The τ-model of Bose-Einsten Correlations

Some Recent Results

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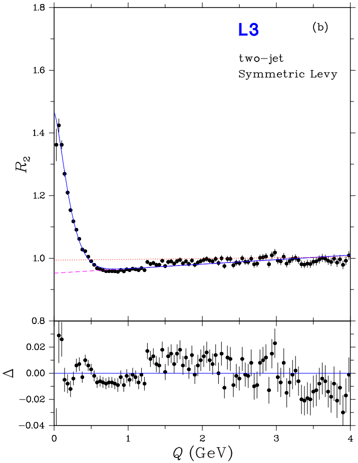
BEC – ‘Classic’ Parametrizations

$$R_2 = \frac{\rho(p_1,p_2)}{\rho_0(p_1,p_2)} = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$ $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^\alpha\right)$, $0 < \alpha \leq 2$
  α is index of stability
  Gaussian if $\alpha = 2$ $\alpha = 1.34 \pm 0.04$

Cannot accomodate the the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

L3, EPJC 71 (2011) 1648

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:

$$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}$$

where for 2-jet events, $a = 1/m_{\mathrm{t}}$

$\tau = \sqrt{\bar{t}^2 - \bar{r}_z^2}$ is the "longitudinal" proper time
and $m_{\mathrm{t}} = \sqrt{E^2 - p_z^2}$ is the "transverse" mass

- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of production points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is

$$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$

- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.

$$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$

- Assume $\delta_\Delta(x^\mu - \overline{x}^\mu)$ is very narrow — a δ-function. Then

$$R_2(p_1,p_2) = 1 + \lambda\,\mathrm{Re}\widetilde{H}\left(\frac{a_1Q^2}{2}\right)\widetilde{H}\left(\frac{a_2Q^2}{2}\right), \qquad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction, $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\tfrac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\tfrac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right]\ , \qquad \alpha \neq 1$$
  where
  - α is the index of stability, $0 < \alpha \leq 2$;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$\begin{aligned} R_2(Q, a_1, a_2) \quad = \quad & \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right. \\ & \left.\cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right\}\cdot(1 + \epsilon Q) \end{aligned}$$

BEC in the τ-model

$$R_2(Q, a_1, a_2) = \gamma \left\{1 + \lambda \cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right]\right\} \cdot (1+\epsilon Q)$$

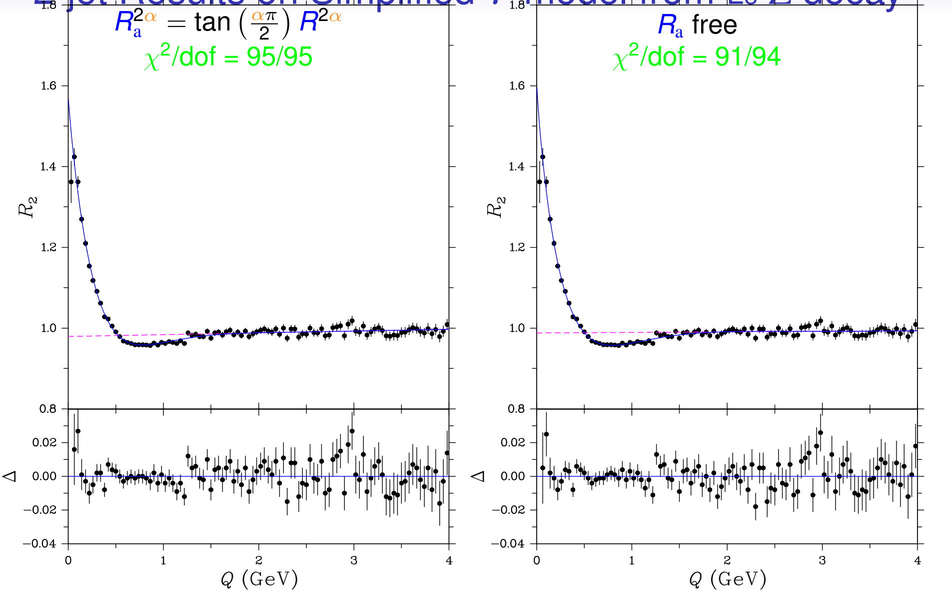
Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^{\alpha}+a_2^{\alpha}}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then
$$R_2(Q) = \gamma\left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right] \cdot (1+\epsilon Q)$$
where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:
$$R_2(Q) = \gamma\left[1 + \lambda \qquad \exp\left(-|rQ|^{\alpha}\right)\right](1+\epsilon Q)$$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$

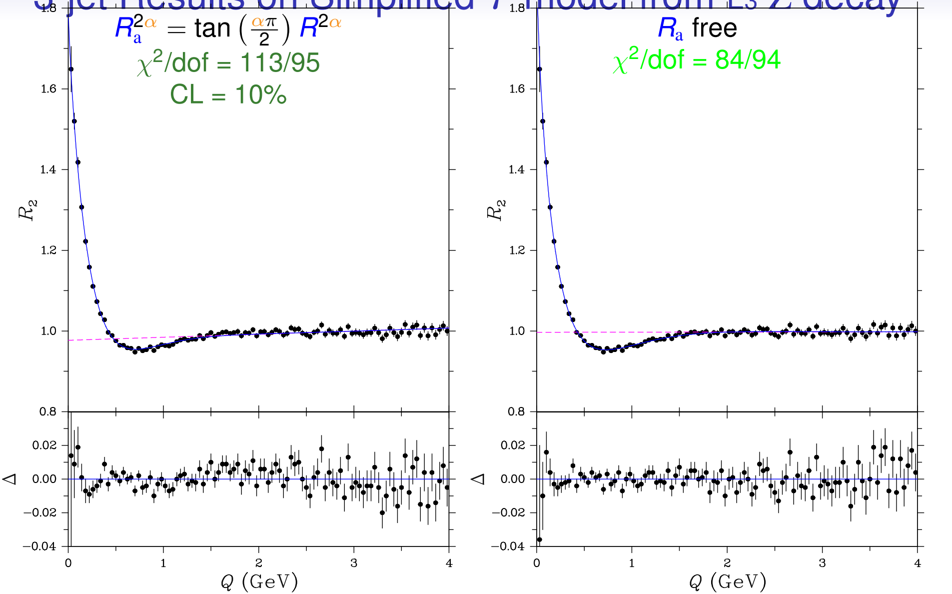
2-jet Results on Simplified τ-model from L3 Z decay



$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ agrees well with data

L3, EPJC 71 (2011) 1648

3-jet Results on Simplified τ-model from L3 Z decay



$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ agrees less well with data

L3, EPJC 71 (2011) 1648

BEC in $\mathrm{e}^+\mathrm{e}^-$ and $\mathbf{pp}$

Use (mostly) simplified τ-model with $\tau_0 = 0$

- L3: $\mathrm{e}^+\mathrm{e}^-$ at $\sqrt{s} = M_\mathrm{Z}$
  - $0.8 \cdot 10^6$ events
  - Durham $y_\mathrm{cut} = 0.006$:
    $0.5 \cdot 10^6$ 2-jet events
    $0.3 \cdot 10^6 > 2$ jets, “3-jet”
  - mixed event ref. sample
- ATLAS: pp at $\sqrt{s} = 7\,\mathrm{TeV}$
  Astaloš thesis http://hdl.handle.net/2066/143448
  - 10^7 min. bias events
  - $|\eta| < 2.5$
  - opposite hemisphere ref. sample

Results are preliminary (unpublished)
and not approved by the collaborations

BOSE-EINSTEIN CORRELATIONS
IN 7 TEV PROTON-PROTON COLLISIONS
IN THE ATLAS EXPERIMENT

Doctoral thesis

to obtain the degree of doctor

from Radboud University Nijmegen
on the authority of the Rector Magnificus prof. dr. Th.L.M. Engelen,
according to the decision of the Council of Deans
and
from Comenius University Bratislava
on the authority of the Rector Magnificus prof. RNDr. Karol Mičieta, PhD.

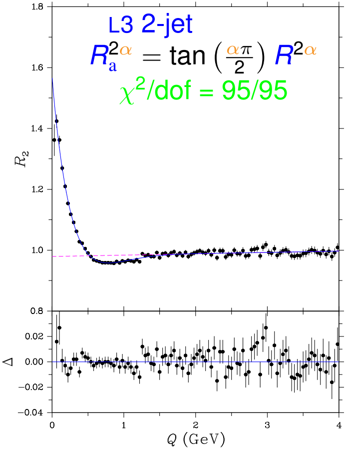
to be defended in public on Wednesday, September 30, 2015
at 10:30 hrs

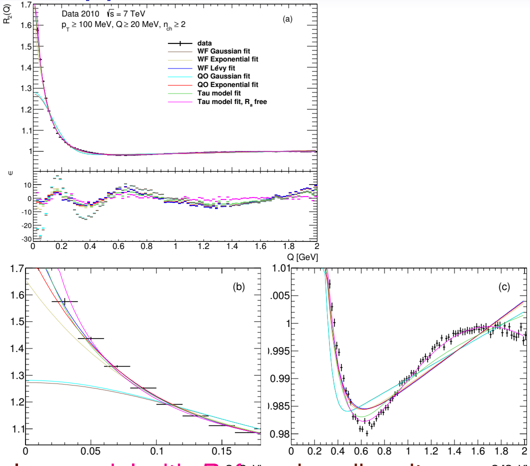
by

Róbert Astaloš

born on December 28, 1985
in Banská Bystrica, Slovakia

2-jet e^+e^- – All pp min. bias





anticorrelation region also in pp – only τ-model with R_a free describes it

BEC peak best described by τ-model with R_a free and sym. Lévy

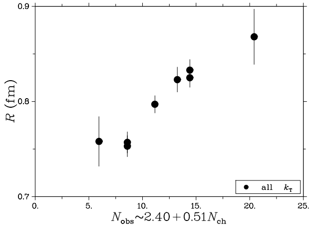
BEC peak next best described by a quantum optical exponential parametrization

and by τ-model $\chi^2(Q \leq 0.36) = 115, 116, 157, 186$

Only τ-model with R_a free describes entire range of Q

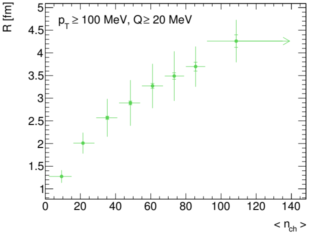
Multiplicity dependence

$\mathrm{e^+e^-}$ simplified τ-model

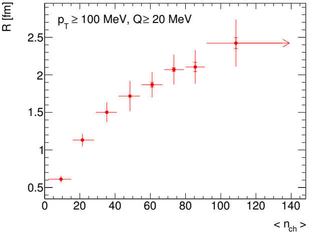


pp, various parametrizations

sym. Lévy



quantum optical exp



pp: shape of R *vs.* N_{ch} nearly independent of parametrization

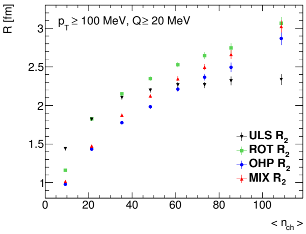
pp: value of R is different – different meaning in parametrizations

shape in $\mathrm{e^+e^-}$ similar to pp

different ref. samples give similar shapes

except ULS flattens at high N_{ch}

exponential

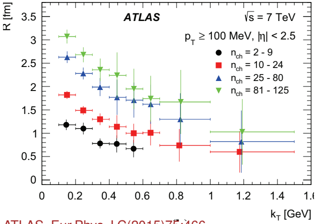


Dependence on $k_t = p_{t\,\mathrm{pair}}/2 = |\vec{p_{t1}} + \vec{p_{t2}}|/2$

pp – conventional parametrizations

Un-Like Sign ref. sample

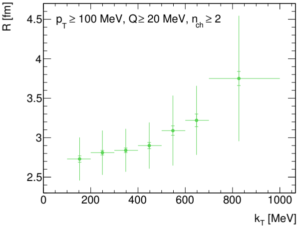
exponential



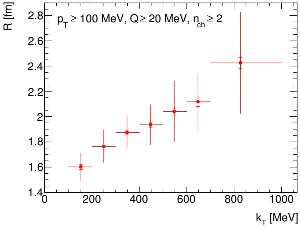
ATLAS, Eur.Phys.J.C(2015)75:466

OHP ref. sample

sym. Lévy

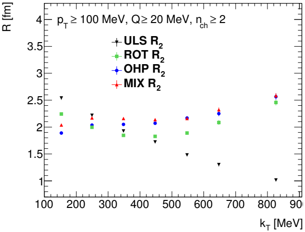


optical exponential



Astaloš thesis

exponential



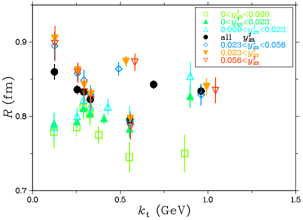
- exponential parametrization:
  ULS ref. sample: *R* decreases with k_t (all N_{ch})
  with other ref. samples, *R* is first constant, then increasing with k_t
- other 'classic parametrizations': *R* increases with k_t

$k_t = |\vec{p}_{t1} + \vec{p}_{t2}|/2$ dependence

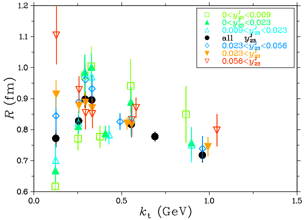
simplified τ-model

e^+e^-, w.r.t. thrust axis

R_a constrained

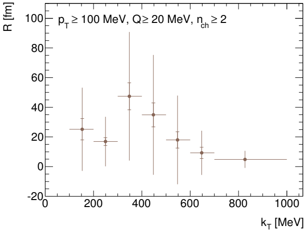


e^+e^-, R_a free



pp, w.r.t. beam direction

R_a free



in e^+e^- dependence of R on k_t depends on 'jettiness'

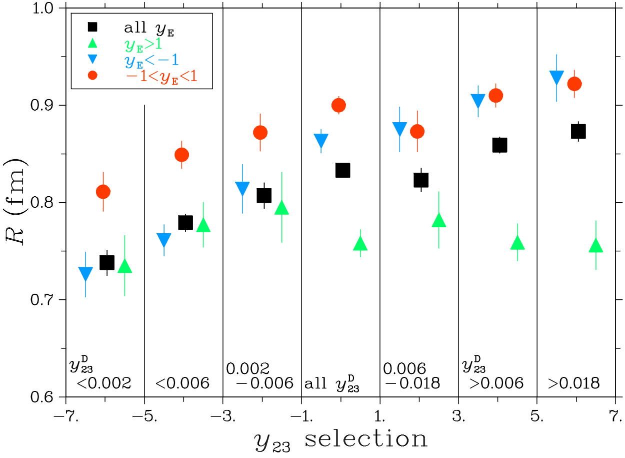
in e^+e^- 3-jet R decreases with k_t.

in e^+e^- 2-jet and in pp R first increases slightly and then falls with k_t.

k_t dependence of R is dependent on parametrization and on ref. sample

Dependence on the Rapidity of the pair

e^+e^-, + thrust axis = hemisphere of jet with highest E



- $R_{\text{all } y}$ and $R_{y<-1}$ increase with y_{23}
- but $R_{y>1}$ constant with y_{23}
- 2-jet: $R_{y>1} = R_{y<-1}$

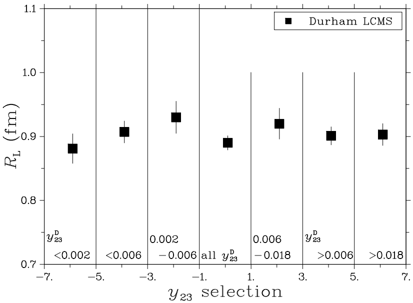
Conclusion: Increase in R is mainly due to gluon

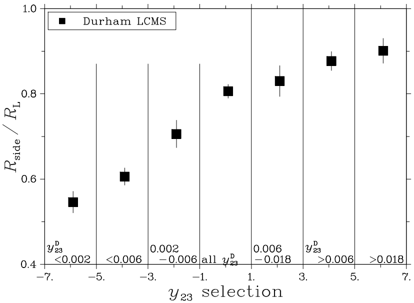
$R_{-1<y<1} > R_{y>|1|}$ for 2-jet

gluon mini-jet in '2-jet sample'?

Simplified τ-model in LCMS

In τ-model: $R^2 Q^2 \Longrightarrow R_\mathrm{L}^2 Q_\mathrm{L}^2 + R_\mathrm{side}^2 Q_\mathrm{side}^2 + R_\mathrm{out}^2 Q_\mathrm{out}^2$





- R_L constant with y_{23}

R_side increases with y_{23}

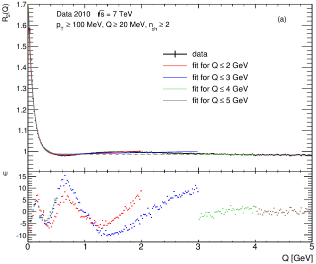
Conclusion: Increase in R is mainly due to increase in transverse plane

Agrees with conclusion that increase is mainly due to harder gluon:

Gluon makes event 'fatter'

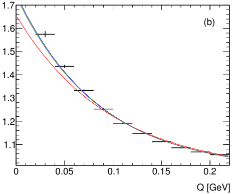
Effect of fit range

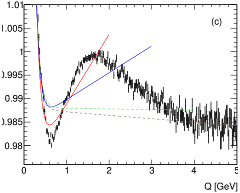
Besides ref. sample, another large systematic effect is the choice of fit range



Using the opposite hemisphere ref. sample, and Exponential parametrization:

Q_{U} (GeV)	Q excl.	R (fm)	λ
2	-	2.02 ± 0.01	0.70 ± 0.01
3	-	2.28 ± 0.01	0.78 ± 0.01
4	0.5–3.0	2.30 ± 0.02	0.77 ± 0.01
5	0.5–4.0	2.29 ± 0.02	0.76 ± 0.01





$Q_{\mathrm{U}} = 2$, 3: baseline tries to describe anticorrelation

Q_{U} larger, with excluded regions can lead to stable results,

but this is simply *bricolage*

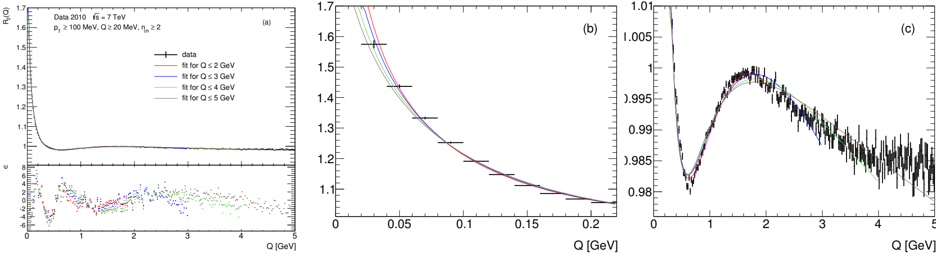
and it is a long extrapolation

Parametrization is just wrong

Effect of fit range

Better to use a parametrization that fits (better): τ-model with R_{a} free and Q_{U} sufficiently beyond the anticorrelation region

Using the opposite hemisphere ref. sample,



Q_{U}	2 GeV	3 GeV	4 GeV	5 GeV
α	0.108 ± 0.001	0.186 ± 0.005	0.235 ± 0.003	0.261 ± 0.003
R (fm)	17.8 ± 0.7	6.7 ± 0.5	4.1 ± 0.2	3.3 ± 0.1
R_{a} (fm)	43.4 ± 1.2	3.0 ± 0.2	1.80 ± 0.04	1.52 ± 0.02
λ	3.08 ± 0.05	1.91 ± 0.10	1.36 ± 0.05	1.15 ± 0.03

- much less dependent on fit range than other parametrizations
- α quite different from $\mathrm{e}^+\mathrm{e}^-$ 2-jet value of $0.41 \pm 0.02^{+0.04}_{-0.06}$

Anti-Correlation Region

In addition to L3 observation of anticorrelation in e^+e^- ($E_{cms} = M_Z$),
CMS has observed it in pp min. bias at 7 TeV JHEP 05 (2011) 29

Now it is also seen in ATLAS data (Astaloš thesis)

In addition to / Instead of the space-momentum correlation of the τ-model,
anti-correlation can also be caused by the finite size of pions Białas, Zalewski

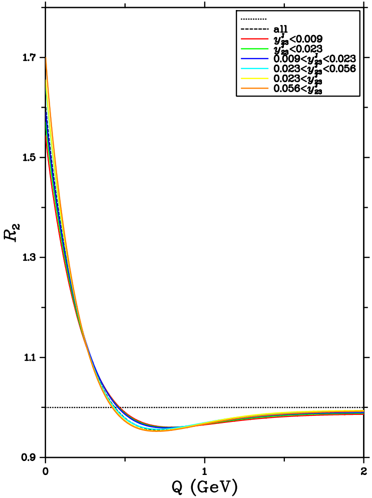
Detailed investigation of the anti-correlation may resolve this.

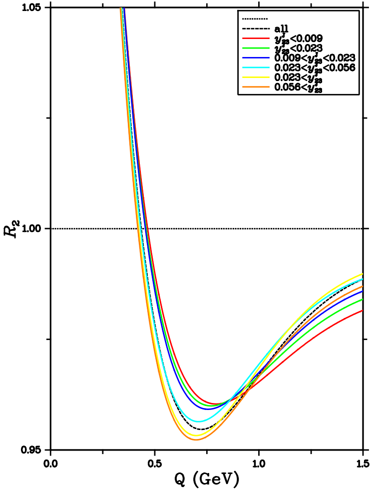
Use simplified τ-model, $\tau_0 = 0$ to investigate anticorrelation region
in e^+e^-: L3 data, $0.8 \cdot 10^6$ events,
in pp: ATLAS data (Astaloš thesis) 10^7 events, $|\eta| < 2.5$

We compare fit results with R_a free, since they provide the best description of the anti-correlation dip.

Anti-Correlation Region

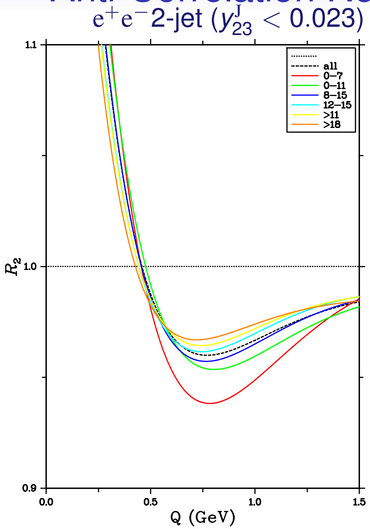
e^+e^- jet dependence:

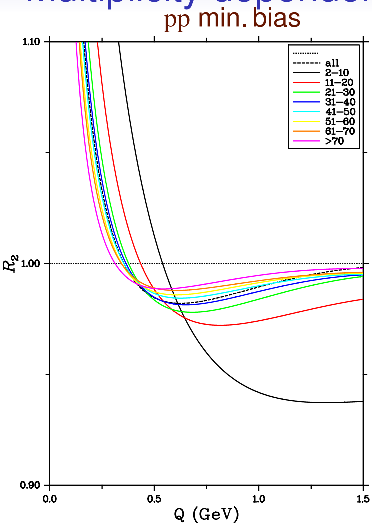




Going from narrow 2-jet to wide 3-jet,
anticorrelation region becomes deeper and moves slightly lower in Q

Anti-Correlation Region – Multiplicity dependence



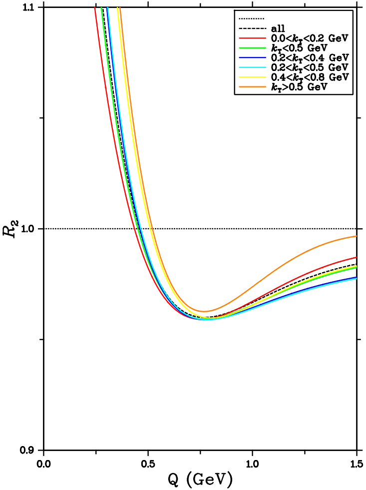


anticorrelation region is deeper and at higher Q in e^+e^- than in pp

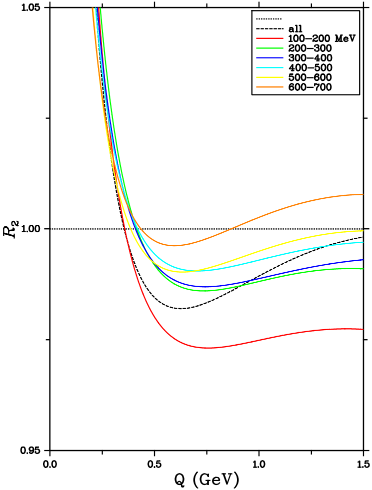
with increasing N minimum moves to lower Q (effect is larger in pp than in e^+e^-) and becomes less deep (also seen by CMS)

Anti-Correlation Region – $\boldsymbol{k}_\mathrm{t}$ dependence

$\mathrm{e}^+\mathrm{e}^-$ 2-jet ($y_{23}^{\mathrm{J}} < 0.023$)



pp min. bias



in $\mathrm{e}^+\mathrm{e}^-$ anticorrelation region shows little or no dependence on k_t

in pp the depth decreases with k_t

but the position of the minimum is approx. constant

Conclusions/Comments/Lessons

1. τ-model
   - τ-model is closely related to a string picture
     - strong *x*-*p* correlation
     - fractal - Lévy distribution
   - BEC in pp (CMS, Astaloš ATLAS thesis) are described by simplified τ-model formula
     Of all parametrizations tried, only τ-model with R_a free describes the data.
   - suggests that BEC in pp is (mostly) from string fragmentation
2. Ref. sample is important
   - Comparison of results using different ref. samples is very problematic
   - Agreement among LHC expts. would facilitate comparisons, *e*.g.,
     - central rapidity *vs*. forward rapidity
     - pp, pA, AA
3. Anticorrelation region is important
   - To study it
     - Look beyond $Q = 2\,\mathrm{GeV}$ – at least to 3, preferably to 4 or 5 GeV
     - Do not use Unlike-Sign ref. sample
   - It depends on N_{ch}, k_{t}, jets. What else ?
   - Is space-momentum correlation, as in the τ-model, the correct explanation?
4. *R* depends on
   - in $\mathrm{e^+e^-}$: N_{jets}, N_{ch}, k_{t}, rapidity
   - in pp: N_{ch}, k_{t}.
     Also on (mini)jets, rapidity, color reconnection, N_{strings}, color ropes?

ADDITIONAL MATERIAL

Quantum Optics parametrizations

In addition to 'classic' and τ-model parametrizations,
Róbert Astaloš's thesis includes fits of parametrizations based on a quantum optical approach

Weiner, Phys. Rep. 327 (2000) 249

- Gaussian

$$R_2(Q) \propto 1 + 2p(1-p)\exp(-R^2Q^2) + p^2\exp(-2R^2Q^2)$$

- Lorentzian in R, exponential in Q

$$R_2(Q) \propto 1 + 2p(1-p)\exp(-RQ) + p^2\exp(-2RQ)$$

p is the degree of chaoticity of the pion emission

Note that for $p = \lambda = 1$ these reduce to the 'classical' Gaussian and exponential parametrizations

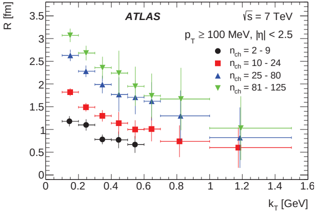
Like the 'classical' parametriazations, these parametrizations cannot accomodate anticorrelation

Dependence on $k_t = p_{t\,\mathrm{pair}}/2 = |\vec{p}_{t1} + \vec{p}_{t2}|/2$

pp – conventional parametrizations

Un-Like Sign ref. sample

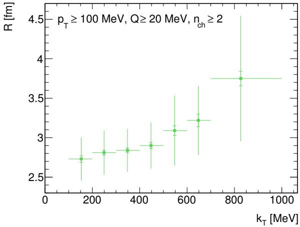
exponential



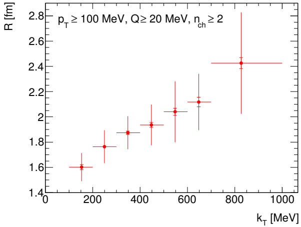
ATLAS, Arxiv.1502.07947v1

OHP ref. sample

sym. Lévy

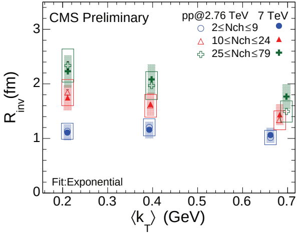


optical exponential



Astaloš thesis

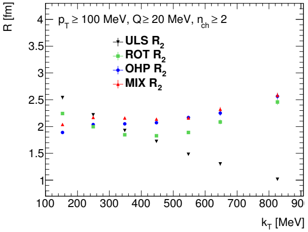
Mix S. Padula, WPCF2014, ArXiv:1502.05757



with ULS ref. sample, R decreases with k_t (all N_{ch})

with other ref. samples, R increases with k_t

exponential

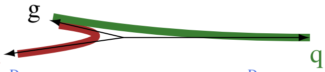


Jets and Rapidity

order jets by energy: $E_1 > E_2 > E_3$

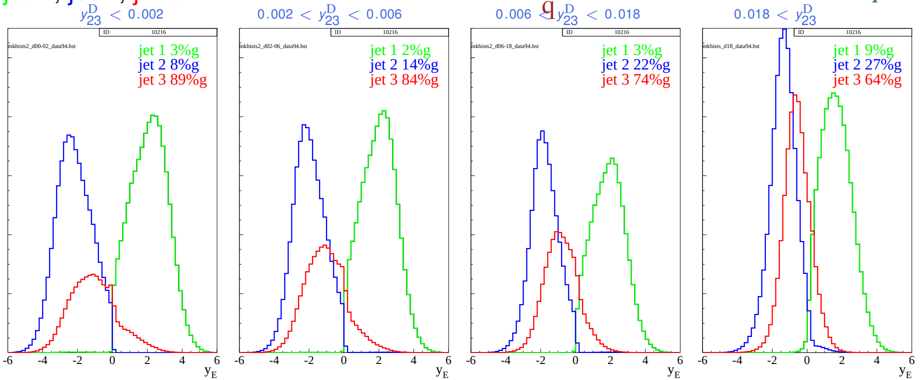
Note: thrust only defines axis $|\vec{n}_{\mathrm{T}}|$, not its direction.

Choose positive thrust direction such that jet 1 is in positive thrust hemisphere



rapidity, y_{E}, of particles from

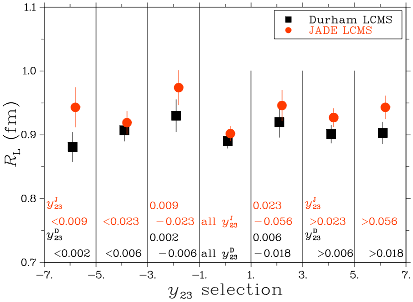
jet 1, jet 2, jet 3:

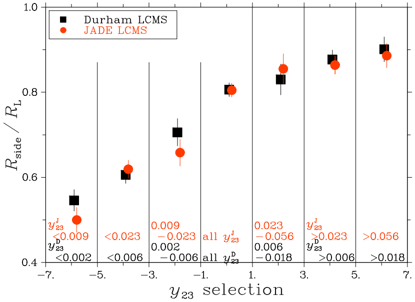


- $y_{\mathrm{E}} > 1$ almost all jet 1 — almost all quark
- $y_{\mathrm{E}} < -1$ mostly jet 2, some jet 3 — mostly quark
- $-1 < y_{\mathrm{E}} < 1$ jet-3 enriched — largely gluon

Simplified τ-model in LCMS

In τ-model: $R^2 Q^2 \Longrightarrow R_{\mathrm{L}}^2 Q_{\mathrm{L}}^2 + R_{\mathrm{side}}^2 Q_{\mathrm{side}}^2 + R_{\mathrm{out}}^2 Q_{\mathrm{out}}^2$





- Durham, JADE agree
- R_{L} constant with y_{23}

R_{side} increases with y_{23}

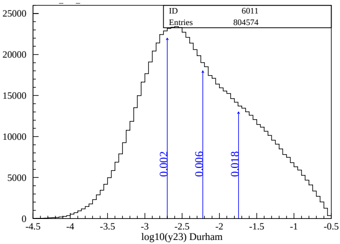
Conclusion: Increase in R is mainly due to increase in transverse plane

Agrees with conclusion that increase is mainly due to harder gluon:
Gluon makes event ‘fatter’

Jets

Jets — JADE and Durham algorithms

- force event to have 3 jets:
  - normally stop combining when all ‘distances’ between jets are $> y_{\mathrm{cut}}$
  - instead, stop combining when there are only 3 jets left
  - y_{23} is the smallest ‘distance’ between any 2 of the 3 jets
- y_{23} is value of y_{cut} where number of jets changes from 2 to 3

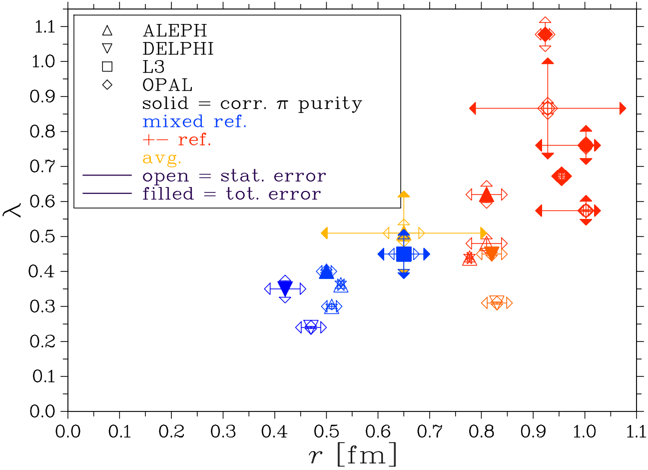


define regions of y_{23}^{D} (Durham):

$y_{23}^{\mathrm{D}} < 0.002$	narrow two-jet	or		
$0.002 < y_{23}^{\mathrm{D}} < 0.006$	less narrow two-jet		$y_{23}^{\mathrm{D}} < 0.006$	two-jet
$0.006 < y_{23}^{\mathrm{D}} < 0.018$	narrow three-jet		$0.006 < y_{23}^{\mathrm{D}}$	three-jet
$0.018 < y_{23}^{\mathrm{D}}$	wide three-jet			

and similarly for y_{23}^{J} (JADE): 0.009, 0.023, 0.056

Results from R_2, $\sqrt{s} = M_Z$ (Gaussian parametrization)



– correction for π purity increases λ

– mixed ref. gives smaller λ, r than $+-$ ref. – Average means little