FINITE SIZE OF HADRONS AND BOSE-EINSTEIN CORRELATIONS

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B-E CORRELATIONS; STANDARD APPROACH

BOSE-EINSTEIN CORRELATION BETWEEN MOMENTA OF TWO IDENTICAL HADRONS

$$C(p_1, p_2) \equiv \frac{N(p_1, p_2)}{N(p_1)N(p_2)} \qquad (1)$$

IS USUALLY ANALYZED USING THE FORMULA

$$C(p_1, p_2) = 1 + \frac{\tilde{w}(P_{12}; Q)\tilde{w}(P_{12}; -Q)}{w(p_1)w(p_2)} = 1 + \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} \geq 1 \quad (2)$$

HERE $w(p, x)$ IS THE SINGLE-PARTICLE "DISTRIBUTION" (WIGNER FUNCTION) AND

$$\tilde{w}(P_{12}; Q) = \int dx\; e^{iQx} w(P_{12}; x); \qquad w(p) = \int dx\; w(p; x)$$
$$P_{12} = (p_1 + p_2)/2; \qquad Q = p_1 - p_2,$$

ONE SEES THAT FROM $C(p_1, p_2)$ ONE CAN GET INFORMATION ON THE DISTRIBUTION IN x.

DATA L3(1)

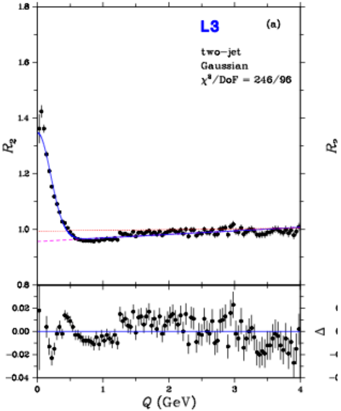


Figure: HBT correlation function from L3; Two-jet events.

DATA L3(2)

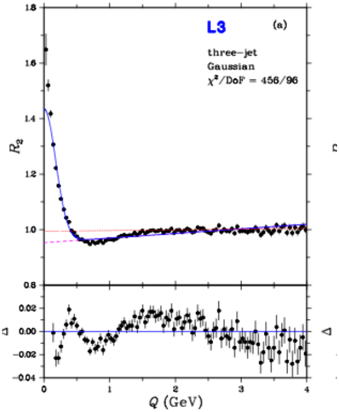


Figure: L3 data for three-jet events.

DATA CMS 1

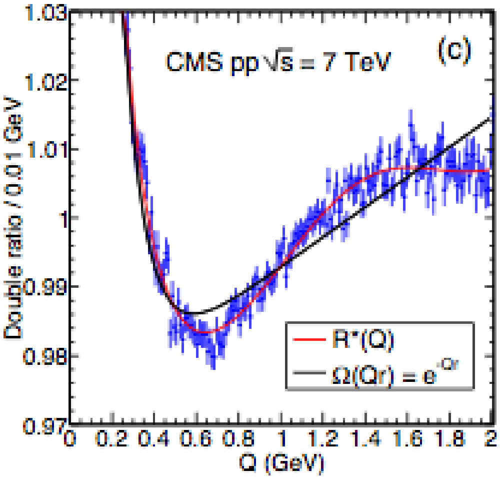


Figure: Two-pion correlation function from CMS (pp at 7 TeV)

DATA CMS 2

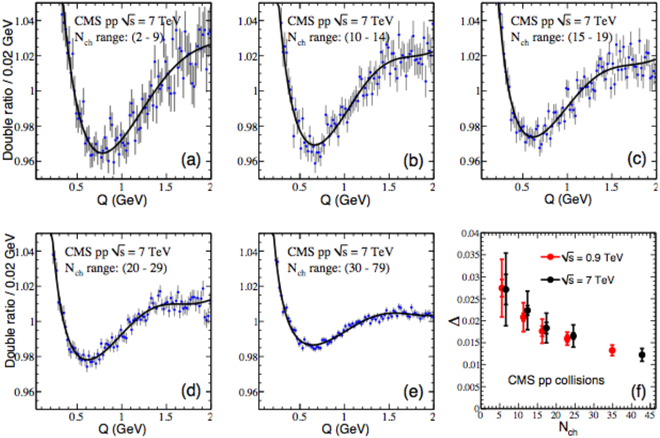


Figure: Two-pion correlation function for various multiplicities from CMS (pp at 7 TeV)

GENERAL TWO PARTICLE CORRELATIONS

LET $W(p_1, p_2; x_1, x_2)$ **BE THE MOMENTUM AND SPACE "DISTRIBUTION" OF TWO PARTICLES ("SOURCE FUNCTION"). IF PARTICLES ARE IDENTICAL, THE OBSERVED MOMENTUM DISTRIBUTION IS**

$$\Omega(p_1, p_2) = \int dx_1 dx_2 W(p_1, p_2; x_1, x_2) + \\ + \int dx_1 dx_2 e^{i(x_1 - x_2)Q} W(P_{12}, P_{12}; x_1, x_2) \equiv \\ \equiv \Omega_0(p_1, p_2) C(p_1, p_2) \tag{3}$$

WHERE $P_{12} = (p_1 + p_2)/2$, $Q = p_1 - p_2$, **AND**

$$\Omega_0(p_1, p_2) = \int dx_1 dx_2 W(p_1, p_2; x_1, x_2) \tag{4}$$

NO CORRELATIONS BETWEEN PARTICLES

IF THERE ARE NO CORRELATIONS BETWEEN PARTICLES,

$$W(p_1, p_2; x_1, x_2) = w(p_1, x_1) w(p_2, x_2)$$

THEN $\Omega(p_1, p_2) = w(p_1) w(p_2) + |\tilde{w}(P_{12}, Q)|^2$**,**

WHERE $\tilde{w}(P_{12}, Q) = \int dx \; w(P_{12}, x) e^{ixQ}$.

THUS THE CORRELATION FUNCTION IS

$$C_2(p_1, p_2) = \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1) w(p_2)} \geq 1!!!! \tag{5}$$

THIS IS THE COMMONLY USED FORMULA.

FROM $\tilde{w}(P_{12}, Q)$ **ONE CAN RECOVER** $w(P_{12}, x)$**.**

BUT: *THIS IS VALID ONLY IF THERE ARE NO INTER-PARTICLE CORRELATIONS.*

CORRELATIONS IN SPACE (1)

IDEA: WHEN PIONS ARE TOO CLOSE TO EACH OTHER THEY ARE *NOT* PIONS ANYMORE!!!
(BECAUSE THEIR CONSTITUENTS ARE MIXING AND THEIR WAVE FUNCTIONS ARE NOT WELL-DETERMINED).

SINCE HBT EXPERIMENTS MEASURE QUANTUM INTERFERENCE BETWEEN THE WAVE FUNCTIONS OF PIONS, THEY CANNOT SEE PIONS WHICH ARE TOO CLOSE TO EACH OTHER.

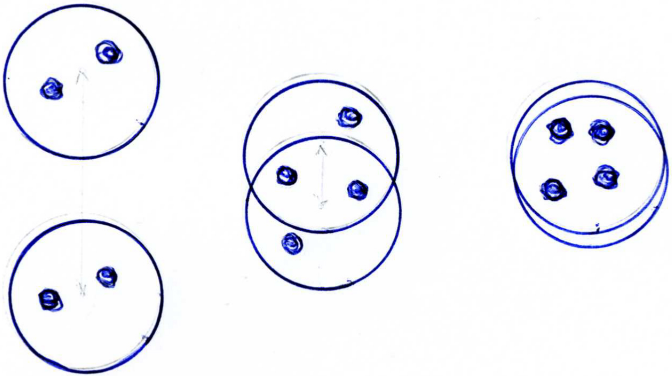
THEREFORE THE TWO-PARTICLE DISTRIBUTION

$$W(P_{12}, P_{12}; x_1, x_2)$$

MUST VANISH AT SMALL $|x_1 - x_2|$, IMPLYING *CORRELATION* **BETWEEN POSITIONS OF TWO PIONS.**

PICTURE

MIXING OF QUARKS



CORRELATIONS IN SPACE (2)

Repeat: $W(P_{12}, P_{12}; x_1, x_2)$ **MUST VANISH AT** $|x_1 - x_2| \approx 0$**, MEANING** *CORRELATION* **BETWEEN POSITIONS OF TWO PIONS. THIS IS THE** *NECESSARY* **CONSEQUENCE OF THE FUNDAMENTAL PROPERTY OF HADRONS: THEY ARE** *NOT* **POINT-LIKE.**

THUS THE TWO-PION DISTRIBUTION IS OF THE FORM

$$W(P_{12}, P_{12}; x_1, x_2) = w(P_{12}; x_1) w(P_{12}; x_2)[1 - D(x_1 - x_2)]. \quad (6)$$

WHERE THE "CUT-OFF" FUNCTION $D(x_1 - x_2)$ **EQUALS 1 AT SMALL** $(x_1 - x_2)$ **(BELOW, SAY, 1 fm) AND VANISHES AT LARGER DISTANCES.**

CORRELATIONS IN SPACE (3)

THE HBT CORRELATION FUNCTION BECOMES:

$$C(P_{12},Q) = 1 + \frac{|\tilde{w}(P_{12},Q)|^2}{w(p_1)w(p_2)} - C_{corr}(p_1,p_2);$$

$$C_{corr} = \frac{\int dx_1 dx_2 e^{i(x_1-x_2)Q} w(P_{12};x_1)w(P_{12};x_2)D(x_1-x_2)}{w(p_1)w(p_2)} \qquad (7)$$

ONE SEES THAT THE CORRELATED PART IS NEGATIVE. MOREOVER, SINCE IT GETS CONTRIBUTION ONLY FROM THE REGION OF SMALL x**, IT EXTENDS TO LARGER** Q **THAN THE FIRST, UNCORRELATED, PART. CONSEQUENTLY, THE TOTAL HBT CORRELATION FUNCTION IS EXPECTED TO BE** *NEGATIVE* **AT LARGE ENOUGH** Q**.**

EXAMPLE: AB&KZ, PLB727(2013)182

FOR ILLUSTRATION, TAKE

$$\Delta(x_1 - x_2) = \Theta[r_{cut}^2 - |\vec{x}_1 - \vec{x}_2|^2 - (t_1 - t_2)^2];$$

$$w(P, x) = e^{-|\vec{x}|^2/R^2} e^{-t^2/\tau^2} f(P)$$

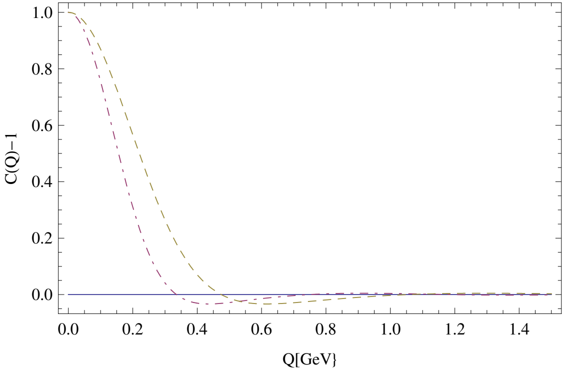


Figure: Oscillating two-pion correlation function. $R = r_{cut} = \tau = 1$ fm.

MORE REALISTIC ESTIMATE: Blast wave model

STEP 1: Describe the $p_\perp$ and multiplicity dependence of the measured HBT radii in terms of the assumed size and shape of the emission volume, *ignoring* the space-time correlations. This allows to determine the parameters of the model.
The assumed shape: (i) fixed proper time $\tau_f = \sqrt{t^2 - z^2}$;
$f(r_\perp) = e^{-(R-r_\perp)^2/d^2}$, corresponding to emission from a "shell" of radius R and the "width" d.
STEP 2: Introduce the space-time correlation (cut-off) and evaluate the full correlation function.
We tried: (i) a Gaussian $D = e^{-d^2/\Delta_G^2}, \Delta_G = 1$ fm (where d is the space-time distance between particles),

$$d^2 = d_\perp^2 + d_{||}^2;\ d_{||} = \tau_f(\eta_1 - \eta_2)$$

and (ii) a sharp cut-off in d at $\Delta_s = 0.75$ fm; There results are very close.
STEP 3: verify that the description of the HBT radii is unaffected. Checked, OK

Blast-wave model description of the HBT radii at 7 TeV

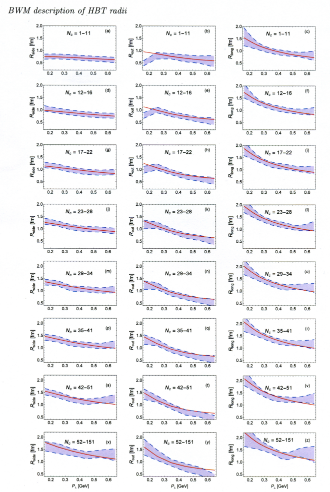


Figure: HBT radii [evaluated as the inverse slope of the correlation function at $Q^2 = 0$] compared to the ALICE data in pp collisions at 7 TeV [BFZ, JOPhys. G42 (2015) 045001]

RESULTS: LONG

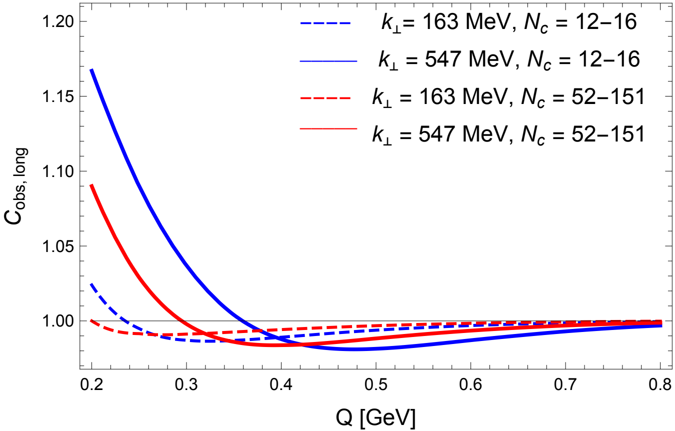


Figure: (Color online) Correlation function $C_{\rm obs}$ for the *long* direction in the interval 0.2 GeV $\leq Q \leq$ 0.8 GeV (normalized to 1 at Q= 1GeV). The dashed lines describe the results for $k_\perp = 163$ MeV and the two multiplicity classes: $N_c = 12$–16 and $N_c = 52$–151. The solid lines describe the results for $k_\perp = 547$ MeV and the same two multiplicity classes.

RESULTS: SIDE

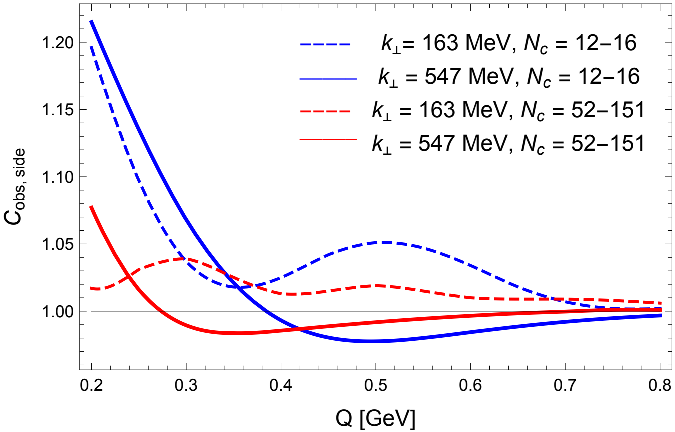


Figure: The same as Fig. 1 but for the side direction.

RESULTS: OUT

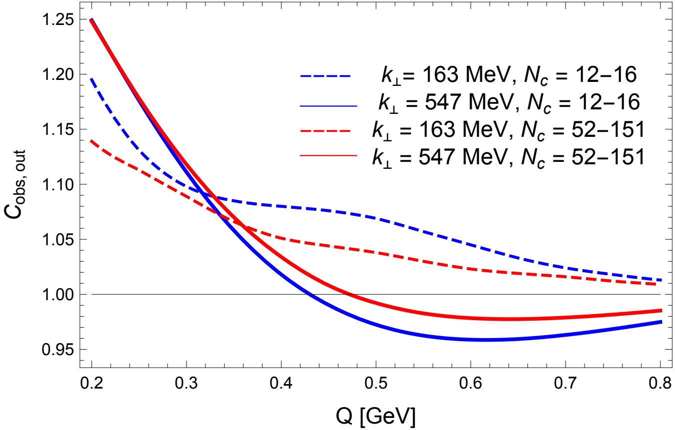


Figure: The same as Fig. 1 but for the out direction.

COMMENTS

The presented argument shows that the (observed) values of the HBT correlation function BELOW 1 are not accidental but reflect the fundamental fact that hadrons are NOT POINT-LIKE. Therefore this region of Q^2 deserves special attention.
Our calculations indicate that measurements of the correlation functions in all three directions may reveal an interesting pattern of complicated behaviour which hardly can be described by simple gaussian fits. A detailed experimental investigation of this region should allow to determine (a) the validity of the presented approach and (b) the size of the space-time cut-off, i.e. the distance at which the hadron structure is affected by its neighbours.
Consequently, one can also learn about the DENSITY at which the hadron gas starts melting into quarks and gluons.

DERIVATION OF THE HBT FORMULA (I)

Density matrix in momentum space:

$$\rho(p_1, p_2; p_1', p_2') =$$
$$= \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_1' x_1' + p_2' x_2')} \rho(x_1, x_2; x_1', x_2') \quad (8)$$

The particle distribution is

$$\Omega(p_1, p_2) = \rho(p_1, p_2; p_1, p_2) \quad (9)$$

The Wigner function:

$$W(p_1, p_2; x_1^+, x_2^+) = \int dx_1^- dx_2^- e^{i(p_1 x_1^- + p_2 x_2^-)} \rho(x_1, x_2; x_1', x_2') \quad (10)$$

with

$$x^+ = (x + x')/2; \quad x^- = x - x' \quad (11)$$

DERIVATION OF THE HBT FORMULA (II)

Symmetrization:

$$\rho(p_1, p_2; p_1', p_2') \to \rho(p_1, p_2; p_1', p_2') + \rho(p_1 p_2; p_2', p_1') \tag{12}$$

$$\Omega(p_1, p_2) = \rho(p_1, p_2; p_1, p_2) + \rho(p_1 p_2; p_2, p_1) =$$

$$= \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_1 x_1' + p_2 x_2')} \rho(x_1, x_2; x_1', x_2') +$$

$$+ \int dx_1 dx_2 e^{i(p_1 x_1 + p_2 x_2)} \int dx_1' dx_2' e^{-i(p_2 x_1' + p_1 x_2')} \rho(x_1, x_2; x_1', x_2') \tag{13}$$

$$dx_1 dx_1' = dx_1^+ dx_1^-; \quad dx_1 dx_1' = dx_2^+ dx_2^-$$

$$p_1 x_1 + p_2 x_2 - p_1 x_1' - p_2 x_2' = p_1 x_1^- - p_2 x_2^-$$

$$p_1 x_1 + p_2 x_2 - p_2 x_1' - p_1 x_2' = P_{12} x_1^- + P_{12} x_2^- + Q(x_1^+ - x_2^+) \tag{14}$$

$$P_{12} = (p_1 + p_2)/2; \quad Q = p_1 - p_2$$

DERIVATION OF THE HBT FORMULA (III)

$$\Omega(p_1,p_2)=\int dx_1^+dx_2^+\int dx_1^-dx_2^-e^{i(p_1x_1^- - p_2x_2^-)}\rho(x_1,x_2;x_1',x_2')+$$
$$+\int dx_1^+dx_2^+e^{iQ(x_1^+-x_2^+)}\int dx_1^-dx_2^-e^{i(P_{12}x_1^-+P_{12}x_2^-)}\rho(x_1,x_2;x_1',x_2')=$$
$$=\int dx_1^+dx_2^+W(p_1,p_2;x_1^+,x_2^+)+$$
$$+\int dx_1^+dx_2^+e^{iQ(x_1^+-x_2^+)}W(P_{12},P_{12};x_1^+,x_2^+) \qquad (15)$$

If particles are uncorrelated, i.e.

$$W(p_1,p_2;x_1,x_2)=W(p_1,x_1)W(p_2,x_2)$$

one obtains

$$\Omega(p_1,p_2)=\Omega(p_1)\Omega(p_2)+\tilde{W}(P_{12},Q)\tilde{W}^*(P_{12},Q) \qquad (16)$$