Weighted GMD model for multiplicity distributions

A new distribution at LHC energies

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Abstract

A new distribution is obtained from the Generalised Multiplicity Distribution (GMD). This distribution is derived by a weighted sum of the GMD over the initial number of gluons, removing the dependence of the model on a specific initial state for Quantum Chromodynamics evolution. The distribution is shown to describe charged-particle multiplicity distributions at various LHC energies, providing information on gluon production.

Introduction

The study of charged-particle multiplicity distributions reveal correlations and dynamics of particle production. For example, Independent emission of single particles lead to a Poisson multiplicity distribution (PD). Cascades (or showers) of particles originating from an initial number of ancestor particles produced after the collision lead to a broader distribution like the Negative Binomial Distribution (NBD) [1].

The NBD and the Generalised Multiplicity Distribution (GMD) can be derived from the differential equation describing the Markov branching processes of quarks and gluons [2, 3, 4, 5]. Given an initial m and k' number of quarks (q) and gluons (g) respectively, the probability of obtaining n final state particles is given by the GMD:

$$P_{GMD}(n;p,k,k') = \frac{\Gamma(n+k)}{\Gamma(n-k'+1)\,\Gamma(k'+k)}(1-p)^{n-k'}(p)^{k'+k}, \quad (1)$$

where $k = m\tilde{A}/A$ and $p = \exp(-At)$. $A\Delta t$ and $\tilde{A}\Delta t$ are the branching probabilities of the processes $g \to g+g$ and $q \to q+g$ respectively, in the infinitesimal interval of the evolution parameter t. The GMD reduces to the Fury-Yule Distribution [6] when $k = 0$ and the NBD when $k' = 0$. Furthermore, the NBD converges to the PD for $k \to \infty$.

One difficulty of the GMD in describing multiplicity distributions comes from the divergent factor of $1/\Gamma(n-k'+1)$ in equation 1 for $n < k'$. This is due to the model's initial condition of k' gluons that branch into n final state gluons, with each gluon hadronising into a final state particle. This poses limits in the description of measured charged-particle multiplicity distributions with non-zero probability of having 0 or a few particles.

Main Objectives

1. **Build a more realistic scenario into the GMD.**
2. **Create a model independent of the initial gluon number condition and able to describe low multiplicity data.**
3. **Apply the model to LHC data to extract information about the model's parameters.**

Results

The top left plot in the figure exhibits a comparison of 3 different PGMD with parameters $k = 0.1$, 1, and 5. The other parameters are fixed at $p = 0.1$ and $\overline{k'} = 2$. For a smaller k, the distribution also peaks at smaller n. The distribution develops a spike at $n = 0$ for small k, with $k = 0.01$ shown as an example. For a relatively larger k, varying the parameters p and $\overline{k'}$ do not reproduce the spike while for a small k, the variation of p and $\overline{k'}$ only modulates the height of the spike.

The right panes and bottom left plots in the figure, show the multiplicity distributions measured by the CMS collaboration [7] at centre-of-mass energies of 0.9, 2.36, and 7 TeV. Comparison is made with the best fit PGMD and GMD. The interior point algorithm [8, 9] is used to find the parameters that give the minimum χ^2 fit to the data, excluding point $n = 0$. The parameters that give the best fit distributions are shown in table 1.

The χ^2/dof show that the PGMD describes the multiplicity distributions better than the GMD. The PGMD describes the tail ends of the distribution better than the GMD, but not as well at low multiplicities.

From table 1, it is evident that the GMD that best describes the data reduces to the NBD ($k' = 0$). Given the computation of χ^2 which considers the points $n \geq 1$, we have the constrain of $k' \leq 1$. The characteristic feature of the GMD having an initial condition of k' number of gluons that branch and eventually hadronise, ensure that there are at least the same number of n hadrons as the initial number of gluons. This constraint limits the applicability of the GMD in finding an optimised solution.

	$\sqrt{s}$ (TeV)	0.9	2.36	7
PGMD	p	0.135	0.0904	0.0614
	k	1.59	1.50	1.07
	$\overline{k'}$	1.07	0.741	0.865
	χ^2/dof	36.7/65	36.9/67	93.4/124
GMD	$\overline{n}$	18.4	23.4	31.0
	k	2.02	1.71	1.41
	k'	0.00	0.00	0.00
	χ^2/dof	47.1/65	41.3/67	136/124

Table: Summary of the best fit parameter values.

The PGMD does not have this constraint since it considers a spread of initial conditions described by the Poisson process which produce the gluons. At 0.9, 2.36, and 7 TeV, the model gives 1.07, 0.741, and 0.865 gluons respectively, on average. For increasing centre-of-mass energies, the parameter p decreases monotonically from 0.135 to 0.0614. From the definition of p, the model describes an increasing initial parton invariant mass as a function of the centre-of-mass energy, given a fixed branching probability $A\Delta t$. The corresponding mean hadron numbers 18.1, 23.3, and 30.4 calculated using equation 6 is compatible with the experimental values measured by the CMS collaboration [7].

Conclusion

The WGMD and PGMD are derived, providing a more realistic description of collision and production conditions where the initial gluon number fluctuates on an event by event basis. The normalisation and mean of the distribution are derived. The PGMD is applied to describe charged-particle multiplicity distributions at LHC energies and shown to describe the data better than pure GMD.

The weighted GMD model

The weighted GMD (WGMD) model is proposed as a more realistic model that generalises the GMD to cases where the number of gluons produced after a pp collision fluctuates on an event by event basis. The WGMD is therefore a weighted sum of the GMD with weights distributed in the gluon number k' as

$$P_{WGMD}(n;x_1,...,x_r,p,k) = \sum_{k'=0}^{n} P(k';x_1,...,x_r) \times P_{GMD}(n;p,k,k'). \quad (2)$$

The parameters $x_1, ..., x_r$ are statistical quantities describing the ensemble of initial states of gluons produced after collision. The distribution $P(k';x_1,...,x_r)$ represents the event to event fluctuation of initial gluon numbers.

The Poisson weighted GMD model

The simplest manifestation of the WGMD is with a Poisson weight:

$$P_{WGMD}(n;x_1,...,x_r,p,k) = \sum_{k'=0}^{n} \frac{\overline{k'}^{k'}\exp\left(-\overline{k'}\right)}{k'!} \times P_{GMD}(n;p,k,k'). \quad (3)$$

The Poisson weighted GMD (PGMD) represents the independent production of single gluons in the initial state of QCD evolution after a pp collision and the parameter $\overline{k'}$ represents the average gluon number over all collision events.

Normalisation and mean

Although the WGMD may not seem normalised by its construction in that the summation over k' runs only up to n and not ∞, it can be shown that the normalisation condition holds, i.e.

$$\sum_{n=0}^{\infty} P_{WGMD}(n;x_1,...,x_r,p,k) = 1. \quad (4)$$

Similarly the mean of the WGMD, $\overline{n} = \sum_{n=0}^{\infty} nP_{WGMD}(n;x_1,...,x_r,p,k)$, has a simple form:

$$\overline{n} = \frac{\langle k' \rangle + k}{p} - k, \quad (5)$$

where $\langle k' \rangle = \sum_{k'=0}^{\infty} k'P(k';x_1,...,x_r)$. This has the exact same form as the mean of the GMD [3, 4, 5] but with k' replaced by $\langle k' \rangle$.

Mean of the PGMD

With the PGMD, the mean is given by:

$$\overline{n} = \frac{\overline{k'} + k}{p} - k, \quad (6)$$

Plots

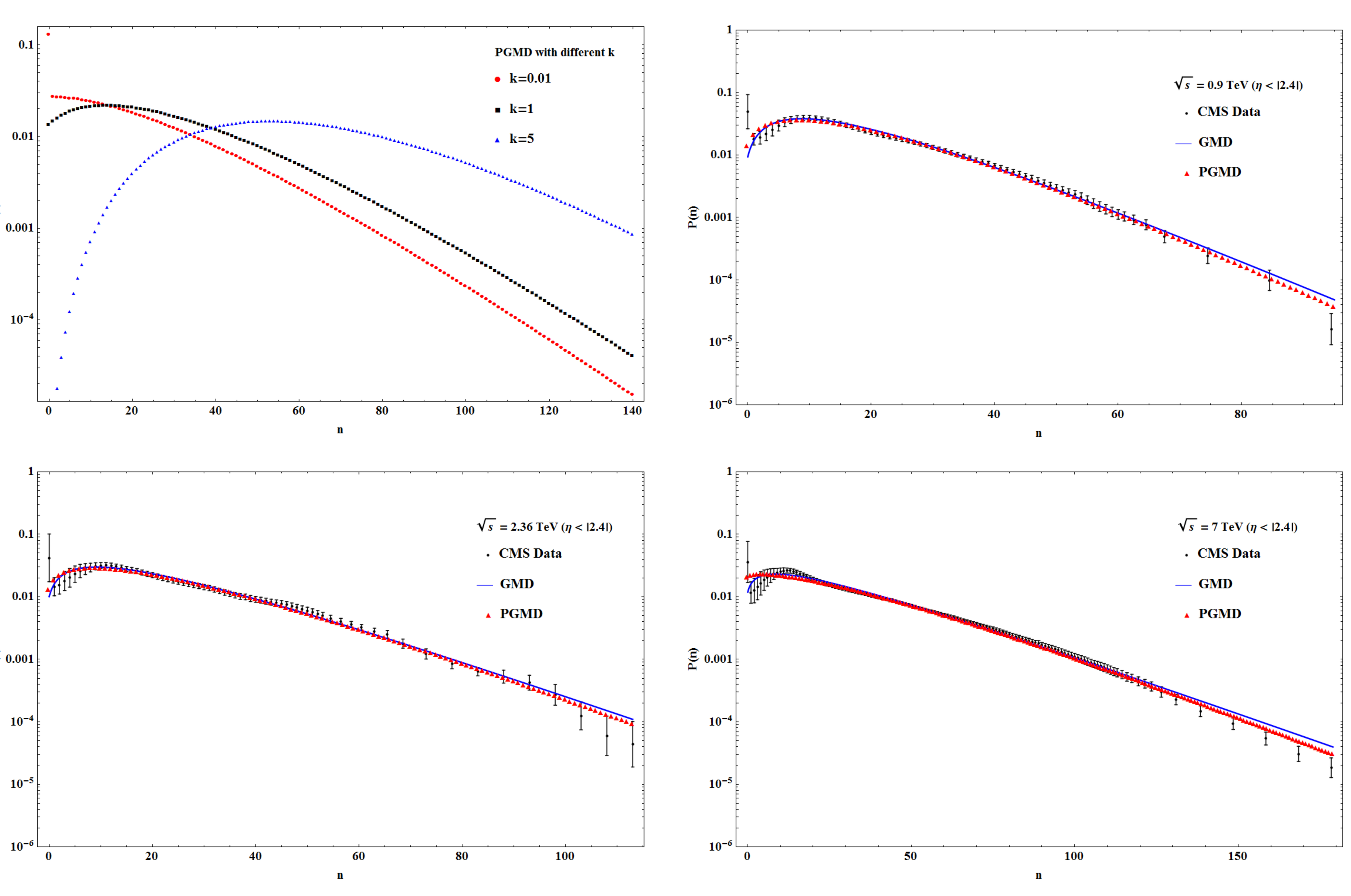

Figure: Comparison of PGMD with different k values (top left), and the charged-particle multiplicity distribution data at $\sqrt{s} = 0.9$ (top right), 2.36 (bottom left), and 7 TeV (bottom right) with $|\eta| < 2.4$ compared to the best fit PGMD (triangle) and GMD (line). The vertical lines in the data points represent the statistical errors and systematic uncertainties added in quadrature.

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