A Model for Soft Interactions based on the CGC/Saturation Approach

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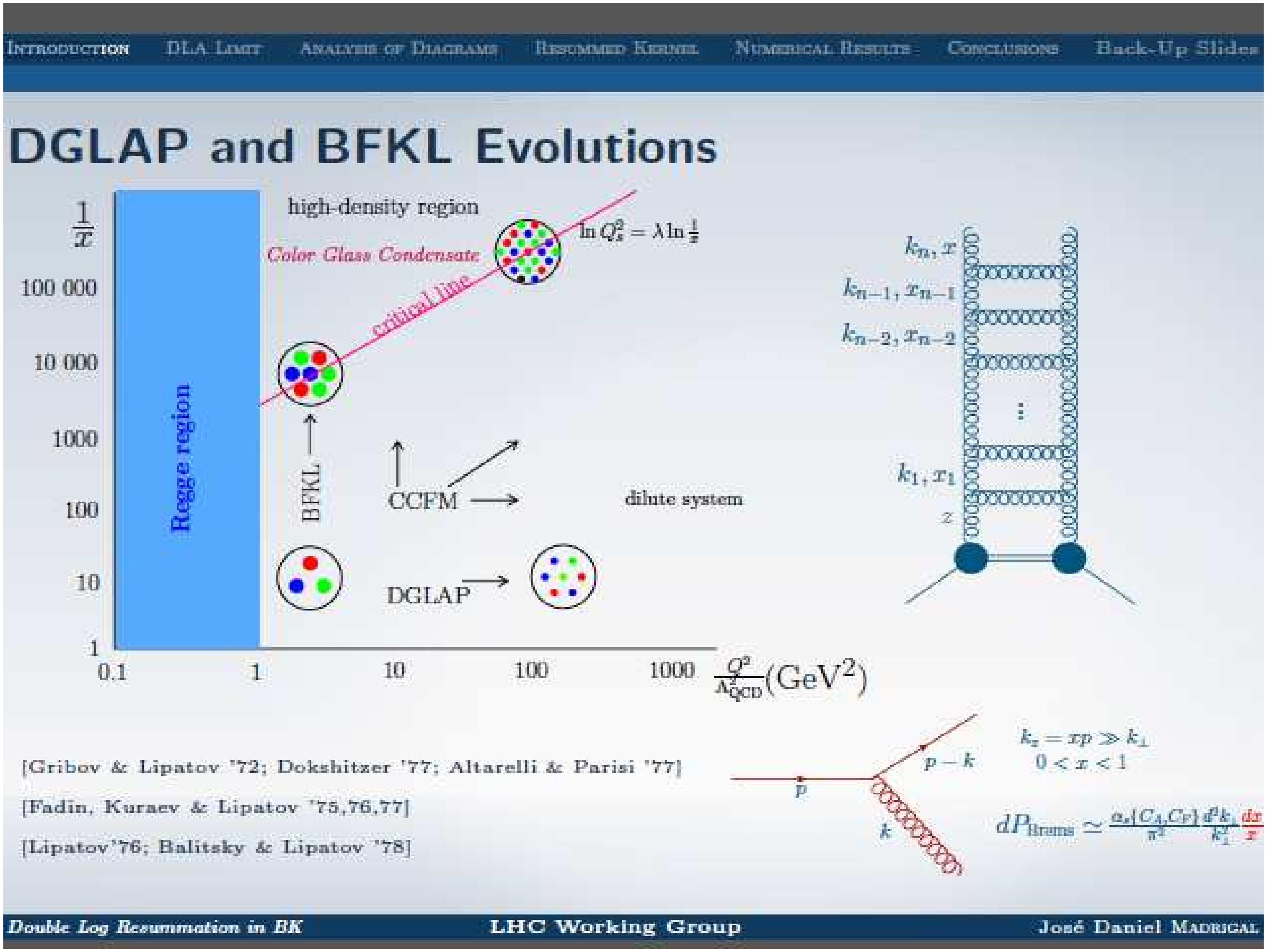
(work done with Genya Levin and Uri Maor)

- Develop a model for SOFT interactions at high energy based on the BFKL Pomeron and the CGC/saturation approach
- Intially, constucted a ONE CHANNEL eikonal type model whose opacity is determined by the exchange of a dressed Pomeron
- Green's function for the Pomeron was calculated in framework of the CGC/saturation approach - replacing Pomeron Calculus
- Our results compared to experimental data were found wanting (shadowing corrections too large)
- To overcome this, reformulated the problem in a TWO channel framework using the B-K equation for calculating the single diffractive scattering

Guide to the Various Regions



Scattering near the Unitarity Limit

1. In the Regge limit of pQCD, when $s \gg \Lambda_{hard}$, as the energy increases the parton density becomes more dense, and the scattering amplitude A(s,t) grows.

2. As long as densities are NOT TOO HIGH, growth is described by BFKL evolution equation.

3. Density becomes higher as A(s,t) $\rightarrow$ 1, and one enters a regime called SATURATION, where the BFKL evolution FAILS.

4. NON LINEARITIES lead to SATURATION + UNITARIZATION of A(s,t).

5. Balitsky-Kovchegov equation is the simplest and most accurate way to describe the saturation regime of QCD. It is non-linear and resums QCD fan diagrams in the LLA.

Our approach to describe Pomeron interactions based on CGC/saturation

[*Eur. Phys. J.* **C75**, 18 (2015)]

Following the treatment of Levin JHEP 1311, 093 (2013)

We would like to find a solution to the B-K non-linear equation:

The solution has different forms in the three kinematic regions:

1. $r^2 Q_s^2(Y, b) \ll 1$, where Q_s denotes the saturation scale. The non-linear corrections are small, and the solution is the BFKL Pomeron, which has the following form:

$$G_{I\!P}(Y, r, R; b) = (w\, w^*)^{\frac{1}{2}} \sqrt{\frac{\pi}{4\, D\, Y}}\, e^{\Delta_{\text{BFKL}} Y - \frac{\ln^2 w\, w^*}{4\, D\, Y}}$$

with $\Delta_{\text{BFKL}} = 2 \ln 2\, \bar{\alpha}_S$ and $D = 14\zeta(3)\bar{\alpha}_S = 16.828\, \bar{\alpha}_S$.

$Y = \ln s$, where $s = W^2$, W denotes the energy of the interaction.
$\bar{\alpha}_S = \frac{\alpha_S N_C}{\pi}$ is the QCD coupling, r and R are the sizes of two interacting dipoles, and b is the impact parameter for the scattering amplitude of two dipoles.

$$w\, w^* = \frac{r^2\, R^2}{\left(\vec{b} - \frac{1}{2}(\vec{r} - \vec{R})\right)^2 \left(\vec{b} + \frac{1}{2}(\vec{r} - \vec{R})\right)^2}.$$

$G_{I\!P}(Y, r, R; b)$ which denotes the BFKL Pomeron Green's function is NOT a pole in angular momentum, but a branch cut.

Our approach to describe Pomeron interactions based on CGC/saturation (continued)

2. $r^2 Q_s^2(Y,b) \sim 1$ (vicinity of the saturation scale). The scattering amplitude has the following form:

$$A \equiv G_{I\!P}(z) = \mathsf{C}\left(r^2 Q_s^2(Y,b)\right)^{1-\gamma_{cr}},$$

where C denotes a constant, and the critical anomalous dimension γ_{cr}, can be found from $\frac{\chi(\gamma_{cr})}{1-\gamma_{cr}} = -\frac{d\chi(\gamma_{cr})}{d\gamma_{cr}} \;(= \kappa)$

The BFKL kernel $\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$ where $\psi(x) = \frac{d \ln \Gamma(x)}{dx}$

3. $r^2 Q_s^2(Y,b) > 1$ (inside the saturation domain).

$$A = 1 - \text{Const}\exp\left(-\frac{z^2}{2\kappa}\right)$$

The Saturation domain is $z > 0\,(r^2 Q_s^2 \geq 1)$, while pQCD domain is $z < 0\,(r^2 Q_s^2 \leq 1)$

4. For a simplified BFKL kernel (Levin JHEP 1311, 039 (2013), found an approximation to the numerical solution of the BK equation:

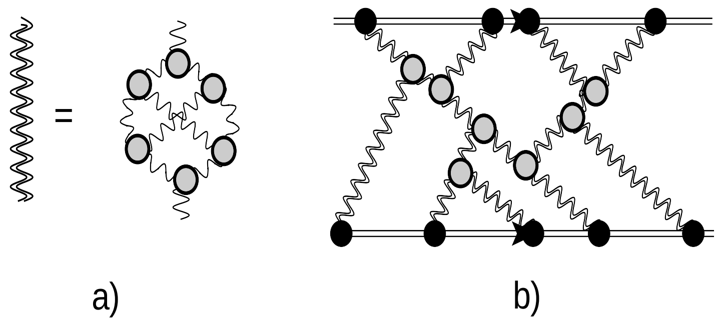
$$N^{BK}(G_{I\!P}(z)) = a\,(1-\exp(-G_{I\!P}(z))) + (1-a)\,\frac{G_{I\!P}(z)}{1+G_{I\!P}(z)},$$

with $a = 0.65$ and $z = \ln\left(r^2 Q_s^2(Y,b)\right)$ and $r = R$.

5. Altinoluk et al JHEP 1404, 075 (2014) have proved the equivalence of the CGC/saturation approach and the BFKL Pomeron calculus for a wide range of rapidities

$$Y \leq \frac{2}{\Delta_{\mathsf{BFKL}}}\ln\left(\frac{1}{\Delta^2_{\mathsf{BFKL}}}\right).$$

Dressed Pomeron in MPSI approximation



a) Dressed Pomeron in MPSI approximation
and
b) Sum of net diagrams
Wavy lines describe BFKL Pomerons.
The grey blobs stand for triple Pomeron vertices,
while black blobs show the hadron-Pomeron vertex g(b).

Since the typical rapidity is $O(Y - Y_i) \approx \frac{1}{\Delta_{BFKL}}$, only large Pomeron loops with rapidity $O(Y)$ contribute at high energies $\longrightarrow$ can sum such loops using MPSI approximation.

Dressed Pomeron in MPSI approximation (continued)

The resulting Green's function of the Dressed Pomeron is given by:

$$G_{I\!P}^{\text{dressed}}\left(Y-Y_0,r,R,b\right) \; = \;$$

$$a^2\left\{1 \; - \; \exp\left(-T\left(Y-Y_0,r,R,b\right)\right) \; \right\} + \; 2a(1-a)\frac{T\left(Y-Y_0,r,R,b\right)}{1 \; + \; T\left(Y-Y_0,r,R,b\right)}$$

$$+ \; (1-a)^2\left\{1-\exp\left(\frac{1}{T\left(Y-Y_0,r,R,b\right)}\right)\frac{1}{T\left(Y-Y_0,r,R,b\right)}\Gamma\left(0,\frac{1}{T\left(Y-Y_0,r,R,b\right)}\right)\right\}$$

$$with \;\; T\left(Y-Y_0,r,R,b\right) \; = \; \frac{\bar{\alpha}_S^2}{4\pi}G_{I\!P}\left(z\rightarrow 0\right) \; = \; \phi_0\left(r^2Q_s^2\left(R,Y,b\right)\right)^{1-\gamma_{cr}}$$

$$= \; \phi_0 S\left(b\right)e^{\lambda(1-\gamma_{cr})Y}$$

Parameters of the Model

Parameters associated with $T\left(Y-Y_0,r,R,b\right) \;=\; \phi_0 S\left(b\right) e^{\lambda(1-\gamma_{cr})Y}$
We used two inputs: $r=R$ and $Q^2_s \;=\; \left(1/(m^2R^2)\right)\; S\left(b\right)\,\exp\left(\lambda\,Y\right)$.
The parameter $\lambda \;=\; \bar{\alpha}_S\chi\left(\gamma_{cr}\right)/(1-\gamma_{cr})$ in leading order of perturbative QCD
($\lambda = 0.2$ to 0.3)

The phenomenological profile function
$S\left(b\right) \;=\; \frac{m^2}{2\pi}\,e^{-m\,b}$ with normalization $\int d^2b\,S\left(b\right) \;=\; 1$

The parameter m represents the inverse size of the dipole $m\sim 1/r = 1/R$
We introduce two constants: g and m_1, to describe the vertex of the hadron-Pomeron interaction

$$g\left(b\right) \;=\; g\,S_{I\!P}\left(b\right) \text{ with } S_{I\!P}\left(b\right) \;=\; \frac{m_1^3\,b}{4\pi}\,K_1\left(m_1 b\right),\; S_{I\!P}\left(b\right) \xrightarrow{\text{Fourier image}} \left(\frac{m_1^2}{q^2+m_1^2}\right)^2$$

$$\Omega\left(Y;b\right) \;=\; \int d^2b'\;\frac{g\left(\vec{b}'\right)\,g\left(\vec{b}-\vec{b}'\right)\,\bar{G}^{\text{dressed}}_{I\!P}\left(Y\right)}{1\,+\,1.29\,\bar{G}^{\text{dressed}}_{I\!P}\left(Y\right)\left[g\left(\vec{b}'\right)+g\left(\vec{b}-\vec{b}'\right)\right]},$$

where $\bar{G}^{\text{dressed}}_{I\!P}\left(Y\right) \;=\; \int d^2\,b''\,G^{\text{dressed}}_{I\!P}\left(Y;b''\right)$.

The elastic amplitude $A_{el} \;=\; i(1-e^{-\Omega(Y;b)})$

Two channel model based on CGC/saturation and BFKL Pomeron

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Main ingredients:

- Include contribution of low mass sector for diffractive processes
- CGC/saturation effective theory for high energy QCD
- BFKL Pomeron that describes both hard AND soft interactions at high energy
- Shadowing corrections from eikonal rescattering and interaction of BFKL Pomeron
- Include a new formula for single diffraction based on Balitsky-Kovchegov equation

Basic formalism for Two Channel Model

Following Good-Walker the observed physical hadronic and diffractive states are written

$$\psi_h \;=\; \alpha\,\Psi_1 + \beta\,\Psi_2\,; \qquad \psi_D \;=\; -\beta\,\Psi_1 + \alpha\,\Psi_2; \qquad \text{where} \qquad \alpha^2 + \beta^2 \;=\; 1.$$

Functions ψ_1 and ψ_2 form a complete set of orthogonal functions $\{\psi_i\}$ which diagonalize the interaction matrix $\mathbf{T}$

$$A^{i'k'}_{i,k} = <\psi_i\,\psi_k|\mathbf{T}|\psi_{i'}\,\psi_{k'}> = A_{i,k}\,\delta_{i,i'}\,\delta_{k,k'}.$$

The unitarity constraints can be written as

$$2\,\mathrm{Im}\,A_{i,k}\,(s,b) = |A_{i,k}\,(s,b)\,|^2 + G^{in}_{i,k}(s,b)$$

At high energies a simple solution to this equation is

$$A_{i,k}(s,b) = i\left(1 - \exp\left(-\frac{\Omega_{i,k}(s,b)}{2}\right)\right)$$

$$G^{in}_{i,k}(s,b) = 1 - \exp\left(-\Omega_{i,k}(s,b)\right).$$

$G^{in}_{i,k}(s,b)$ denotes the contribution of all non-diffractive inelastic processes

Physical Observables for Elastic, and Low Mass Diffraction

elastic amplitude : $a_{el}(s,) \;=\; i\left(\alpha^4 A_{1,1} \;+\; 2\alpha^2\beta^2 A_{1,2} \;+\; \beta^4 A_{2,2}\right);$

elastic observables : $\sigma_{tot} \;=\; 2\int d^2b\, a_{el}\,(s,b)\,; \quad \sigma_{el} \;=\; \int d^2b\,|a_{el}\,(s,b)\,|^2;$

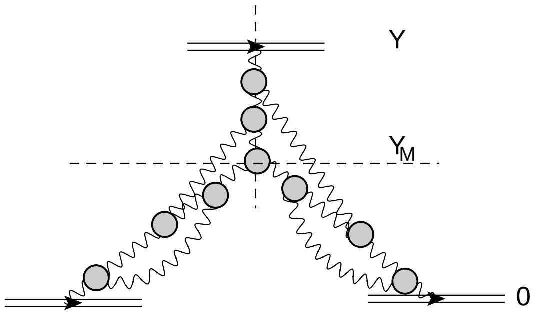
optical theorem : $2\,\mathrm{Im}A_{i,k}(s,t=0) \;=\; 2\int d^2b\,\mathrm{Im}A_{i,k}(s,b) \;=\; \sigma_{el}+\sigma_{in} \;=\; \sigma_{tot}$

single diffraction : $\sigma^{GW}_{sd} \;=\; \int d^2b\,\left(\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2-\beta^2)A_{1,2} + \beta^2 A_{2,2}\}\right)^2;$

double diffraction : $\sigma^{GW}_{dd} \;=\; \int d^2b\;\alpha^4\beta^4\,\{A_{1,1} \;-\; 2\,A_{1,2} \;+\; A_{2,2}\}^2\,.$

'GW' denotes the Good -Walker component, that is responsible for diffraction in the small mass region.

Diffractive Scattering based on the Levin-Kovchegov Equation



MPSI approximation: the simplest diagram for single diffraction production.
The wavy lines describe BFKL Pomerons. The blobs stand for triple Pomeron vertices.
The dashed line denotes the cut Pomeron. $Y_M = \ln\left(M^2/s_0\right)$, where M is the mass of produced particles and s_0 is the scale taken to be of the order of $1\,GeV^2$.

L-K equation has same form as the B-K equation (Nucl.Phys. B577 (2000) 221) for the function

$$G\left(Y, Y_0, r, b\right) \;=\; 2\,N\left(Y, Y_0, r, b\right) \;-\; N_{SD}\left(Y, Y_0, r, b\right),$$

$N\left(Y, Y_0, r, b\right)$ is the imaginary part of the elastic amplitude

The cross section for diffraction production is:

$$\sigma_{diff}\left(Y, Y_0, r\right) \;=\; \int d^2b\, N_{SD}\left(Y, Y_0, r, b\right)$$

Single Diffractive Scattering in the region of Large Mass

The large Mass contribution for single diffraction is:

$$\sigma_{sd}^{\text{large mass}} = 2 \int d^2 b$$

$$\Big\{ \alpha^6 A^{sd}_{1;1,1}\, e^{-2\,\Omega^D_{1,1}(Y;b)} + \alpha^2 \beta^4 A^{sd}_{1;2,2}\, e^{-2\,\Omega^D_{1,2}(Y;b)} + 2\,\alpha^4 \beta^2 A^{sd}_{1;1,2}\, e^{-\left(\Omega^D_{1,1}(Y;b)+\Omega^D_{1,2}(Y;b)\right)}$$

$$+ \beta^2 \alpha^4 A^{sd}_{2;1,1}\, e^{-2\,\Omega^D_{1,2}(Y;b)} + 2\,\beta^4 \alpha^2 A^{sd}_{2;1,2}\, e^{-\left(\Omega^D_{1,2}(Y;b)+\Omega^D_{2,2}(Y;b)\right)} + \beta^6 A^{sd}_{2;2,2}\, e^{-2\,\Omega_{2,2}(Y;b)} \Big\}$$

$$\Omega^D_{i,k}(Y;b) = \int d^2 b' \, \frac{g_i\left(\vec{b}'\right) g_k\left(\vec{b}-\vec{b}'\right) \bar{G}^{\,\text{dressed}}(T)}{\left(1 + 1.29\,\bar{G}^{\,\text{dressed}}(T)\left[g_i\left(\vec{b}'\right) + g_k\left(\vec{b}-\vec{b}'\right)\right]\right)^2}$$

$$A^{sd}_{i;k,l}(Y, Y_{max}, Y_{min}; b) = \int d^2 b' \, \sigma_{diff}(Y, Y_{max}, Y_{min}, 1/m)\,.$$

$$g_i g_k g_l S_{I\!P}(b', m_i)\; S_{I\!P}\left(\vec{b}-\vec{b}', m_k\right)\; S_{I\!P}\left(\vec{b}-\vec{b}', m_l\right),$$

and $S_{I\!P}(b', m_i) = \frac{1}{4\pi} m_i^3\, b'\, K_1(b', m_i)$

Double Diffractive Scattering in the region of Large Mass

Unitarity constraints for the dressed Pomeron takes the form:

$$2\,G^{\text{dressed}}\left(T\left(Y,b\right)\right) \;=\; G^{\text{dressed}}\left(2\,T\left(Y;b\right)\right) \;+\; N_{DD}\left(Y;b\right)$$

where $G^{\text{dressed}}\left(2\,T\left(Y;b\right)\right)$ describes all inelastic processes that are generated by the exchange of the dressed Pomeron.

$$\sigma_{dd} \;=\; \int d^2b\left\{2G^{\text{dressed}}\left(T\left(Y,b\right)\right) \;-\; G^{\text{dressed}}\left(2\,T\left(Y;b\right)\right)\right\}$$

For the double diffraction production at large mass we have

$$\sigma_{dd}^{\text{large mass}} \;=\; \int d^2b\,\left\{\alpha^4\,A^{dd}_{1,1}\,e^{-2\Omega^D_{1,1}(Y;b)} \;+\; 2\alpha^2\,\beta^2\,A^{dd}_{1,2}\,e^{-2\Omega^D_{1,2}(Y;b)} \;+\; \beta^4\,A^{dd}_{2,2}\,e^{-2\Omega^D_{2,2}(Y;b)}\right\}.$$

where

$$A^{dd}_{i,k} \;=\; \int d^2\,b\,g_i\,g_k\,S^{i,k}_{DD}\left(b\right)\sigma_{dd}\left(Y\right)$$

$$S^{i,k}_{DD}\left(b\right) \;=\; \int d^2b'\,S_{I\!P}\left(b',m_i\right)\;S_{I\!P}\left(\vec{b}-\vec{b}',m_k\right)$$

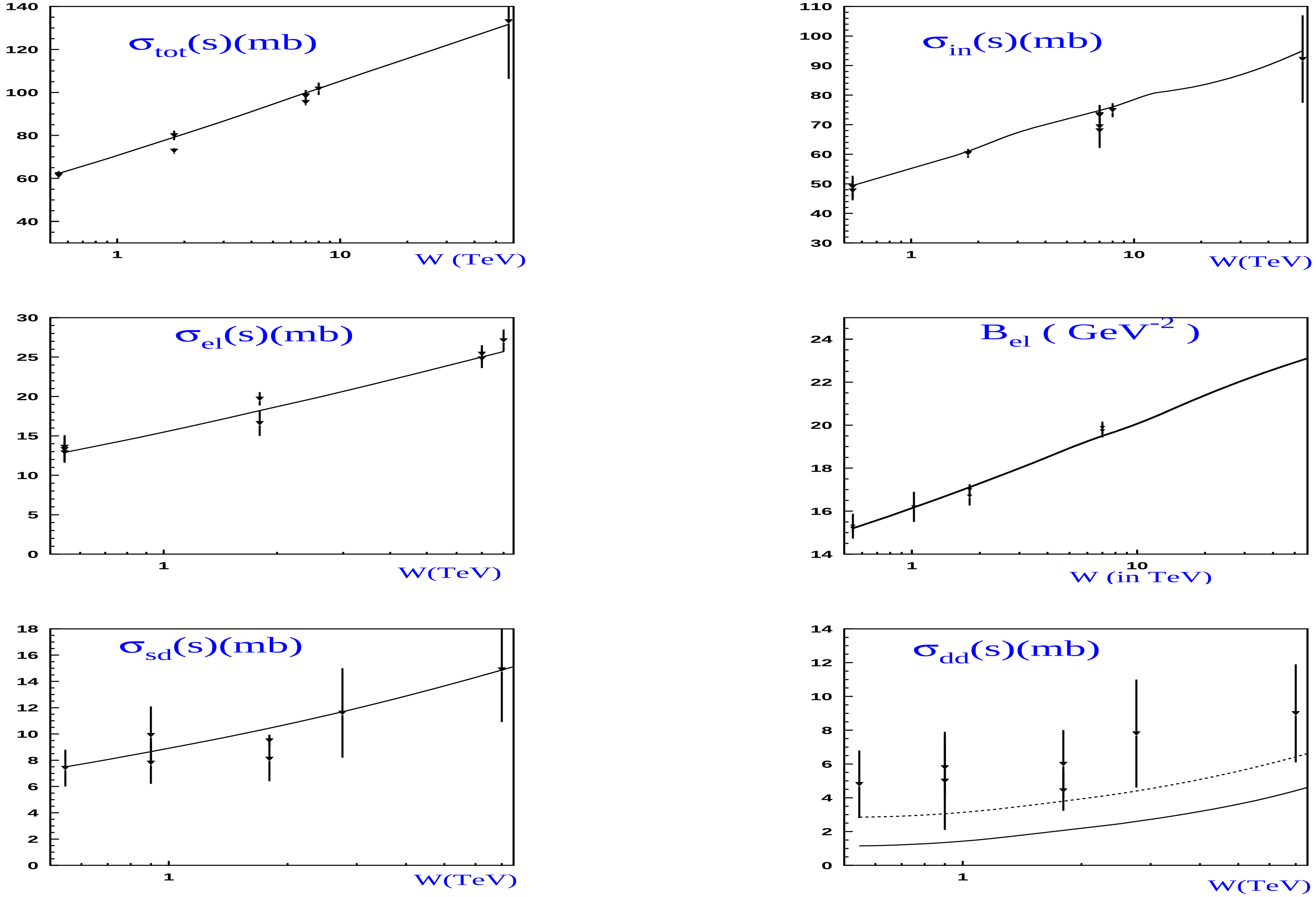
and $S_{I\!P}\left(b',m_i\right) \;=\; \frac{1}{4\pi}\,m_i^3\,b'\,K_1\left(b',m_i\right)$

Parameters and Predictions for Two channel Model

model	λ	ϕ_0	$g_1\ (GeV^{-1})$	$g_2\ (GeV^{-1})$	$m(GeV)$	$m_1(GeV)$	$m_2(GeV)$
2 channel	0.38	0.0019	110.2	11.2	5.25	0.92	1.9
1 channel	0.323	0.019	25.7	n/a	6.35	0.813	n/a

W (TeV)	σ_{tot} (mb)	σ_{el}(mb) (mb)	B_{el} (GeV^{-2})	single σ_{sd}^{LM} (mb)	diffraction σ_{sd}^{HM} (mb)	double σ_{dd}^{LM} (mb)	diffraction σ_{dd}^{HM} (mb)
0.546	62.3	12.9	15.2	5.64	1.85	0.7	0.46
0.9	69.2	15	16	6.25	2.39	0.77	0.67
1.8	79.2	18.2	17.1	7.1	3.35	0.89	1.17
2.74	85.5	20.2	17.8	7.6	4.07	0.97	1.62
7	99.8	25	19.5	8.7	6.2	1.15	3.27
8	101.8	25.7	19.7	8.82	6.55	1.17	3.63
13	109.3	28.3	20.6	9.36	8.08	1.27	5.11
14	110.5	28.7	20.7	9.44	8.34	1.27	5.4
57	131.7	36.2	23.1	10.85	15.02	1.56	13.7

Results for Two channel Model



Conclusions

- Constructed a model based on the BFKL Pomeron and the CGC/saturation approach, which successfully describes data in the Regge region, for high energy hadron scattering.
- Do not require that the soft Pomeron to appear as a Regge pole.
- Suggest a procedure where the matching with long distance physics (where confinement of quarks and gluons is essential) can be reached within the CGC/saturation approach.
- Model also successfully describes:
  Inclusive production [Physics Lett.B746 (2015) 154]
  Long range rapidity correlations [arXiv: 1508.04236]

Diffractive Scattering for Two channel model

The initial condition, for $G\left(Y, Y_0, r, b\right)$ is given by the following equation:

$$G\left(Y, Y_0\;=\;Y, r, b\right)\;=\;2\,N\left(Y, r, b\right)\;-\;N^2\left(Y, r, b\right)$$

The cross section for the production of a bunch of hadrons with a mass from M_{min} to M_{max} can be written as

$$\sigma_{diff}\left(Y, Y_{max}, Y_{min}, r\right)\;=\;\int d^2b\,\tilde{N}_{SD}\left(Y, Y_{max}, Y_{min}, r; b\right)$$

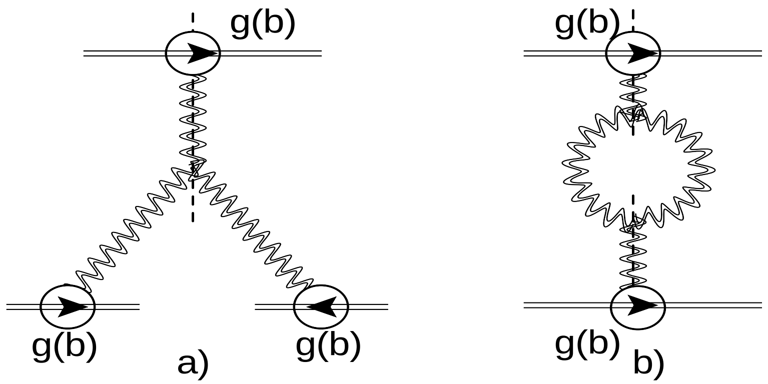
$$\tilde{N}_{SD}\left(Y, Y_{max}, Y_{min}, r; b\right)\;=\;N\left(TT\left(Y, Y_{max}, b\right)\right)\;-\;N\left(TT\left(Y, Y_{min}, b\right)\right)$$

where $Y_{max} = \ln\left(M^2_{max}/s_0\right)$ and $Y_{min} = \ln\left(M^2_{min}/s_0\right)$.

and

$$TT\left(Y, Y_M, b\right)\;=\;\left(2\,G^{\,\text{dressed}}\left(T\left(Y - Y_M, b\right)\right)\;-\;\left(G^{\,\text{dressed}}\left(T\left(Y - Y_M, b\right)\right)\right)^2\right).e^{(1-\gamma_{cr})\,\lambda\left(Y_M\right)}.$$

Diffractive Production in the one channel CGC/saturation Model



a) Single and b) Double diffraction production.

The double wavy lines describe dressed Pomerons. The double wavy line crossed by the dashed one stands for the dressed Pomeron structure, in terms of produced particles.

The blobs stand for the hadron-Pomeron interaction $g\,(b)$ vertices.

The cross section for single diffraction can be written as

$$\sigma_{SD}\,(Y)\;=\;2\int d^2b\frac{d\sigma_{SD}}{db^2}\;=\;\int d^2bg^3S_{SD}\,(b)\,\bar{N}_{SD}\,(Y)\;e^{-2\,\Omega(Y,b)},$$

The full expressions for $\bar{N}_{SD}\,(Y)$ and $S_{SD}(b)$ are given in [E.Levin, JHEP 1311, 039 (2013)]

Double Diffractive in the one channel CGC/saturation Model

The double diffraction cross section has the form

$$\sigma_{DD}(Y) \;=\; \int d^2 b \frac{d\sigma_{DD}}{db^2} \;=\; \int d^2\, b g^2\, S_{DD}(b)\, \bar{N}_{DD}(Y)\; e^{-2\,\Omega(Y,b)}$$

$$\bar{N}_{DD}(Y) \;=\; \int d^2 b'\, N_{DD}\left(Y, b'\right)$$

and $N_{DD}\left(Y, b'\right)$ can be determined from the simple expression

$$N_{DD}\left(Y, b'\right) \;=\; 2\, G^{\text{dressed}}_{I\!P}(T) \;-\; G^{\text{dressed}}_{I\!P}(2\,T)\,,$$

The profile $S_{DD}(b)$ is given by

$$S_{DD}(b) \;=\; \int d^2 b' S_{I\!P}\left(\vec{b} - \vec{b}'\right)\, S_{I\!P}\left(b'\right) \;=\; \frac{m_1^5 b^3}{96\pi} K_3\left(m_1 b\right).$$

The values for the parameters were determined by fitting all available data for $\sigma_{tot}, \sigma_{el}, \sigma_{sd}, \sigma_{dd}$ and B_{el} for $0.546\, TeV \leq W \leq 8\, TeV$.

λ	ϕ_0	g (GeV^{-1})	m (GeV)	$m_1(GeV)$	$\chi^2/d.o.f.$
0.323	0.019	25.7	6.35	0.813	1.98

Results of One-channel CGC/saturation Model