Quantum properties of QCD string fragmentation
(Šárka Todorova-Nová, Charles University, Prague)

- (Lund) string fragmentation in 1 and 3 dimensions
- Causal constraint & quantum properties
- Light meson spectra & QCD field properties
- 2-particle correlations in coherent hadron production (+ comparison with data)

NO TIME TO TALK ABOUT:
- inclusive \((p_T)\) spectra
- azimuthal ordering of hadrons
- polarization effects
- topological QCD
**Lund string fragmentation model**


**Semi-classical model:**
- confinement modeled by relativistic
  massless string with constant string tension ( $\kappa \sim 1 \text{ GeV/fm}$ )

- massless partons moving in light-cone coordinates

- the area span of the string (A) corresponds to the action

- homogenous QCD field with probability of breakup $\sim \exp(-b\Gamma)$

**Related to Wilson loops**

$M \sim \exp(i \xi A)$
**Mass and transverse momentum generation**

*strongly influenced by the absence of cross-talk between adjacent string breakups:*

**QUANTUM TUNNELING**

*evoked by the model:*

- -> to generate intrinsic $p_T$
- -> to ensure proper mass spectrum

A new $qq$ pair is created from ‘vacuum’ and assigned a randomly sampled transverse momentum

( *local charge and momentum conservation holds*)

*Two adjacent string break-ups define a HADRON with*$ \vec{p}_T = \vec{p}_1 - \vec{p}_2$

$\vec{p}_1 = \kappa (t_1 - t_2)$

$E = \kappa (x_1 - x_2)$

( *breakups separated by space-like distance*)
QCD field is not 1-dimensional ....

most likely, a thin vortex tube
( superconductor type II )
[B. Andersson:”The Lund Model”, Cambridge 1998]

... what about working with a 3 dimensional string?

Helix string concept [JHEP09(1998)14.]

B. Andersson et al.: “Is there a screwiness at the end of partons showers? “

- introduced on the basis of helicity conservation to stabilize the (soft) end of parton cascades
i.e., we can IMPOSE time-like distance (↔ cross-talk) between string breakup vertices which define the hadron

Let’s concentrate on the case when information (about string break-up) travels along the string only, together with the massless parton:

**QCD field (string tension) transforms into:** longitudinal quark momentum

\[ p_{\parallel} = \kappa \beta c \Delta t \]

+ transverse quark momentum

\[ |p_T| = 2 \kappa R \sin\left(\frac{\Delta \Phi}{2}\right) \]

+ effective quark mass

\[ m_T = \sqrt{m^2 + p_T^2} = \kappa R \Delta \Phi \]

\[ \beta^2 = 1 - \left(\frac{R \omega}{c}\right)^2 , \quad \omega = \frac{\Delta \Phi}{\Delta t} \]
-> propagating quark “triggers” the next breakup & creation of hadron with mass

\[ m_S = \kappa R \sqrt{(\Delta \Phi)^2 - (2 \sin \Delta \Phi / 2)^2} \]

( depends on TRANSVERSE shape of string only, longitudinal momentum and transverse mass decouple )

-> these should be narrow resonant states

? Quantization in \( \Delta \Phi (m_T) \), does it reproduce hadron spectra?

Let’s consider light pseudoscalar mesons: \( \pi, \eta(547), \eta'(948) \)
( they fit well in the picture of chains of ground state hadrons )

• \( \pi \) ground state (lightest hadron) ; \( \eta \rightarrow 3 \pi \) ; \( \eta' \rightarrow 5 \pi \)
• quantization: \( E_T (n=1,3,5) = n \kappa R \Delta \Phi \)
  \( p_T (n=1,3,5) = 2 \kappa R | \sin( n \Delta \Phi / 2 ) | \)
  \( m = \sqrt{E_T^2 - p_T^2} \)

2 parameters to constrain : \( \kappa R \) [GeV], \( \Delta \Phi \) [rad]
The fit is overconstrained - but there IS a solution which agrees with the experimental data with precision of ~3%.

### TABLE I

<table>
<thead>
<tr>
<th>meson</th>
<th>PDG mass [MeV]</th>
<th>model estimate [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>135 - 140</td>
<td>137</td>
</tr>
<tr>
<td>$\eta$</td>
<td>548</td>
<td>565</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>958</td>
<td>958</td>
</tr>
</tbody>
</table>

Best fit of the parameters of the pion ground state obtained from the mass spectrum of light pseudoscalar mesons. The $\eta$ mass is reproduced within a 3% margin which serves as the base of uncertainty for $R, \Delta \Phi$ parameters.

arXiv:1309.6761
**UNEXPECTED CONNECTION to topological QCD : fitted scales in excellent agreement**

[arXiv:1309.6761]  
**estimate of radius of helix-like QCD string**  
\[ \kappa R = 68 \pm 2 \text{ MeV} \]  
*obtained from the fit of mass of light pseudoscalar mesons*

\[ E_T(n) = \kappa R \cdot n \Delta \Phi \quad \text{[helical QCD string]} \]

[arXiv:1212.1500]  
**estimate of the confinement scale**  
\[ \Lambda_{\text{tube}} = 65.16 \pm 0.61 \text{ MeV} \]  
*obtained from the fit of J++ spectra modelled as tight topological QCD knots*

\[ E(K) = \Lambda_{\text{tube}} \varepsilon(K) \quad \text{[knotted flux tube]} \]
- **intrinsic** $p_T$ of (ground state) pions $\sim 0.13$ GeV
  Just about what is needed to “regularize”
  the soft particle production in the helix string model?

- **minimal distance between adjacent hadrons (+- pairs)**
  “Q threshold” at $\sim 0.26$ GeV
  (ground state pions) -> direct impact on the correlation pattern

  $Q = \sqrt{- (p_i - p_{i+1})^2} \approx |p_{Ti} - p_{Ti+1}|$
  $\approx 2 p_T^{\text{thr}} |\sin(\Delta\Phi/2)|$

- **Correlations for like-sign pairs**
  with $Q^{++,-,-} \approx 2 p_T^{\text{thr}} |\sin(2\Delta\Phi/2)| \approx 0.09$ GeV
**How to extract the adjacent pairs from the combinatorial background?**

MC truth (PYTHIA8): bin per bin, nearly perfect subtraction of non-adjacent pair contribution

- rank>1: non-adjacent (subtracted)
- rank=1: adjacent direct (correlated)
- rank=0: common direct mother (resonant spectrum)
- rank=-1: different strings (random, subtracted)
Can be analysed in 2 ways

- assuming incoherent particle production
  -> ratio -> correlation function (BE effect)
- assuming coherent particle production
  -> subtraction -> (resonant) shape

\[ \Delta Q = Q(+-) - Q(\pm\pm) \]

The shape of subtracted Q distributions agrees well with the expectations of the quantized helix string model:

Threshold for adjacent, rank 1 (+-) pairs production \(~ 0.26\) GeV

rank 2 (++,--) around \(Q \sim 0.09\) GeV
Helical QCD string or Bose-Einstein effect?

Coherent or incoherent origin of hadron correlations?

Inevitably, we approach the confrontation of the two concepts - competing over the same data (ALL hadronic data)  HELIX vs. BE

The question can be settled experimentally:

A/ in the coherent scenario, close like-sign pairs belong to a triplet ‘chains’ with mass $M_n < 0.6$ GeV

B/ no correlations between hadrons from colour disconnected sources

LEP2 : WW studies:
no correlations among “mixed” pairs
-> coherent origin favoured
HELIX – BE (1 : 0)

Elimination of free parameters from the modelling (hadron spectra can be deduced from correlation shape)
HELIX – BE (2 : 0)

LHC : minimum bias:
shape of correlations in agreement with model prediction
-> coherent origin favoured

Source size considerations:
-> coherent origin favoured
If the model survives the experimental scrutiny, it has a huge potential for further development:

Quantized QCD helix (hadronization)

Spin physics
( helical OCD field generates intrinsic $p_T$ AND intrinsic angular momentum )

Topological QCD
striking similarities between numerical results obtained with quantized helix and knotted flux tubes

Quantum theory of motion
(P. Holland, Bohm-de Broglie)

Non-classical component of the particle trajectory:
postulated /quantum potential/
derived from parton-field interaction

CAUSAL QUANTUM THEORY
with dynamically generated mass and spin effects
Summary

• Phenomenological (data driven) approach to the quantization of QCD field
• Helical QCD string seems to be an excellent approximation of the effective QCD field
• Causality provides the key to the study of the mass spectrum of light mesons, coupled with dynamical correlations
• Number of observable model predictions up for validation:
  - inclusive low $p_T$ ( < 200 MeV )
  - 2-particle correlations
  - azimuthal ordering
  - polarization studies

PHENOMENOLOGY:

2 degrees of freedom removed from the string fragmentation model & number of free parameters reduced
( ++ : numerical agreement with topological QCD studies )
- better hadronization model
back-up slides
**Documentation**

- Bo Andersson et al., JHEP 09 (1998) 014. [phenomenology of helix]
- Š. Todorova, arXiv:1012.5778 [hep-ph] [helix-tune with LEP data]
- [http://projects.hepforge.org/helix/] [PYTHIA-compatible code]
- Š. Todorova, Phys.Rev.D89, 015002 (2014) [quantization of helix]
- ATLAS Coll., ATL-COM-PHYS-2013-1657 (private) [study of hadron chains]
**Helix string concept** [JHEP09(1998)14.]

(introduced to stabilize the end of parton shower)

3dim string tension: let string transverse shape define the hadron $p_T$

$$|p_T| = 2\kappa R \sin (\Delta\phi/2)$$

$\kappa R$ helix radius

$\phi$ helix phase

LEP data show some correlations between longitudinal ($x_p$) and transverse $<p_T>$ momentum component ....


The removal of 2 degrees of freedom associated with the intrinsic $p_T$ generation leads to a significant improvement in the description of hadronic $Z^0$ data

[arXiv:1012.5778]
Indirect experimental evidence: tuning on $Z^0$ data

(DELPHI_1996_S3430090)

Helix string fragmentation improves “goodness of fit” ($\chi^2/N_{dof}$)

6.6 $\rightarrow$ 4.0
( Pythia 6
DELPHI 1996 tune)

4.0 $\rightarrow$ 2.4
(Pythia 6 with ARIADNE parton shower )

10/5/15
Azimuthal ordering of hadrons [Phys.Rev.D86,052005 (2012)]

The helix-like shape structure of the QCD field should be visible in the azimuthal ordering of hadrons along the string

The exact form of the helix structure not predicted.

With the help of power spectra, we test two (weakly correlated) hypotheses

A/ $\Delta \Phi \sim \Delta \eta$

$$S_{\eta}(\xi) = \frac{1}{N_{\text{ev}}} \sum_{n_{\text{ch}}} \left| \sum_{j} \exp(i(\xi \eta_j - \phi_j)) \right|^2$$

B/ $\Delta \Phi \sim \Delta X$ (energy-distance - amount of energy stored in the string/ ordered hadron chain - experimentally: ordered in pseudorapidity)

$$S_{E}(\omega) = \frac{1}{N_{\text{ev}}} \sum_{n_{\text{ch}}} \left| \sum_{j} \exp(i(\omega X_j - \phi_j)) \right|^2$$

Search for resonant behaviour -> density of helix winding
Azimuthal ordering

The model predicts azimuthal ordering of hadrons

\[ S_E(\omega) = \frac{1}{N_{\text{ev}} N_{\text{event}}} \frac{1}{n_{\text{ch}}} \left| \sum_j \exp(i(\omega X_j - \phi_j)) \right|^2 \]

\( \Delta \Phi \sim \Delta X \) (energy-distance: amount of energy stored in the string / pseudorapidity ordered hadron chain)


The model drastically reduces “randomness” in the intrinsic pT sector yet describes the relevant hadronic data better over large span of energies ....
Min. bias inclusive (jet dominated)

ATLAS

$\sqrt{s} = 7$ TeV

$n_{ch}>10$, $\max(p_T)<10$ GeV

$p_T>100$ MeV, $|\eta|<2.5$

- Data 2010
- PHOJET
- PYTHIA8 4C
- PYTHIA6 AMBT2b
- HERWIG++ UE7-2 soft

Min. bias low $p_T$ selection (dominated by fragmentation)

ATLAS

$\sqrt{s} = 7$ TeV

$n_{ch}>10$, $\max(p_T)<1$ GeV

$p_T>100$ MeV, $|\eta|<2.5$

- Data 2010
- PHOJET
- PYTHIA8 4C
- PYTHIA6 AMBT2b
- HERWIG++ UE7-2 soft
.. the model is not complete ...

The azimuthal ordering is not well described if the helix model operates on direct hadrons only -> decay of short-lived resonances incorporated in the model

.. but the enhanced helix model produces way too many soft particles (Z⁰ data no longer so well described)

p_T threshold needed?

Quantum effects need to be understood and included!

Resonances & correlations part of the model

The string area can be interpreted in terms of action, the tuning of helical string parameters suggest \( \kappa R \Delta \varphi c \sim \hbar \)
How to extract the adjacent pairs from the combinatorial background?

“standard” string fragmentation preserves 4-momentum, flavour and charge at each breakup vertex -> adjacent hadrons correlated + strong charge-combination asymmetry (adjacent charged like-sign pairs forbidden) -> non-adjacent direct hadrons uncorrelated

! Q (+- -) Q (++,--) removes very efficiently the non-adjacent hadron pairs!

Probability to find pair of hadrons with rank 2 about the same for like-sign and unlike-sign pair:

```
Rank: +0  +1  +2
  0 0 -
 +0 - +0
```

PYTHIA8 minbias (GENEV1)
- true resolved hadron chain
- [+-] - [++,--] (all pairs)

$p_t \geq 100 \text{ MeV}$

$| \eta | < 2.5$

S.Todorova-Nova, ISMD 2015
The residual shape after subtraction reflects the spectrum of resonances (+ adjacent pairs)
The residual shape after subtraction reflects the spectrum of resonances (+ adjacent pairs)

Low $Q$ dominated by $\eta, \eta', \Phi$ ($K \to \pi$ in our approximation)
Effect of the quantization on the subtracted 2-particle densities

Combined effect of **quantum threshold for adjacent unlike-sign pairs**
and the **enhanced**
production of
close like-sign
pairs (of rank 2)
leads to
emergence
of BE-like
correlation
pattern

(enhanced production
of like-sign pairs)

Source of correlations: **coherent** particle emission
Comparison with generator level
Pythia8 minbias sample
shows the Q threshold is violated in MC
for both rank 0 pairs (resonance decays)
and rank 1 pairs (adjacent)

The “resonant” spectrum
dominated by

\[ \eta, \eta', \omega, \rho, K^*, \Phi \]

(\(K^0\) neglected here)

In the absence of particle identification,
the spectrum can be roughly splitted
into 3 “peaks”: 1/ \(\eta+\eta'+\Phi\)
2/ \(\omega (+K)\)
3/ \(\rho+K^*\)
**Enforcing Q threshold**

- Estimate: take distribution from Pythia8, fit with Landau/Gaus, adjust width so that low edge does not overflow Q threshold “too much”

For pseudoscalar mesons, the Q should coincide with the threshold value. η' should produce close like-sign pairs (jointly with rank 1 distribution) to be restored (2-body ω decay).
The effect of enforced $Q$ threshold is spectacular

Adjusted spectra- estimate
- ATLAS data
- $\eta,\eta',\phi$ with $Q$ threshold
- $\omega$ with $Q$ threshold
- rank 1 with $Q$ threshold
- $0.8\rho + 1.8K^0$
- $[++]$ (pseudo)scalar
- $[+-]$ ground state

$1/N_{pair} \frac{dQ}{dQ} \Delta Q$

Q [GeV]
It is possible to distinguish the coherent particle source

by study of the contribution from n-hadron chains (n > 2)

i.e., for each pair of close like-sign particles, take into account the “connecting” unlike-sign hadron (there must be one, due to the local charge conservation)

Remember:
a chain of 
n ground state 
pions has mass

\[ M(n) \leq n \times 0.2 \text{ GeV} \]

\[ M(\text{triplet}) \leq 0.6 \text{ GeV} \]

\[ \Delta Q = N^- - N^{++} \]

\[ Q = \sqrt{- (p_1 - p_2)^2} \text{ [GeV]} \]

? Do these chains generate enough correlations to describe the inclusive \( \Delta Q \)?

If not, how much space is left for incoherent sources?
**Light meson spectra in quantized helix string model (fit, prediction)**

<table>
<thead>
<tr>
<th>Meson Type</th>
<th>Quantum Number</th>
<th>Mass (MeV)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudoscalar</td>
<td>$N = 1$</td>
<td>$\pi$</td>
<td>135-140, 137±4</td>
</tr>
<tr>
<td></td>
<td>$N = 2$</td>
<td>$K$</td>
<td>493-497, 495 ± 15</td>
</tr>
<tr>
<td></td>
<td>$N = 3$</td>
<td>$\eta$</td>
<td>548, 565 ± 17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\omega$</td>
<td>782, 799 ± 24</td>
</tr>
<tr>
<td></td>
<td>$N = 4$</td>
<td>$\rho$</td>
<td>769, 766 ± 23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K^*$</td>
<td>892, m($\rho$) + 120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\phi$</td>
<td>1020, m($\rho$) + 2*120</td>
</tr>
<tr>
<td></td>
<td>$N = 5$</td>
<td>$\eta'$</td>
<td>948, 948 ± 30</td>
</tr>
</tbody>
</table>

Beyond scheme: $f_0(s)$

~ incoherent fragmentation

10/5/15

S.Todorova-Nova, ISMD 2015
Cross-talk: what if the curly field interacts across loops ("knotting")

The mass of the multiple string loop correlates with the emergence of new quantum numbers

 histórico

- topological origin of mass hierarchy?
- increasingly complex knots generate new properties?
- unknotting (of oriented knots) requires parity violating operation (~ weak decay), or becomes impossible (proton – trefoil?)
Trajectory of parton under helix-string potential satisfies the uncertainty principle:

The particle appears at the classical trajectory

\[ \vec{x}(t) = \vec{x}(0) + \beta \, c \, t \, \hat{n} \]

just once per the period for \( \omega \, \Delta t = k \, 2\pi \), \( k = 1, 2, 3 \ldots \)

And the corresponding action is

\[ x(\Delta t) \, p(\Delta t) - x(0) \, p(0) = \kappa c \, (\Delta t / \gamma)^2 = \kappa c \, \tau^2 = \kappa \, (0.183 \pm 0.01) \, \text{fm}^2/c \approx \hbar \]

\( \tau \) is the invariant period of parton rotation (spin)
So called uncertainty principle cannot be broken in any sound description of the data

- The non-commutativity of $x$, $p$ stems directly from the fact that $p$ is a differential ($\sim d/dt x$): the measurement of the momentum cannot be done instantly, one has to measure twice:

  check the position of the measured object at time $t_1$
  check the position of the measured object at time $t_2 = t_1 + \Delta t$
  deduce velocity $\rightarrow$ momentum

  position change $\rightarrow$ non-commutativity IS a PURE MATH!

- PHYSICS steps in with the EMPIRICAL observation that the $\Delta t$ cannot be infinitesimally small: minimal action $\sim \hbar$
  ALWAYS INVOLVED

ANY THEORY COMPLYING with the MINIMAL ACTION REQUIREMENT is all right (AND infrared safe!)