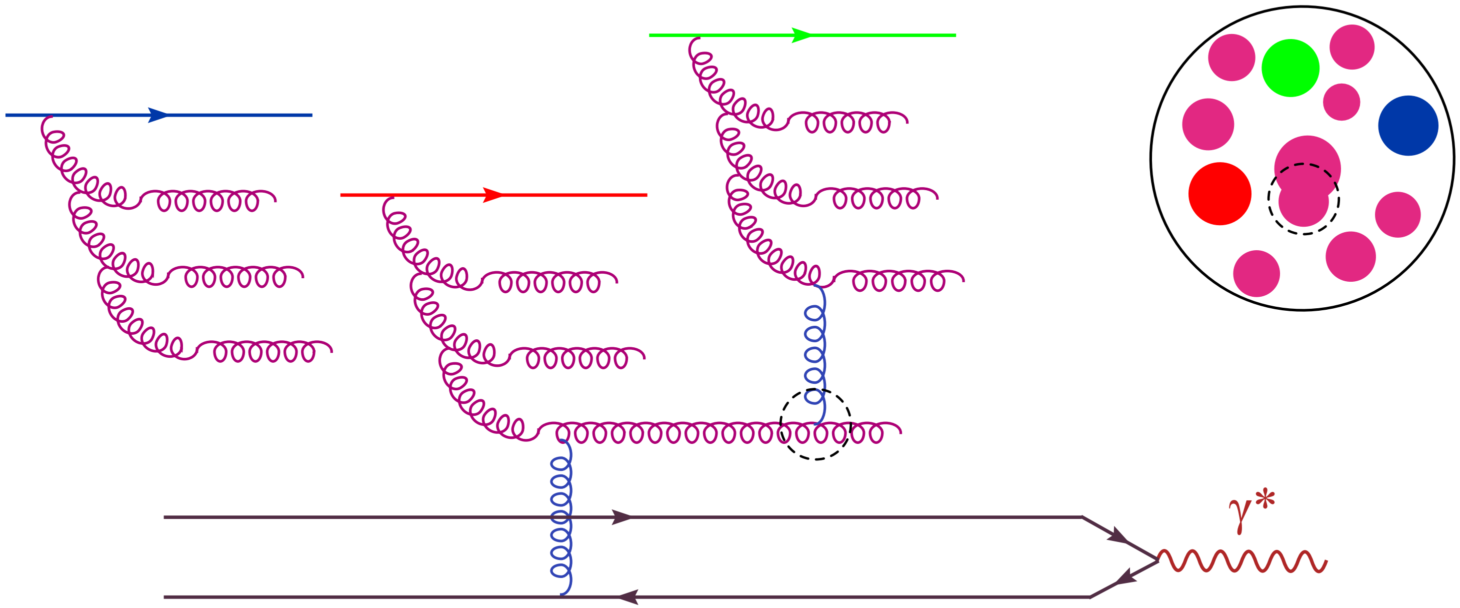
- High density: recombination processes leading to saturation



$$\frac{\partial n}{\partial Y} \simeq \alpha_s \partial_\rho^2 n + \alpha_s n - \alpha_s^2 n^2 = 0 \quad \text{when} \quad n \sim \frac{1}{\alpha_s} \gg 1$$

- Non–linear equation $\Longrightarrow$ stable fixed point at high energy !
- Unitarity restoration & Hard momentum scale ($Q_s(Y)$)

Non–linear evolution: Saturation



$$\partial_Y n(\rho, Y) = \alpha_s \partial_\rho^2 n + \alpha_s n - \alpha_s^2 n^2$$

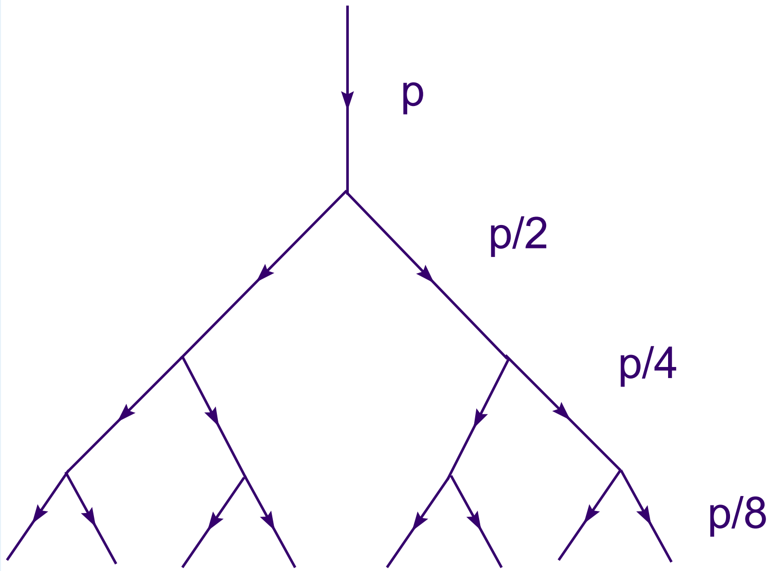
- Cartoon version of BK *(Balitsky–Kovchegov)* equation (99)
- Mean field (large–N_c) approx. to JIMWLK equation (CGC)
  (Jalilian-Marian, E.I., McLerran, Weigert, Leonidov, and Kovner, 97–00)
- Derived to leading–order in perturbative QCD

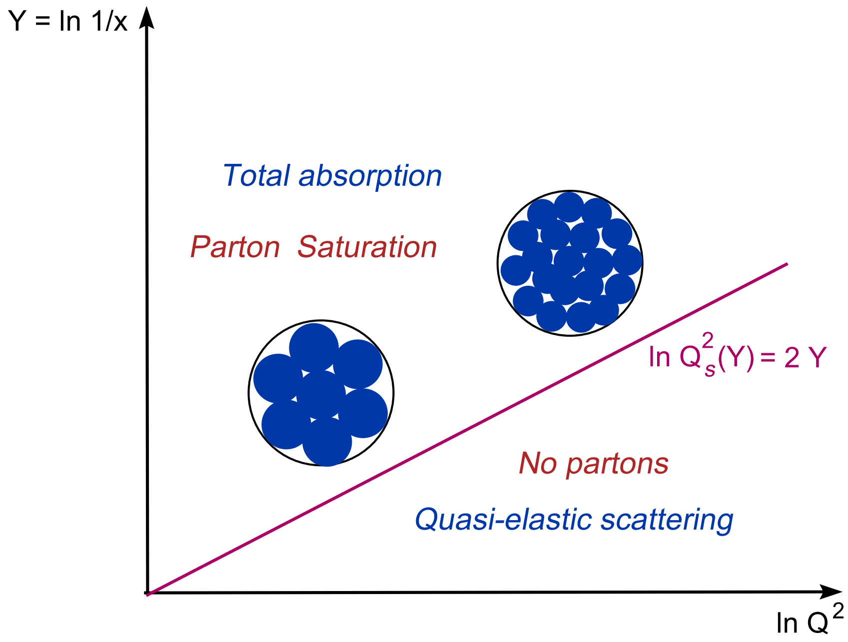
DIS at strong coupling

(Polchinki, Strassler, 02; Hatta, E.I., Mueller, 07) *see talk by R. Peschanski*

- $\lambda \equiv g^2 N_c \gg 1$ with $g^2 \ll 1 \Longrightarrow$ AdS/CFT correspondence
- $\mathcal{N} = 4$ SYM $\Longleftrightarrow$ classical gravity in the $AdS_5 \times S^5$
- Parton branching at strong coupling :

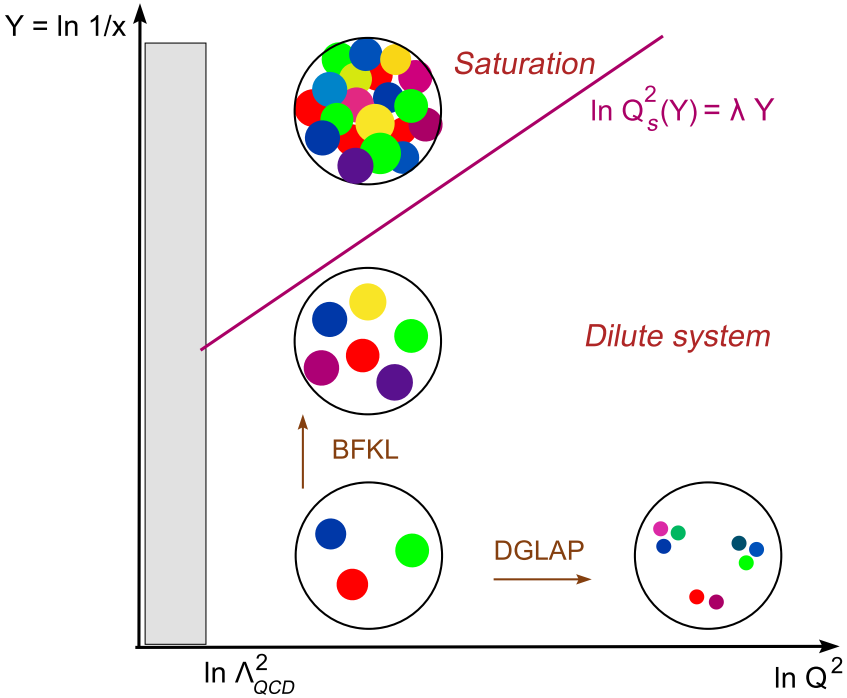
  No reason to favour special corners of phase–space !

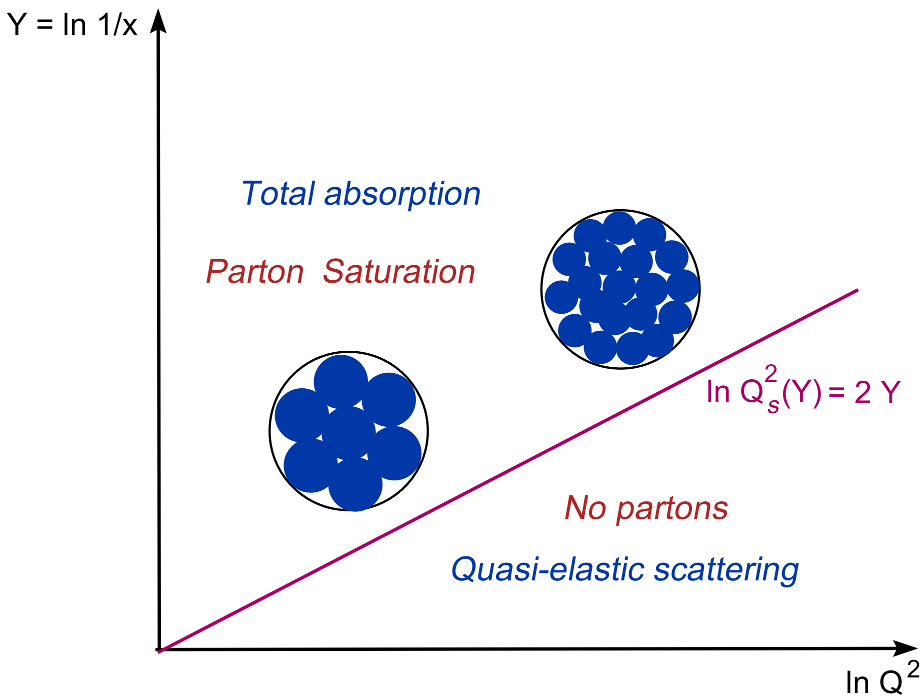




- All partons have branched down to small values of x !

Saturation line: weak vs. strong coupling





- No 'leading–twist' (no pdf's !) at $Q^2 > Q_s^2(x)$

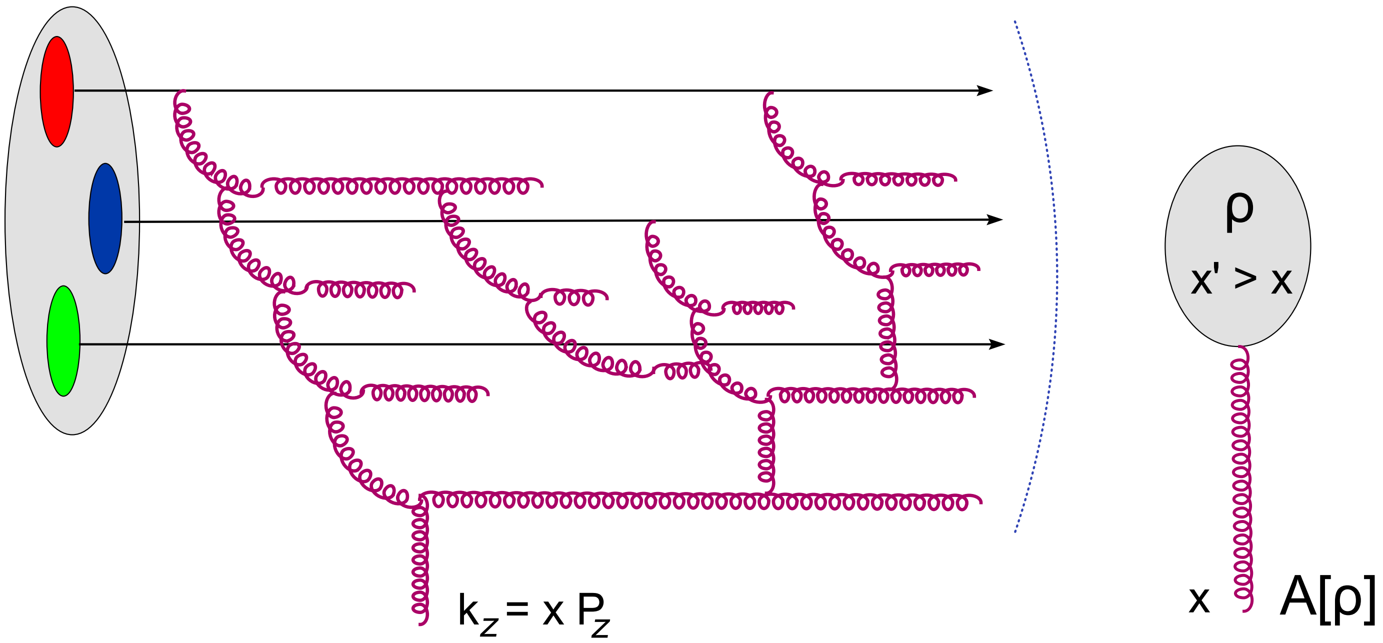
  all partons lie within the CGC with occupancy $n \sim \mathcal{O}(1)$

- Saturation exponent : $Q_s^2(x) \propto 1/x^{\lambda_s} \equiv \mathrm{e}^{\lambda_s Y}$
  - weak coupling : $\lambda_s \approx 0.4\, g^2 N_c$ (LO BFKL Pomeron)
  - strong coupling : $\lambda_s = 1$ (graviton)

The Color Glass Condensate

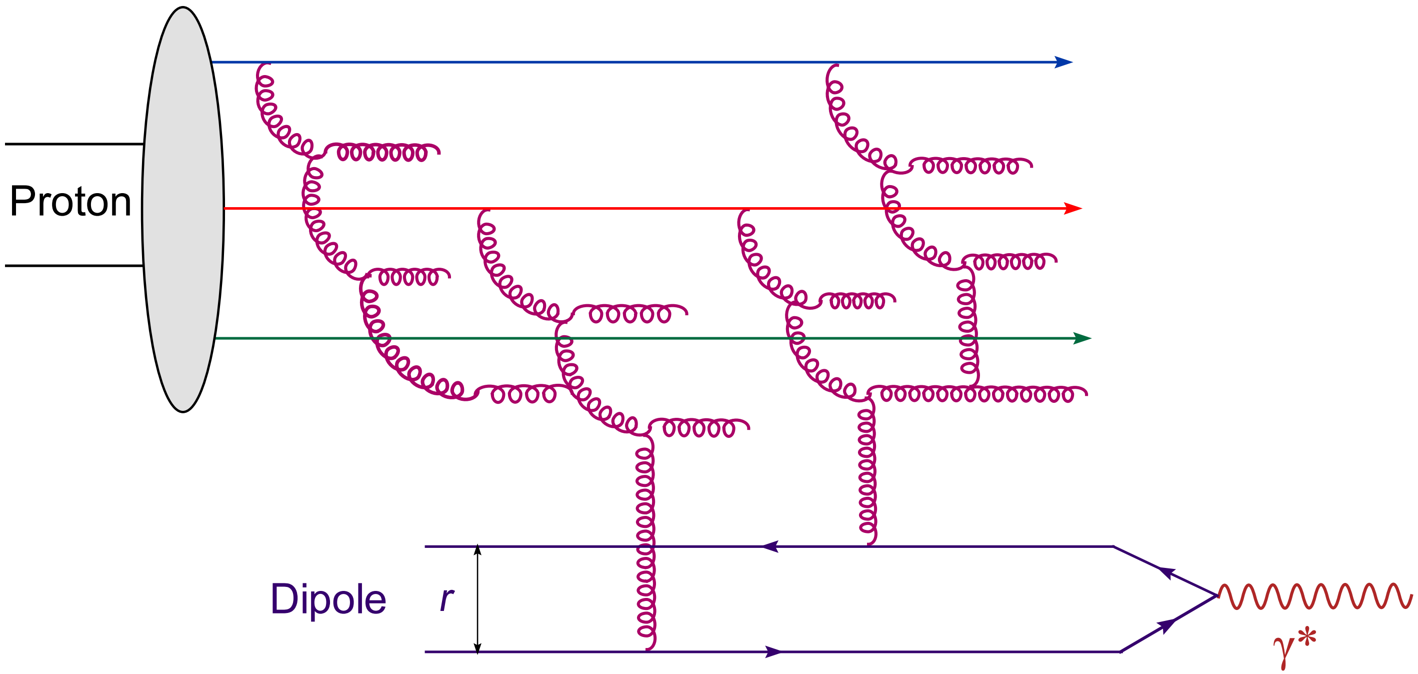
(McLerran, Venugopalan, 1994; E.I., Leonidov, McLerran, 2000)

- An effective theory for the evolution towards saturation



- Small–x gluons: Classical color fields radiated by fast color sources ($x' \gg x$) ‘frozen’ in some random configuration ρ_a
- $W_Y[\rho]$: Probability distribution for the color charge density
- Functional evolution equation for $W_Y[\rho]$: JIMWLK

Deep Inelastic Scattering off the CGC



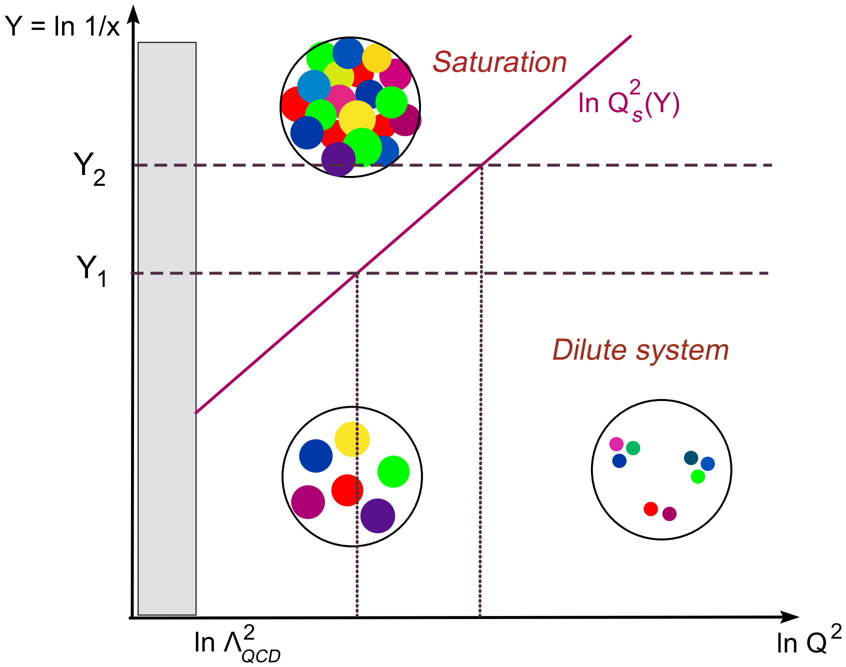
- $T(r)[\rho]$: scattering off a given configuration ρ of the color sources (multiple scattering in the eikonal approximation)
- Average over ρ with weight function $W_Y[\rho]$ (glass)

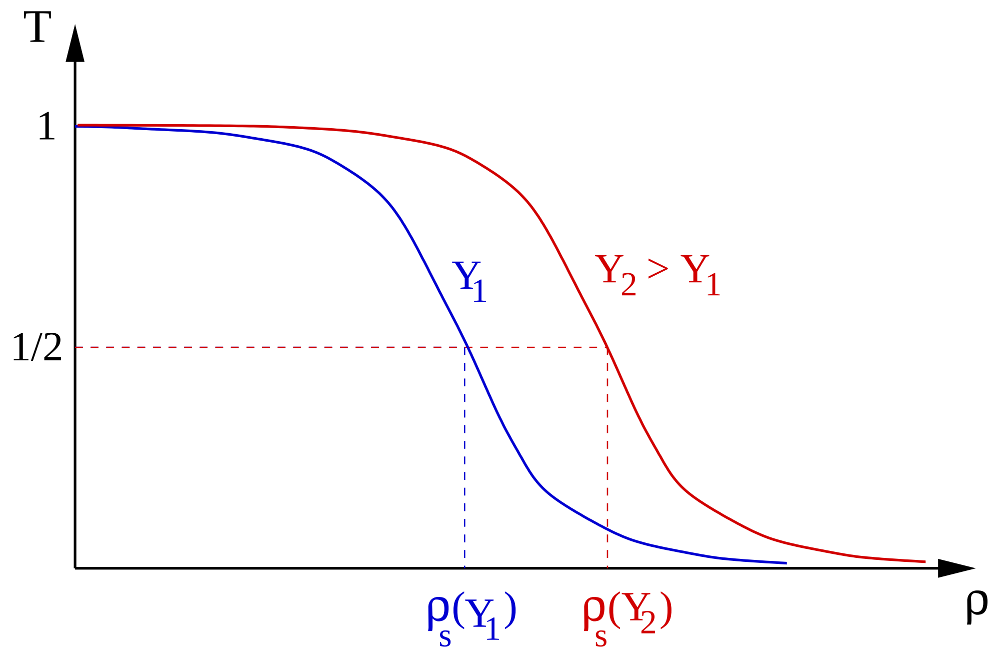
$$\langle T(r)\rangle_Y = \int \mathrm{D}[\rho]\; W_Y[\rho]\; T(r)[\rho]$$

Saturation momentum

- Saturation front : $T(\rho, Y)$ with $\rho = \ln(1/r^2)$

  $\Longrightarrow$ a front interpolating between $T = 0$ and $T = 1$





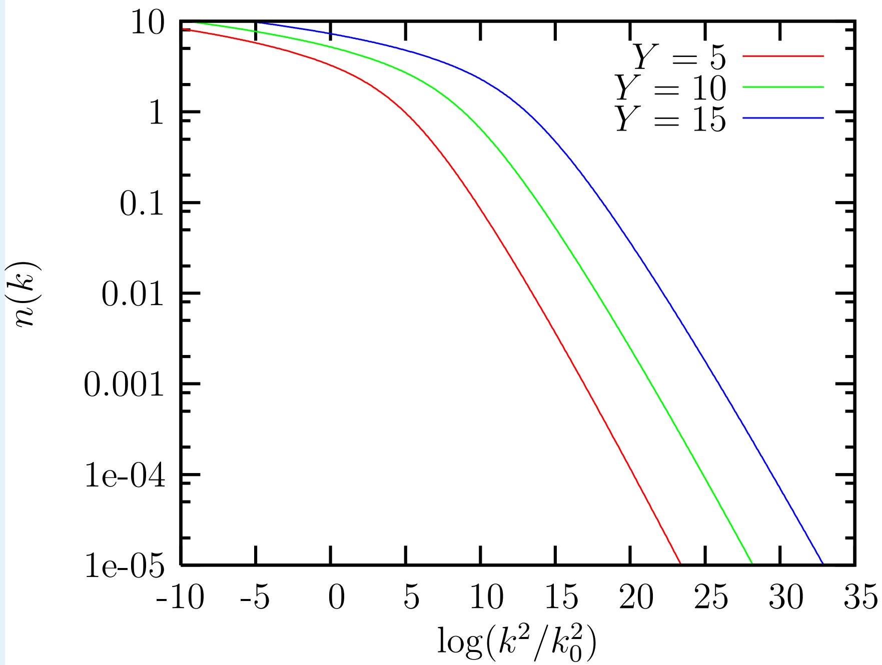
- The position $\rho_s(Y)$ of the front $\Longrightarrow$ saturation momentum

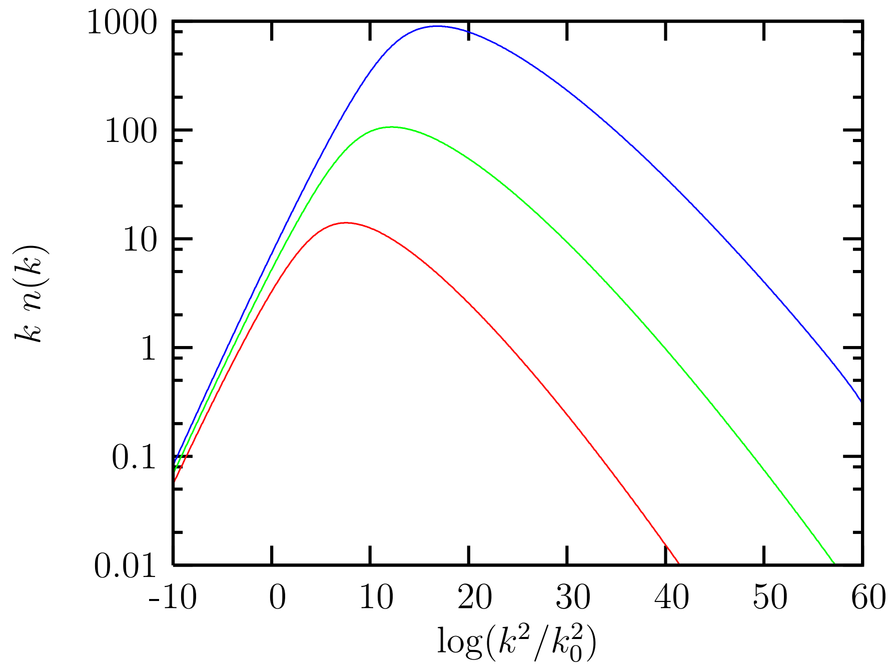
  BK $\Longrightarrow$ $\rho_s(Y) \equiv \ln Q_s^2(Y) \approx \lambda Y$ with $\lambda \approx 4.88\bar{\alpha}_s \sim 1$

Gluon occupation number

- A similar front holds for the ‘unintegrated gluon distribution’

$$xG(x,Q^2) = \int d^2b \int^Q dk\, k\, n(x,k)$$

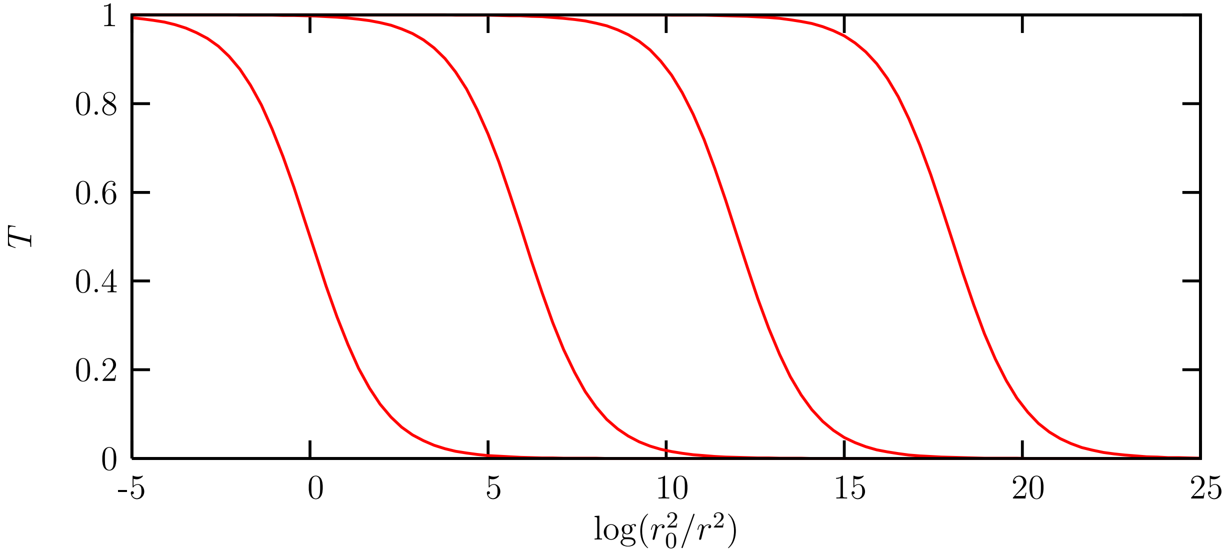




- The typical transverse momentum of the gluons is $\sim Q_s(Y)$

BK equation: The traveling wave

- The shape of the front is not altered by the evolution



$$T(\rho, Y) \simeq \mathcal{T}(\rho - \rho_s(Y)) \equiv \mathcal{T}\big(r^2 Q_s^2(Y)\big)$$

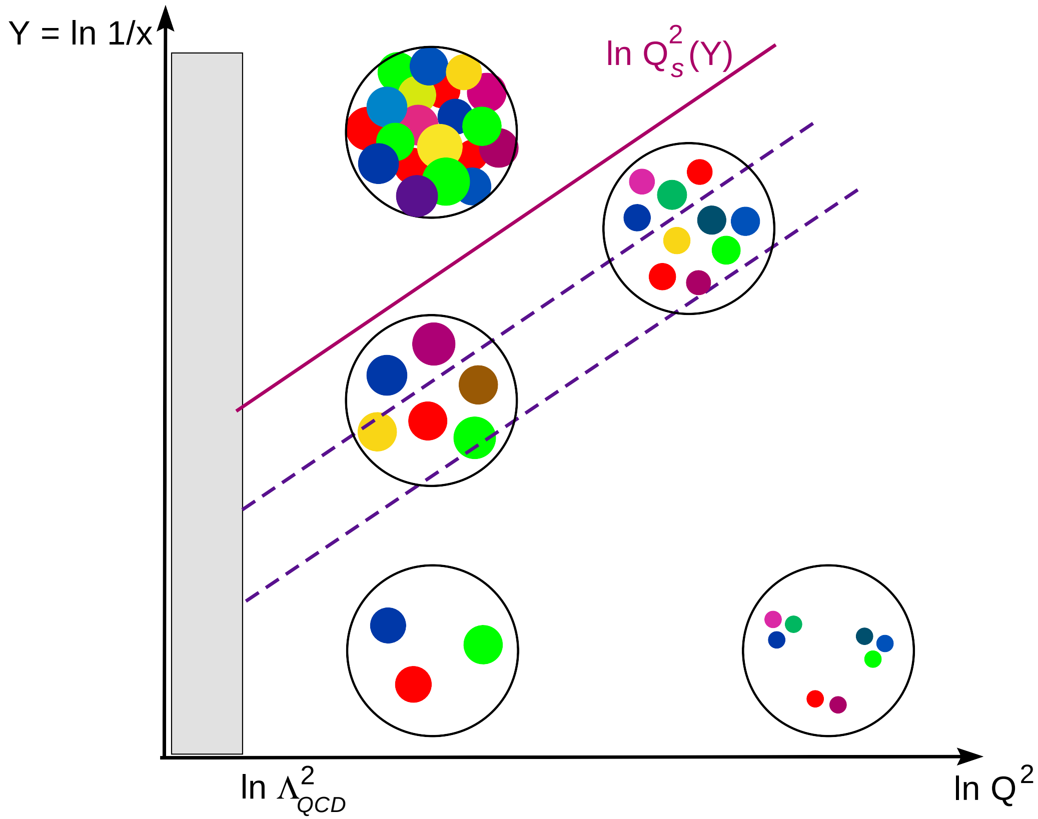
- ‘Geometric scaling’

  E.I., Itakura, McLerran (02) ; Mueller, Triantafyllopoulos (02)

- Traveling wave picture : *Munier, Peschanski (03)*

- Relation to Stat Phys : ‘reaction–diffusion’ $A \rightleftharpoons 2A$

Geometric scaling

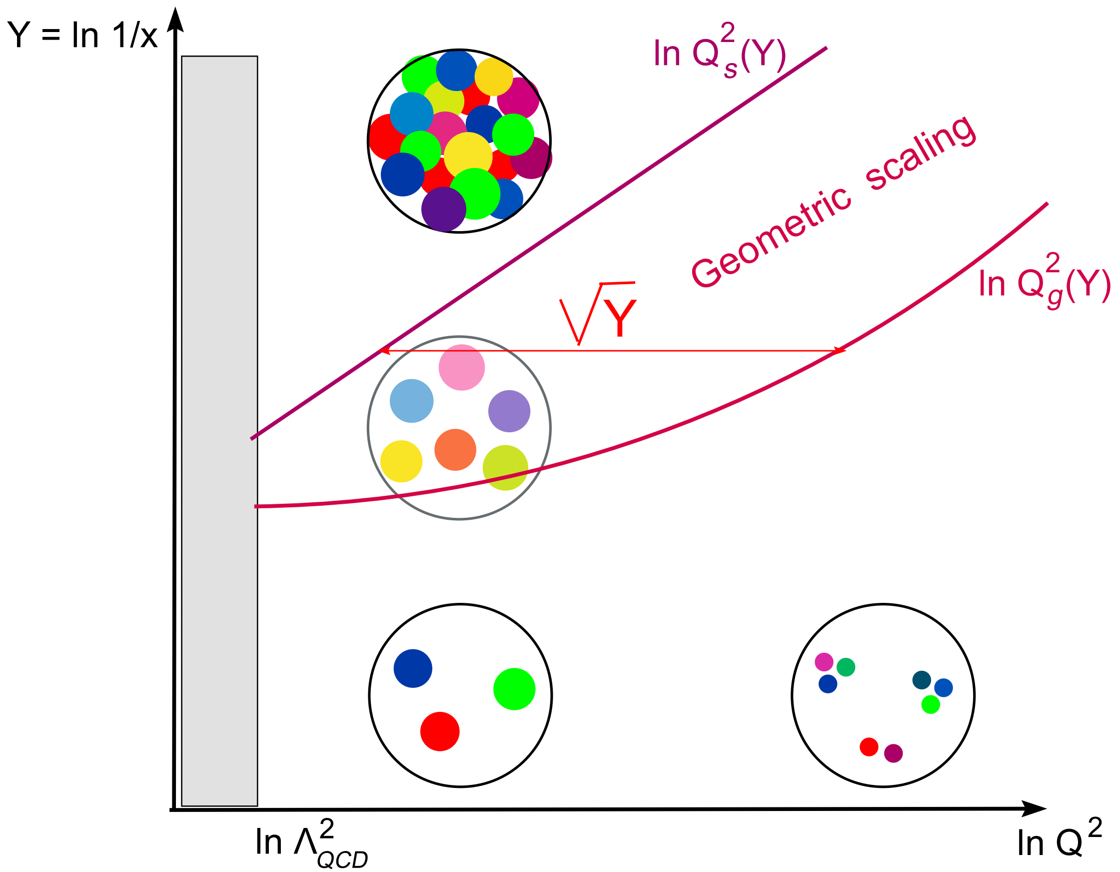


- $\rho - \rho_s(Y) = const$: A line of constant gluon occupancy

  $\Longrightarrow$ physics must be invariant along any such a line !

- Saturation makes itself felt in the dilute regime $(Q^2 > Q_s^2)$

Geometric scaling



- Strictly true only within a finite 'scaling window' above Q_s, which extends with Y : $\ln Q_g^2(Y) - \ln Q_s^2(Y) \propto \sqrt{\alpha_s Y}$

Geometric Scaling at HERA

(Staśto, Golec-Biernat and Kwieciński, 2000)

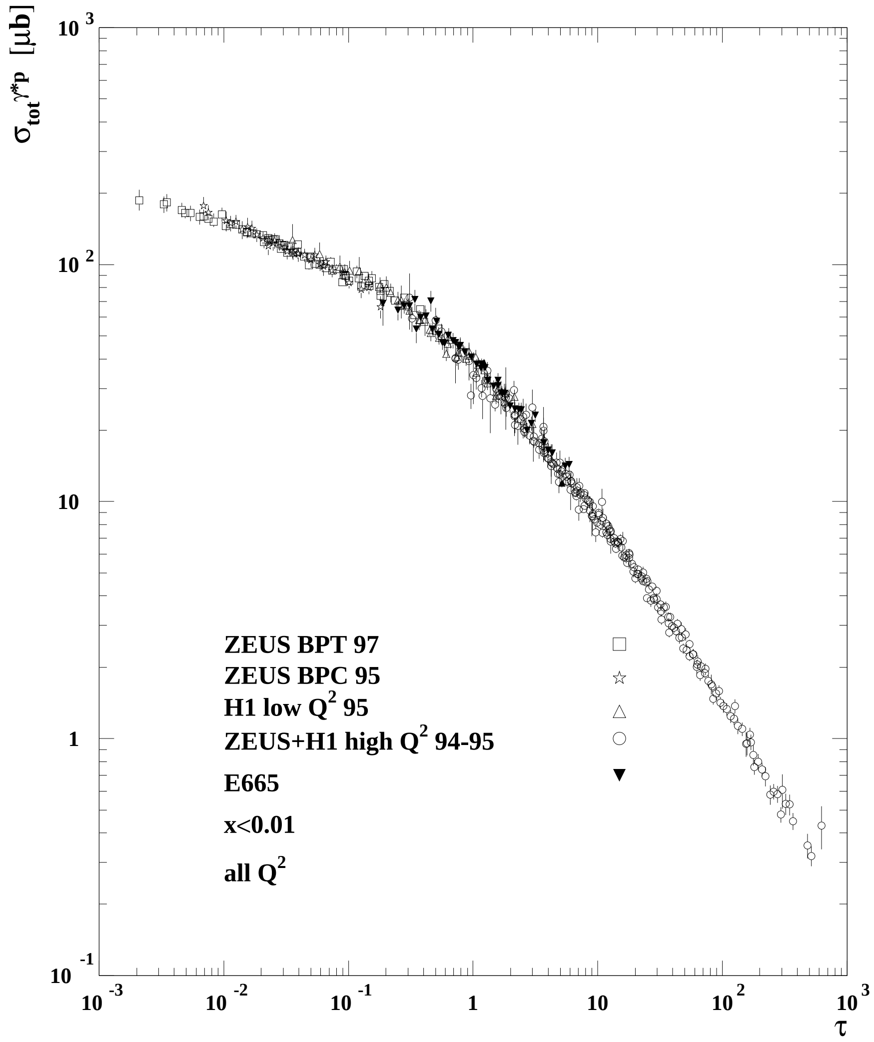
$$\sigma(x, Q^2) \approx \sigma(\tau) \quad \text{with} \quad \tau \equiv Q^2/Q_s^2(x), \quad Q_s^2(x) = (x_0/x)^\lambda \,\text{GeV}^2\,, \quad \lambda \simeq 0.3$$

$x \leq 0.01$

$Q^2 \leq 450$ GeV^2

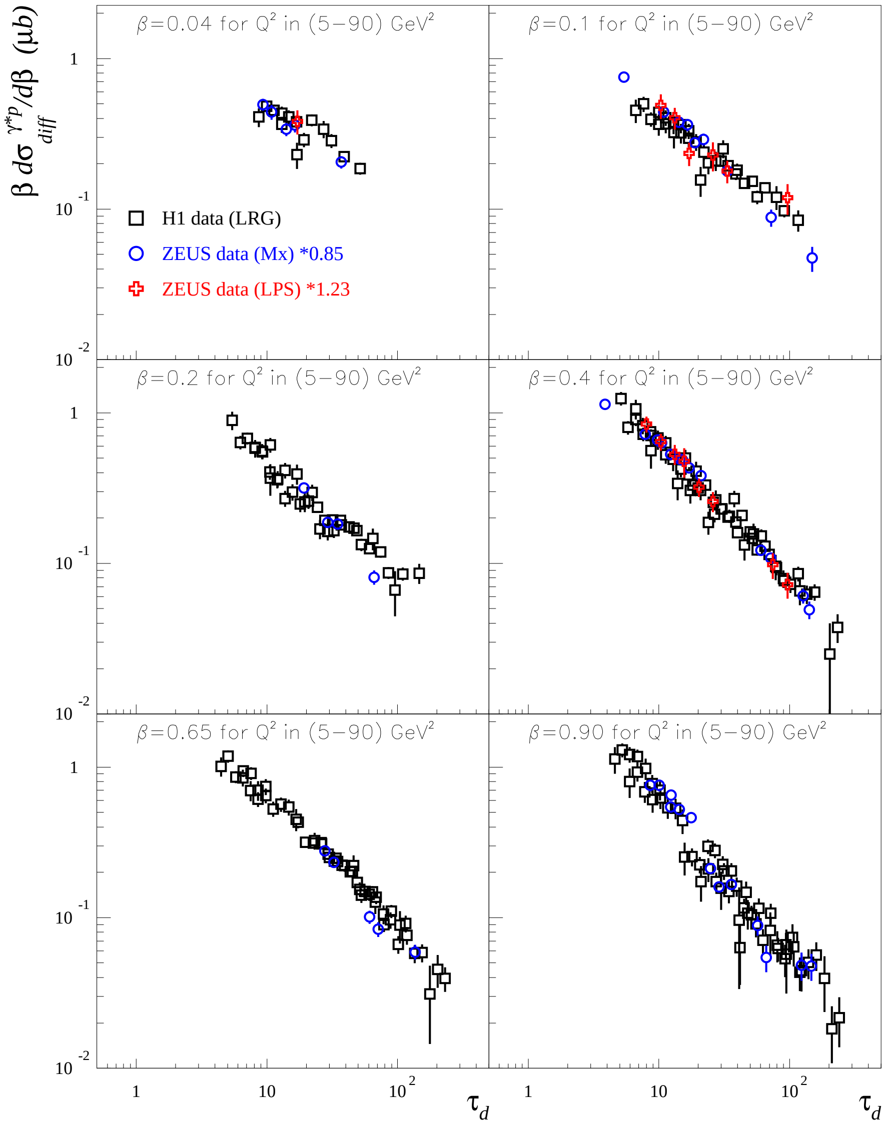
$Q_s^2 \sim 1$ GeV^2

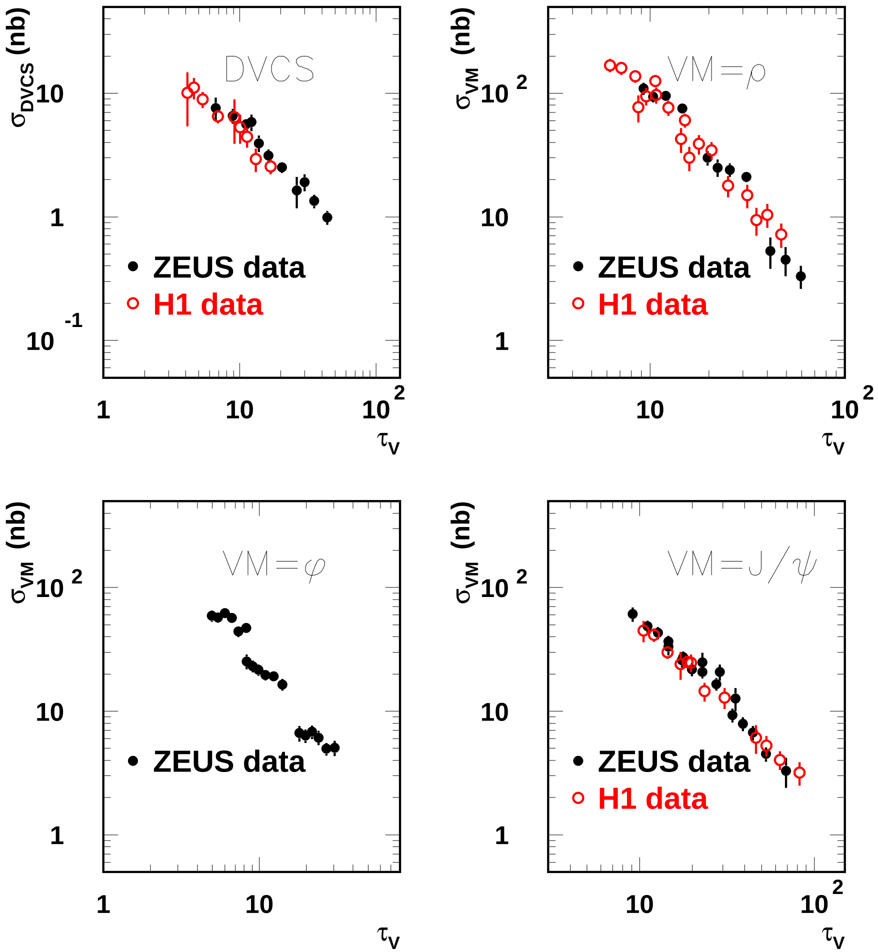
for $x \sim 10^{-4}$



Geometric Scaling at HERA (2)

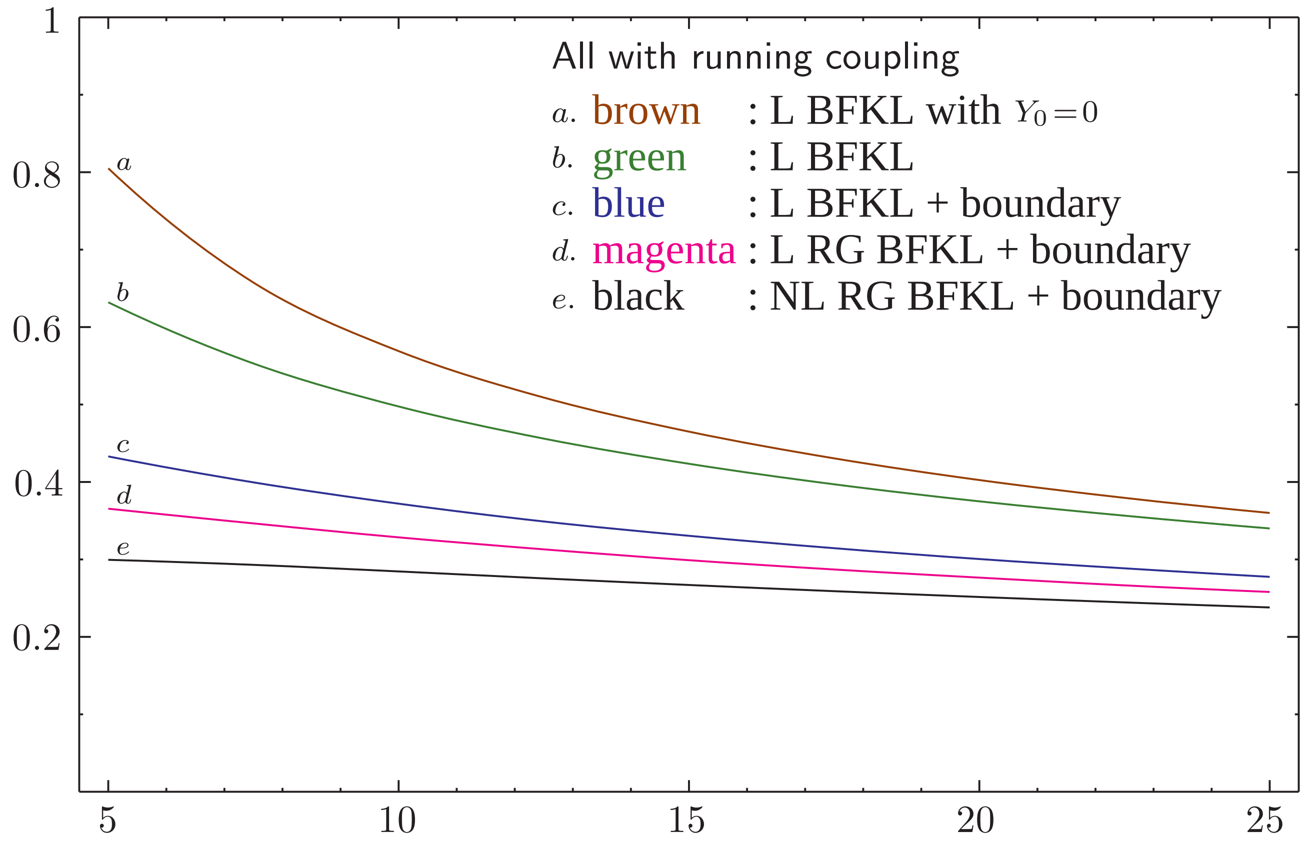
(Marquet and Schoeffel 2006)





Saturation exponent at NLO

D.N. Triantafyllopoulos, 2002



$$\lambda(Y) \equiv \frac{\mathrm{d}\ln Q_s^2(Y)}{\mathrm{d}Y} \approx 0.3$$

- NLO BFKL + Collinear resummation + Saturation Boundary

The unreasonable effectiveness of the 'saturation models'

- "Saturation models" $\equiv$ QCD–inspired models for $\sigma_{\rm dipole}$ involving saturation and a reasonable # of free parameters
- The parameters are fixed by fits to the F_2 data alone !
- Satisfactory description of the ensemble of HERA data at $x \le 0.01$

  All other observables (F_2^D, F_L, F_2^c, ρ, J/ψ, DVCS, ...) emerge as 'predictions'.

- Important qualitative predictions of the theory which appear to be consistent with the data.

  geometric scaling, the transition towards low Q^2 for F_2, a nearly constant $\sigma_{\rm diff}/\sigma_{\rm tot}$ ratio ...

- A similar success for the relevant data at RHIC

  high–$p_\perp$ suppression in forward d–Au collisions ('R_{pA}')

Saturation models

- The Golec-Biernat and Wüsthoff model (1999)

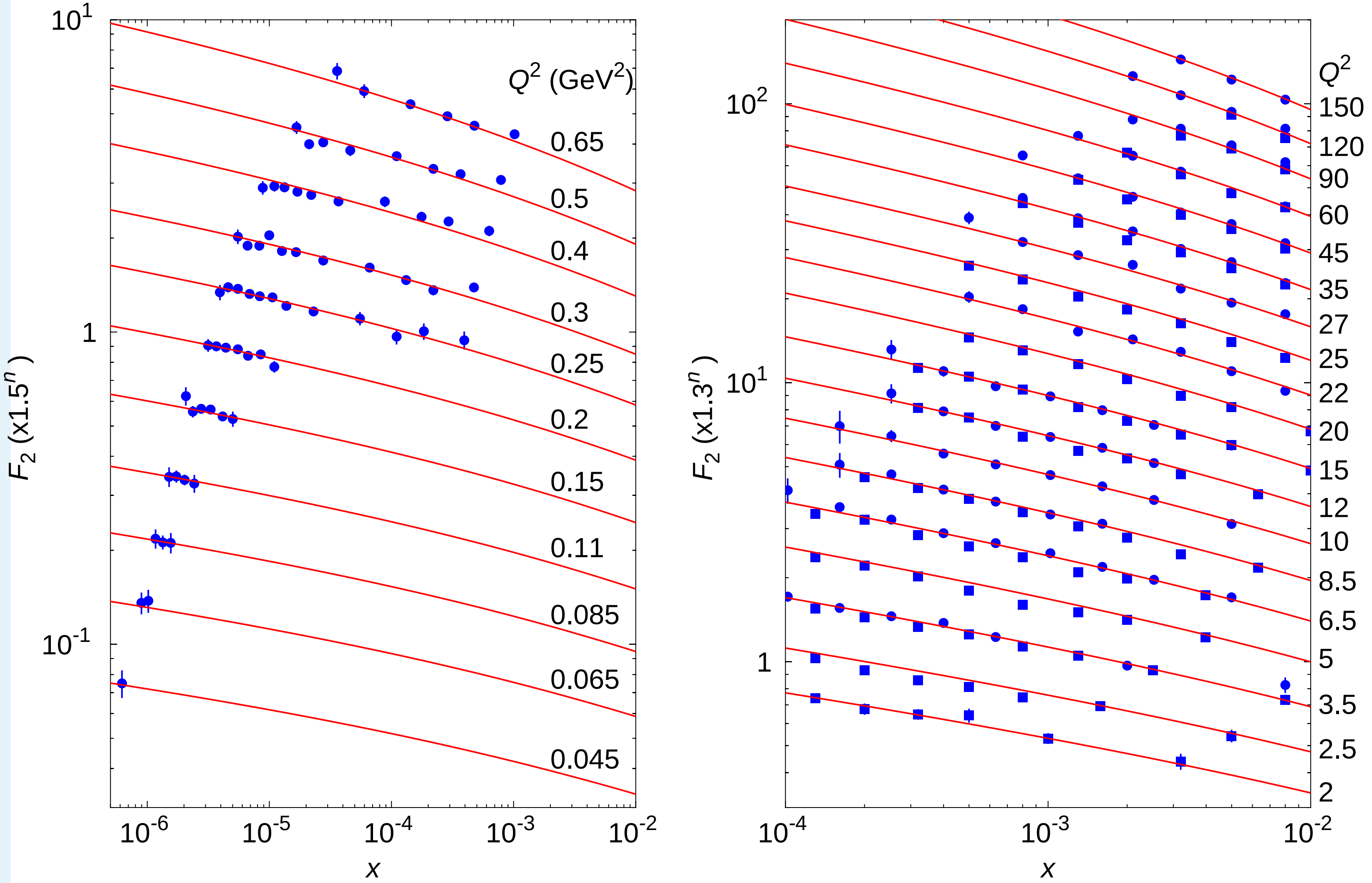
$$\sigma_{\rm dipole}^{\rm GBW}(x,r) = 2\pi R^2\left(1 - \mathrm{e}^{-r^2 Q_s^2(x)}\right), \quad Q_s^2(x) = (x_0/x)^\lambda\,\mathrm{GeV}^2$$

  - Good fit to the early HERA data with only 3 parameters
  - Exact ‘geometric scaling’ built in : $\sigma^{\rm GBW}(r^2 Q_s^2(x))$

- More sophisticated models (pQCD evolution, geometric scaling violations)
  - DGLAP–like (also with b dependence) *Bartels, Golec-Biernat, Kowalski (02), Kowalski, Teaney (03), Kowalski, Motyka, Watt (06)*
  - CGC model (BK eq.) *E.I., Itakura, Munier (03)* : 3 light quarks
  - Improvements of CGC model: heavy quarks, b–dependence *Kowalski, Motyka, Watt (06), Soyez (07)*
  - FS04 saturation model *Forshaw, Shaw (04)*

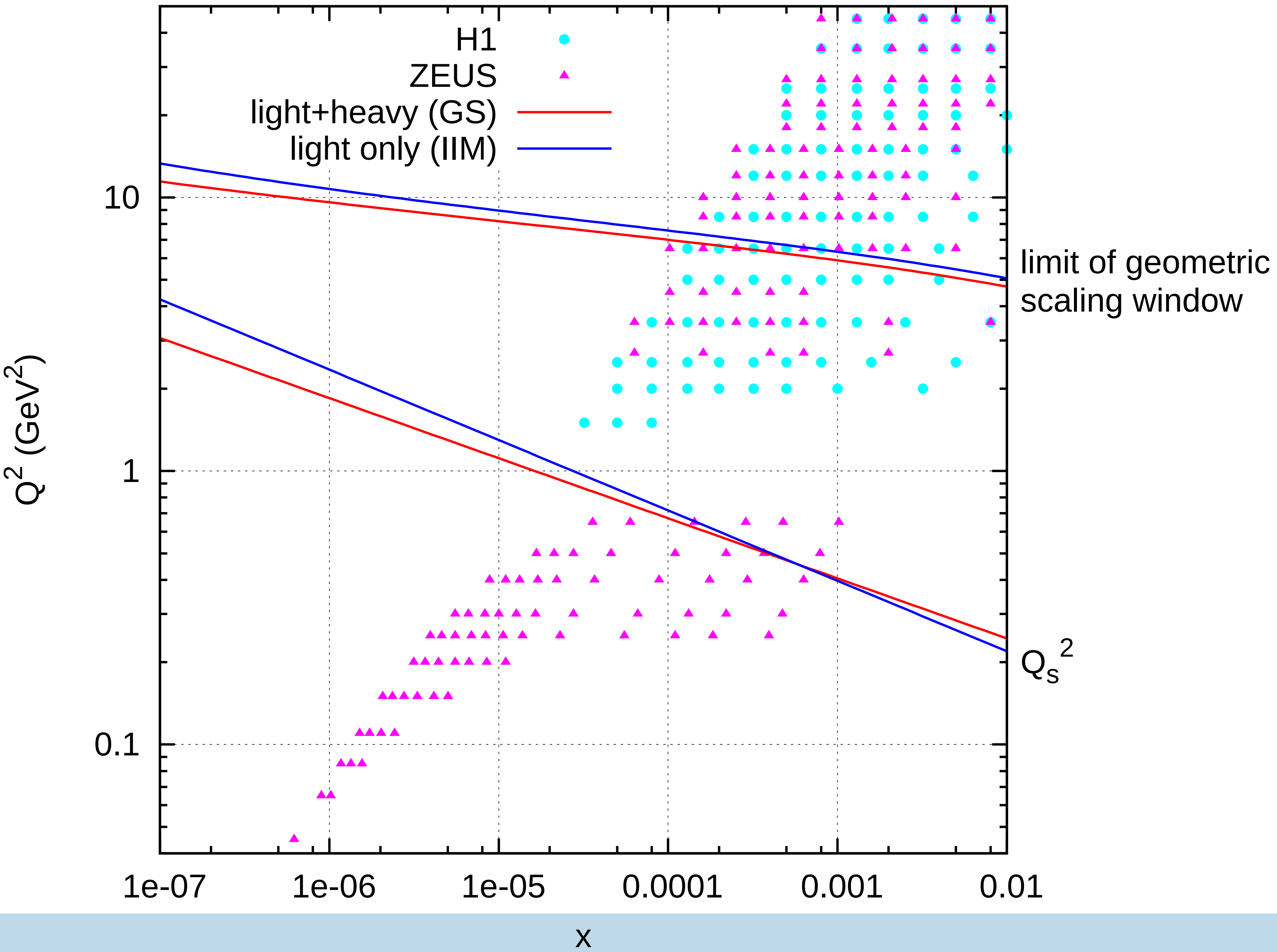
A CGC fit to F_2 *(G. Soyez, 2007)*

$x \leq 10^{-2}$, $Q^2 \leq 150\,\mathrm{GeV}^2$ (281 data points, ZEUS & H1)



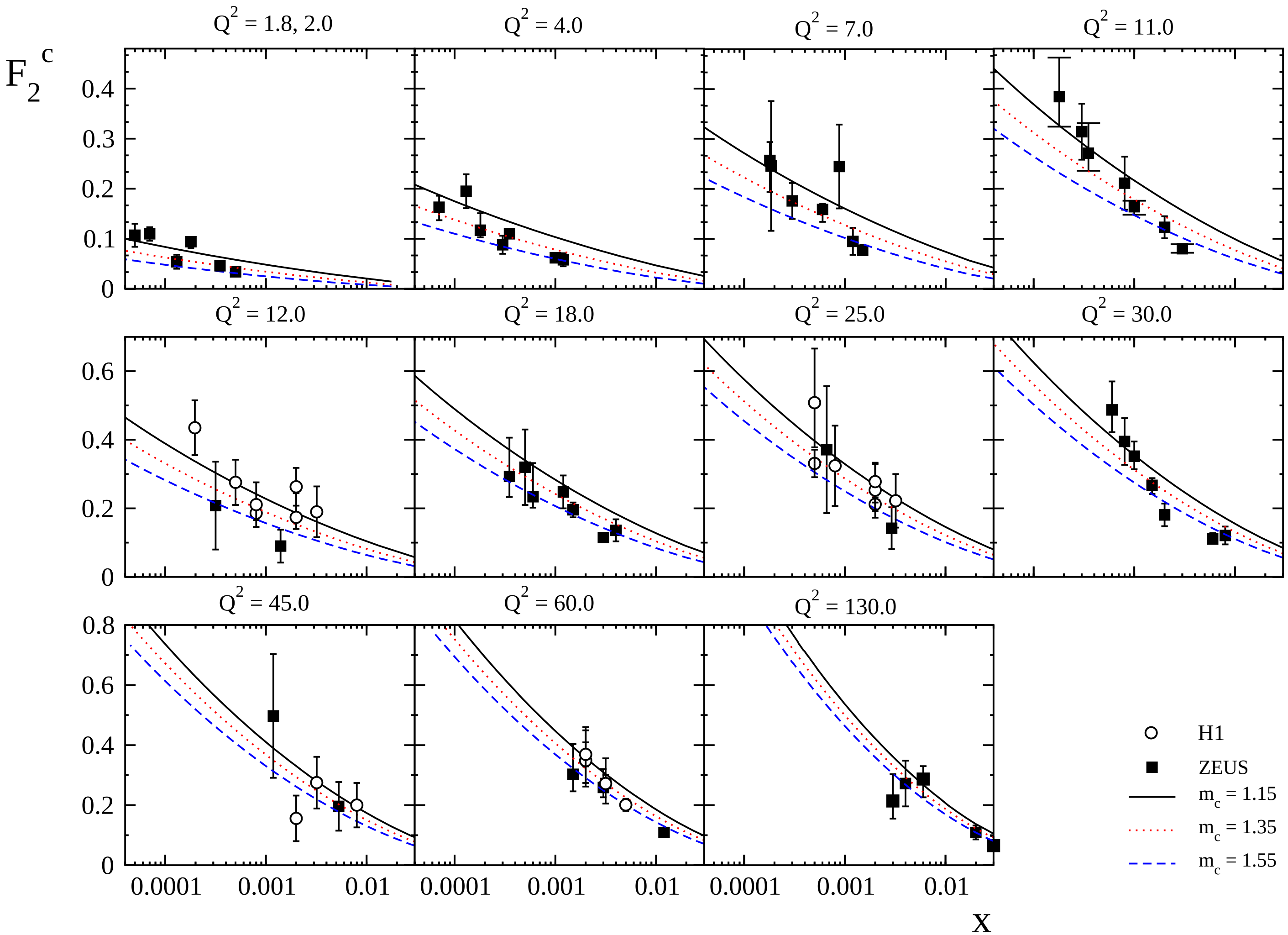
4 parameters: R, $x_0 \approx 2 \times 10^{-5}$, $\gamma \approx 0.26$ *and* $\lambda \approx 0.22$ $(\chi^2 = 0.90)$

A CGC fit to F_2 *(G. Soyez, 2007)*



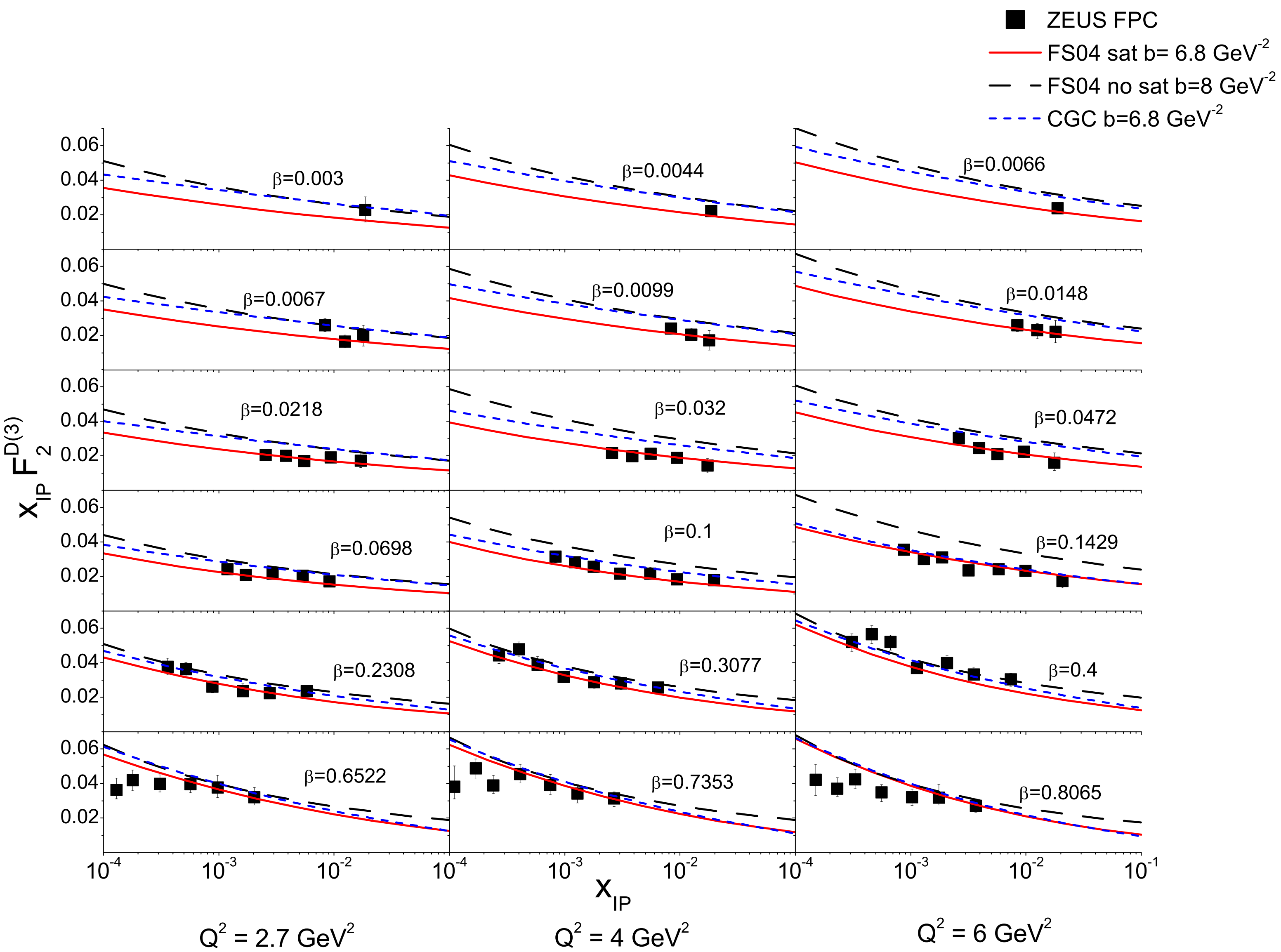
A CGC fit to F_2^c

Forshaw, Sandapen and Shaw (06)



Saturation fits for Diffraction

Forshaw, Sandapen and Shaw (06) Low Q^2



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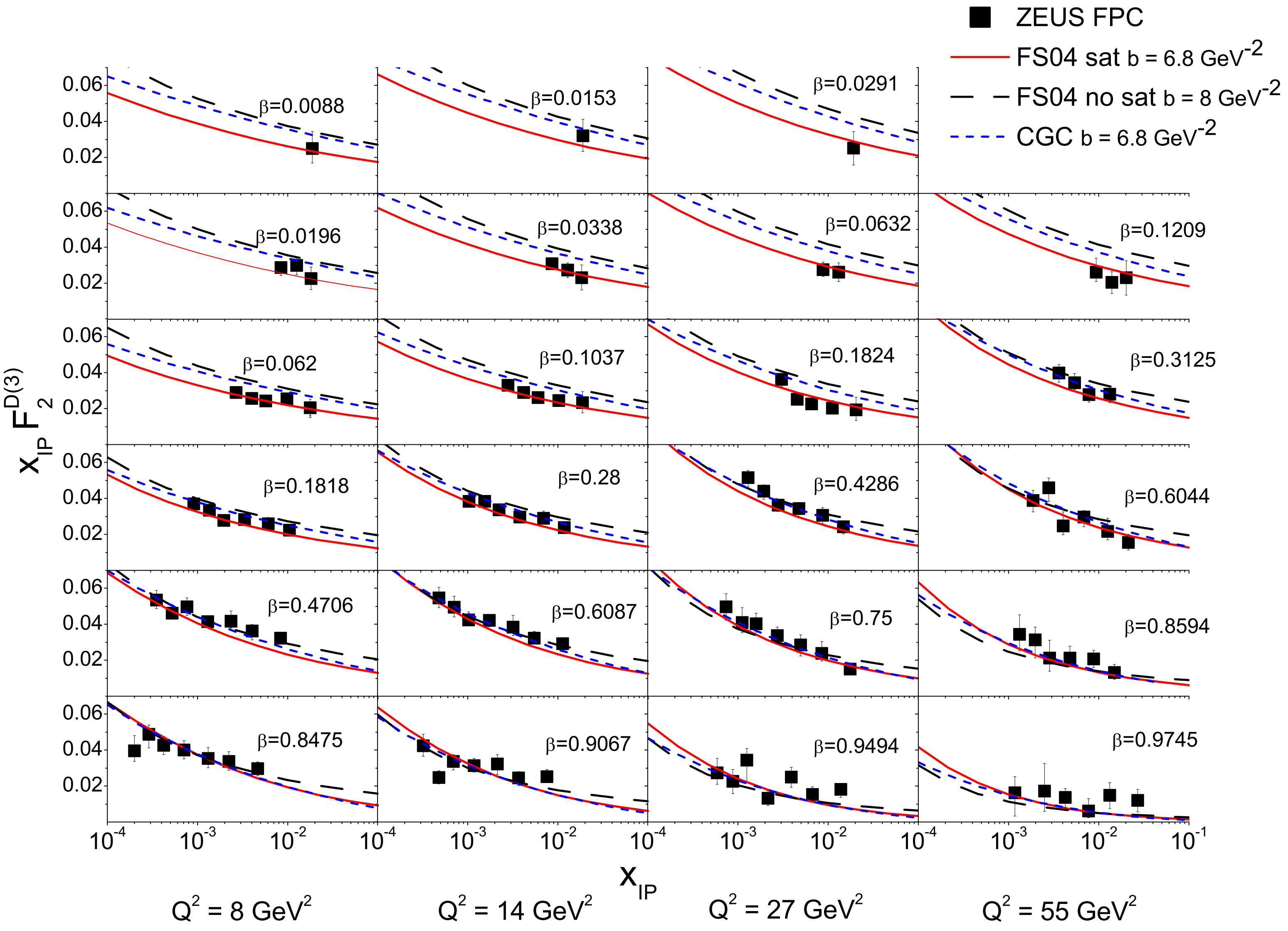
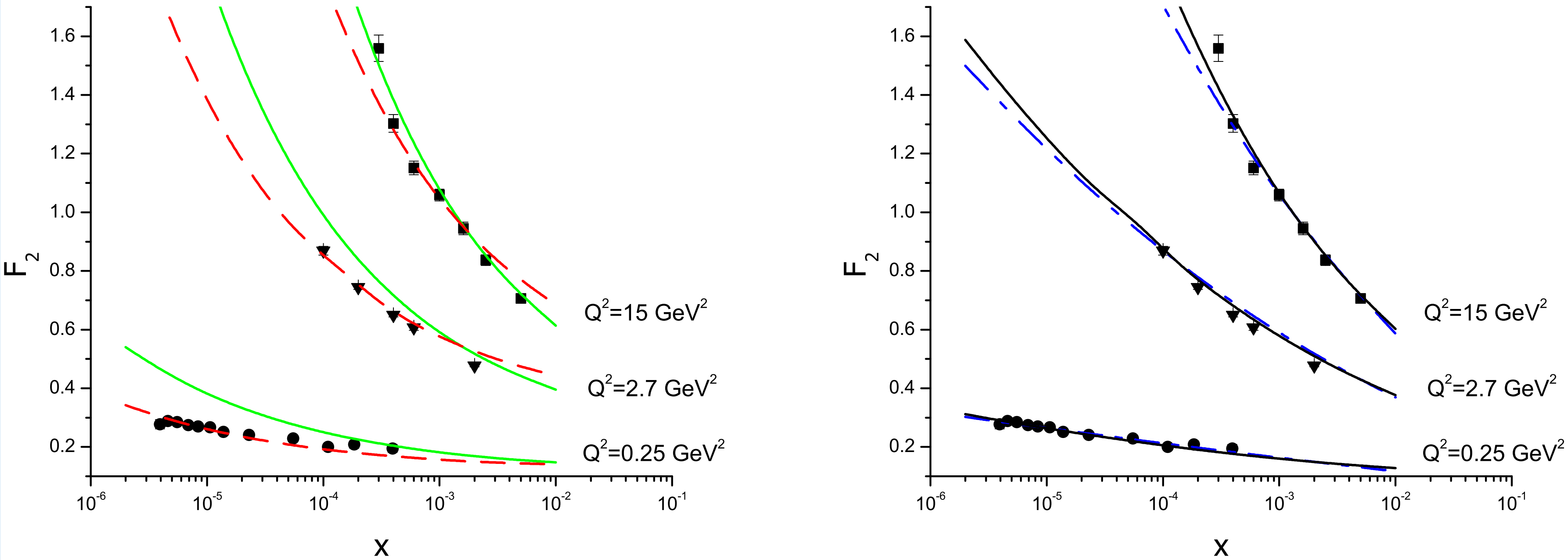


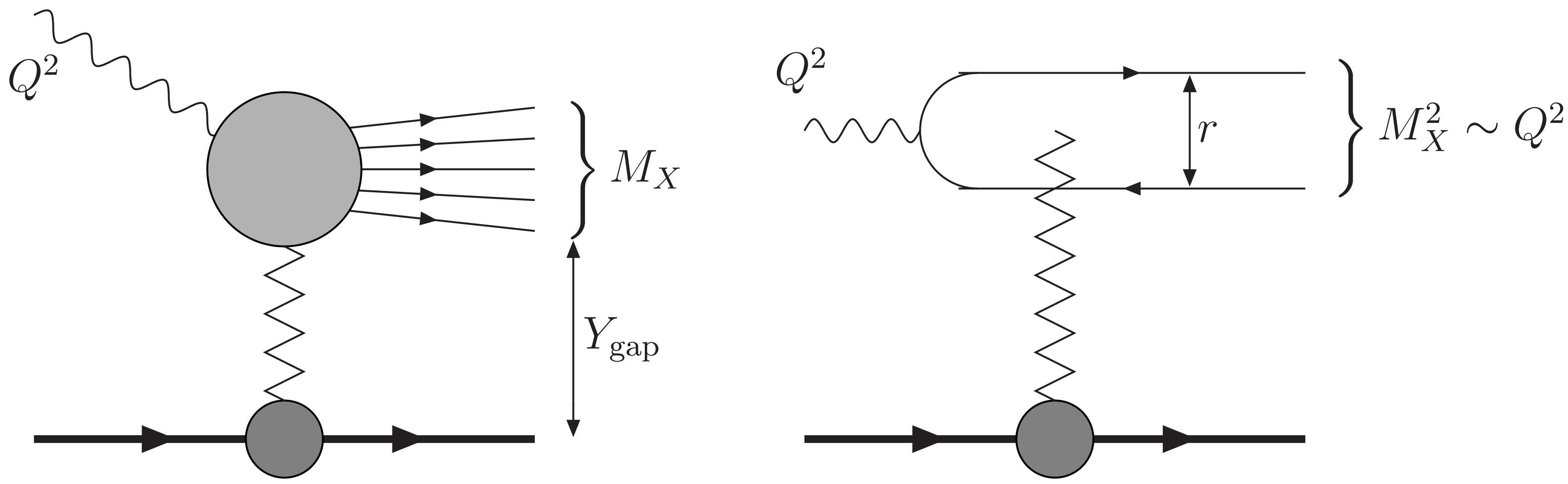
Saturation vs. Regge fits for F_2

Forshaw and Shaw (04) Relatively low Q^2



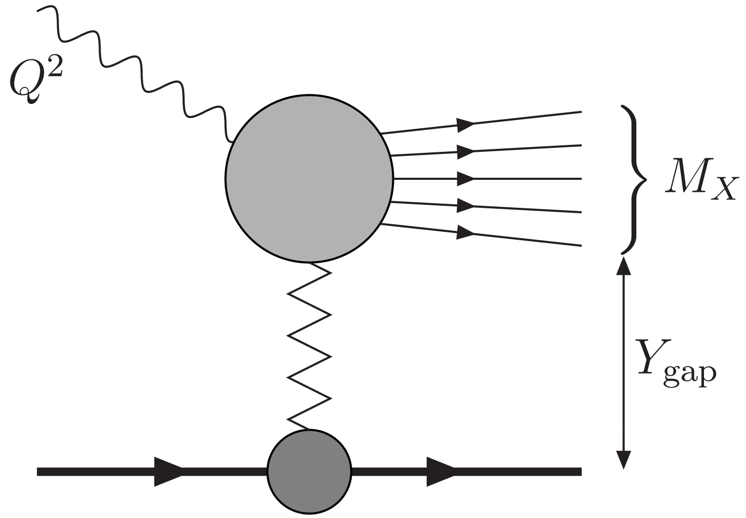
- **Left:** Regge fits (a sum of 'soft' + 'hard' Pomerons)
- **Right:** 2 saturation fits (FS04 and CGC)
- Data appear to prefer saturation !

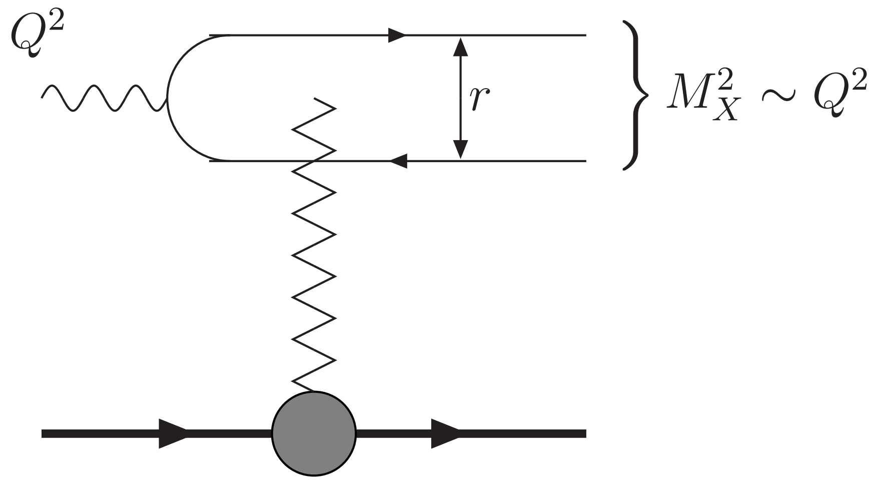
DIS Diffraction



- An ideal laboratory to study saturation/unitarity effects
  - sensitive to relatively large dipole sizes
  - sensitive to theoretical models (or prejudices)

DIS Diffraction

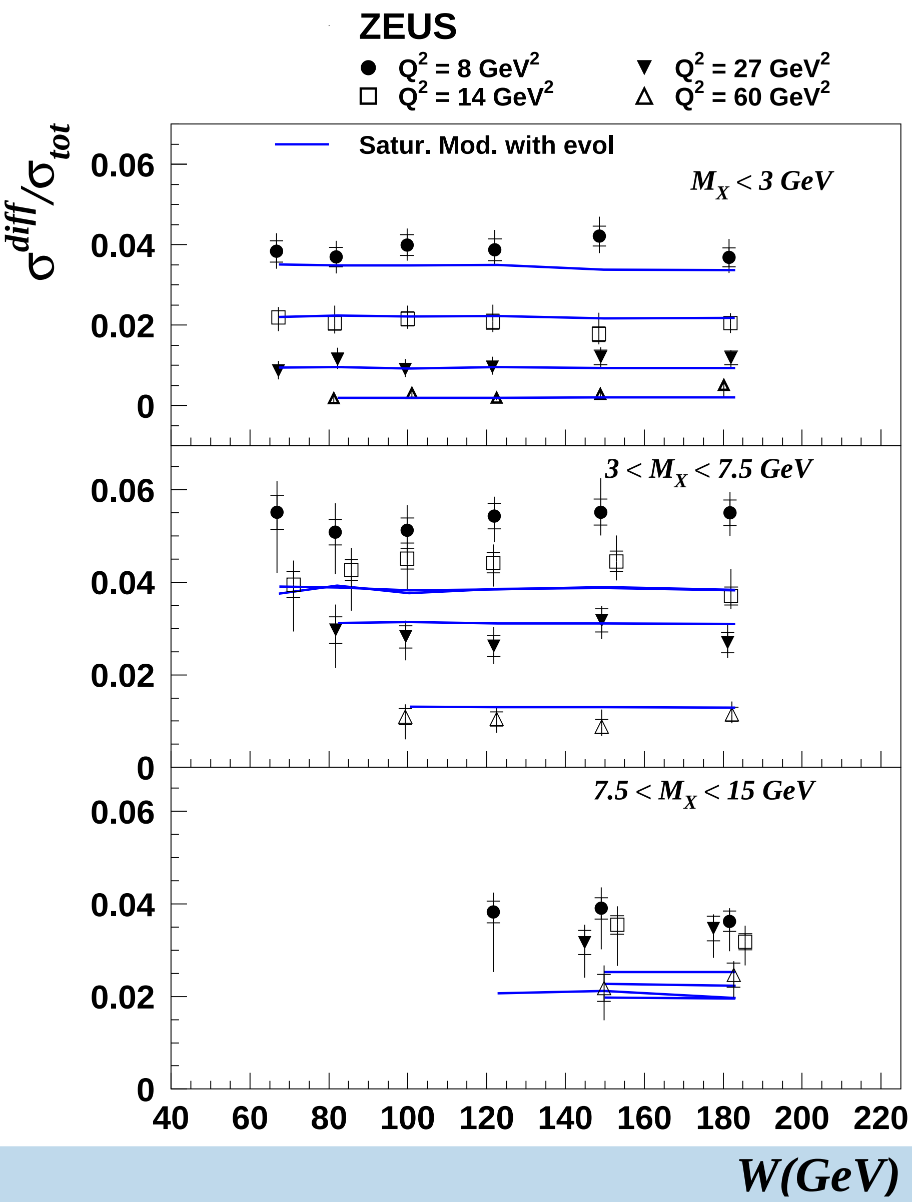




- An ideal laboratory to study saturation/unitarity effects
  - sensitive to relatively large dipole sizes
  - sensitive to theoretical models (or prejudices)
- Original prejudice: "Even for large Q^2, diffraction is soft"

$\sigma_{\text{diff}} \propto x^{-2(\alpha_{\mathbb{P}}-1)}$ and hence $\dfrac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \sim x^{-(\alpha_{\mathbb{P}}-1)}$ at small x ✓

Golec-Biernat, Wüsthoff (99) ; Bartels, Golec-Biernat & Kowalski (02)



Diffractive dissociation of the virtual photon

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} = \int \mathrm{d}z\,\mathrm{d}^2\boldsymbol{r}\,|\Psi_\gamma(z,r;Q)|^2\,\Big(T(r,Y)\Big)^2$$

- The photon wavefunction favors small dipoles ($r \sim 1/Q$)

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} \sim \frac{1}{Q^2}\int_{1/Q^2}^{\infty}\frac{\mathrm{d}r^2}{r^4}\Big(T(r,Y)\Big)^2$$

- The dipole amplitude favors relatively large dipoles :

$$T(r) \propto r^2 \qquad \text{(single scattering)}$$

- “The integral is dominated by large, non–perturbative, dipoles with size $r \sim 1/\Lambda_{\mathrm{QCD}}$, hence the soft pomeron ! ”

Hardening the diffraction

- At sufficiently high energy, gluon saturation cuts off the large dipoles already on the 'semi–hard' scale $1/Q_s$!

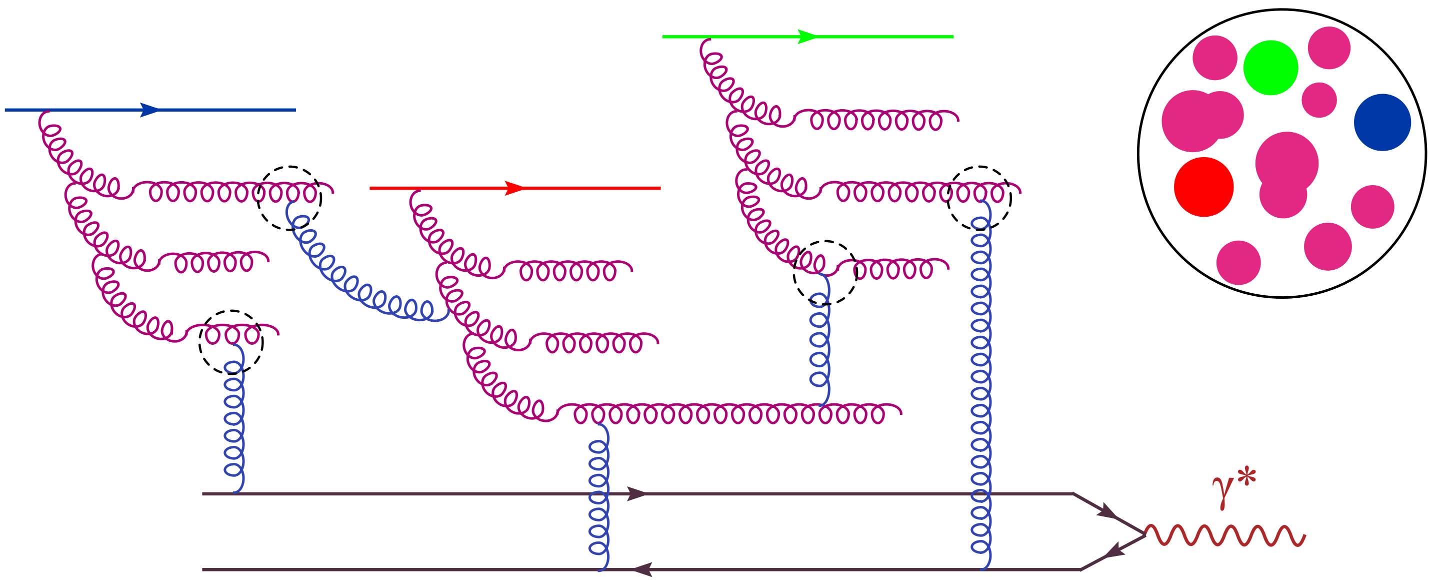
$$\frac{d\sigma_{\mathrm{diff}}}{d^2 b} \sim \frac{1}{Q^2} \int_{1/Q^2}^{1/Q_s^2} \frac{dr^2}{r^4} \left(r^2 Q_s^2(x) \right)^2 \sim \frac{Q_s^2(x)}{Q^2} \propto x^{-\lambda}$$

  - σ_{diff} is dominated by dipole sizes $r \sim 1/Q_s(x)$!
  - $\sigma_{\mathrm{diff}} \propto x^{-\lambda}$: single, hard pomeron increase with $1/x$ (instead of double soft !)
  - $\sigma_{\mathrm{diff}}/\sigma_{\mathrm{tot}} \approx$ constant ! ✓

- 'Semi–hard diffraction' ... at intermediate energies !

Stochastic aspects of high–energy QCD

- Classical fields (JIMWLK) : no gluon–number fluctuations
- Gluons in the same cascade are correlated with each other
- Saturation & multiple scattering could probe the correlations

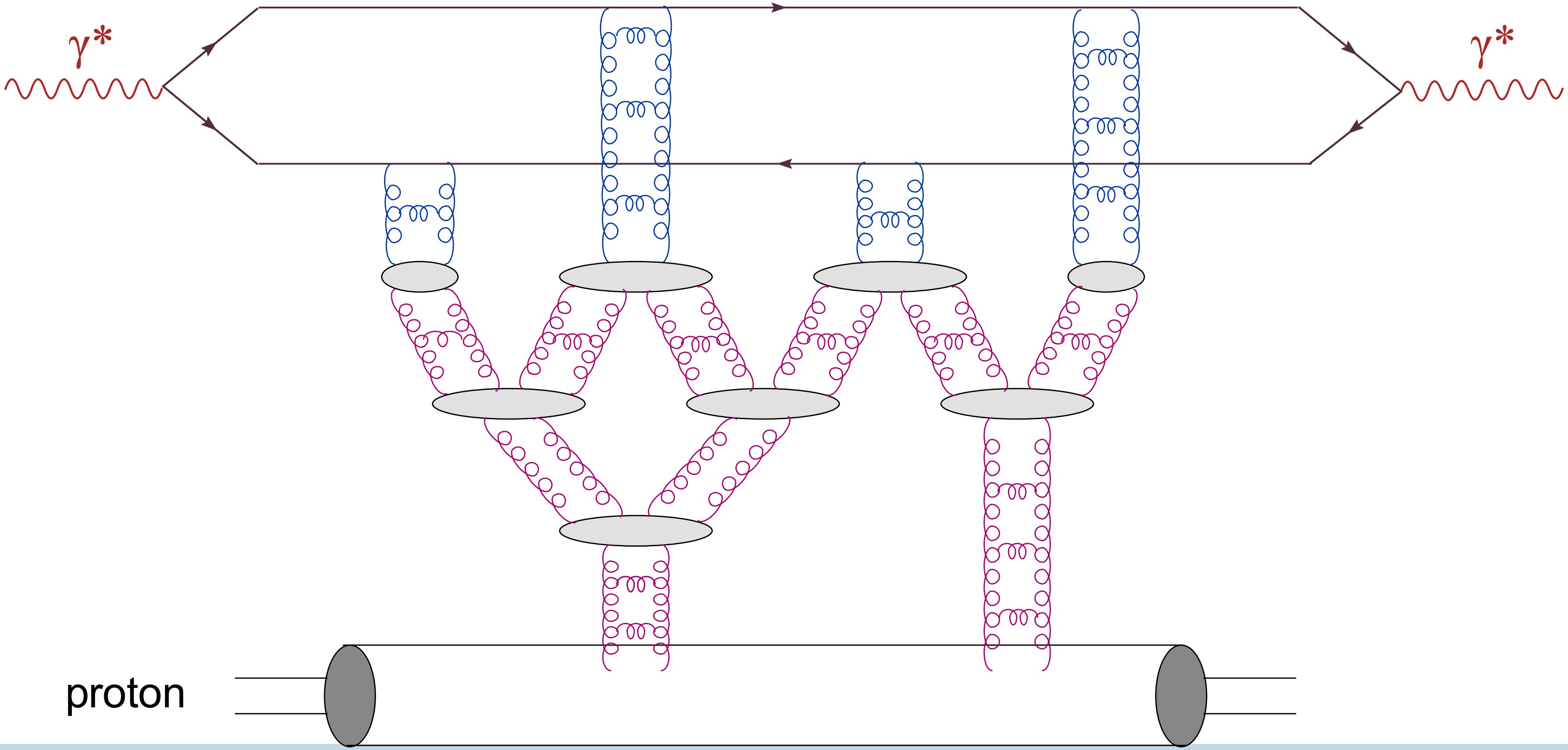


- 'Reaction–diffusion' $A \rightleftharpoons 2A$ *Munier, Peschanski (03)*

$$\partial_t n(x,t) = \underbrace{\partial_x^2 n(x,t)}_{\text{diffusion}} + \underbrace{\alpha\, n(x,t)}_{\text{growth}} \underbrace{- \beta\, n^2(x,t)}_{\text{recombination}}$$

DIS with Pomeron loops

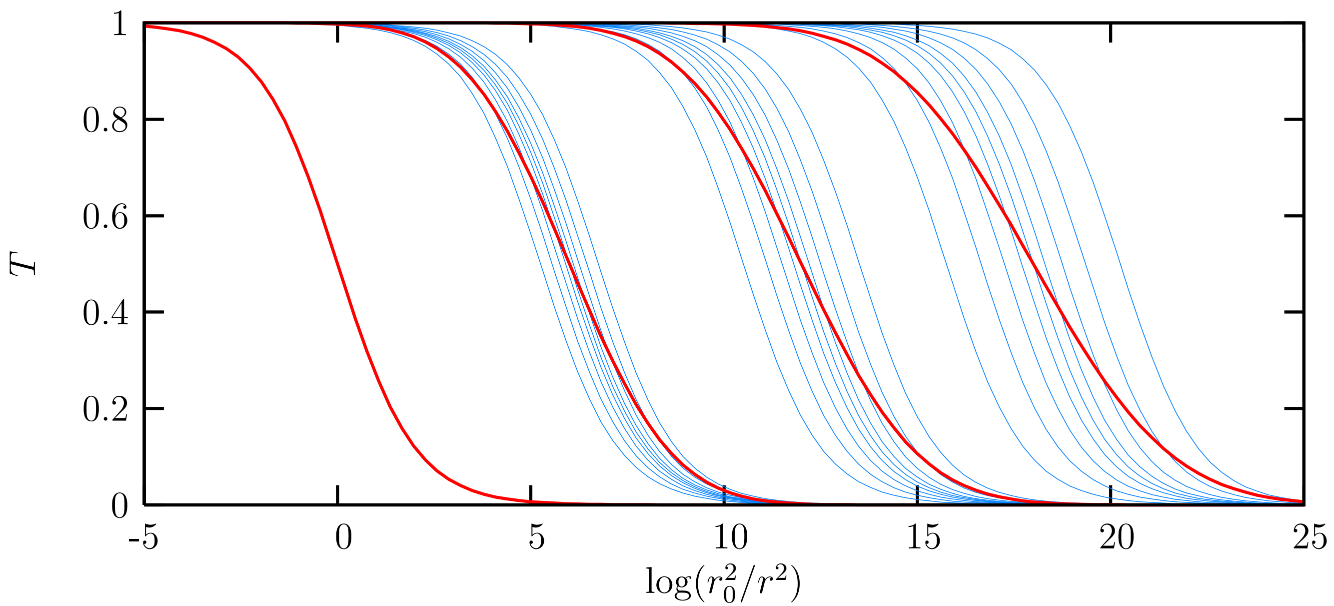
- Statistical physics: The effects of fluctuations are dramatic !
  (The front is pulled by the dynamics in its dilute tail)
- Important consequences for high–energy QCD
  (Mueller, Shoshi; E.I., Mueller, Munier; E.I., D. Triantafyllopoulos, 2004)



Front diffusion through fluctuations

- The stochastic evolution generates un ensemble of fronts which differ by their saturation momentum $\rho_s \equiv \ln Q_s^2$

$$\langle \rho_s(Y) \rangle = \lambda Y, \qquad \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = DY, \qquad D \sim \frac{1}{\ln^3(1/\alpha_s)}$$

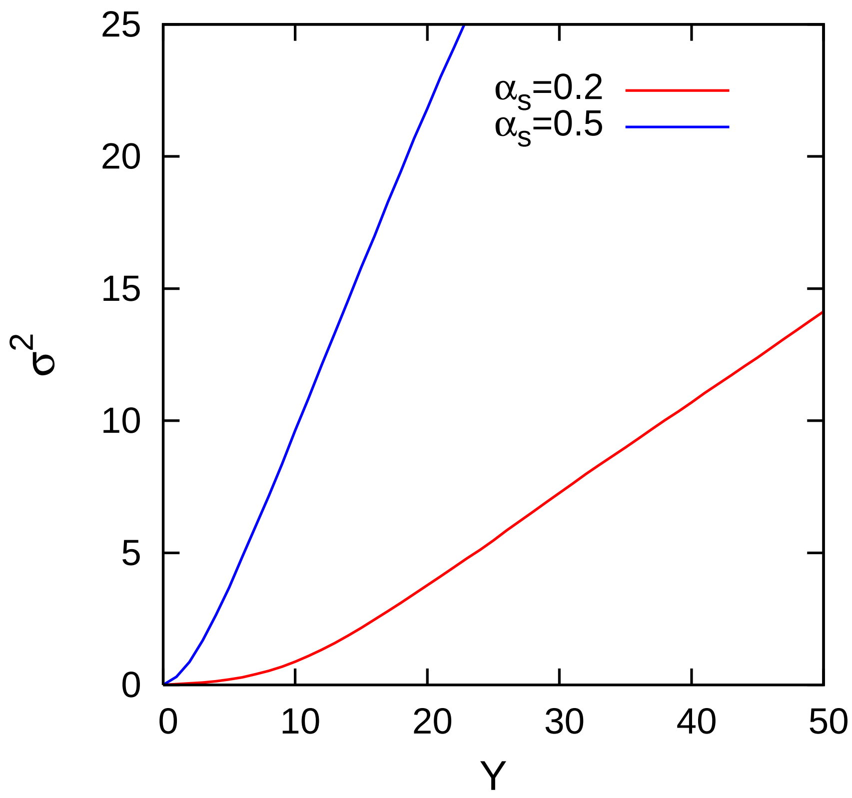


- With increasing energy, the fronts spread from each other

  $\Longrightarrow$ geometric scaling is progressively washed out !

Dispersion: Fixed coupling

- $\sigma^2(Y) \simeq D\bar{\alpha}_s Y$ with $D \sim \mathcal{O}(1)$



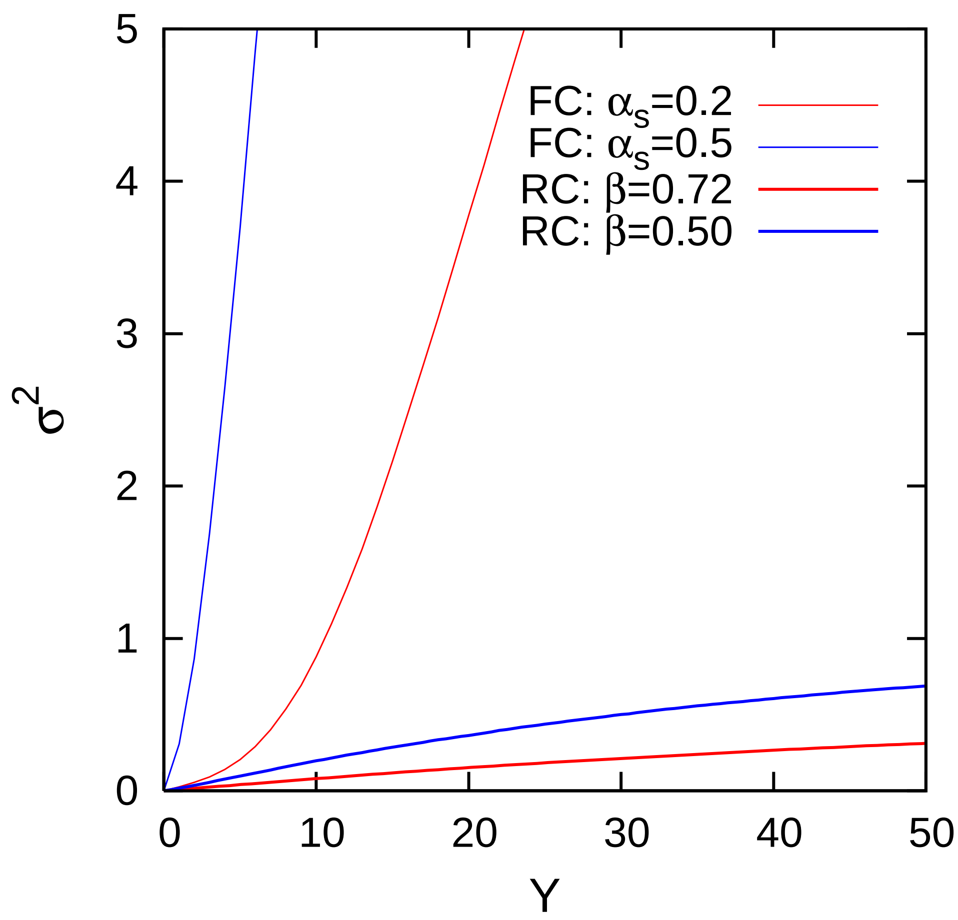
- Fluctuations effects are clearly important ...

$$\alpha_s = 0.5 \implies \sigma^2(Y) \simeq 10 \quad \text{for} \quad Y = 10$$

- $\sigma^2(Y) \gtrsim 1 \implies$ a totally new picture : ‘diffusive scaling’

Dispersion: Running coupling

- The dispersion keeps rising with Y ...

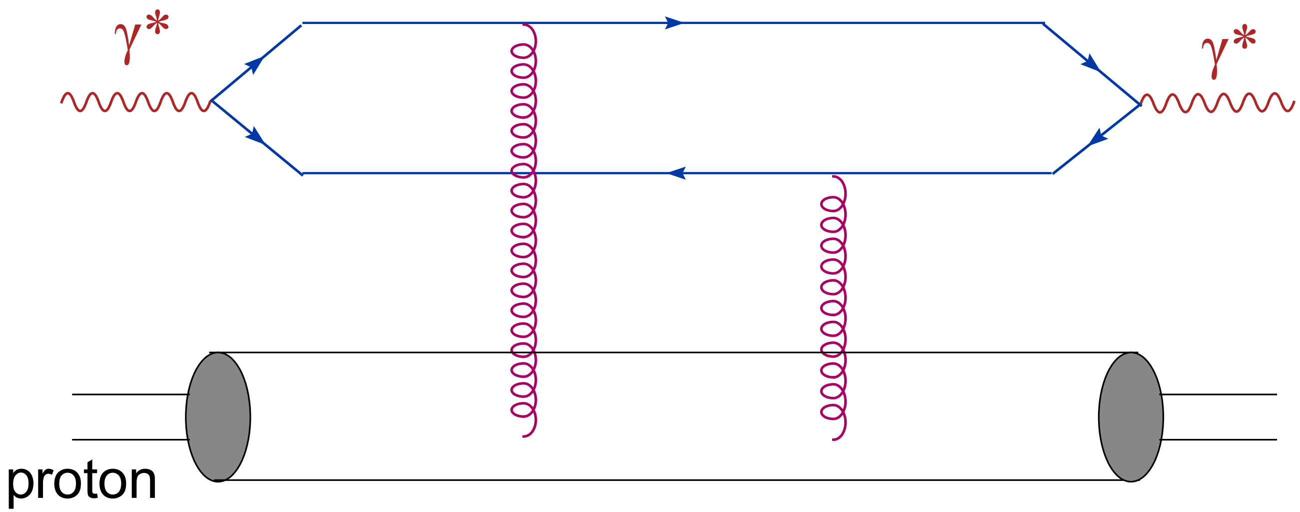


- ... but now it is tremendously smaller ! (by a factor ~ 100)
- No physical effect up to unrealistically large Y !

Single scattering: 2–gluon exchange

- The dipole scatters off the gluon field in the target

$$V(\boldsymbol{r}) \simeq g t^a \boldsymbol{r} \cdot \boldsymbol{E}_a \implies T(x, r, b) \propto g^2 r^2 \langle \boldsymbol{E}_a \cdot \boldsymbol{E}_a \rangle_x$$



$$T(x, r, b) \simeq \alpha_s\, r^2 \, \frac{xG(x, 1/r^2)}{\pi R^2} \equiv \alpha_s\, n(x, Q^2 \sim 1/r^2)$$

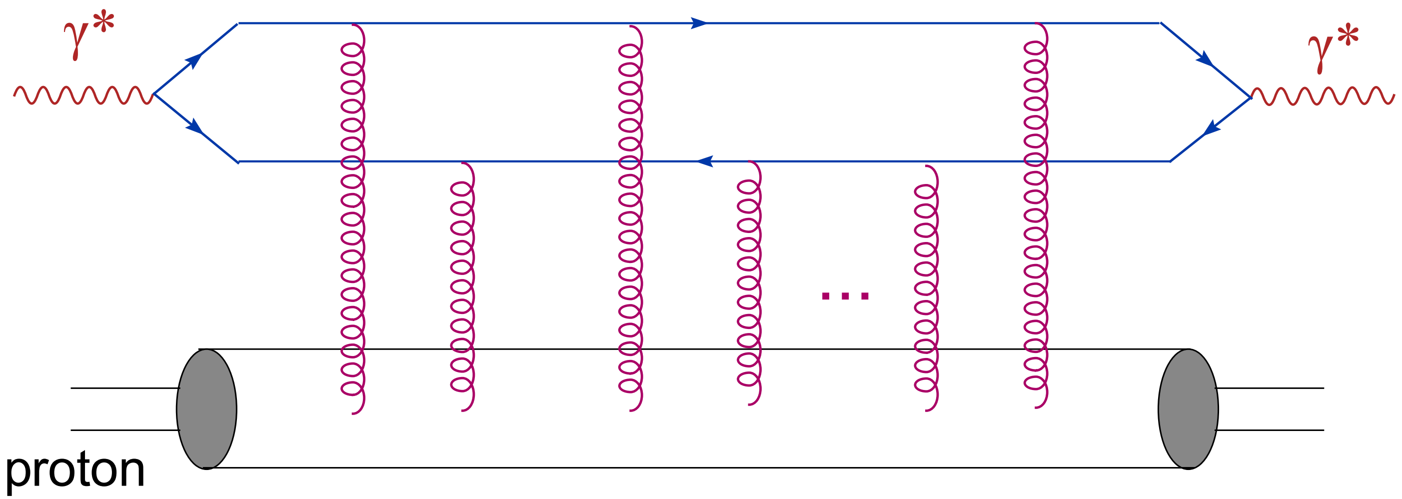
Weak scattering ($T \ll 1$) $\iff$ Low gluon occupation ($n \ll 1/\alpha_s$)

Multiple scattering: Unitarization

- When decreasing x at fixed r : $xG(x, 1/r^2) \sim 1/x^\lambda$

  $\Longrightarrow$ Unitarity is eventually violated ! $(T \gtrsim 1)$

- Multiple scattering becomes important and restores unitarity

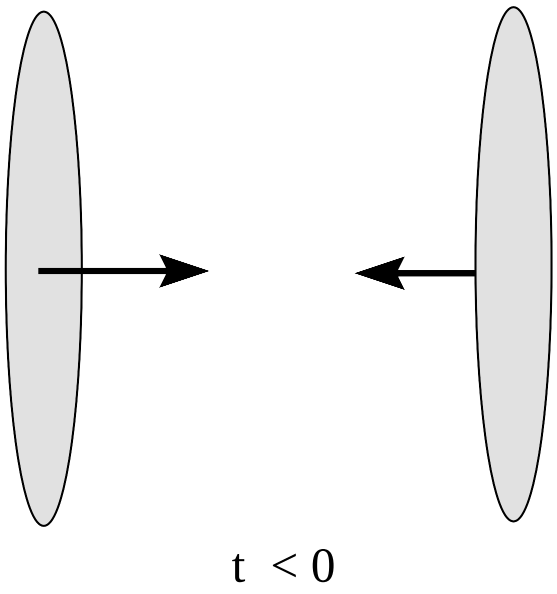


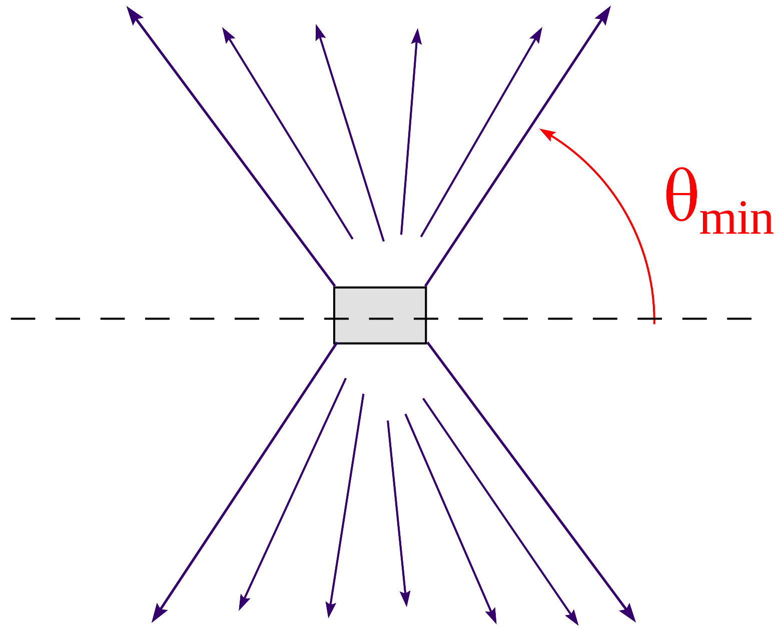
- Eikonal approximation + Incoherent scattering $\Longrightarrow$

$$T(x,r) \simeq 1 - \exp\left\{-\alpha_s\, r^2\, \frac{xG(x,1/r^2)}{\pi R^2}\right\}$$ ("Glauber–Mueller")

No forward jets !

- No large–x partons $\Longrightarrow$ no forward/backward jets in a hadron–hadron collision at strong coupling





- 'The Nightmare of CMS'

$$|\eta| \lesssim \eta_{\rm max}(Q) = \ln\frac{\sqrt{s}}{Q} - \ln\frac{1}{x_s(Q)}\,, \qquad x_s(Q) \sim \frac{\Lambda^2}{Q^2 N_c^2} \ll 1$$