International Symposium on Multiparticle Dynamics, ISMD 2015

Diffractive phenomena & Pomeron physics @ LHC

László L. Jenkovszky
jenk@bitp.kiev.ua
\[ \sigma_t(s) = \frac{4\pi}{s} \text{Im} A(s, t = 0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2; \quad n(s); \]

\[ \sigma_{el} = \int_{t_{\text{thr.}}}^{t_{\text{min}}} \frac{d\sigma}{dt} \, dt; \quad \sigma_{\text{in}} = \sigma_t - \sigma_{el}; \quad B(s, t) = \frac{d}{dt} \ln \left( \frac{d\sigma}{dt} \right); \]

\[ A_{pp}^p(s, t) = P(s, t) \pm O(s, t) \pm f(s, t) \pm \omega(s, t) \rightarrow_{\text{LHC}} \approx P(s, t) \pm O(s, t), \]

where \( P, \; O, \; f, \; \omega \) are the Pomeron, odderon and non-leading Reggeon contributions.

<table>
<thead>
<tr>
<th>( \alpha(0) \backslash C )</th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>O</td>
</tr>
<tr>
<td>1/2</td>
<td>f</td>
<td>( \omega )</td>
</tr>
</tbody>
</table>

\textit{NB: The S-matrix theory (including Regge pole) is applicable to asymptotically free states only (not to quarks and gluons)!}
Total Cross-Section

Inversely:

\[ \sigma_{tot} \propto (\log s)^\gamma \]

\[ \sigma_{tot}(\text{LHC}) \sim 110 \text{ mb} \quad (\gamma=2; \text{ best-fit}) \]

\[ \sigma_{tot}(\text{LHC}) \sim 95 \text{ mb} \quad (\gamma=1) \]

Luminosity-independent measurement via optical-theorem \( \rightarrow \) simultaneous evaluation of forward elastic and inelastic rate (TOTEM)

- elastic rate down to \(|t|=10^{-3}\) GeV\(^2\) to keep extrapolation error small (1-2%)
- Sufficient \( \eta \) coverage to access Nel+Ninel

\[ \sigma_{tot} = \frac{16\pi \left( \frac{dN}{dt} \right)_{t=0}}{1 + \rho^2} \times \frac{N_{el} + N_{inel}}{L} \]

\[ L\sigma_{tot} = N_{elastic} + N_{inelastic} \]

\[ (\sigma_{tot} + \frac{dN}{dt}|_{t=0}) \quad (\Delta L/L > \sim 2 \Delta \sigma_{tot}/\sigma_{tot}) \]

\[ (L + \frac{dN}{dt}|_{t=0}) \quad (\Delta \sigma_{tot}/\sigma_{tot} > \sim \frac{1}{2} \Delta L/L) \]

\[ L\sigma_{tot}^2 = \frac{16\pi}{1 + \rho^2} \times \frac{dN}{dt}|_{t=0} \]
Elastic Scattering

\[ \sqrt{s} = 14 \text{ TeV prediction of BSW model} \]

\[ \rho = \frac{\text{Re}(f_{el}(t))}{\text{Im}(f_{el}(t))} \bigg|_{t \to 0} \]

- Momentum transfer \(-t \sim (p\theta)^2\)
- \(\theta = \text{beam scattering angle}\)
- \(p = \text{beam momentum}\)

\[ d\sigma/dt \text{ (mb/GeV}^2) \]

\[ \frac{dN}{dt} \bigg|_{t=CNI} = L\pi|f_C + f_N|^2 \approx L\pi \left| \frac{2\alpha_{EM}}{t} + \frac{\sigma_{tot}}{4\pi} (i + \rho)e^{-\frac{b|t|}{2}} \right|^2 \]

CNI region: \(|f_C| \sim |f_N| \to \text{ @ LHC: } -t \sim 6.5 \times 10^{-4} \text{ GeV}^2; \ \theta_{\text{min}} \sim 3.4 \ \mu\text{rad} \)

(\(\theta_{\text{min}} \sim 120 \ \mu\text{rad @ SPS}\)
CERN LHC, TOTEM Collab., June 26, 2011:
Fine structure of the Pomeron (TOTEM)
Low- \( |t| \) (fine-) structure of the cone (Pomeron?) the LHC:

Tiny oscillations on the cone?

ISR 32 GeV
bin length 8
7 point overlap
The Pomeran trajectory

The Pomeran trajectory has threshold singularities, the lowest one being due to the two-pion exchange, required by the \( t \)-channel unitarity. There is a constraining (Barut, Zwanziger; Gribov) from the \( t \)-channel unitarity, by which

\[
\Im \alpha(t) \sim (t - t_0)^{\Re \alpha(t_0) + 1/2}, \quad t \to t_0,
\]

where \( t_0 \) is the lightest threshold. For the Pomeron trajectory it is \( t_0 = 4m_\pi^2 \), and near the threshold:

\[
\alpha(t) \sim \sqrt{4m_\pi^2 - t}. \quad (1)
\]
Representative examples of the Pomeron trajectories: 1) Linear; 2) With a square-root threshold, required by $t$–channel unitarity and accounting for the small-$t$ “break” as well as the possible “Orear”, $e^{\sqrt{-t}}$ behavior in the second cone; and 3) A logarithmic one, anticipating possible “hard effects” at large $|t| < 8 \text{ GeV}^2$.

\[ \alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t, \quad \text{(TR.1)} \]

\[ \alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t - \alpha_{2P} \left( \sqrt{4\alpha_{3P}^2 - t - 2\alpha_{3P}} \right), \quad \text{(TR.2)} \]

\[ \alpha_P \equiv \alpha_P(t) = 1 + \delta_P - \alpha_{1P} \ln (1 - \alpha_{2P} t). \quad \text{(TR.3)} \]
Linear particle trajectories

Plot of spins of families of particles against their squared masses:

- $\alpha(t)$ vs. $t$ (GeV$^2$)
- $f_6$, $a_6$ (right upper corner)
- $\rho_5$ (top middle)
- $a_4$, $f_4$ (top middle)
- $\omega_3$, $\rho_3$ (middle)
- $f_2$, $a_2$ (middle)
- $\rho$, $\omega$ (bottom middle)
The slope of the cone for a single pole is: $B(s,t) \sim \alpha'(t) \ln s$. The Regge residue $e^{b\alpha(t)}$ with a logarithmic trajectory $\alpha(t) = \alpha(0) - \gamma \ln(1 - \beta t)$, is identical to a form factor (geometrical model).
$P$ and $f$ (second column) have positive $C$-parity, thus entering in the scattering amplitude with the same sign in $pp$ and $\bar{p}p$ scattering, while the Odderon and $\omega$ (third column) have negative $C$-parity, thus entering $pp$ and $\bar{p}p$ scattering with opposite signs, as shown below:

$$A(s, t)_{\bar{p}p} = A_P(s, t) + A_f(s, t) \pm [A_\omega(s, t) + A_O(s, t)], \quad (1)$$

where the symbols $P$, $f$, $O$, $\omega$ stand for the relevant Regge-pole amplitudes and the super(sub)script, evidently, indicate $\bar{p}p(pp)$ scattering with the relevant choice of the signs in the sum.

$$A_P(s, t) = \frac{d}{d\alpha_P} \left[ e^{-i\pi\alpha_P/2} G'(\alpha_P) \left( s/s_0 \right)^{\alpha_P} \right] =$$

$$e^{-i\pi\alpha_P(t)/2} \left( s/s_0 \right)^{\alpha_P(t)} \left[ G'(\alpha_P) + \left( L - i\pi/2 \right) G(\alpha_P) \right].$$
The Pomeron is a dipole in the $j$–plane

$$A_P(s, t) = \frac{d}{d\alpha_P} \left[ e^{-i\pi\alpha_P/2} G(\alpha_P) \left( \frac{s}{s_0} \right)^{\alpha_P} \right] =$$

$$e^{-i\pi\alpha_P(t)/2} \left( \frac{s}{s_0} \right)^{\alpha_P(t)} \left[ G'(\alpha_P) + \left( L - i\pi/2 \right) G(\alpha_P) \right].$$

Since the first term in squared brackets determines the shape of the cone, one fixes

$$G'(\alpha_P) = -a_P e^{b_P[\alpha_{P-1}]},$$

where $G(\alpha_P)$ is recovered by integration, and, as a consequence, the Pomeron amplitude can be rewritten in the following “geometrical” form

$$A_P(s, t) = i \frac{a_P}{b_P} \frac{s}{s_0} \left[ r_1^2(s) e^{r_1^2(s)[\alpha_{P-1}]} - \varepsilon_P r_2^2(s) e^{r_2^2(s)[\alpha_{P-1}]} \right],$$

where $r_1^2(s) = b_P + L - i\pi/2$, $r_2^2(s) = L - i\pi/2$, $L \equiv \ln(s/s_0)$. 
Energy variation of the relative importance of the Pomeron with respect to contributions from the secondary trajectories and the Odderon:

\[ R(s, t = 0) = \frac{\Im m(A(s, t) - A_P(s, t))}{\Im A(s, t)}, \]  

where the total scattering amplitude \( A \) includes the Pomeron contribution \( A_P \) plus the contribution from the secondary Reggeons and the Odderon.

Starting from the Tevatron energy region, the relative contribution of the non-Pomeron terms to the total cross-section becomes smaller than the experimental uncertainty and hence at higher energies they may be completely neglected, irrespective of the model used.

\[ R(s, t) = \frac{|(A(s, t) - A_P(s, t)|^2}{|A(s, t)|^2}. \]
Factorization (nearly perfect at the LHC!)

\[(g_1 g_2)^2 = \frac{(g_1 f_1)^2 (f_1 g_2)^2}{(f_1 f_2)^2} .\]

Hence

\[\frac{d^3 \sigma}{dtdM_1^2 dM_2^2} = \frac{d^2 \sigma_1}{dtdM_1^2} \frac{d^2 \sigma_2}{dtdM_2^2} \frac{d\sigma_{el}}{dt} .\]

Assuming exponential cone, \( t^{bt} \) and integrating in \( t \), one gets

\[\frac{d^2 \sigma_{DD}}{dM_1^2 dM_2^2} = k \frac{1}{\sigma_{el}} \frac{d\sigma_1}{dM_1^2} \frac{d\sigma_2}{dM_2^2} ,\]

where \( k = r^2/(2r - 1) \), \( r = b_{SD}/b_{el} \).

Further integration in \( M^2 \) yields \( \sigma_{DD} = k \frac{\sigma_{SD}^2}{\sigma_{el}} \).
Low-mass diffraction dissociation at the LHC

L. Jenkovszky, O. Kuprash, Risto Orava, A. Salii, arXiv:1211.584,
Low missing mass, single- and double diffraction dissociation at the LHC

Experimentally, diffraction dissociation in proton-proton scattering was intensively studied in the ’70-ies at the Fermilab and the CERN ISR. In particular, double differential cross section $\frac{d\sigma}{dt dM^2_X}$ was measured in the region $0.024 < -t < 0.234$ (GeV/c)$^2$, $0 < M^2 < 0.12s$, and $(105 < s < 752)$ GeV$^2$, and a single peak in $M^2_X$ was identified.

Low-mass single diffraction dissociation (SDD) of protons, $pp \rightarrow pX$ as well as their double diffraction dissociation (DDD) are among the priorities at the LHC. For the CMS Collaboration, the SDD mass coverage is presently limited to some 10 GeV. With the Zero Degree Calorimeter (ZDS), this could be reduced to smaller masses, in case the SDD system produces very forward neutrals, i.e. like a $N^*$ decaying into a fast leading neutron. Together with the T2 detectors of TOTEM, SDD masses down to 4 GeV could be covered.
\[ -q^2 = Q^2 \]

\[ \alpha^{1/2} \gamma \]

\[ \text{Unitarity}_{t=0} \]

\[ \text{Veneziano duality} \]
Similar to the case of elastic scattering, the double differential cross section for the SDD reaction, by Regge factorization, can be written as

\[
\frac{d^2\sigma}{dtdM_X^2} = \frac{9\beta^4[F^p(t)]^2}{4\pi \sin^2[\pi \alpha_P(t)/2]} \left(\frac{s}{M_X^2}\right)^{2\alpha_P(t)-2} \times \left[\frac{W_2}{2m} \left(1 - \frac{M_X^2}{s}\right) - mW_1(t + 2m^2)/s^2\right],
\]

where \(W_i, \quad i = 1, 2\) are related to the structure functions of the nucleon and \(W_2 \gg W_1\). For high \(M_X^2\), the \(W_{1,2}\) are Regge-behaved, while for small \(M_X^2\) their behavior is dominated by nucleon resonances. The knowledge of the inelastic form factors (or transition amplitudes) is crucial for the calculation of low-mass diffraction dissociation.
In the LHC energy region it simplifies to:

\[
\frac{d^2\sigma}{dtdM_X^2} \approx \frac{9\beta^4[F^p(t)]^2}{4\pi} \left(\frac{s}{M_X^2}\right)^{2\alpha_P(t)-2} \frac{W_2}{2m}.
\]  

(1)

These expressions for SDD do not contain the elastic scattering limit because the inelastic form factor \(W_2(M_X, t)\) has no elastic form factor limit \(F(t)\) as \(M_X \to m\). This problem is similar to the \(x \to 1\) limit of the deep inelastic structure function \(F_2(x, Q^2)\). The elastic contribution to SDD should be added separately.
The $pp$ scattering amplitude

$$A(s, t)_P =$$

$$- \beta^2 [f^u(t) + f^d(t)]^2 \left( \frac{s}{s_0} \right)^{\alpha_P(t) - 1} \frac{1 + e^{-i\pi \alpha_P(t)}}{\sin \pi \alpha_P(t)},$$

where $f^u(t)$ and $f^d(t)$ are the amplitudes for the emission of $u$ and $d$ valence quarks by the nucleon, $\beta$ is the quark-Pomeron coupling, to be determined below; $\alpha_P(t)$ is a vacuum Regge trajectory. It is assumed that the Pomeron couples to the proton via quarks like a scalar photon.

A single-Pomeron exchange is valid at the LHC energies, however at lower energies (e.g. those of the ISR or the SPS) the contribution of non-leading Regge exchanges should be accounted for as well.

Thus, the unpolarized elastic $pp$ differential cross section is

$$\frac{d\sigma}{dt} = \frac{[3\beta F^p(t)]^4}{4\pi \sin^2[\pi \alpha_P(t)/2]} \left( \frac{s}{s_0} \right)^{2\alpha_P(t) - 2}.$$

(2)
The final expression for the double differential cross section reads:

\[ \frac{d^2\sigma}{dt dM_X^2} = \]

\[ A_0 \left( \frac{s}{M_X^2} \right)^{2\alpha_p(t)-2} \frac{x(1-x)^2 \left[ F^p(t) \right]^2}{(M_x^2 - m^2) \left( 1 + \frac{4m^2x^2}{-t} \right)^{3/2} \times} \]

\[ \sum_{n=1,3} \frac{[f(t)]^{2(n+1)} Im \alpha(M_X^2)}{(2n + 0.5 - Re \alpha(M_X^2))^2 + (Im \alpha(M_X^2))^2}. \]
$$\frac{d^2 \sigma_{SD}}{dtdM_x^2} = F_p^2(t) F(x_B, t) \frac{\sigma_{pp}^p(M_x^2, t)}{2m_p} \left( \frac{s}{M_x^2} \right)^{2(\alpha(t)-1)} \ln \left( \frac{s}{M_x^2} \right)$$

$$\frac{d^3 \sigma_{DD}}{dtdM_1^2 dM_2^2} = C_n F^2(x_B, t) \frac{\sigma_{pp}^p(M_1^2, t)}{2m_p} \frac{\sigma_{pp}^p(M_2^2, t)}{2m_p} \times \left( \frac{s}{(M_1 + M_2)^2} \right)^{2(\alpha(t)-1)} \ln \left( \frac{s}{(M_1 + M_2)^2} \right)$$
“Reggeized (dual) Breit-Wigner” formula:

\[
\sigma_T^p(M_x^2, t) = \text{Im} \ A(M_x^2, t) = \sum_n \frac{A_{N^*}}{n - \alpha_{N^*}(M_x^2)} + B g(t, M_x^2) = \\
= A_n \sum_{n=0,1,...} \frac{[f(t)]^{2(n+1)} \text{Im} \ \alpha(M_x^2)}{(2n + 0.5 - \text{Re} \ \alpha(M_x^2))^2 + (\text{Im} \ \alpha(M_x^2))^2} + B_n e^{b_{int} t} (M_x^2 - M_{p+\pi}^2)^c
\]

\[
F(x_B, t) = \frac{x_B(1-x_B)}{(M_x^2 - m_p^2) \left(1 + 4m_p^2 x_B^2/(-t)\right)^{3/2}} \quad x_B = \frac{-t}{M_x^2 - m_p^2 - t}
\]

\[
F_p(t) = \frac{1}{1 - \frac{t}{0.71}}, \quad f(t) = e^{b_{int} t}
\]

\[
\alpha(t) = \alpha(0) + \alpha' t = 1.04 + 0.25t
\]
SDD cross sections vs. energy.

Normalization to ~14 mb at 7 TeV.
Approximation of background to reference points (t=-0.05)
Approximation of background to reference points \((t=-0.5)\)
Double differential SD cross sections

![Graphs showing double differential SD cross sections.](image)
Single differential integrated SD cross sections

\[ \frac{d\sigma}{dM^2} \text{ (mb/GeV}^2\text{)} \]

\[ \sqrt{s} = 7 \text{ TeV} \]
\[ t: [0.0; 1.0] \]
\[ \alpha' = 0.25, b_{\text{in}} = 0.2 \]

\[ \frac{d\sigma_{SD}}{dt} \text{ (mb/GeV}^2\text{)} \]

\[ \sqrt{s} = 7 \text{ TeV} \]
- \( M^2 \in [1.0; 2.0] \)
- \( M^2 \in [1.0; 2.5] \)
- \( M^2 \in [1.0; 3.2] \)
- \( M^2 \in [1.0; 4.1] \)
- \( M^2 \in [1.0; 6.0] \)
- \( M^2 \in [1.0; 5.0e+08] \)
Integrated DD cross sections

Integrated DD cross sections as a functions of $t$ integrated in different $M_1:M_2$ regions.

Double differential SD cross sections as a function of $M_2$ integrated in region $[0.0:1.0]$ of $t$ value.
Triple differential DD cross sections
The parameters and results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{in}$ (GeV$^{-2}$)</td>
<td>0.2</td>
<td>$\sigma_{SD}$ (mb)</td>
<td>14.13</td>
</tr>
<tr>
<td>$b_{bg}$ (GeV$^{-2}$)</td>
<td>3</td>
<td>$\sigma_{SD}(M &lt; 3.5\text{GeV})$ (mb)</td>
<td>4.68</td>
</tr>
<tr>
<td>$\alpha'$ (GeV$^{-2}$)</td>
<td>0.25</td>
<td>$\sigma_{SD}(M &gt; 3.5\text{GeV})$ (mb)</td>
<td>9.45</td>
</tr>
<tr>
<td>$\alpha(0)$</td>
<td>1.04</td>
<td>$\sigma_{Res}^{SD}$ (mb)</td>
<td>2.48</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>1.03</td>
<td>$\sigma_{Bg}^{SD}$ (mb)</td>
<td>9.45</td>
</tr>
<tr>
<td>$A_n$</td>
<td>18.7</td>
<td>$\sigma_{DD}$ (mb)</td>
<td>10.68</td>
</tr>
<tr>
<td>$B_n$</td>
<td>8.8</td>
<td>$\sigma_{DD}(M &lt; 10\text{GeV})$ (mb)</td>
<td>1.05</td>
</tr>
<tr>
<td>$C_n$</td>
<td>3.79e-2</td>
<td>$\sigma_{DD}(M &gt; 10\text{GeV})$ (mb)</td>
<td>9.63</td>
</tr>
</tbody>
</table>
Prospects:
Central diffractive meson production (double Pomeron exchange);
Antoni Szczurek et al., talk at this Symposium
<table>
<thead>
<tr>
<th>(Im A(a + b \rightarrow c + d) = )</th>
<th>(R)</th>
<th>(\text{Pomeron})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s)-channel</td>
<td>(\sum A_{Res})</td>
<td>Non-resonant background</td>
</tr>
<tr>
<td>(t)-channel</td>
<td>(\sum A_{Regge})</td>
<td>(\text{Pomeron (} I = S = B = 0; \ C = +1))</td>
</tr>
<tr>
<td>Duality quark diagram</td>
<td>Fig. 1b</td>
<td>Fig. 2</td>
</tr>
<tr>
<td>High energy dependence</td>
<td>(s^{\alpha - 1}, \ \alpha &lt; 1)</td>
<td>(s^{\alpha - 1}, \ \alpha \geq 1)</td>
</tr>
</tbody>
</table>
Thank you!

László L. Jenkovszky
jenk@bitp.kiev.ua