c and b quark masses from lattice QCD.

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Intro & Motivation

- Quark masses – fundamental parameters of the Standard Model.
- Many applications to phenomenology and BSM physics. Example: Higgs partial widths.
  - Couplings proportional to quark masses.
  - Main source of uncertainty in partial widths from m_b, m_c, α_s. [1404.0319]
- Focus on recent lattice results for charm and bottom masses.

Outline

- Background
  - Theory background.
  - Lattice determinations.
- Charm mass
  - Time moments of $\langle JJ \rangle$ correlators.
  - Comparison with perturbation theory.
- Bottom mass
  - Different approaches.
  - $\langle JJ \rangle$ moments in NRQCD.
- Mass ratios
  - m_c/m_b
  - m_s/m_c
- Future Work & Conclusions

Quark mass – definitions

- Quarks are not asymptotic (physical) states.
- Quark masses are scheme and scale dependent, $m_q^{\text{scheme}}(\mu)$.
- Generally will quote results $m_q^{\overline{\text{MS}}}(\mu_{\text{ref}})$.

Lattice determination of quark mass

Bare quark masses are input parameters to lattice simulations. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud0} \to m_\pi^2$
- $m_{s0} \to m_K^2$
- $m_{c0} \to m_{\eta_c}$

Tuning performed at multiple lattice spacings, defining a continuum trajectory for which $a^2 \to 0$ limit can be taken.

- Rest of physics is then prediction of QCD.
- Parameters can be varied away from physical values.. understand effect of quark mass, quantify systematics, etc.

c-quark mass

Simulating charm

Heavy quarks are challenging to simulate.

- Requires $am_0 < 1$ to keep discretization effects under control.
- Need large enough box to minimize finite-volume effects $\rightarrow N_{\text{site}}$ large.

These conditions can be satisfied by using a highly improved action (e.g. HISQ).

HISQ action

HISQ action

- No $\mathcal{O}(a^2)$ discretization errors (begin at $\mathcal{O}(\alpha_s a^2)$).
- Significant $\mathcal{O}(\alpha_s a^2)$ effects are in turn suppressed.

$n_f = 4$ simulations

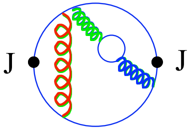
- Charm quarks in the sea.
- Avoid applying perturbation theory at m_c (matching $n_f = 4 \to 3$).

It is increasingly feasible to simulate the b quark relativistically.

Current-current correlators

Calculate time-moments of $J_5 \equiv \bar{\psi}_h \gamma_5 \psi_h$ correlators:

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle J_5(t, \mathbf{x}) J_5(0, 0) \rangle$$



- Currents are absolutely normalized (no Zs required).
- $G(t)$ is UV finite $\rightarrow G(t)_{\text{cont}} = G(t)_{\text{latt}} + \mathcal{O}(a^2)$.

Moments

The time-moments $G_n = \sum_t (t/a)^n G(t)$ can be computed in perturbation theory. For $n \geq 4$,

$$G_n = \frac{g_n(\alpha_{\overline{\mathrm{MS}}}, \mu)}{am_h(\mu)^{n-4}} \, .$$

Basic strategy:

1. Calculate $G_{n,\mathrm{latt}}$ for a variety of lattice spacings and m_{h0}.
2. Compare continuum limit $G_{n,\mathrm{cont}}$ with $G_{n,\mathrm{pert}}$ (at reference scale $\mu = m_h$, say).
3. Determine best-fit values for $\alpha_{\overline{\mathrm{MS}}}(m_h), m_h(m_h)$.

Reduced moments

In practice comparison carried out using reduced moments.

$$
\begin{aligned}
R_4 &= G_4/G_4^{(0)} \\
R_n &= \frac{1}{m_{0c}}(G_n/G_n^{(0)})^{1/(n-4)} \quad (n \geq 6)\,.
\end{aligned}
$$

On the perturbative side,

$$
\begin{aligned}
R_4 &= r_4(\alpha_{\overline{\mathrm{MS}}}, \mu) \\
R_n &= \frac{1}{m_c(\mu)}\, r_n(\alpha_{\overline{\mathrm{MS}}}, \mu) \quad (n \geq 6)\,.
\end{aligned}
$$

Reference scale is taken as $\mu = 3m_h(= m_c \frac{m_{h0}}{m_{c0}})$.

Some details

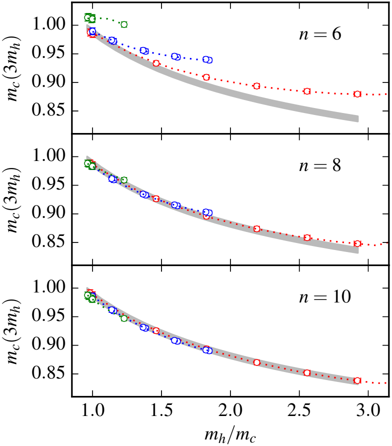
- Calculate moments for $n = 4, 6, 8, 10$.
- Three lattice spacings: $a \approx 0.12, 0.09, 0.06$ fm. (MILC)
- Seven input masses from $m_h = m_c - 0.7 m_b$.

All data points fit simultaneously with perturbative R_n expressions $\rightarrow m_c^{\overline{\mathrm{MS}}}(\mu)$, $\alpha_{\overline{\mathrm{MS}}}(\mu)$ for $\mu \approx 3 - 9$ GeV.

Uncertainties

- Non-perturbative terms/condensates.
  - $r_n(\alpha_{\overline{\rm MS}}) \to r_n(\alpha_{\overline{\rm MS}}) \left[1 + d_n(\alpha)\langle \frac{\alpha G^2/\pi}{(2m_h)^4}\rangle + \cdots\right]$
  - Effects supressed by $(\frac{\Lambda}{2m_h})^4$.
- Truncation of perturbation theory.
  - $r_n = 1 + \sum_j \alpha^j(m_h) r_{nj}$.
  - $j = 1, 2, 3$ known for $n \leq 10$
- Lattice artifacts.
  - Grow like $\alpha_s \times (am_h)^2$.
  - Decrease with increasing n.

Results for $n_f = 4$ [1408.4169]

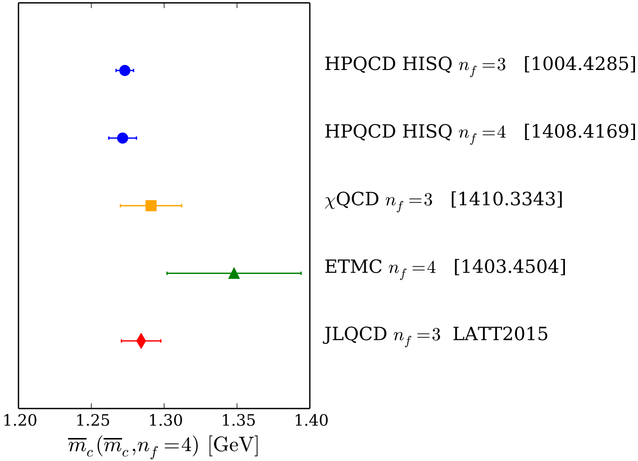


$$m_c(3m_h) = \frac{r_n(\alpha_{\overline{\mathrm{MS}}}, \mu = 3m_h)}{R_n}$$

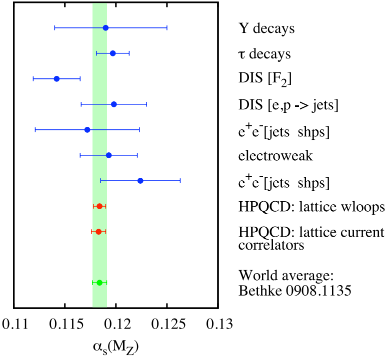
- Discretization effects grow with am_h and decrease with n.
- Grey band shows best-fit $m_c(3m_c)$ evolved perturbatively.

$$m_c^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 0.9851(63)\ \mathrm{GeV}$$

m_c comparison plot



$\alpha_s^{\overline{\mathrm{MS}}}(m_Z)$



HPQCD $\langle JJ \rangle$ result:

- $\alpha_s^{\overline{\mathrm{MS}}}(m_Z) = 0.1182(7)$
- Agrees with $n_f = 3$ result.
- Agrees well with world average.

b-quark mass

Approaches to calculating.

It is challenging to treat b-quark in LQCD calculations.

- Fully relativistic treatment.
  - Requires $am_{b0} \ll 1$.
  - Now becoming possible using highly-improved actions.
- Effect field theories.
  - NRQCD
  - HQET

Focus on NRQCD approach..

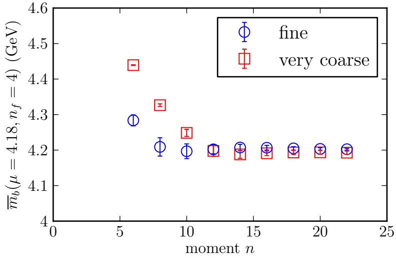
NRQCD approach

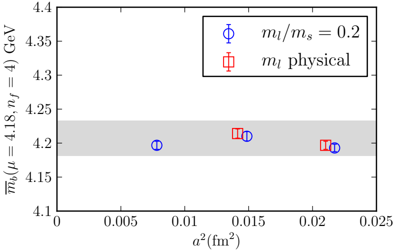
- Expansion in v^2 where $v^2 \sim 0.1$ for Υ.
- Want $am_{b0} > 1$.
- $H = H_0 + \delta H$
- Unlike relativistic case, current needs normalized: $J_\mu^{\mathrm{NRQCD}} = Z_{\mathrm{V}} J_\mu^{\mathrm{cont}}$.
- Effective theory $\to$ no continuum limit.

$$G_n^{\mathrm{NRQCD}} = Z_{\mathrm{V}}^2 \frac{g_n(\alpha_{\overline{\mathrm{MS}}}, \mu)}{am_b(\mu)^{n-2}} \,.$$

NRQCD moments [1408.5768]

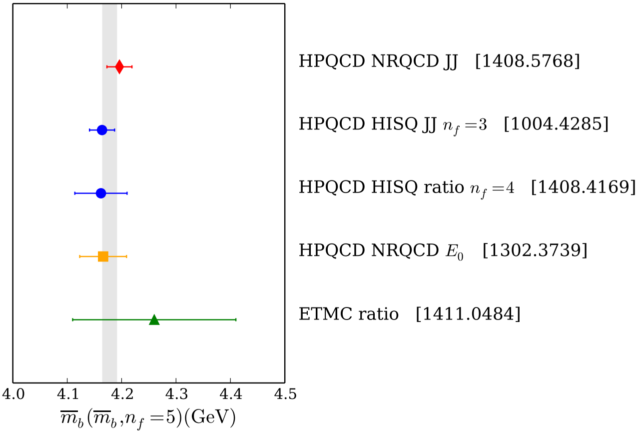
- Study ratios of successive moments to cancel factors of Z_V.
- Look for a "plateau" in the moment number n.



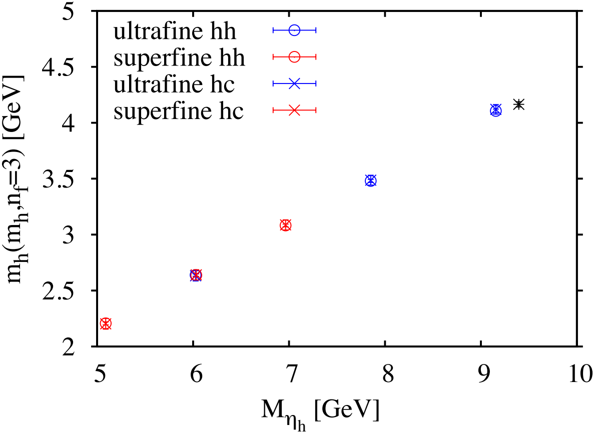


$$m_b^{\overline{\mathrm{MS}}}(m_b, n_f = 5) = 4.196(23)\ \mathrm{GeV}$$

m_b comparison plot



Heavy-charm HISQ moments



Mass ratios

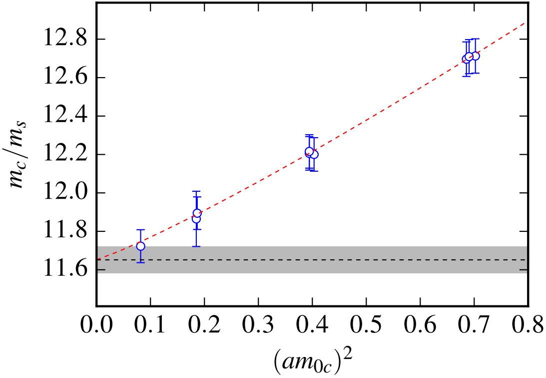
Mass ratios

$$\frac{m1_0}{m2_0} = \frac{m1^{\overline{\mathrm{MS}}}(\mu)}{m2^{\overline{\mathrm{MS}}}(\mu)} + \mathcal{O}(a^2)$$

- Tuning of simulation $\rightarrow$ accurate determination of bare ratios.
- Precise determination of one renormalized mass can be translated to other masses.

m_c/m_s $(n_f = 4)$

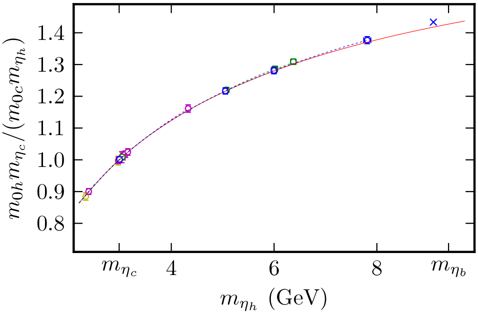
[1408.4169]



$$m_c/m_s = 11.652(65) \rightarrow m_s^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 93.6(8)\mathrm{MeV}.$$

m_b/m_c

HPQCD [1004.4285]



$m_b/m_c = 4.49(4)$ HPQCD [1004.4285]

$m_b/m_c = 4.40(8)$ ETMC [1411.0484]

Conclusions & Future Work

- Accurate determinations of quark masses are of fundamental importance for (B)SM physics.
- LQCD simulations provide an effective and controlled way to determine quark masses.
  - Systematically improveable.
  - Multiple complementary approaches $\rightarrow$ assess systematics, check consistency.
  - Control of input parameters.
- In the future we can expect:
  - Fully relativistic b.
  - Additional approaches:
    Heavy-light $\langle JJ \rangle$, RI/MOM - type determinations..
  - More independent calculations by different groups.

Thank you!

Additional Slides

Error budget – $\langle JJ \rangle$ HISQ

	$m_c(3)$	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$	m_c/m_s	m_b/m_c
Perturbation theory	0.3	0.5	0.0	0.0
Statistical errors	0.2	0.2	0.3	0.3
$a^2 \to 0$	0.3	0.3	0.0	1.0
$\delta m_{uds}^{\mathrm{sea}} \to 0$	0.2	0.1	0.0	0.0
$\delta m_c^{\mathrm{sea}} \to 0$	0.3	0.1	0.0	0.0
$m_h \neq m_c$	0.1	0.1	0.0	0.0
Uncertainty in w_0, w_0/a	0.2	0.0	0.1	0.4
α_0 prior	0.0	0.1	0.0	0.0
Uncertainty in m_{η_s}	0.0	0.0	0.4	0.0
$m_h/m_c \to m_b/m_c$	0.0	0.0	0.0	0.4
δm_{η_c}: electromag., annih.	0.1	0.0	0.1	0.1
δm_{η_b}: electromag., annih.	0.0	0.0	0.0	0.1
Total:	0.64%	0.63%	0.55%	1.20%

Error budget – $\langle JJ \rangle$ NRQCD

Error	$f_\Upsilon \sqrt{M_\Upsilon}$	$\overline{m}_b(10\text{GeV})$
Statistics	0.3	0.0
Z_V/k_1	2.5	0.3
perturbation theory/α_s	-	0.3
uncertainty in a	1.6	0.0
lattice spacing dependence	3.4	0.4
sea-quark mass dependence	1.0	0.0
b-quark mass tuning	1.0	0.0
NRQCD systematics	1.0	0.3
electromagnetism η_b annihilation	0.0	0.0
total	4.8	0.7