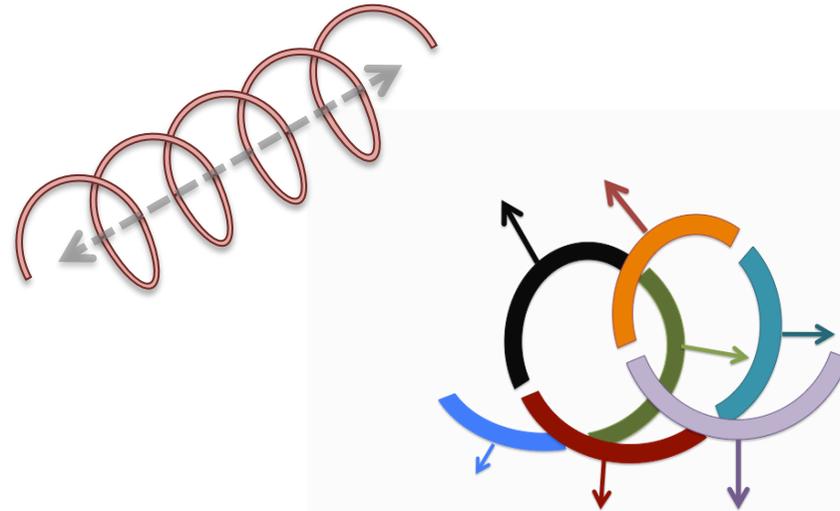


# Quantum properties of QCD string fragmentation

(Šárka Todorova-Nová, Charles University, Prague)

- (Lund) string fragmentation in 1 and 3 dimensions
- Causal constraint & quantum properties
- Light meson spectra & QCD field properties
- **2-particle correlations in coherent hadron production (+ comparison with data)**



**NO TIME TO TALK ABOUT :**

- > inclusive ( $p_T$ ) spectra
- > azimuthal ordering of hadrons
- > polarization effects
- > topological QCD

# Lund string fragmentation model

B. Andersson et al., "Parton fragmentation and string dynamics," *Phys. Rept.* **97, 31 (1983)391**

## **Semi-classical model :**

- confinement modeled by relativistic massless string with constant string tension ( $\kappa \sim 1 \text{ GeV/fm}$ )

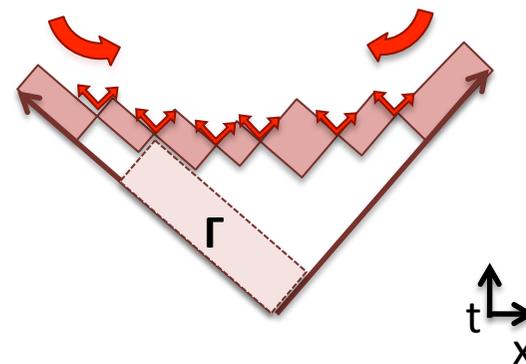
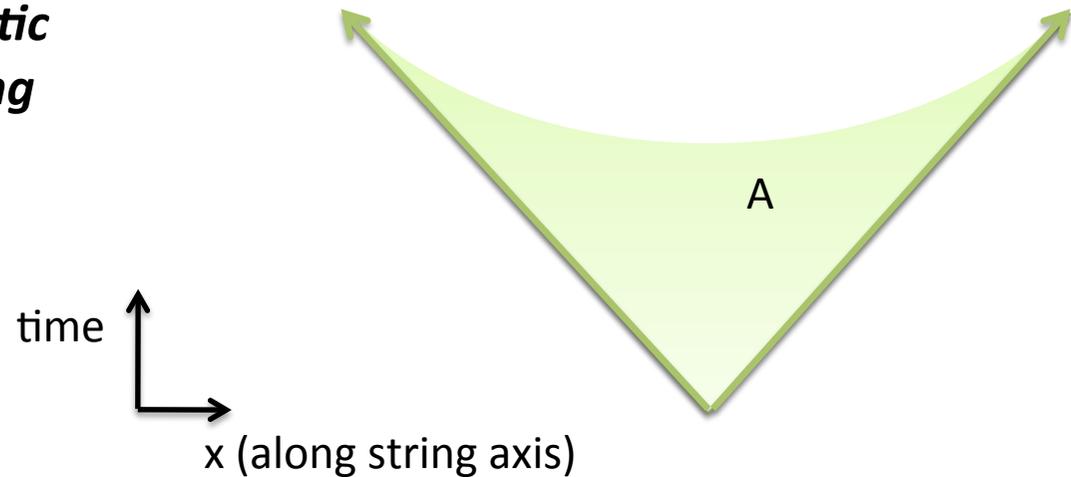
- massless partons moving in light-cone coordinates

- the area span of the string ( $A$ ) corresponds to the action

- homogenous QCD field with probability of breakup  $\sim \exp(-b\Gamma)$

Related to Wilson loops

$$M \sim \exp(i \xi A)$$



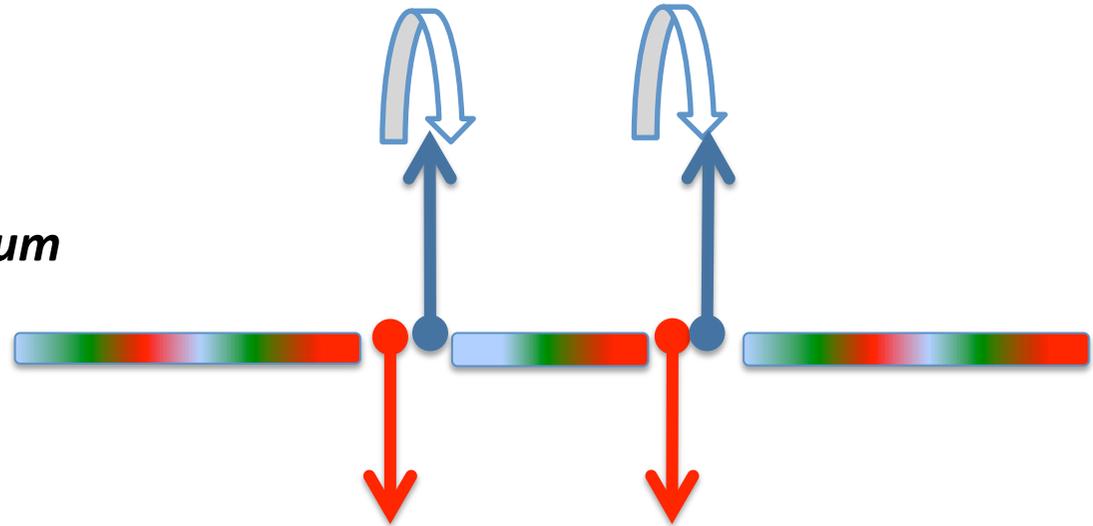
## Mass and transverse momentum generation

*strongly influenced by the absence of cross-talk between adjacent string breakups :*

### **QUANTUM TUNNELING**

*evoked by the model :*

- > to generate intrinsic  $p_T$*
- > to ensure proper mass spectrum*



*A new  $qq^{\sim}$  pair is created from 'vacuum' and assigned a randomly sampled transverse momentum*

*( local charge and momentum conservation holds )*

*Two adjacent string break-ups define a HADRON with*

$$\vec{p}_T = \vec{p}_1 - \vec{p}_2$$
$$p_l = \kappa ( t_1 - t_2 )$$
$$E = \kappa ( x_1 - x_2 )$$

*( breakups separated by space-like distance )*

**QCD field is not 1-dimensional ....**

***most likely , a thin vortex tube  
( superconductor type II )***

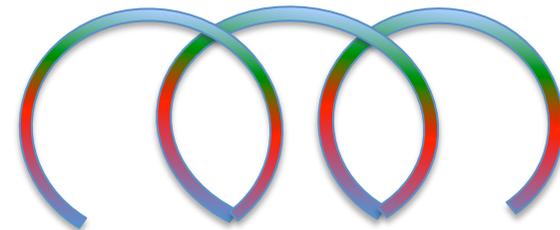
***[B.Andersson:"The Lund Model",Cambridge 1998]***



**... what about working with a 3 dimensional string ?**

**Helix string concept** [JHEP09(1998)14.]

***B.Andersson et al.: "Is there a screwiness  
at the end of partons showers ? "***



***- introduced on the basis of helicity  
conservation to stabilize the (soft) end of parton cascades***

## !!! Causal constraint applicable on twisted / 3dim string !!!

*i.e., we can IMPOSE time-like distance ( $\Leftrightarrow$  cross-talk ) between string breakup vertices which define the hadron*

*Let's concentrate on the case when information ( about string break-up ) travels along the string only, together with the massless parton :*

**QCD field (string tension) transforms into : longitudinal quark momentum**

$$p_{||} = \kappa \beta c \Delta t$$

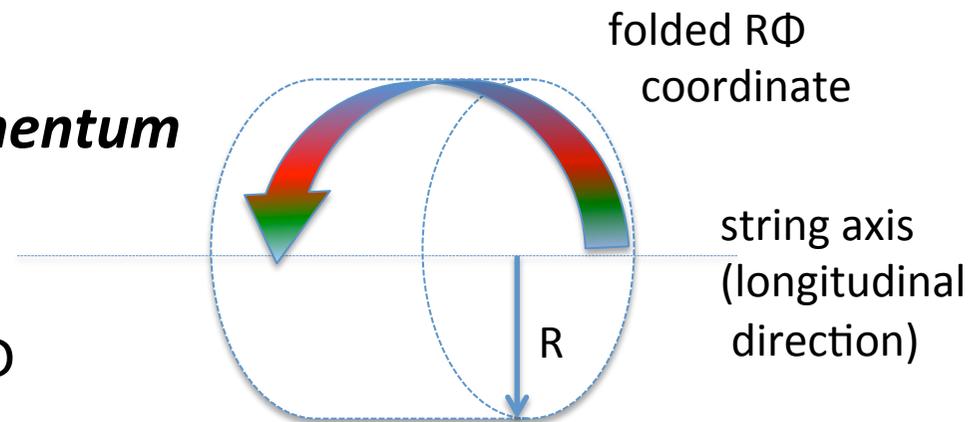
**+ transverse quark momentum**

$$|p_{\perp}| = 2 \kappa R \sin ( \Delta\Phi/2 )$$

**+ effective quark mass**

$$m_{\perp} = \sqrt{m^2 + p_{\perp}^2} = \kappa R \Delta\Phi$$

$$\beta^2 = 1 - ( R\omega/c )^2 , \quad \omega = \Delta\Phi/\Delta t$$



-> propagating quark “triggers” the next breakup & creation of hadron with mass

$$m_S = \kappa R \sqrt{(\Delta\Phi)^2 - (2 \sin \Delta\Phi / 2)^2}.$$

( depends on TRANSVERSE  
shape of string only,

longitudinal momentum and transverse mass decouple )

-> these should be narrow resonant states

**? Quantization in  $\Delta\Phi$  (  $m_T$  ) , does it reproduce hadron spectra ?**

*Let's consider light pseudoscalar mesons :  $\pi$ ,  $\eta(547)$ ,  $\eta'(948)$   
( they fit well in the picture of chains of ground state hadrons )*

•  $\pi$  ground state (lightest hadron) ;  $\eta \rightarrow 3 \pi$  ;  $\eta' \rightarrow 5 \pi$

• quantization:  $E_T (n=1,3,5) = n \kappa R \Delta\Phi$

$$p_T (n=1,3,5) = 2 \kappa R | \sin( n \Delta\Phi / 2 ) |$$

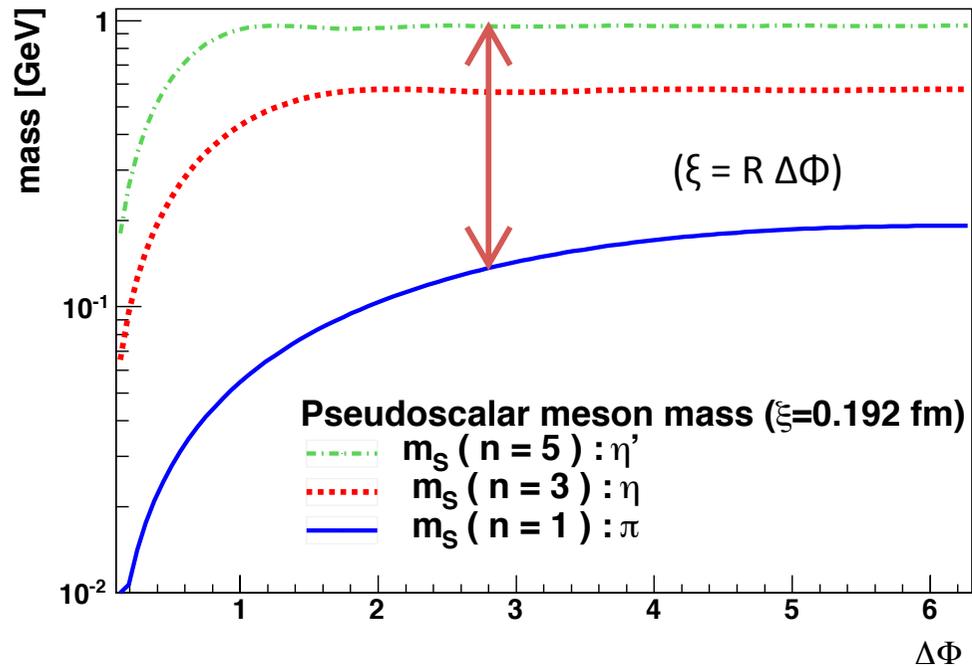
$$m = \text{sqrt} ( E_T^2 - p_T^2 )$$

2 parameters to constrain :  $\kappa R$  [GeV],  $\Delta\Phi$  [rad]

**The fit is overconstrained  
- but there IS a solution  
which agrees with the  
experimental data  
with precision of ~3%**

$\kappa\xi$ [MeV]	$\kappa R$ [MeV]	$\Delta\Phi$
$192.5 \pm 0.5$	$68 \pm 2$	$2.82 \pm 0.06$
meson	PDG mass [MeV]	model estimate [MeV]
$\pi$	135 - 140	137
$\eta$	548	565
$\eta'$	958	958

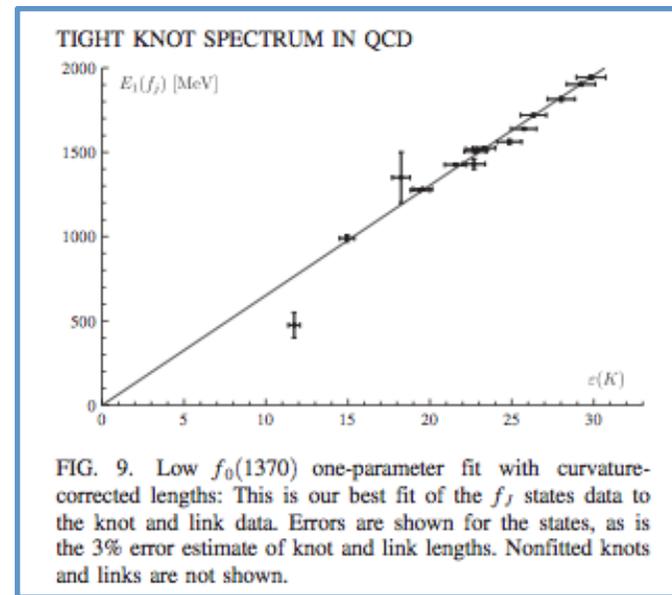
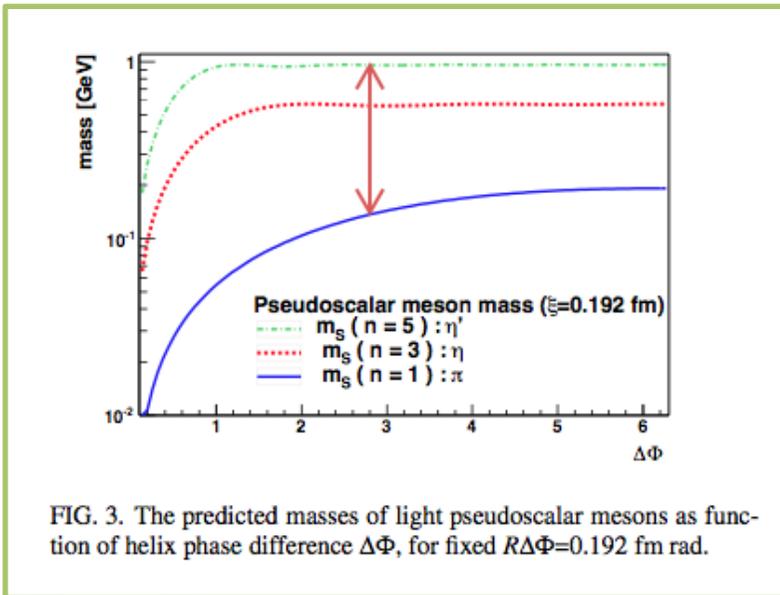
TABLE I. Best fit of the parameters of the pion ground state obtained from the mass spectrum of light pseudoscalar mesons. The  $\eta$  mass is reproduced within a 3% margin which serves as the base of uncertainty for  $R, \Delta\Phi$  parameters.



**Best fit of the pion  
ground state properties  
in the quantized  
helix string model**

Phys.Rev.D89,015002(2014)  
arXiv:1309.6761

**UNEXPECTED CONNECTION to topological QCD : fitted scales in excellent agreement**



Phys.Rev.D89, 015002(2014)  
 [arXiv:1309.6761]  
 estimate of radius of helix-like QCD string

$$\kappa R = 68 \pm 2 \text{ MeV}$$

*obtained from the fit of mass of light pseudoscalar mesons*

$$E_T(n) = \kappa R n \Delta\Phi \quad \text{[helical QCD string]}$$

Phys.Rev.D89, 054513(2014)  
 [arXiv:1212.1500]  
 estimate of the confinement scale

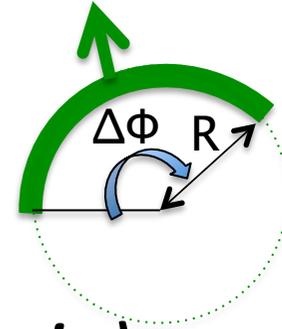
$$\Lambda_{\text{tube}} = 65.16 \pm 0.61 \text{ MeV}$$

*obtained from the fit of  $J^{++}$  spectra modelled as tight topological QCD knots*

$$E(K) = \Lambda_{\text{tube}} \varepsilon(K) \quad \text{[knotted flux tube]}$$

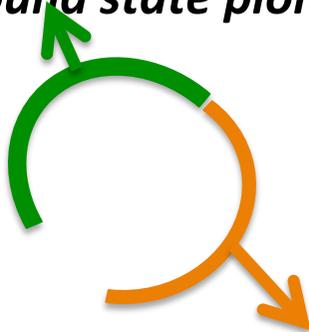
## QCD field parametrization implications (observables)

- **intrinsic  $p_T$  of (ground state) pions  $\sim 0.13$  GeV**  
*Just about what is needed to “regularize”  
the soft particle production in the helix string model ?*



- **minimal distance between adjacent hadrons (+- pairs)**  
**“Q threshold” at  $\sim 0.26$  GeV**

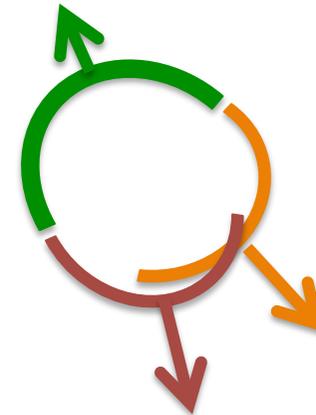
*( ground state pions ) -> direct impact on the correlation pattern*



$$Q = \sqrt{-(\mathbf{p}_i - \mathbf{p}_{i+1})^2} \approx |\vec{p}_{Ti} - \vec{p}_{Ti+1}|$$

$$\approx 2 p_T^{\text{thr}} |\sin(\Delta\Phi/2)|$$

- **Correlations for like-sign pairs**  
**with  $Q^{++,--} \approx 2 p_T^{\text{thr}} |\sin(2\Delta\Phi/2)| \approx 0.09$  GeV**

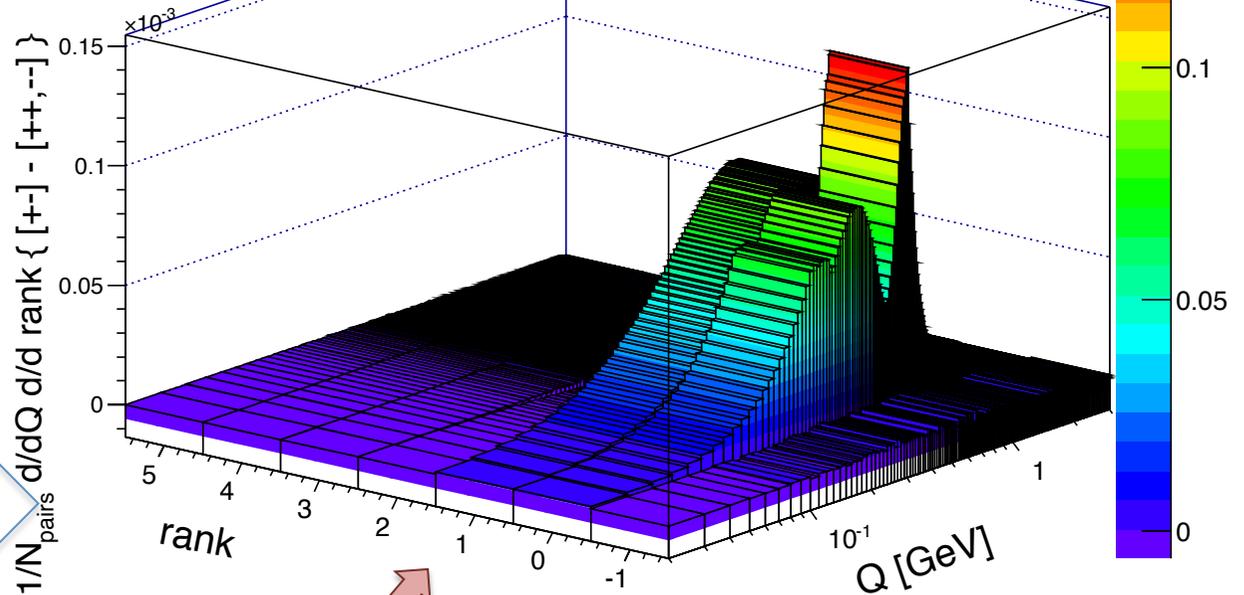


**How to extract the adjacent pairs from the combinatorial background ?**

**Experimental methods**

MC truth (PYTHIA8) :  
bin per bin, nearly perfect subtraction of non-adjacent pair contribution

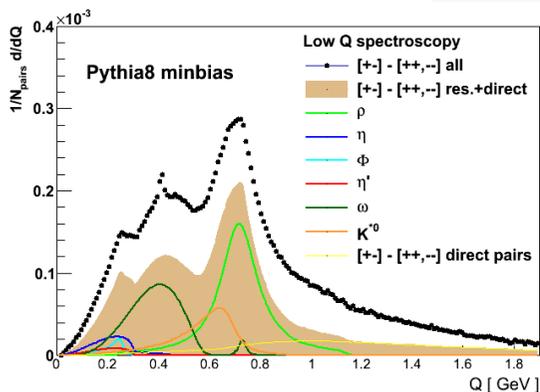
rank > 1 : non-adjacent (subtracted)



rank=1 : adjacent direct (correlated)

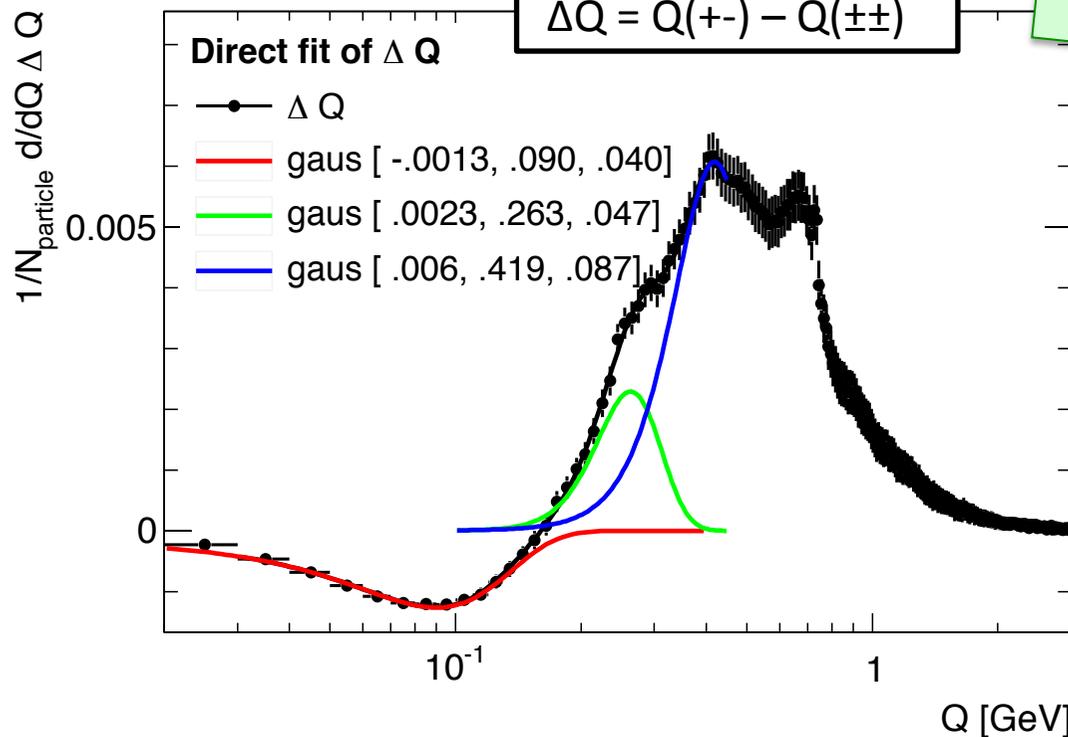
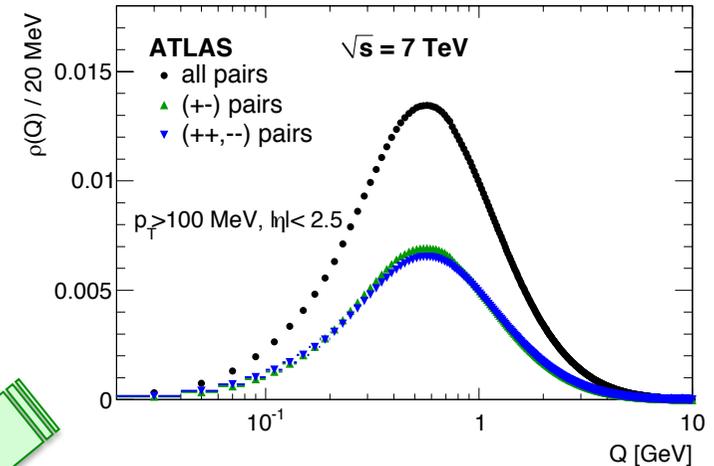
rank=0 : common direct mother (resonant spectrum)

rank=-1 : different strings (random, subtracted)



**Can be analysed in 2 ways**

- **assuming incoherent particle production**  
-> ratio -> correlation function (BE effect)
- **assuming coherent particle production**  
-> subtraction -> (resonant) shape



**The shape of subtracted Q distributions agrees well with the expectations of the quantized helix string model:**

**Threshold for adjacent, rank 1 (+-) pairs production ~ 0.26 GeV**

**rank 2 (++,--) around Q~0.09 GeV**

## Helical QCD string or Bose-Einstein effect ?

### Coherent or incoherent origin of hadron correlations ?

*Inevitably, we approach the confrontation of the two concepts - competing over the same data (ALL hadronic data) HELIX vs. BE*

*The question can be settled experimentally :*

*A/ in the coherent scenario, close like-sign pairs belong to a triplet 'chains' with mass  $M_n < 0.6$  GeV*

*B/ no correlations between hadrons from colour disconnected sources*

LEP2 : WW studies :

no correlations among "mixed" pairs

-> coherent origin favoured

HELIX – BE ( 1 : 0 )

Elimination of free parameters from the modelling ( hadron spectra can be deduced from correlation shape )

HELIX – BE ( 2 : 0 )

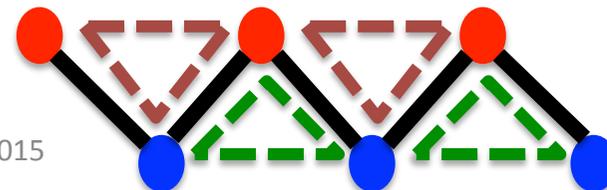
LHC : minimum bias :

shape of correlations in agreement with model prediction

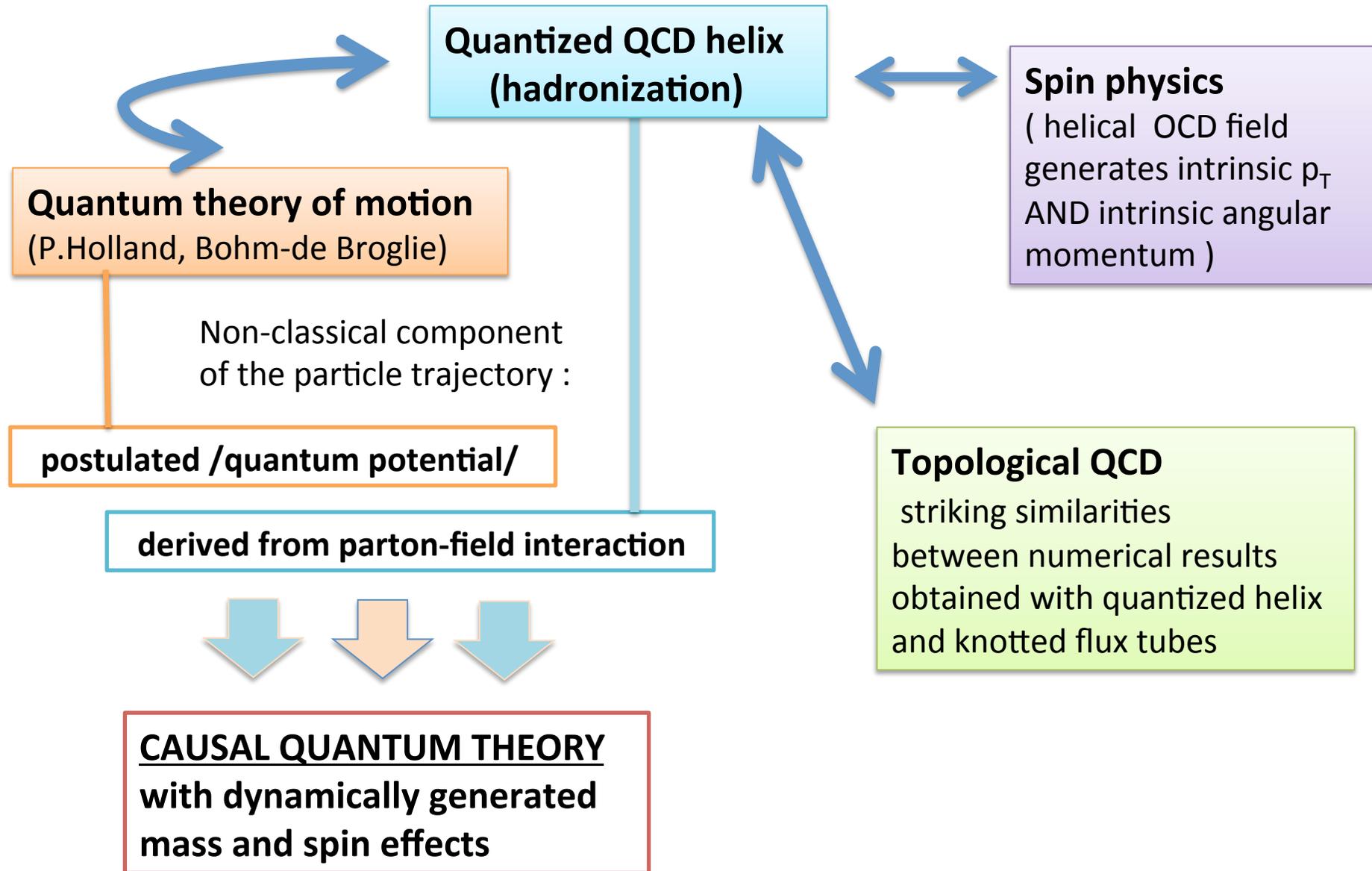
-> coherent origin favoured

Source size considerations :

-> coherent origin favoured



*If the model survives the experimental scrutiny , it has a huge potential for further development:*



## Summary

- **Phenomenological (data driven) approach to the quantization of QCD field**
- **Helical QCD string seems to be an excellent approximation of the effective QCD field**
- **Causality provides the key to the study of the mass spectrum of light mesons, coupled with dynamical correlations**
- **Number of observable model predictions up for validation :**
  - > **inclusive low  $p_T$  ( < 200 MeV )**
  - > **2-particle correlations**
  - > **azimuthal ordering**
  - > **polarization studies**

### **PHENOMENOLOGY :**



**2 degrees of freedom removed from the string fragmentation model & number of free parameters reduced**

**( ++ : numerical agreement with topological QCD studies )**

**-> better hadronization model**

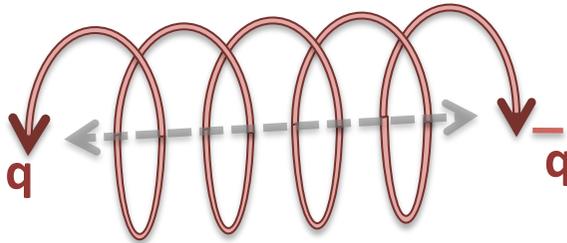
**back-up slides**

## Documentation

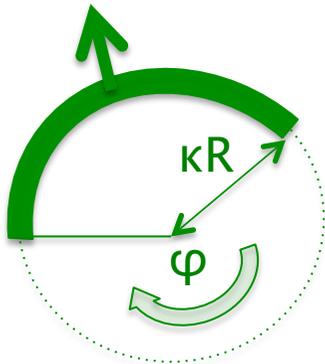
- **Bo Andersson et al., JHEP 09 (1998) 014.** [phenomenology of helix]
- **Š.Todorova, Phys.Rev.D86, 034001 (2012)** [phenomenology of helix]
- **ATLAS Coll., Phys.Rev.D86, 052005 (2012)** [azimuthal ordering]
- **Š.Todorova, arXiv:1012.5778 [hep-ph]** [ helix-tune with LEP data ]
- **<http://projects.hepforge.org/helix/>** [ PYTHIA-compatible code ]
- **Š.Todorova, Phys.Rev.D89, 015002 (2014)** [quantization of helix]
- **Š.Todorova, arXiv:1406.3564 [hep-ph]** [quantization of helix]
- **ATLAS Coll., ATL-COM-PHYS-2013-1657 (private)** [study of hadron chains]

**Helix string concept** [JHEP09(1998)14.]

(introduced to stabilize the end of parton shower)



**3dim string tension : let string transverse shape define the hadron  $p_T$**



$$|p_T| = 2\kappa R \sin(\Delta\phi/2)$$

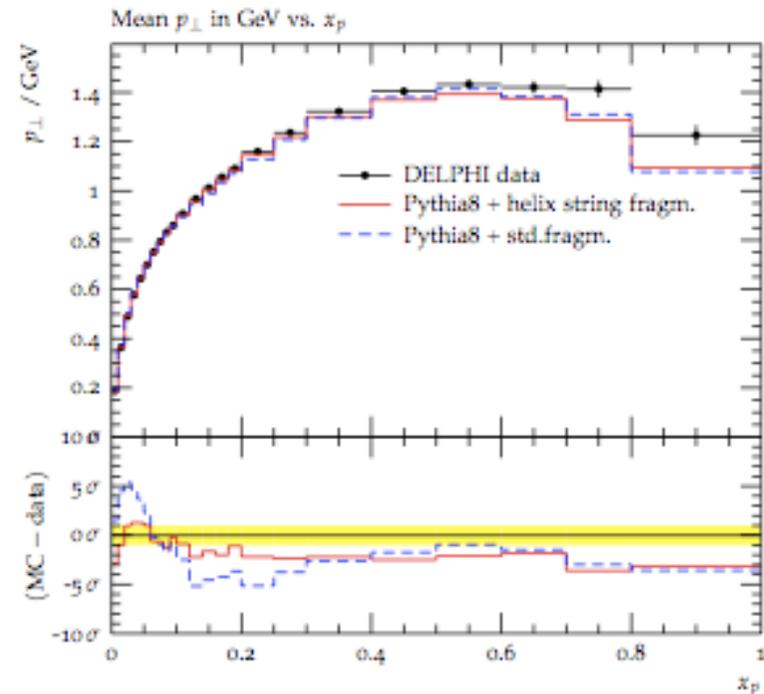
$\kappa R$  helix radius  
 $\phi$  helix phase

**The removal of 2 degrees of freedom associated with the intrinsic  $p_T$  generation leads to a significant improvement in the description of hadronic Z0 data**

**[arXiv:1012.5778]**

**LEP data show some correlations between longitudinal ( $x_p$ ) and transverse  $\langle p_T \rangle$  momentum component ....**

**[Z.Phys.C73(1996)11]**



Indirect experimental evidence: tuning on  $Z^0$  data  
 ( DELPHI\_1996\_S3430090 )

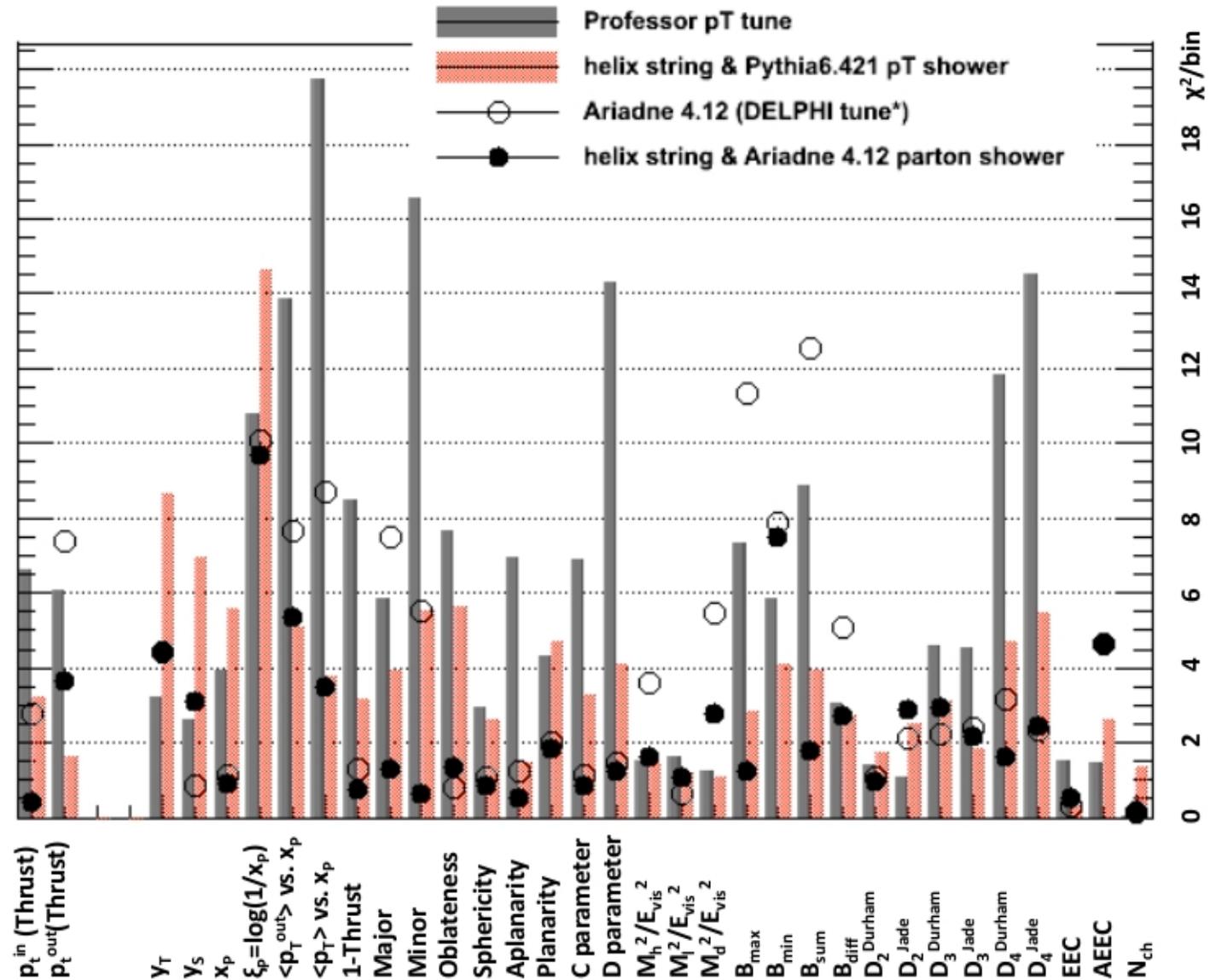
arXiv:1012.5578  
 [hep-ph]

Helix string  
 fragmentation  
 improves  
 “goodness of fit”  
 ( $\chi^2/N_{\text{dof}}$ )

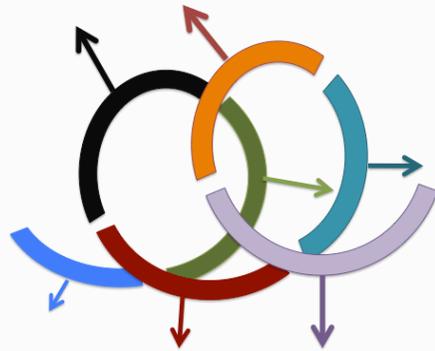
6.6 -> 4.0  
 ( Pythia 6  
 DELPHI 1996  
 tune )

4.0 -> 2.4  
 (Pythia 6 with  
 ARIADNE parton  
 shower )

10/5/15



# Azimuthal ordering of hadrons [Phys.Rev.D86,052005 (2012)]



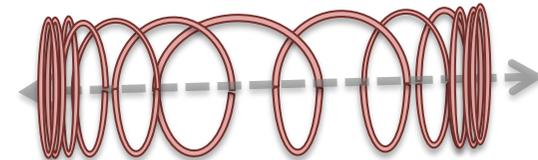
*The helix-like shape structure of the QCD field should be visible in the azimuthal ordering of hadrons along the string*

*The exact form of the helix structure not predicted.*

*With the help of power spectra, we test two (weakly correlated) hypotheses*

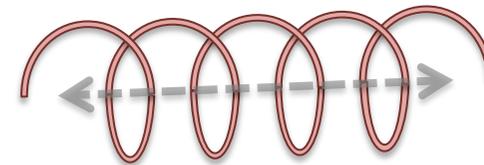
**A/  $\Delta\Phi \sim \Delta\eta$**

$$S_{\eta}(\xi) = \frac{1}{N_{ev}} \sum_{event} \frac{1}{n_{ch}} \left| \sum_j \exp(i(\xi \eta_j - \phi_j)) \right|^2$$



**B/  $\Delta\Phi \sim \Delta X$  (energy-distance - amount of energy stored in the string/ ordered hadron chain - experimentally : ordered in pseudorapidity )**

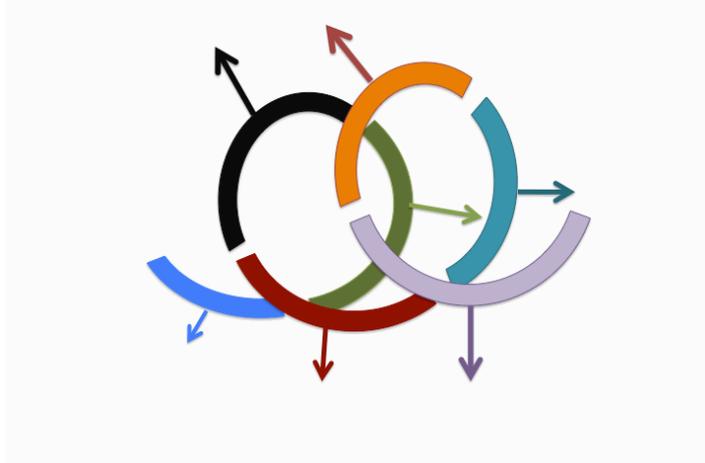
$$S_E(\omega) = \frac{1}{N_{ev}} \sum_{event} \frac{1}{n_{ch}} \left| \sum_j \exp(i(\omega X_j - \phi_j)) \right|^2$$



**Search for resonant behaviour -> density of helix winding**

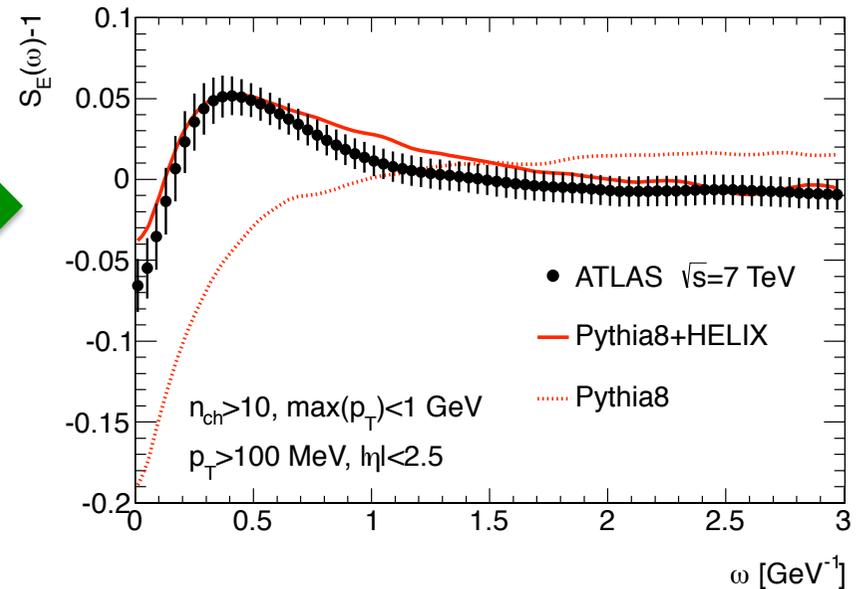
# Azimuthal ordering

The model predicts azimuthal ordering of hadrons  
 [ Phys.Rev.D86, 052005 (2012) ]



$$S_E(\omega) = \frac{1}{N_{ev}} \sum_{event} \frac{1}{n_{ch}} \left| \sum_j \exp(i(\omega X_j - \phi_j)) \right|^2$$

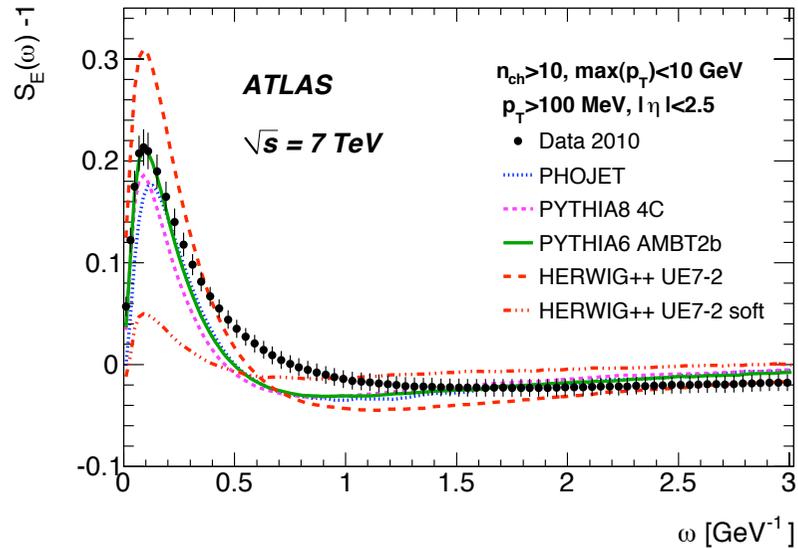
$\Delta\Phi \sim \Delta X$  (energy-distance:  
 amount of energy stored in the string  
 / pseudorapidity ordered hadron chain )



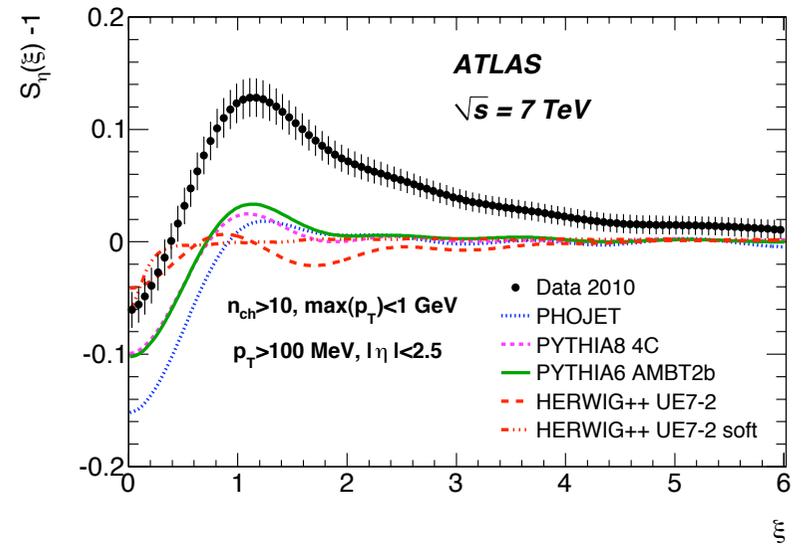
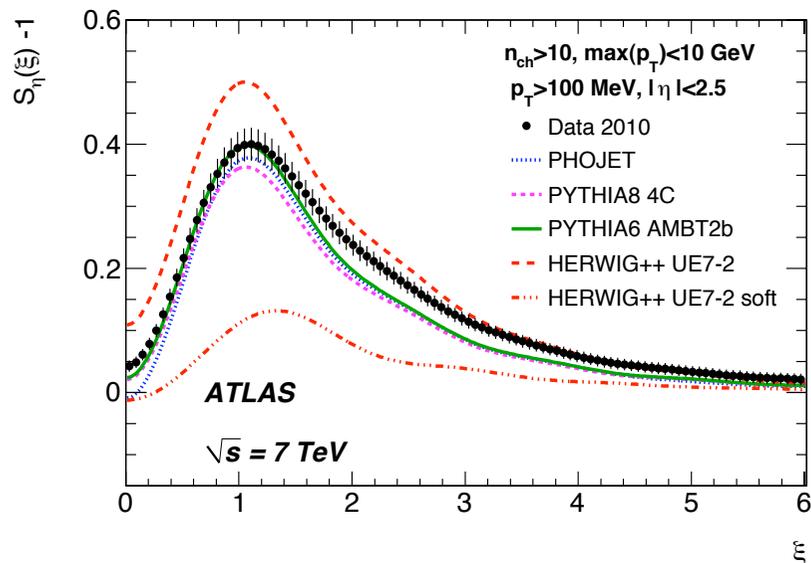
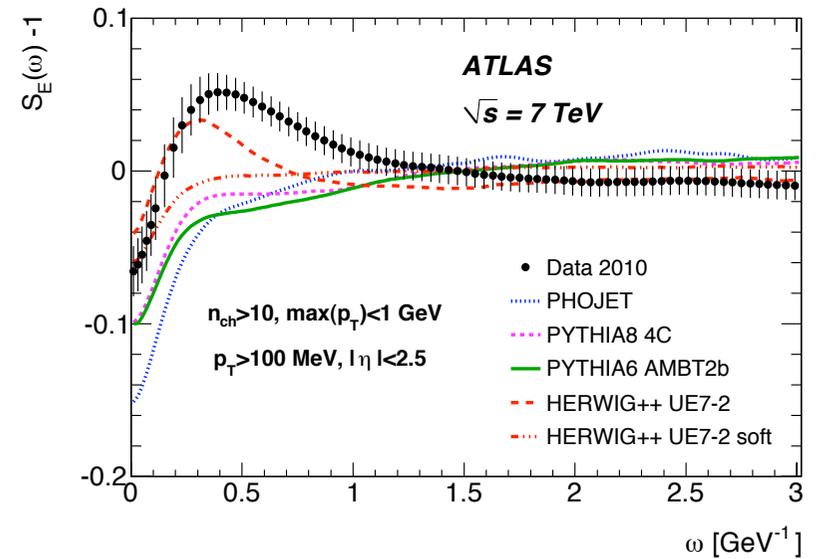
Comparison with model in  
 Phys.Rev.D86,034001(2012)

The model drastically reduces “randomness”  
 in the intrinsic pT sector yet describes the  
 relevant hadronic data better over large span  
 of energies ....

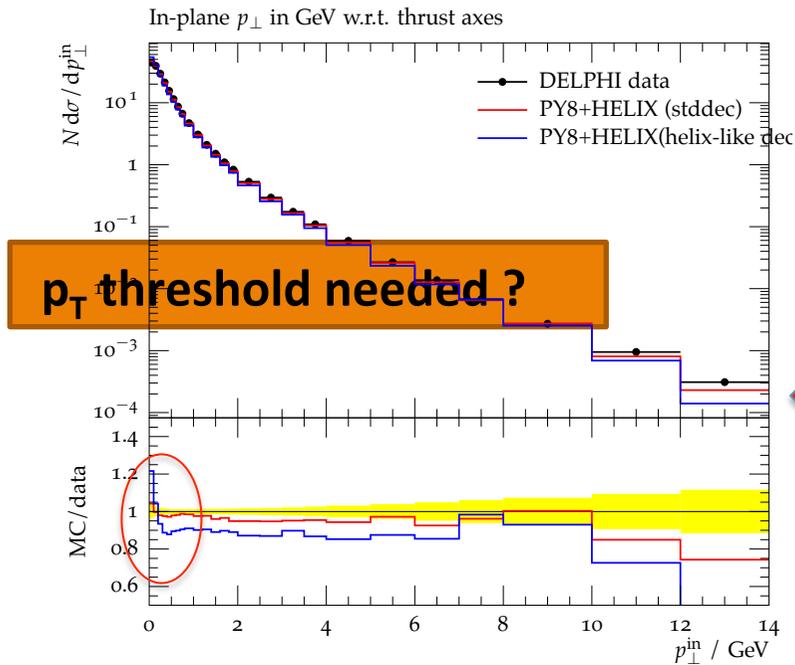
## Min.bias inclusive (jet dominated)



## Min.bias low $p_T$ selection (dominated by fragmentation)



*.. the model is not complete ...*



*The azimuthal ordering is not well described if the helix model operates on direct hadrons only -> decay of short-lived resonances incorporated in the model*

*.. but the enhanced helix model produces way too many soft particles (  $Z^0$  data no longer so well described )*

**Quantum effects need to be understood and included !**

*Resonances & correlations part of the model*

*The string area can be interpreted in terms of action ,  
the tuning of helical string parameters suggest  $\kappa R \Delta \varphi c \sim \hbar$*

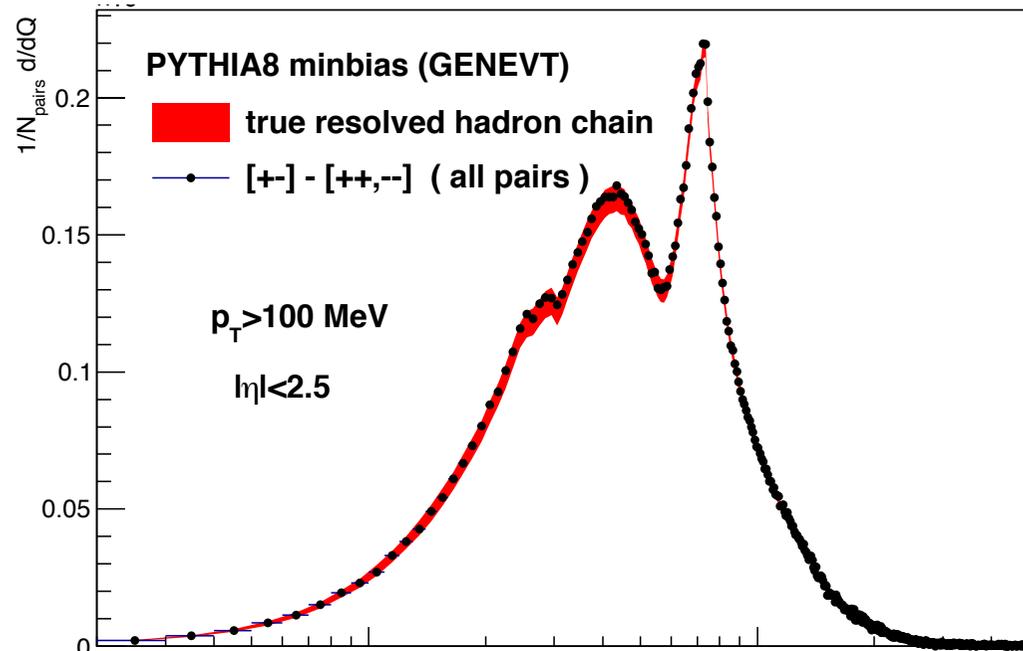
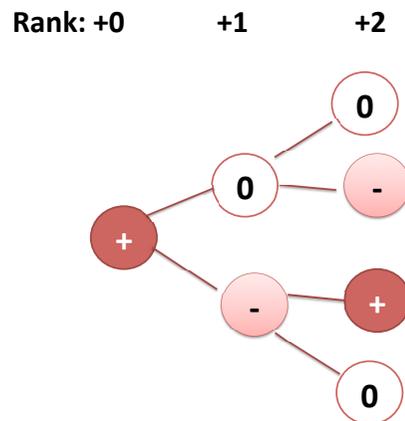
## Experimental methods

## How to extract the adjacent pairs from the combinatorial background ?

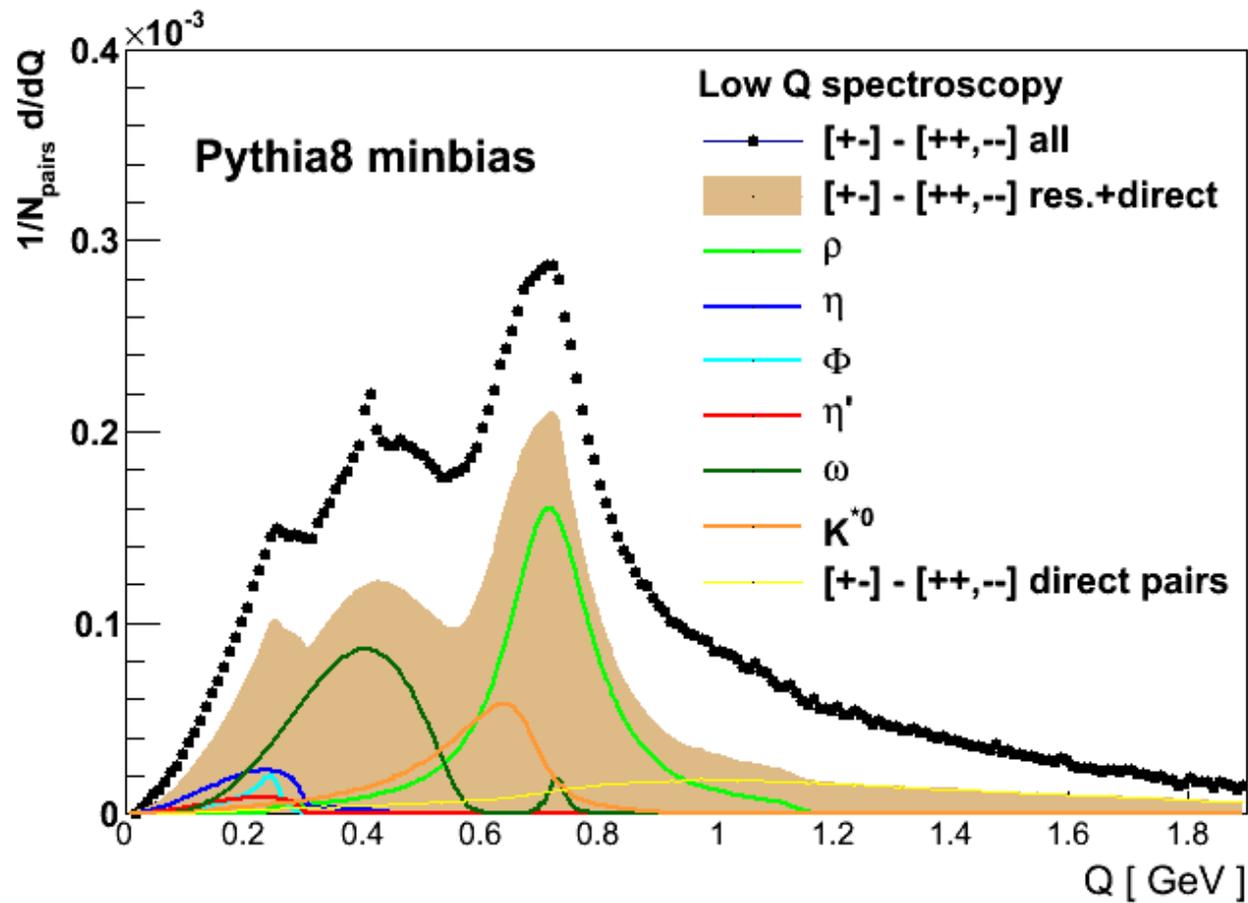
“standard” string fragmentation preserves 4-momentum, flavour and charge at each breakup vertex -> adjacent hadrons correlated + strong charge-combination asymmetry ( adjacent charged like-sign pairs forbidden )  
-> non-adjacent direct hadrons uncorrelated

**! Q (+-) - Q (++,--) removes very efficiently the non-adjacent hadron pairs !**

Probability to find pair of hadrons with rank 2 about the same for like-sign and unlike-sign pair:

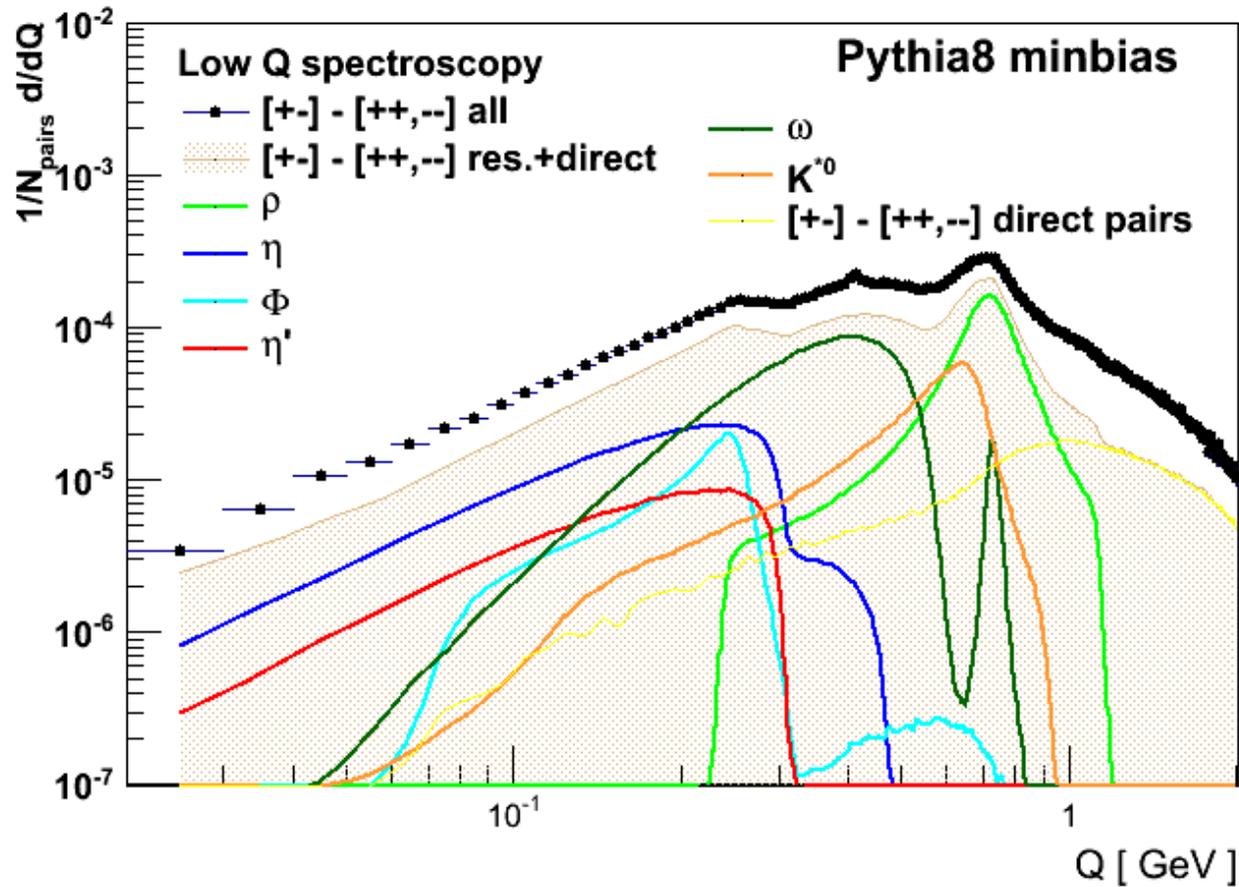


**The residual shape after subtraction reflects the spectrum of resonances ( + adjacent pairs )**



**The residual shape after subtraction reflects the spectrum of resonances ( + adjacent pairs )**

**Low Q dominated by  $\eta, \eta', \Phi$  (  $K \rightarrow \pi$  in our approximation )**

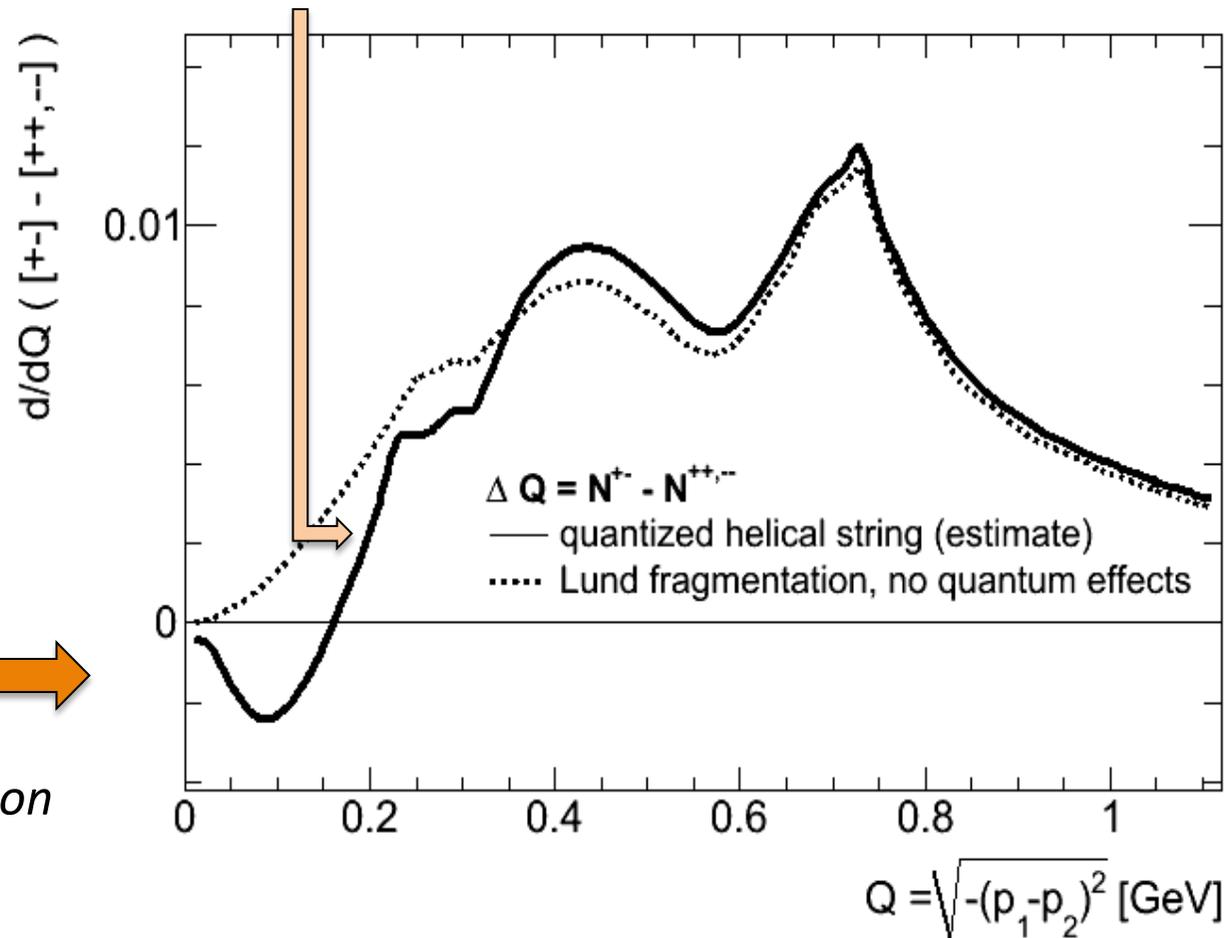


# Effect of the quantization on the subtracted 2-particle densities

Combined effect of quantum threshold for adjacent unlike-sign pairs and the enhanced production of close like-sign pairs (of rank 2) leads to emergence of BE-like correlation pattern



(enhanced production of like-sign pairs)



Source of correlations : coherent particle emission

**Comparison with generator level**

**Pythia8 minbias sample**

**shows the Q threshold is violated in MC for both rank 0 pairs (resonance decays) and rank 1 pairs (adjacent)**

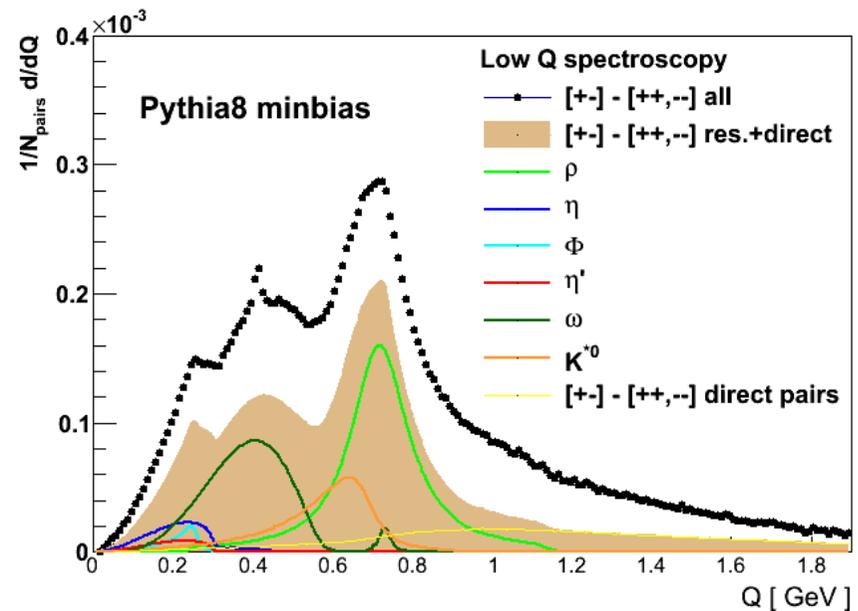
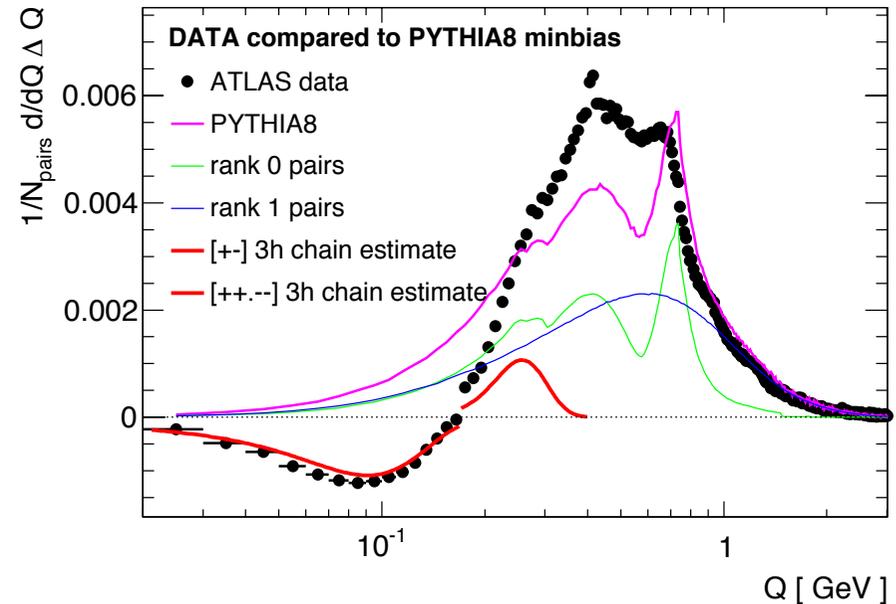
**The “resonant” spectrum dominated by**

$\eta, \eta', \omega, \rho, K^*, \Phi$

**(  $K^0$  neglected here )**

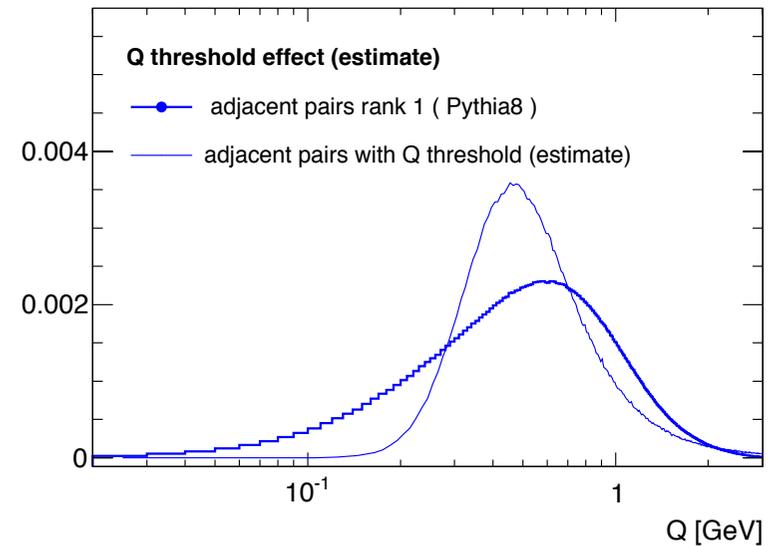
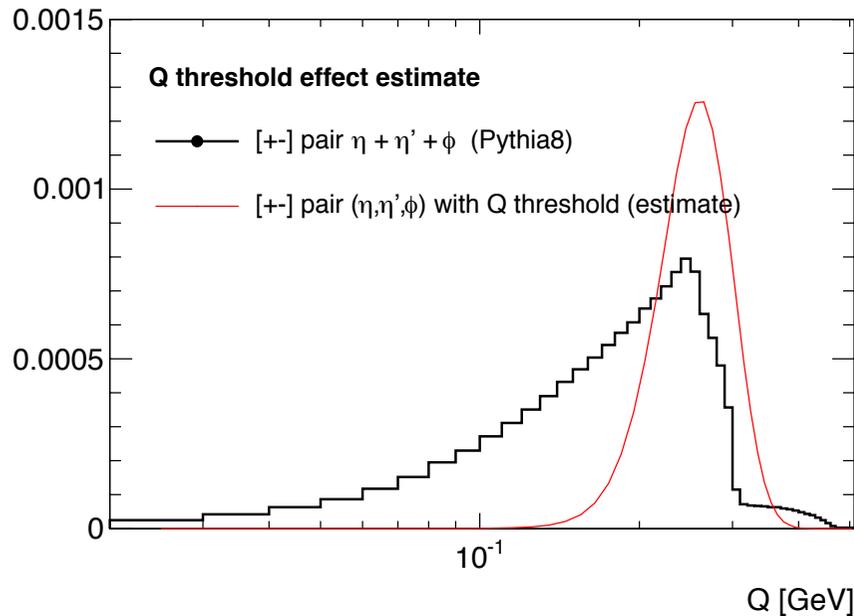
**In the absence of particle identification, the spectrum can be roughly splitted into 3 “peaks” :**

- 1/  $\eta+\eta'+\Phi$**
- 2/  $\omega (+K)$**
- 3/  $\rho+K^*$**

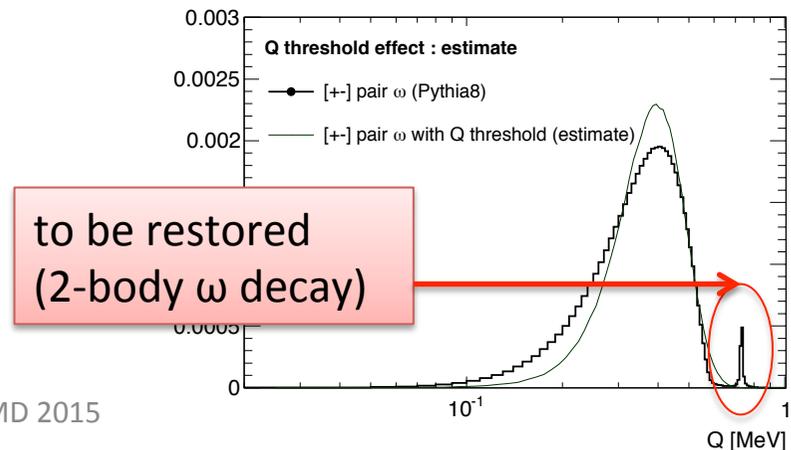


## Enforcing Q threshold

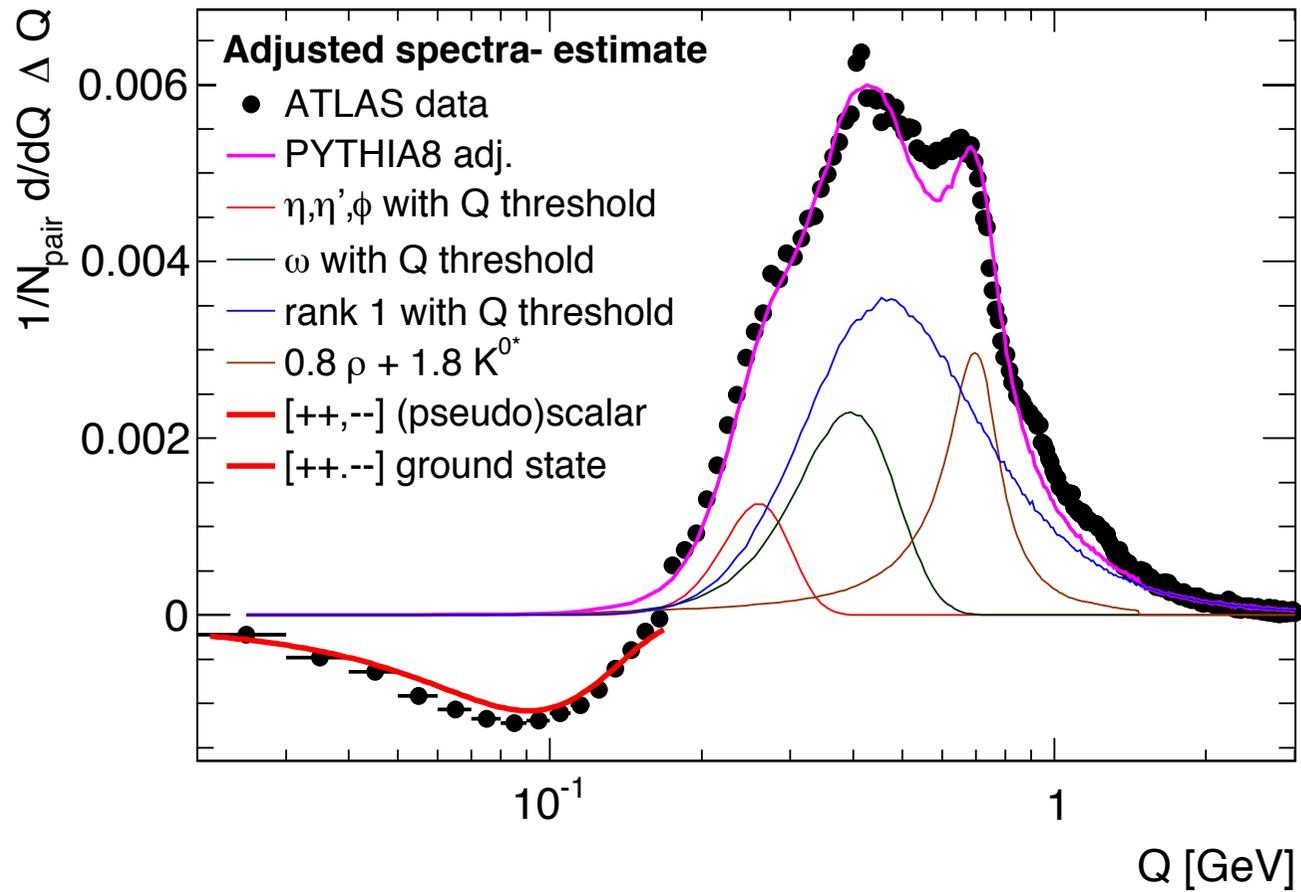
- Estimate** : take distribution from *Pythia8*, fit with *Landau/Gaus*, adjust width so that low edge does not overflow *Q* threshold “too much”



**for pseudoscalar mesons , the *Q* should coincide with the threshold value  $\eta'$  should produce close like-sign pairs ( jointly with rank 1 distribution )**



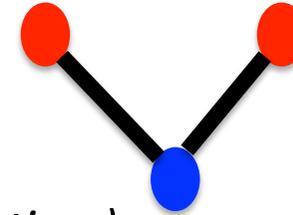
## The effect of enforced Q threshold is spectacular



# *It is possible to distinguish the coherent particle source*

*by study of the contribution from  $n$ -hadron chains ( $n > 2$ )*

*i.e., for each pair of close like-sign particles, take into account the “connecting” unlike-sign hadron (there must be one, due to the local charge conservation)*



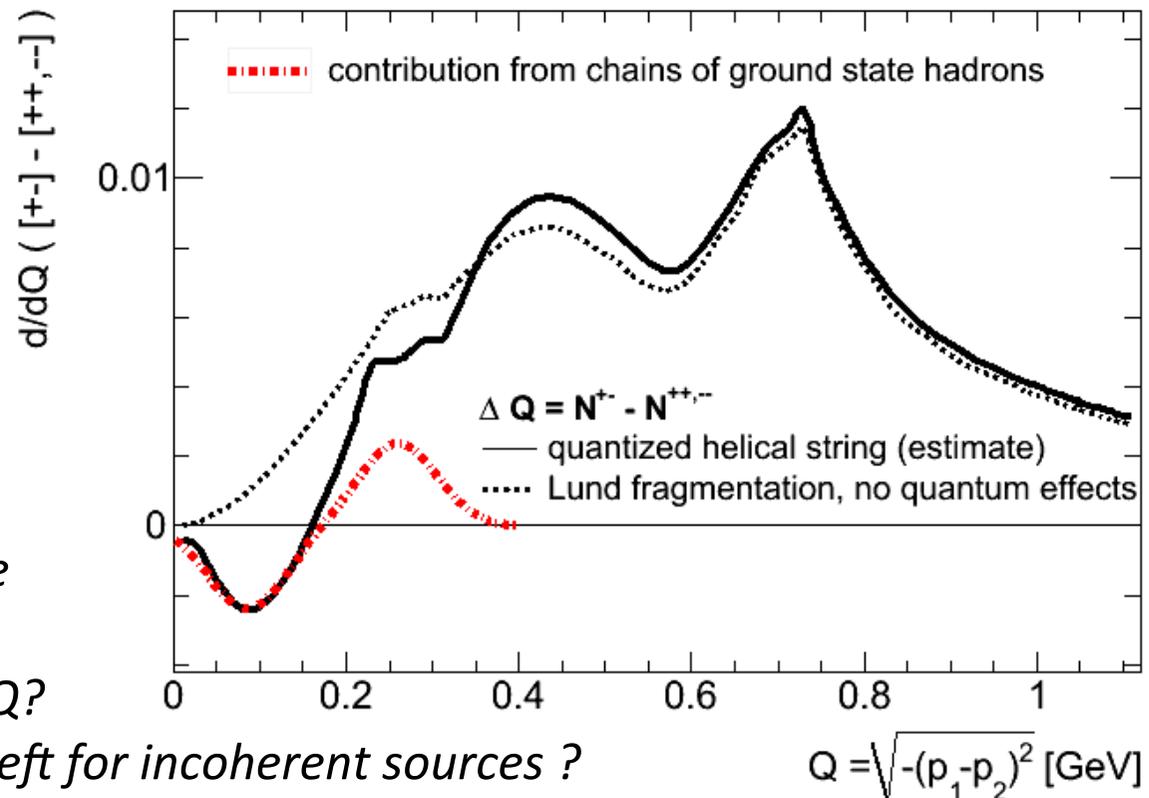
*Remember:*

*a chain of  $n$  ground state pions has mass  $M(n) \leq n \cdot 0.2 \text{ GeV}$*

*$M(\text{triplet}) \leq 0.6 \text{ GeV}$*

*? Do these chains generate enough correlations to describe the inclusive  $\Delta Q$ ?*

*If not, how much space is left for incoherent sources ?*



**Light meson spectra in quantized helix string model (fit, prediction)**

**pseudoscalar mesons**

**N  $\kappa$ R  $\Delta\Phi \Rightarrow$  N pions**

**vector mesons**

**N  $\kappa$ R  $\Delta\Phi \Rightarrow$  M pions ; M<N**

**strange mesons**

**+ 120 MeV**

**( ~ s quark mass )**

**N=1      $\pi$  : 135-140 , **137 $\pm$ 4****

**N=2**

**K : 493-497, **495  $\pm$  15****

**N=3      $\eta$  : 548 , **565  $\pm$  17****

**( $\omega$  : 782 , **799  $\pm$  24** )**

**N=4**

**$\rho$  : 769 , **766  $\pm$  23****

**K\* : 892 , **m( $\rho$ ) + 120****

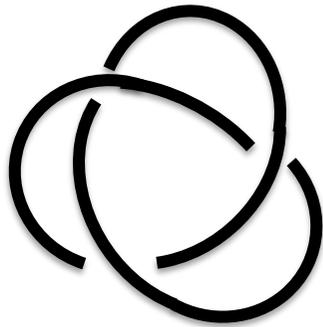
**( $\omega$  : 782 , **766  $\pm$  23** )**

**$\phi$  : 1020 , **m( $\rho$ ) + 2\*120****

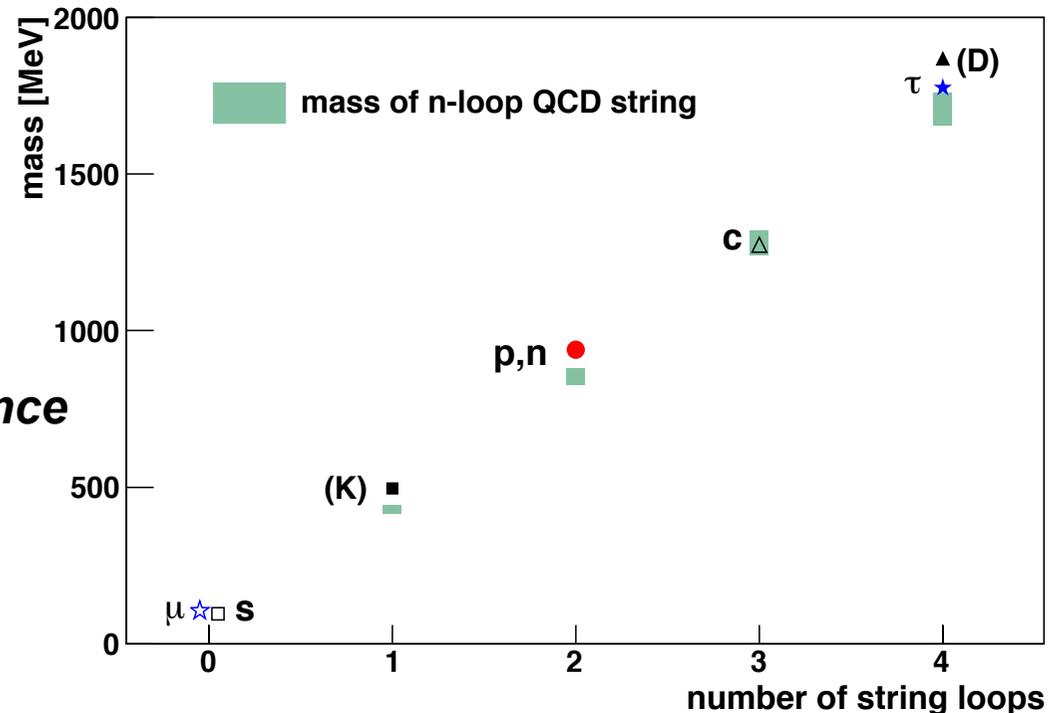
**N=5      $\eta'$ : 948 , **948  $\pm$  30****

Beyond scheme :  $f_0$  (s)  
~ incoherent fragmentation

## ***Cross-talk : what if the curly field interacts across loops ( “knotting” )***



***The mass of the multiple string loop correlates with the emergence of new quantum numbers***



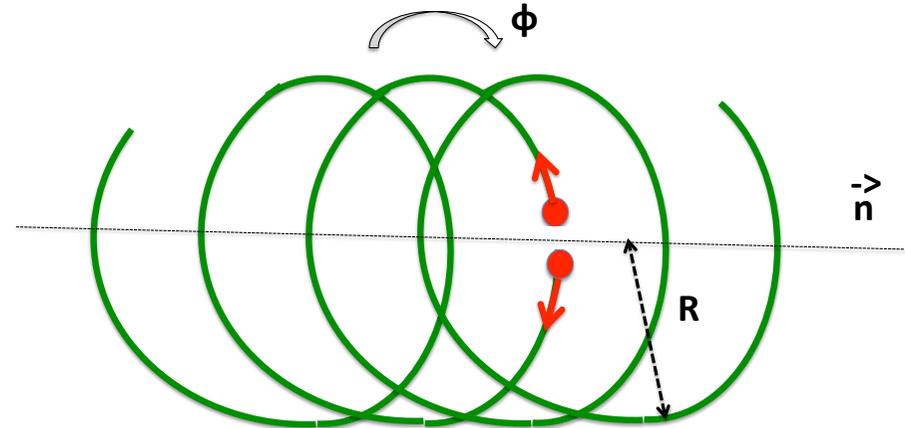
**[arXiv:1406.3564]**

- > topological origin of mass hierarchy ?***
- > increasingly complex knots generate new properties ?***
- > unknotting (of oriented knots) requires parity violating operation ( ~ weak decay ), or becomes impossible ( proton – trefoil ? )***

**Trajectory of parton under helix-string potential satisfies the uncertainty principle:**

**The particle appears at the classical trajectory**

$$\vec{x}(t) = \vec{x}(0) + \beta c t \vec{n}$$



**just once per the period for  $\omega \Delta t = k 2\pi$ ,  $k = 1, 2, 3 \dots$**

**And the corresponding action is**

$$x(\Delta t) p(\Delta t) - x(0) p(0) = \kappa c (\Delta t / \gamma)^2 = \kappa c \tau^2 = \kappa (0.183 \pm 0.01) \text{ fm}^2/c \approx \hbar$$

*$\tau$  is the invariant period of parton rotation (spin)*

**So called uncertainty principle cannot be broken**  
**in any sound description of the data**

- **The non-commutativity of  $x$ ,  $p$  stems directly from the fact that  $p$  is a differential ( $\sim d/dt x$ ) : the measurement of the momentum cannot be done instantly, one has to measure twice:**

**check the position of the measured object at time  $t_1$**   
**check the position of the measured object at time  $t_2 = t_1 + \Delta t$**   
**deduce velocity  $\rightarrow$  momentum**

**position change  $\rightarrow$  non-commutativity IS a PURE MATH !**

- **PHYSICS steps in with the EMPIRICAL observation that the  $\Delta t$  cannot be infinitesimally small : minimal action  $\sim \hbar$**   
**ALLWAYS INVOLVED**

**ANY THEORY COMPLYING with the MINIMAL ACTION REQUIREMENT is all right ( AND infrared safe ! )**