Deconfinement and Equation of State in QCD

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What is deconfinement in QCD ? What is the nature of the deconfined matter ?

Tools: screening of color charges, EoS, fluctuation of conserved quantum numbers

QGP: state of strongly interacting matter for weakly interacting gas of quark and gluons ? $T \gg \Lambda_{QCD}, g \ll 1$

$$2\pi T \gg m_D \sim gT \gg g^2 T$$

EFT approach: EQCD

Magnetic screening scale: non-perturbative

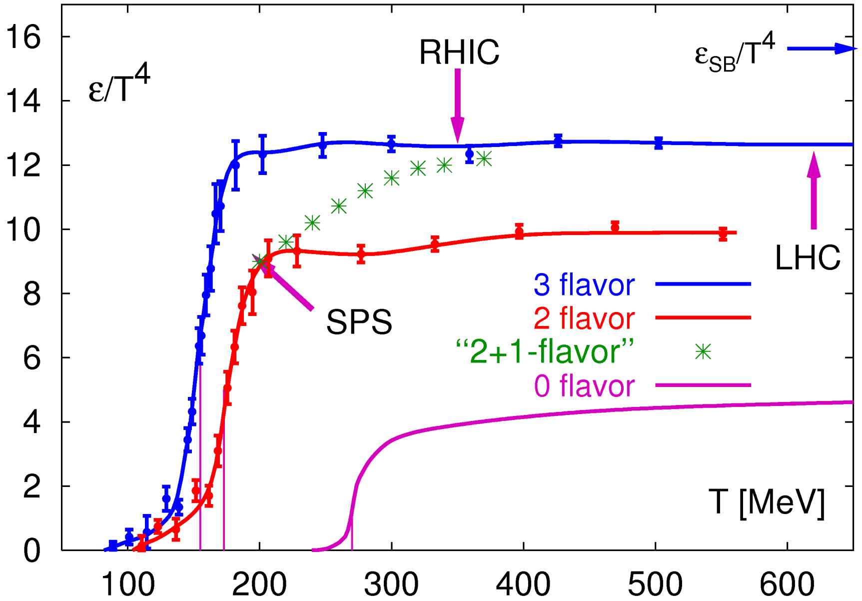
Perturbative series is an expansion is in g and not α_s
Loop expansion breaks down at some order

Problem : $g(\mu = 10^2 \mathrm{GeV}) = \sqrt{4\pi\alpha_s(\mu = 10^2\mathrm{GeV})} \simeq 1$
$g(\mu = 10^{16}\mathrm{GeV}) \simeq 1/2$

→ Lattice QCD

Lattice QCD at T>0 now and then

Lattice QCD calculations at T>0 around 2002:



$T_c \simeq 173\text{MeV}$

for both chiral transition and deconfinement transition (in terms of Polyakov loop)

Problems:

$N_\tau = 4 : a \equiv 1/(N_\tau a) = 1/(4T)$

$m_\pi = (500 - 800)\text{MeV}$

Continuum limit and physical masses are needed $N_\tau \to \infty$ $m_\pi = 140\text{MeV}$

$costs \sim N_\tau^{11}$

$\sim 1/m_\pi^3$

This task can be accomplished using improved staggered fermions actions:

Highly Improved Staggered Quark (HISQ)

Stout action

Fluctuations of conserved charges: new look into deconfinement and QGP properties

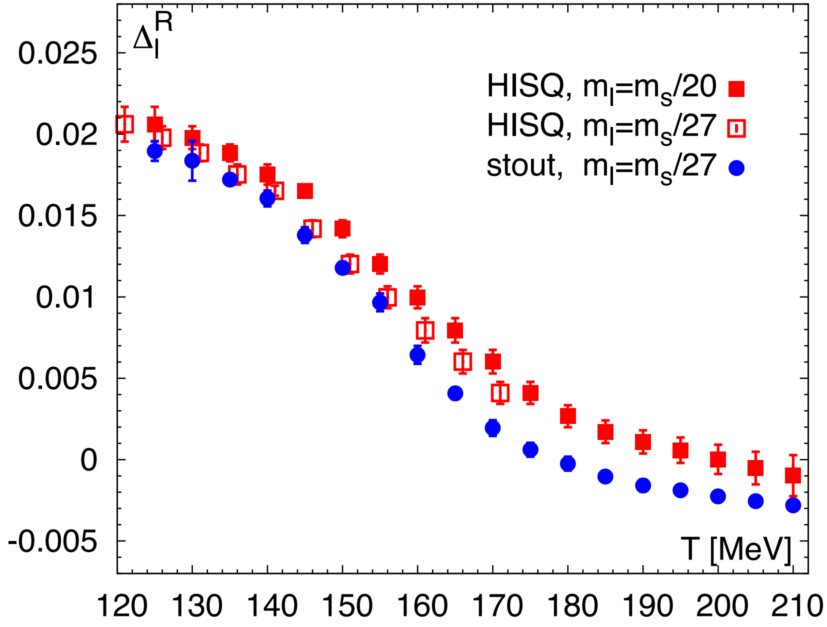
The temperature dependence of chiral condensate

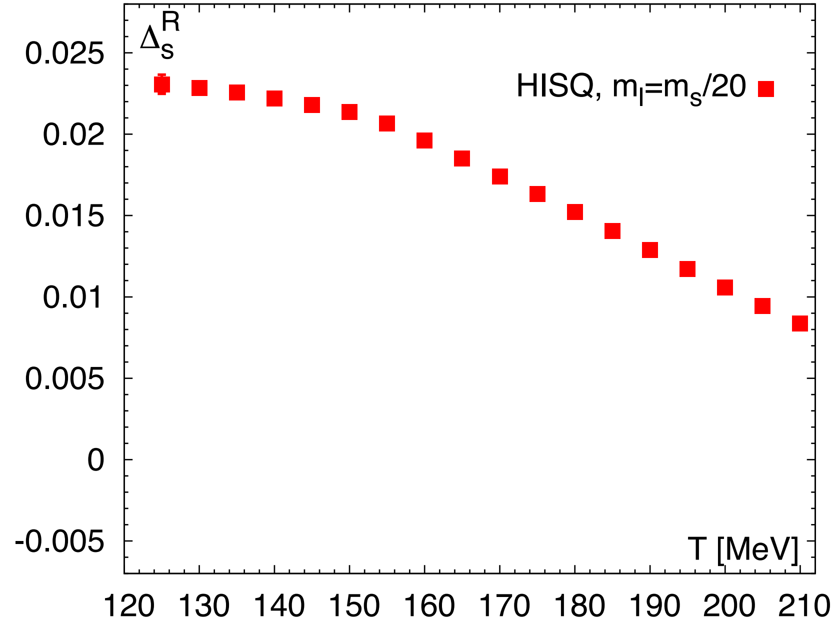
Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar{\psi}\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle\bar{\psi}\psi\rangle_{q,T} - \langle\bar{\psi}\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

With choice : $d = \langle\bar{\psi}\psi\rangle_{m_q=0}^{T=0}$

Bazavov Phys. Rev. D85 (2012) 054503; PRRD 87(2013)094505,
Borsanyi et al, JHEP 1009 (2010) 073





• after extrapolation to the continuum limit and physical quark mass HISQ/tree calculation agree with stout results

$$T_c = (154 \pm 8 \pm 1(scale))\text{MeV}$$

• strange quark condensate does not show a rapid change at the chiral crossover => strange quark do not play a role in the chiral transition

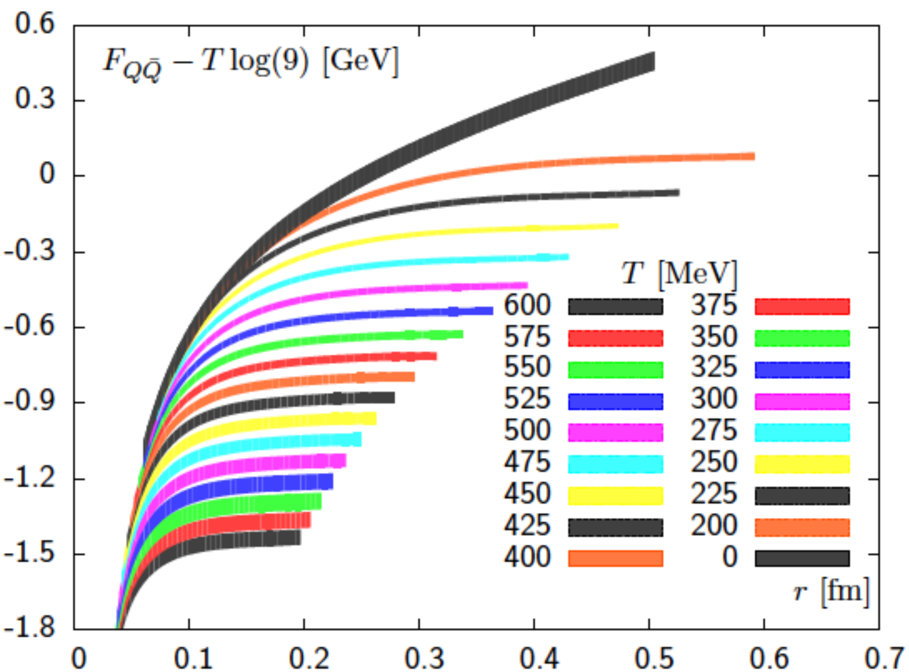
Deconfinement and color screening

Onset of color screening is described by Polyakov loop (order parameter in SU(N) gauge theory)

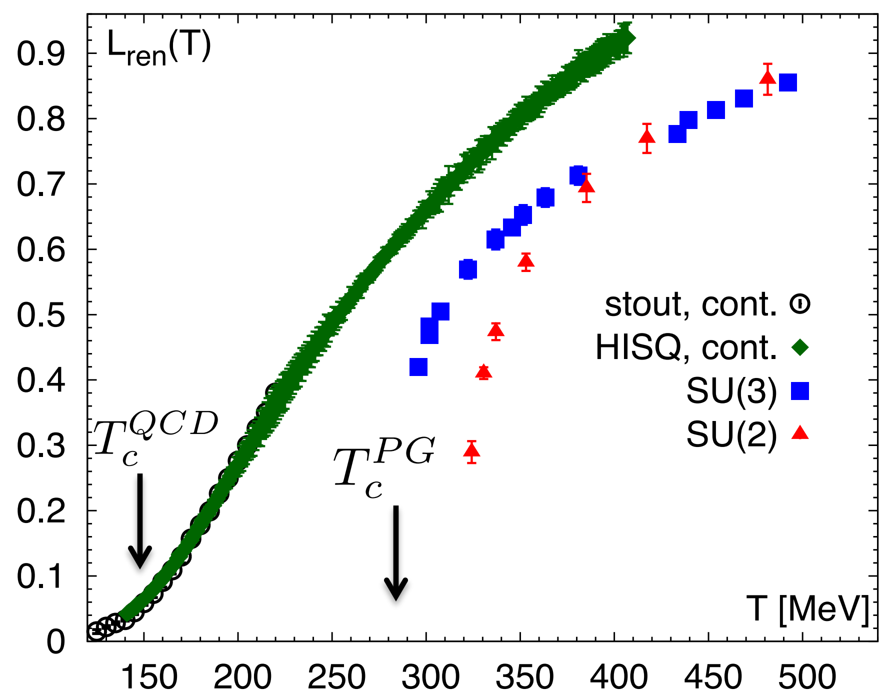
$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \qquad \exp(-F_{Q\bar{Q}}(r,T)/T) = \frac{1}{9}\langle \mathrm{tr}L(r)\mathrm{tr}L^\dagger(0)\rangle$$

$$F_{Q\bar{Q}}(r\to\infty,T) = 2F_Q(T) \quad \Rightarrow \quad L_{ren} = \exp(-F_Q(T)/T)$$

2+1 flavor QCD, continuum extrapolated (work in progress with Bazavov, Weber …)



free energy of static quark anti-quark pair shows Debye screening at high temperatures



SU(N) gauge theory ≠ QCD !

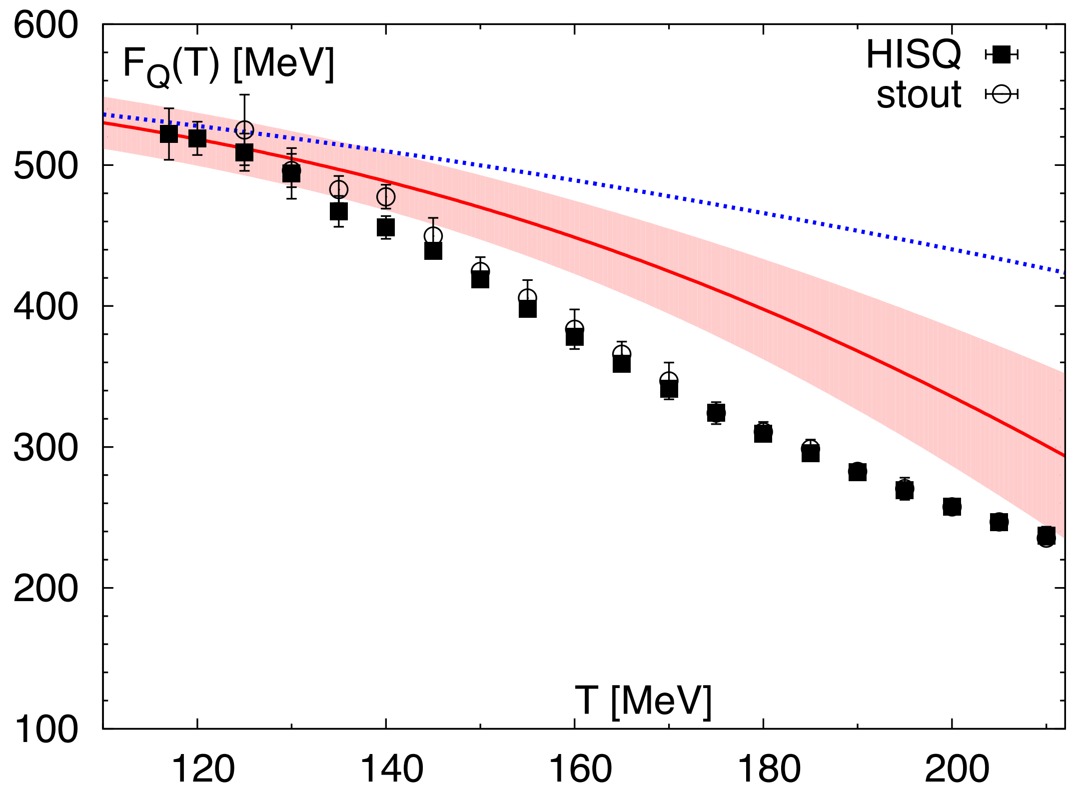
Similar results with stout action Borsanyi et al, JHEP04(2015) 138

Polyakov and gas of static-light hadrons

$$Z_{Q\bar{Q}}(T)/Z(T) = \sum_n \exp(-E_n^{Q\bar{Q}}(r \to \infty)/T)$$

Energies of static-light mesons: $E_n^{Q\bar{Q}}(r \to \infty) = M_n - m_Q$

Free energy of an isolated static quark: $F_Q(T) = -\frac{1}{2}(T \ln Z_{Q\bar{Q}}(T) - T \ln Z(T))$



Megias, Arriola, Salcedo,
PRL 109 (12) 151601

Bazavov, PP, PRD 87 (2013) 094505

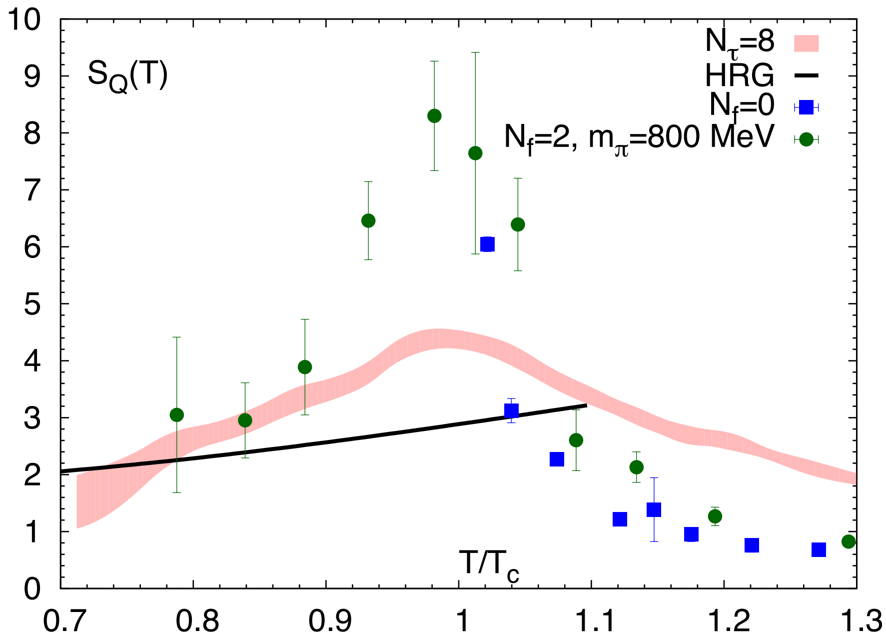
Ground state and first excited states are from lattice QCD
Michael, Shindler, Wagner,
arXiv1004.4235
Wagner, Wiese,
JHEP 1107 016,2011

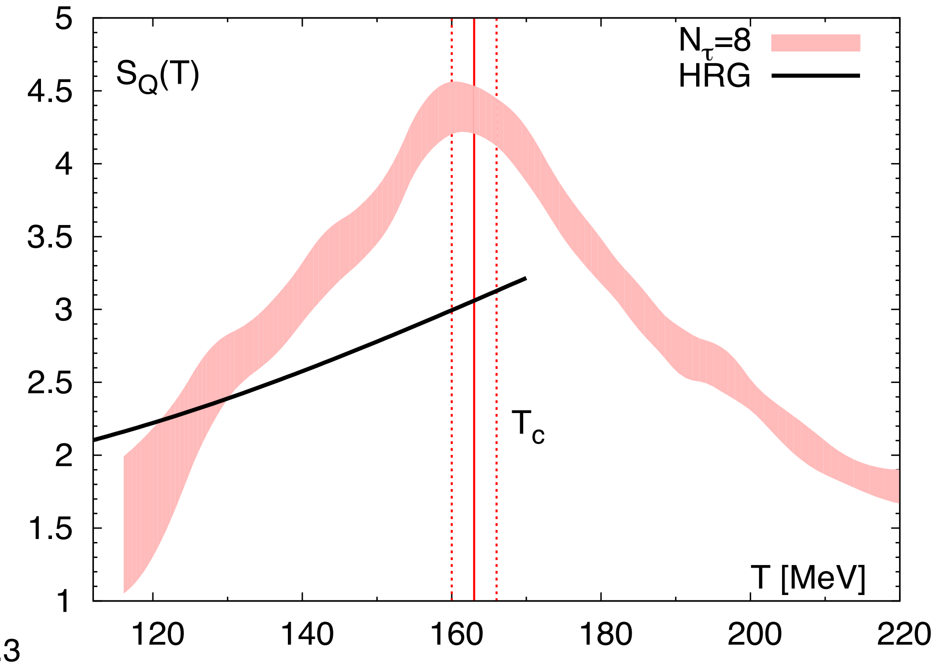
Higher excited state energies are estimated from potential model

Gas of static-light mesons only works for $T < 145$ MeV

The entropy of static quark

$$S_Q = -\frac{\partial F_Q}{\partial T}$$





At low T the entropy S_Q increases reflecting the increase of states the heavy quark can be coupled to

At high temperature the static quark only "sees" the medium within a Debye radius, as T increases the Debye radius decreases and S_Q also decreases

The onset of screening corresponds to peak is S_Q and its position coincides with T_c

Casimir scaling of the Polyakov loop

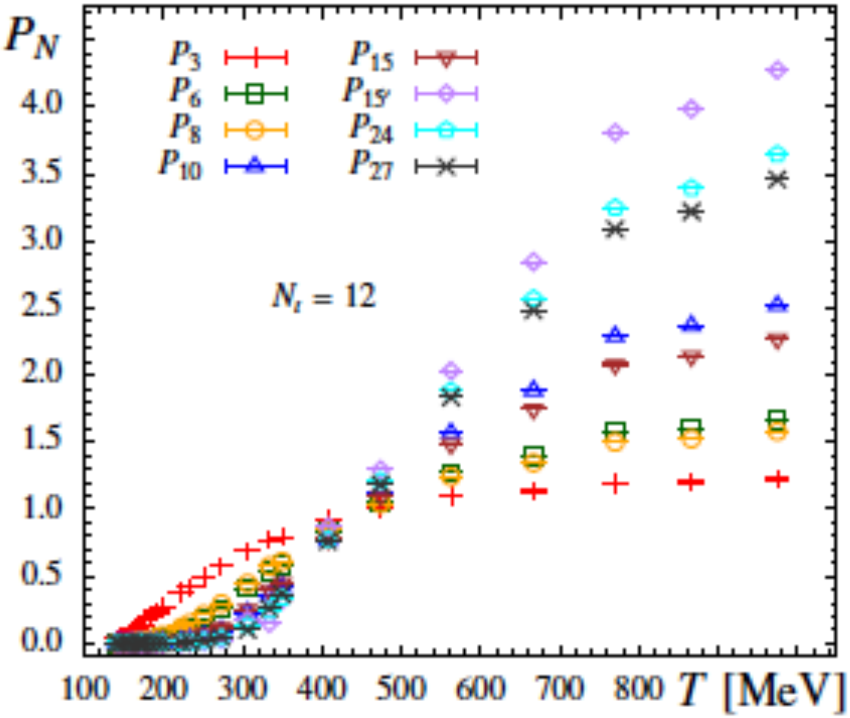
Instead of fundamental representations consider Polyakov loop P_n in arbitrary representation n

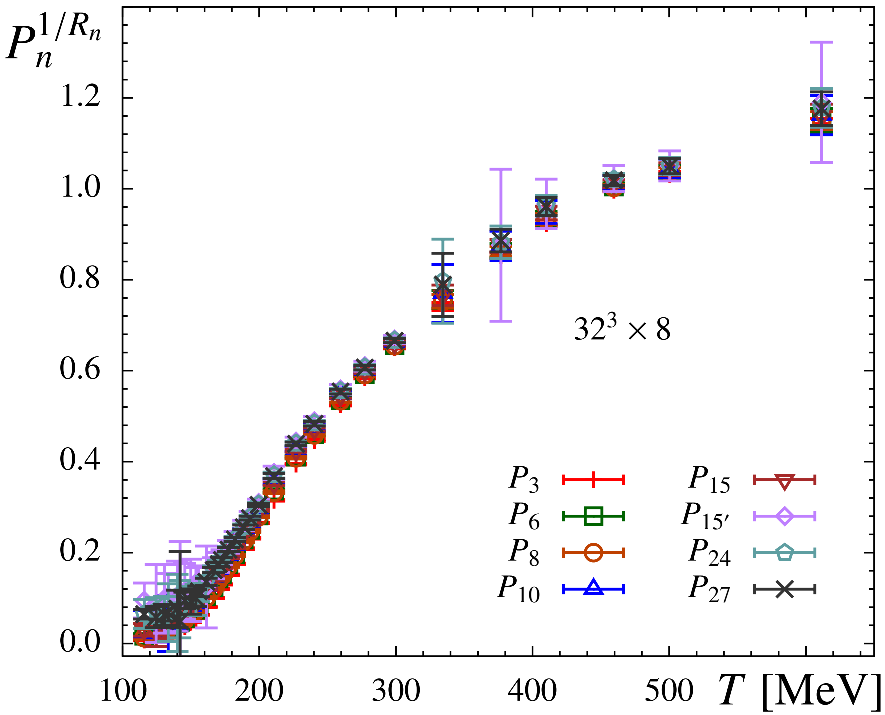
PP, Schadler, arXiv:1509.07874

$$P_3 = L_{ren}$$

Casimir scaling: free energy is proportional to qudratic Casimir operator C_n of rep n

$$R_n = C_n / C_3$$

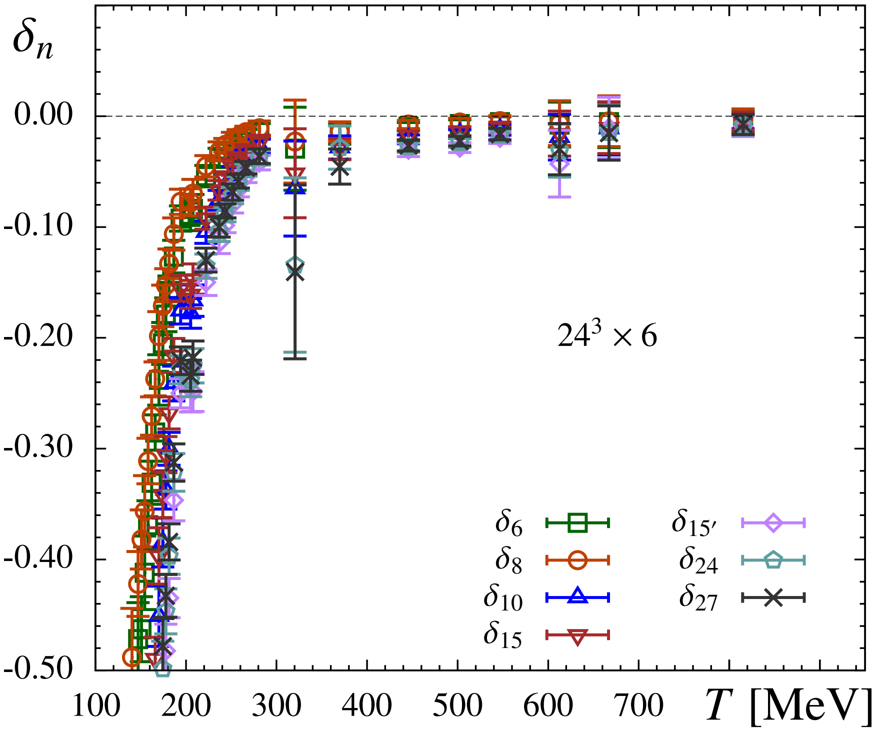


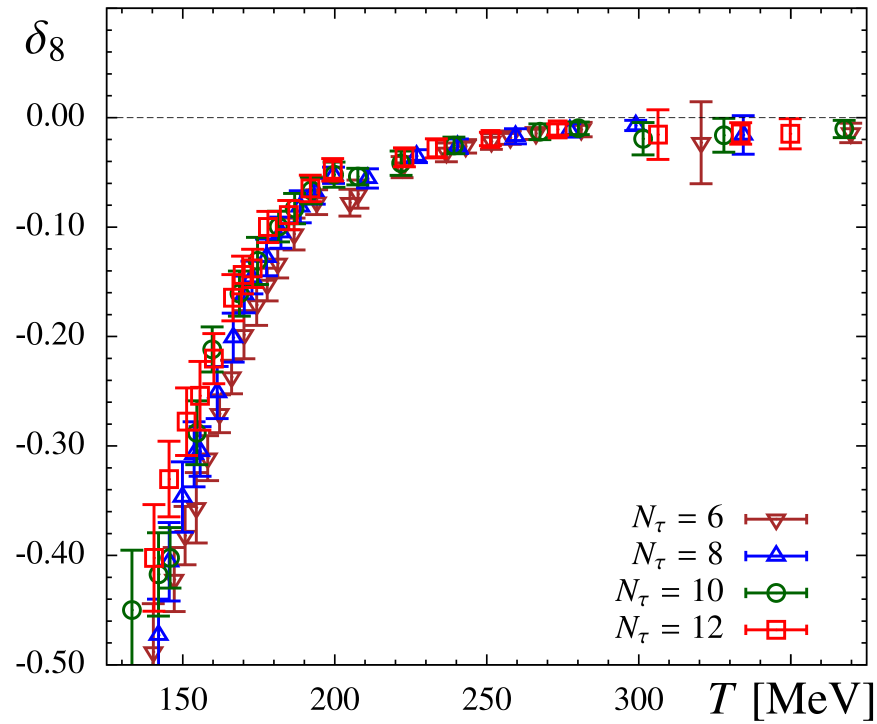


Expected in weak coupling expansion: e.g. at LO $F_Q^n = -C_n \alpha_s m_D$

Casimir scaling of the Polyakov loop (con't)

$$\delta_n = 1 - P_n^{1/R_n}/P_3$$



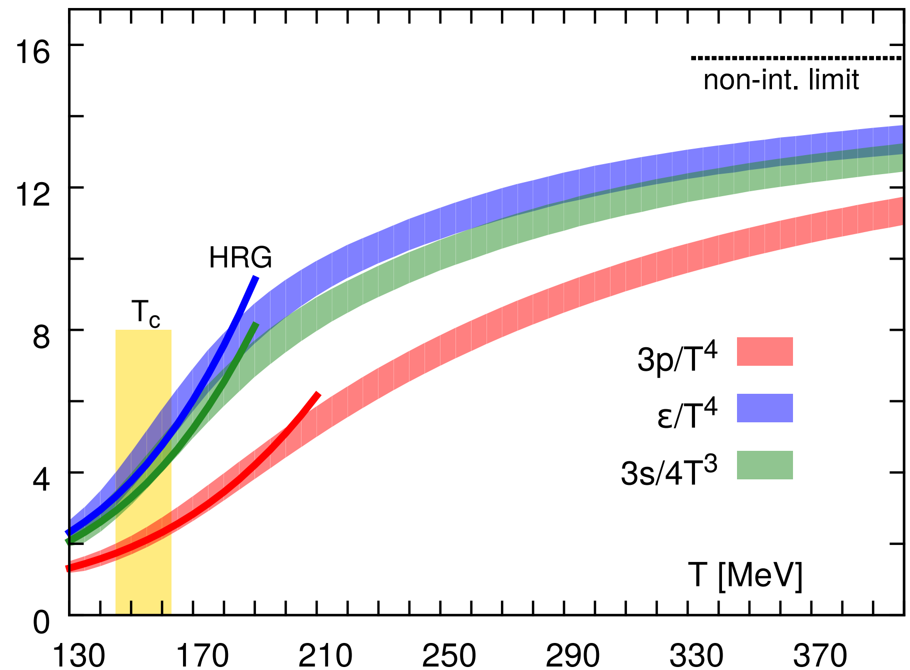


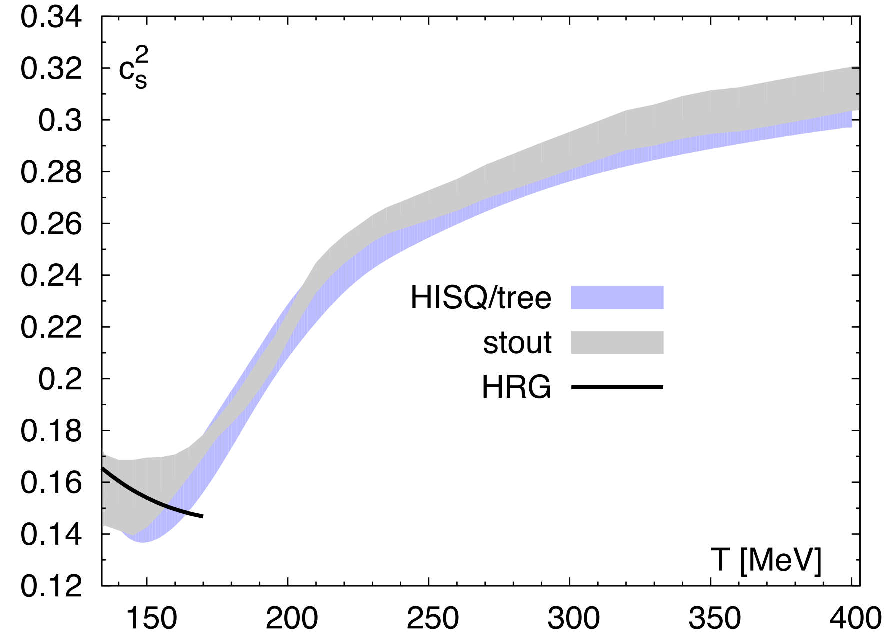
Casimir scaling holds for *T>300* MeV
color screening like in weakly coupled QGP ?

Equation of state in the continuum limit

Equation of state has be calculated in the continuum limit up to *T=400* MeV using two different quark actions and the results agree well

Bazavov et al, PRD 90 (2014) 094503





Hadron resonance gas (HRG):
Interacting gas of hadrons = non-interacting gas of hadrons and hadron resonances
(virial expansion, Prakash & Venugopalan)

HRG agrees with the lattice for *T< 145* MeV

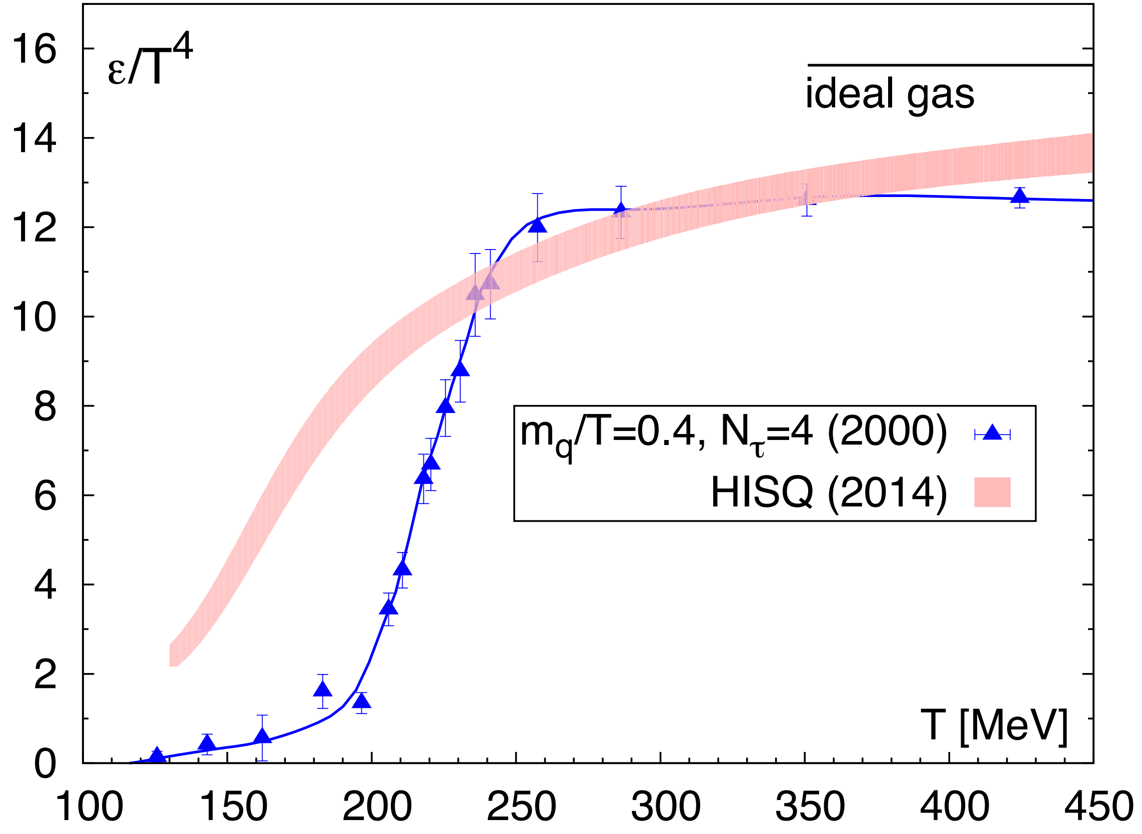
$$T_c = (154 \pm 9)\mathrm{MeV}$$

$$\epsilon_c \simeq 300\mathrm{MeV/fm}^3$$

$$\epsilon_{low} \simeq 180\mathrm{MeV/fm}^3 \leftrightarrow \epsilon_{nucl} \simeq 150\mathrm{MeV/fm}^3$$

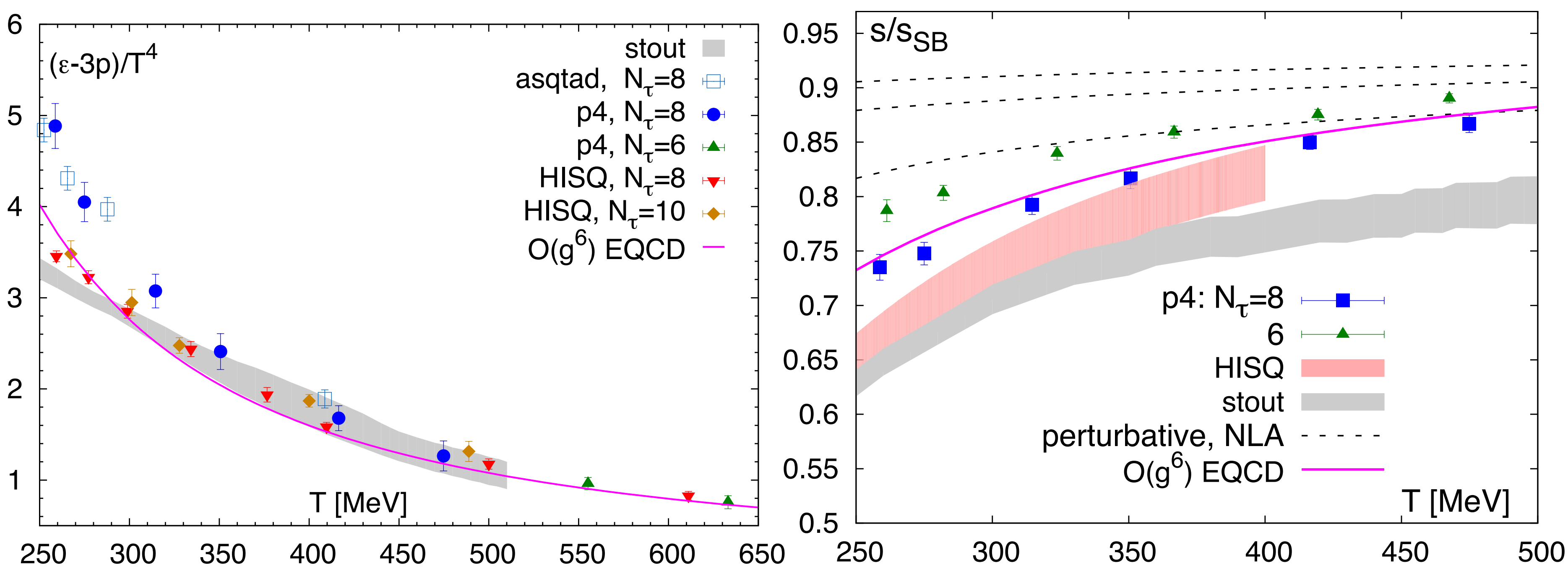
$$\epsilon_{high} \simeq 500\mathrm{MeV/fm}^3 \leftrightarrow \epsilon_{proton} \simeq 450\mathrm{MeV/fm}^3$$

How Equation of state changed since 2002



- Much smoother transition to QGP
- The energy density keeps increasing up to 450 MeV instead of flattening

Equation of State on the lattice and in the weak coupling



The high temperature behavior of the trace anomaly is not inconsistent with weak coupling calculations (EQCD) for $T>300$ MeV

For the entropy density the continuum lattice results are below the weak coupling calculations For $T< 500$ MeV

At what temperature can one see good agreement between the lattice and the weak coupling results ?

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

⬇

information about carriers of the conserved charges (hadrons or quarks)

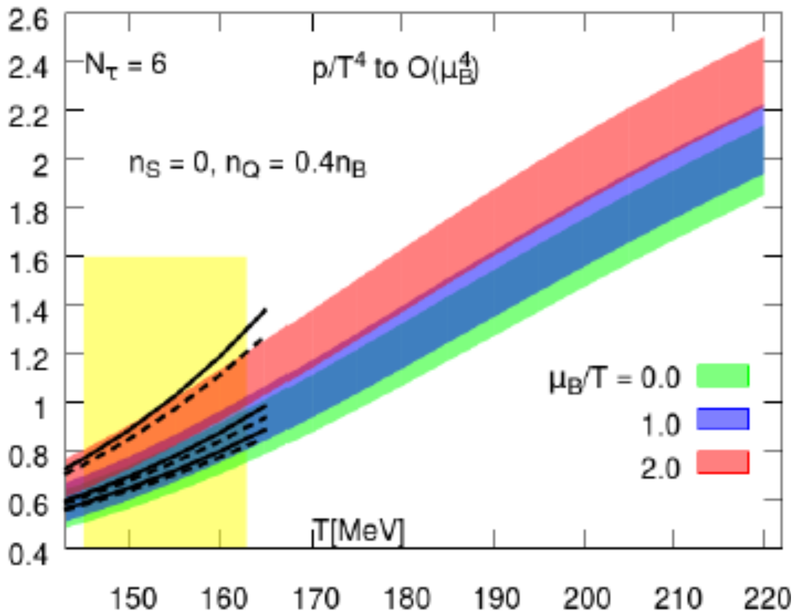
⬇

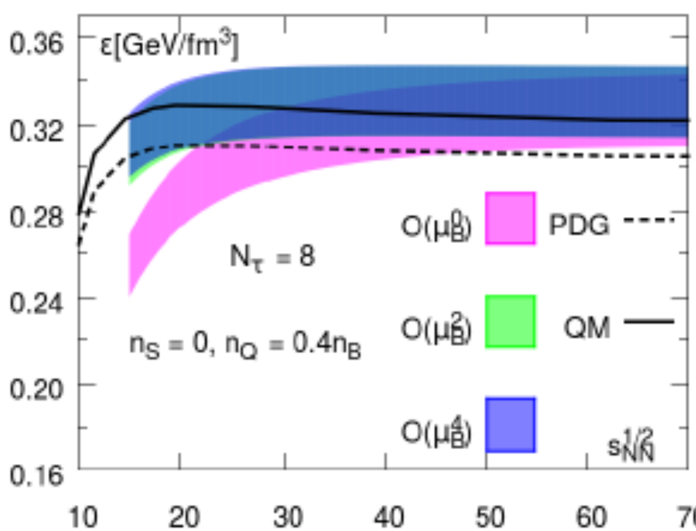
probes of deconfinement

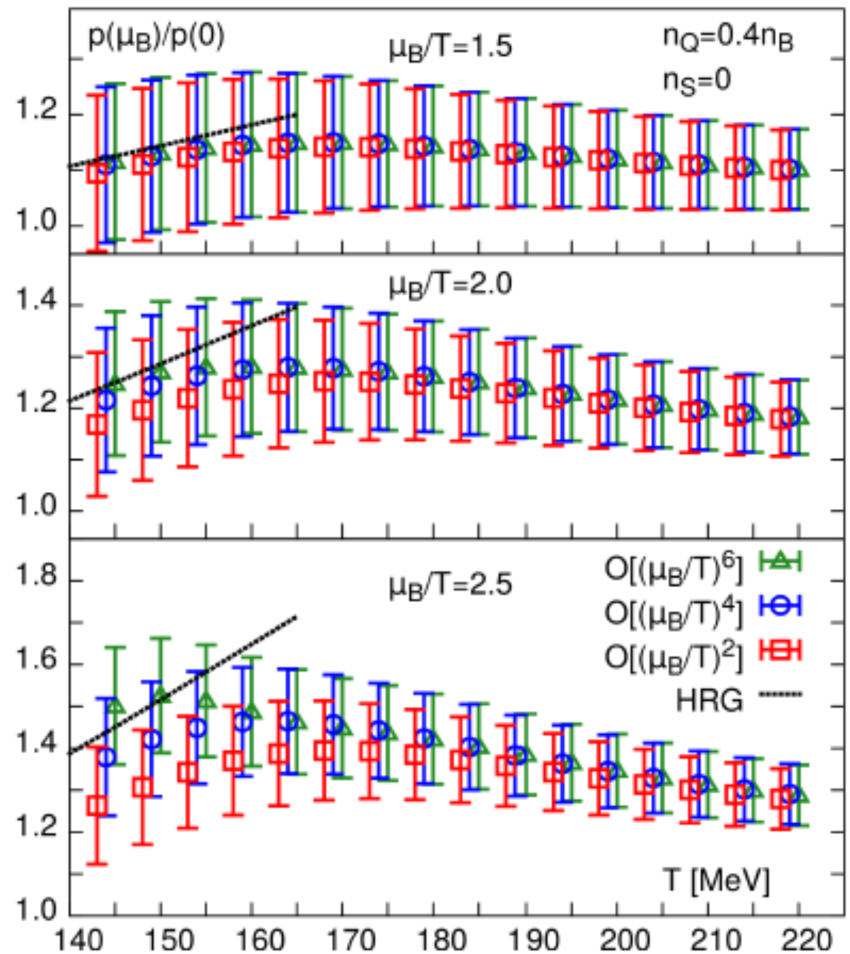
Equation of state at non-zero baryon density

Taylor expansion up to 4th order for net zero strangeness $n_S = 0$ and $r = n_Q/n_B = Z/A = 0.4$

BNL-Bielefeld-CCNU







Moderate effects due to non-zero baryon density up to $\mu_B/T = 2 \leftrightarrow \sqrt{s} \sim 20\mathrm{GeV}$

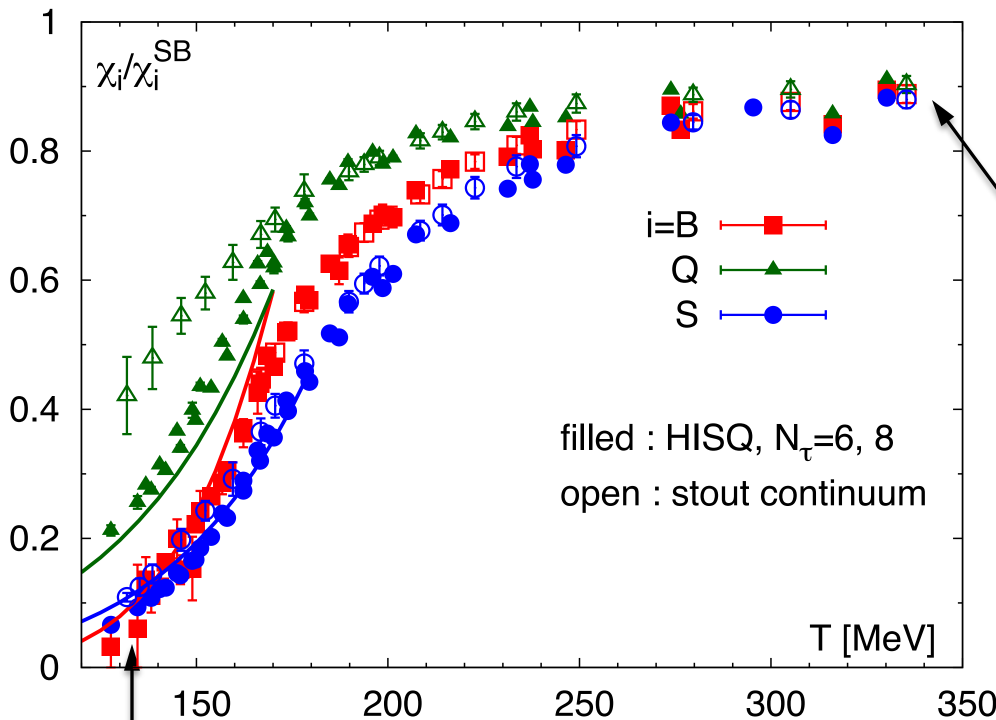
Energy density at freeze-out is independent of μ_B

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_S^{SB} = 1$$

conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T)\cosh\left(\frac{\mu_S}{T}\right) +$$
$$B_{S=1}(T)\cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T)\cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T)\cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

Strange hadrons are heavy treat them
As Boltzmann gas

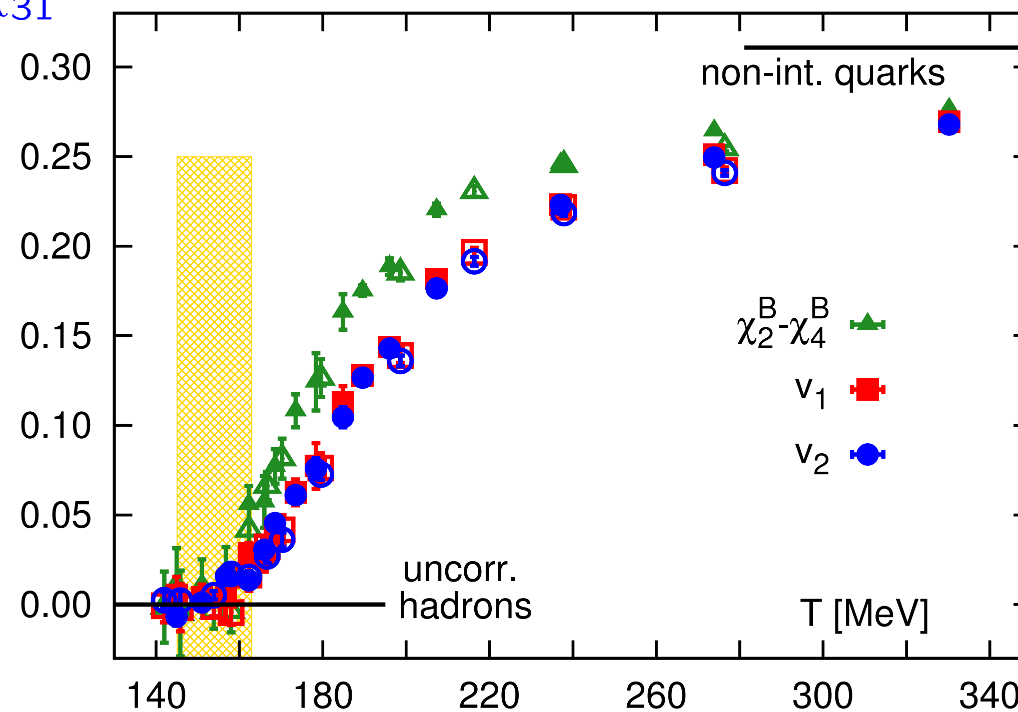
$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$
$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- v_1 and v_2 do vanish within errors at low T

- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

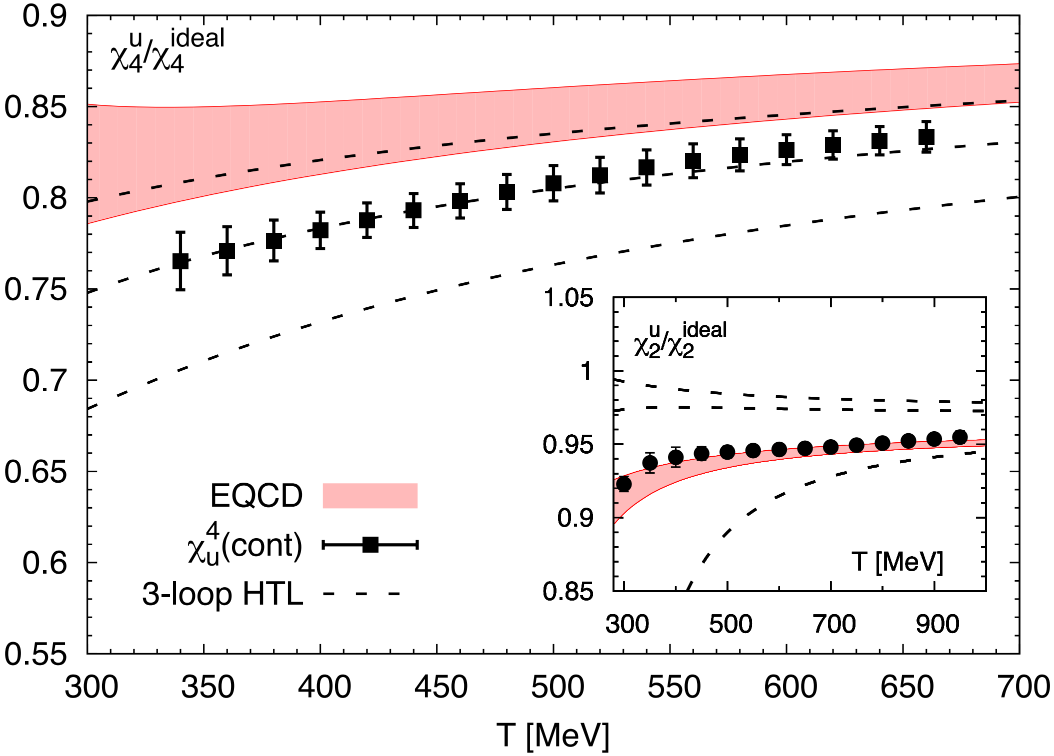
Bazavov et al, PRL 111 (2013) 082301



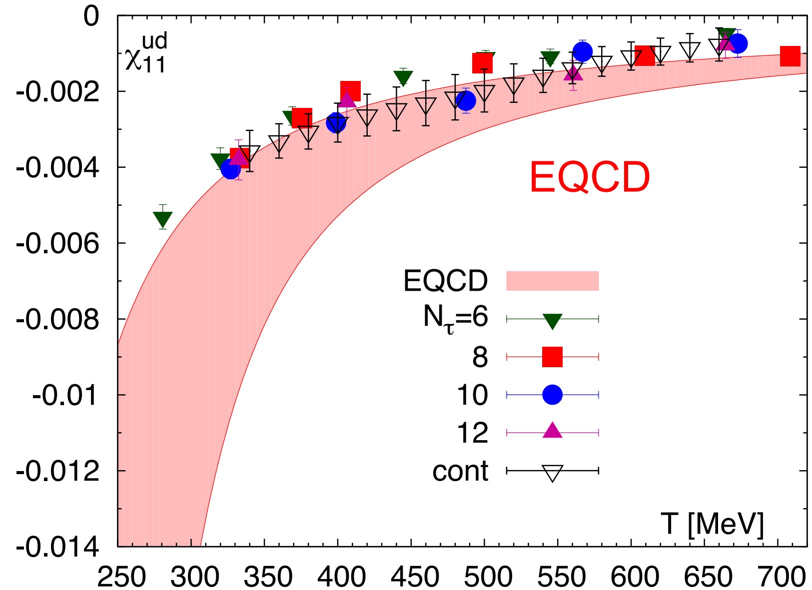
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations



quark number correlations



- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd and 4th order quark number fluctuations as well as for correlations

Bazavov et al, PRD88 (2013) 094021, Ding et at, arXiv:1507.06637

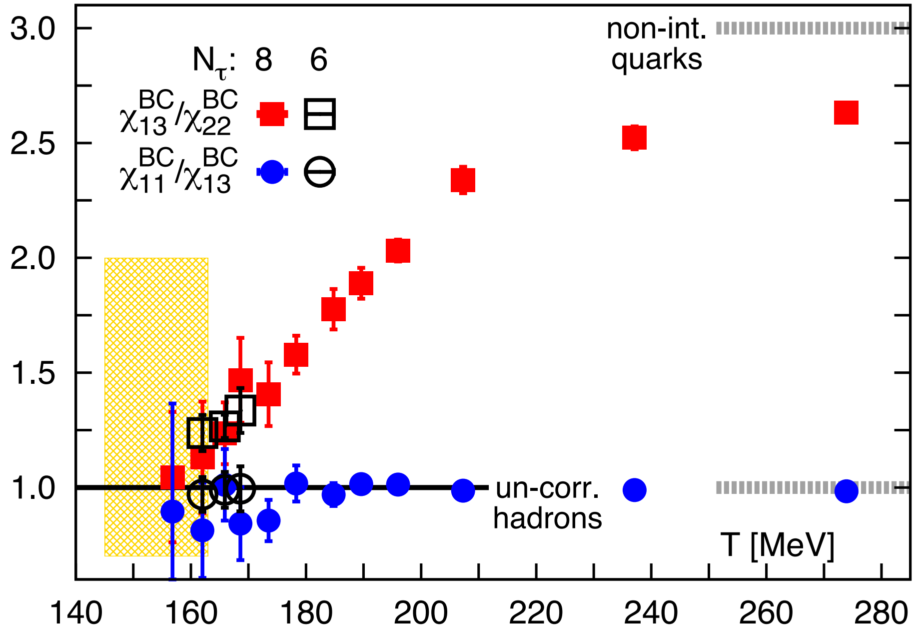
What about charm hadrons ?

$$\chi^{XYC}_{nml} = T^{m+n+l} \frac{\partial^{n+m+l} p(T,\mu_X,\mu_Y,\mu_C)/T^4}{\partial\mu^n_X \partial\mu^m_Y \partial\mu^l_C}$$

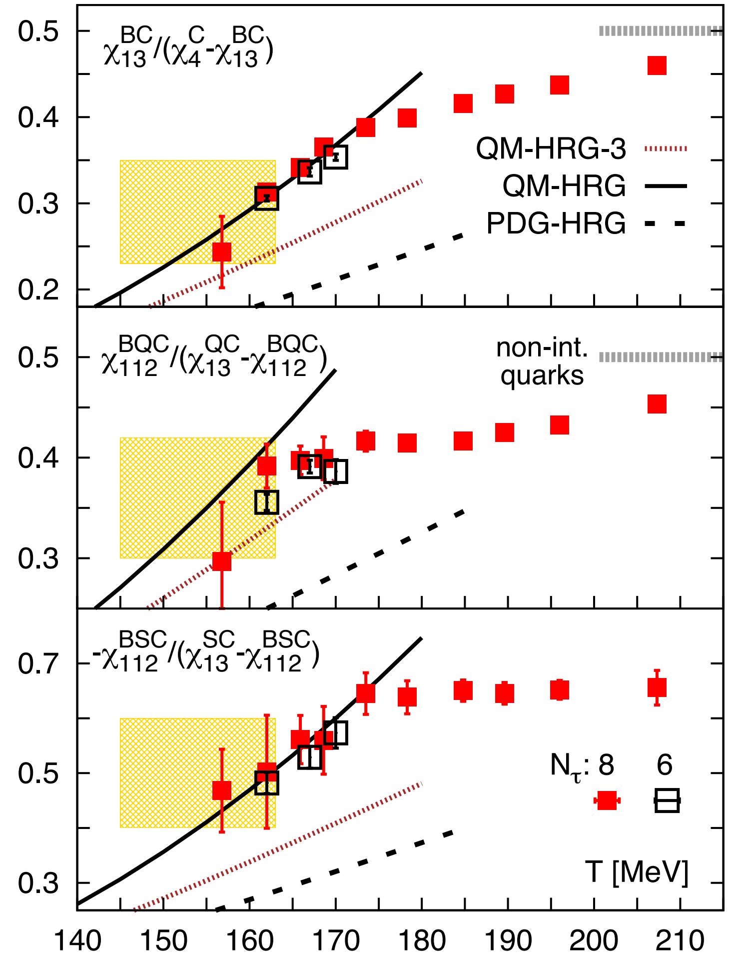
Bazavov et al, Phys.Lett. B737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !



Charm baryon to meson pressure



Hadronic description breaks down just above T_c

⇒ open charn deconfines above T_c

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the “missing” states are included

Quasi-particle model for charm degrees of freedom

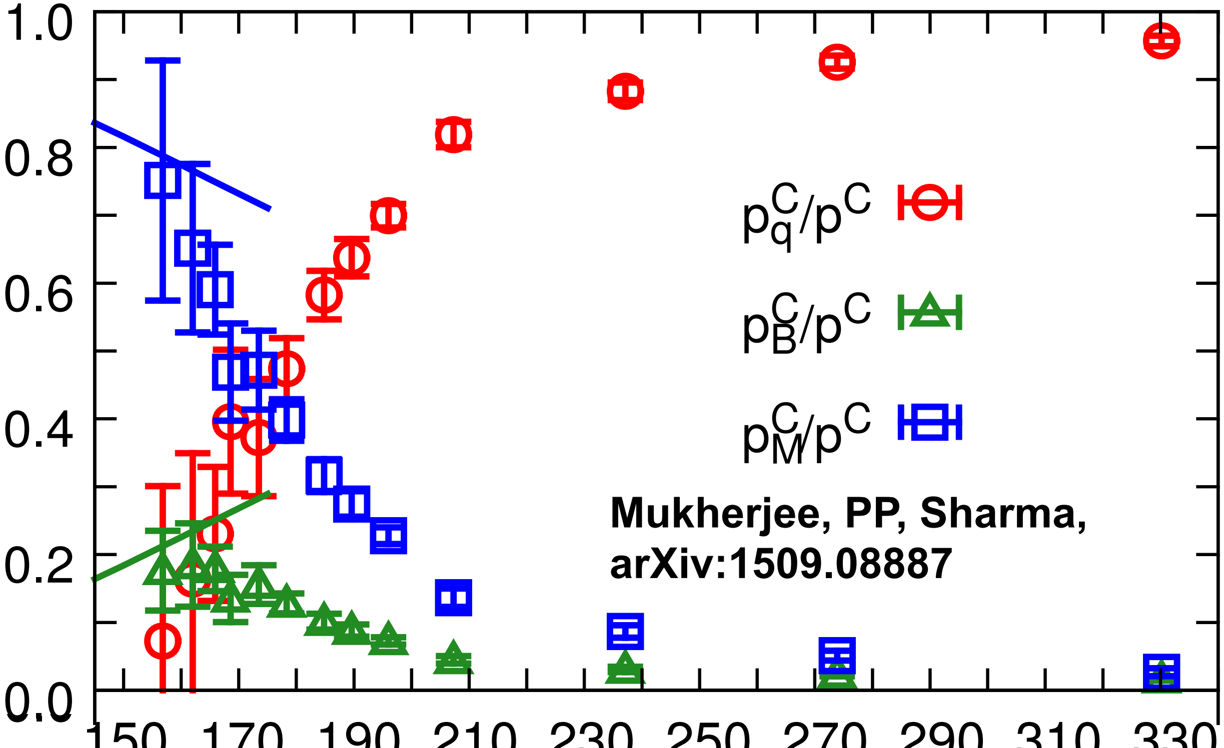
Charm dof are good quasi-particles at all T because $M_c >> T$ and Boltzmann approximation holds

$$p^C(T,\mu_B,\mu_c) = p_q^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T)\cosh(\hat{\mu}_C)$$

$$\hat{\mu}_X = \mu_X/T$$

$$\chi_2^C,\ \chi_{13}^{BC},\ \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T>200$ MeV



Partial pressures drop because hadronic cxcitations become broad at high temperatures (bound state peaks merge with the continuum)

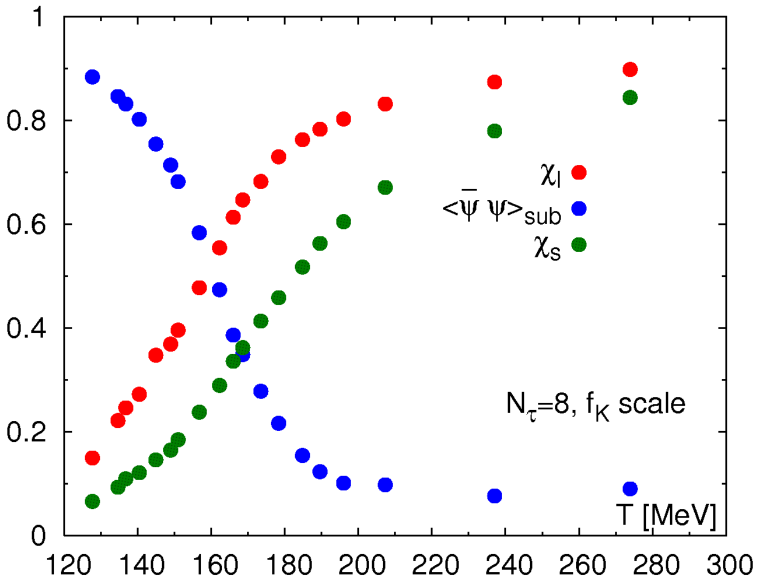
See
Jakovac, PRD88 ('13), 065012
Biro, Jakovac, PRD('14)065012

Vice versa for quarks

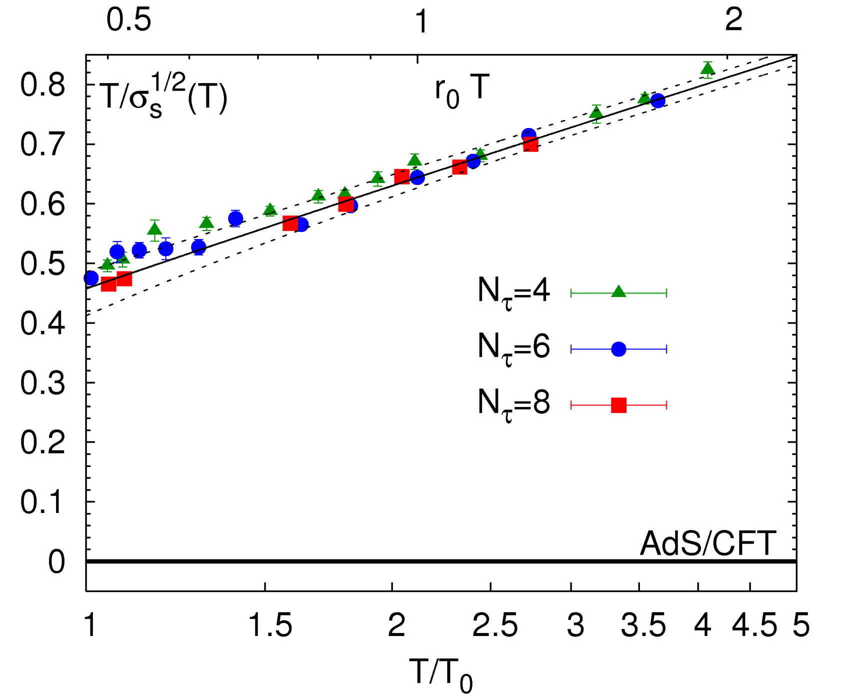
Summary

- The deconfinement transition temperature defined in terms of the free energy of static quark agrees with the chiral transition temperature for physical quark mass
- Equation of state are known in the continuum limit up to *T=400* MeV at zero baryon density and the effect of non vanishing baryon densities seem to be moderate.
- Hadron resonance gas (HRG) can describe various thermodynamic quantities at low temperatures
- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, it manifest itself as a abrupt breakdown of hadronic description that occurs around the chiral transition temperature
- Charm hadrons can exist above T_c and are dominant dof for $T<180$ MeV
- For *T* > *(300-400)* MeV weak coupling expansion works well for certain quantities (e.g. quark number susceptibilities), more work is needed to establish the connection between the lattice and the weak coupling results
- Comparison of lattice and HRG results for certain charm correlations hints for existence of yet undiscovered excited baryons

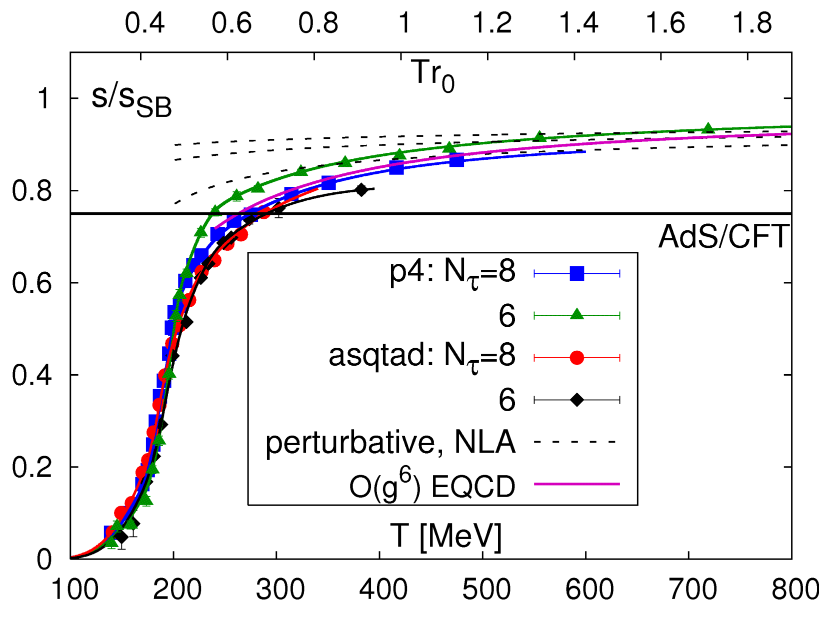
Back-up:



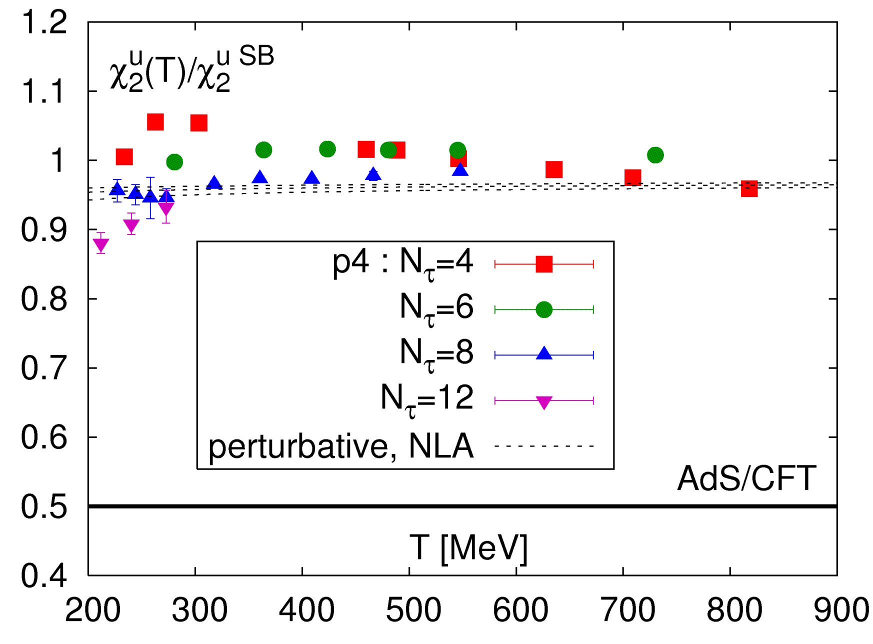
χ_l
$<\bar{\psi}\psi>_{sub}$
χ_s
N_τ=8, f_K scale
T [MeV]



$T/\sigma_s^{1/2}(T)$
r_0 T
N_τ=4
N_τ=6
N_τ=8
AdS/CFT
T/T_0



s/s_{SB}
Tr_0
AdS/CFT
p4: N_τ=8
6
asqtad: N_τ=8
6
perturbative, NLA
O(g^6) EQCD
T [MeV]

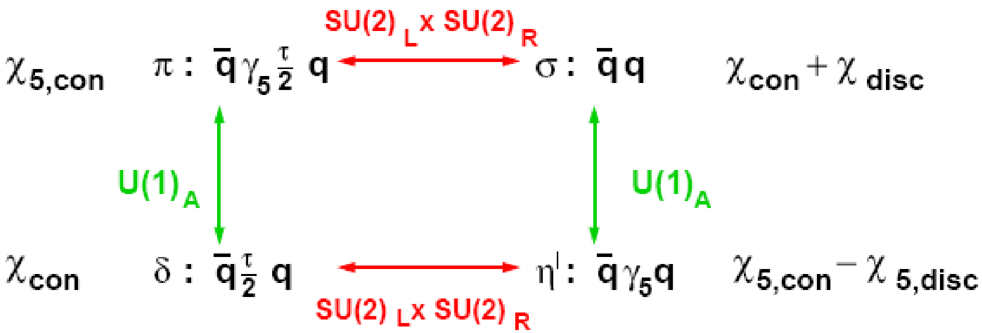


$\chi_2^u(T)/\chi_2^{u\ SB}$
p4 : N_τ=4
N_τ=6
N_τ=8
N_τ=12
perturbative, NLA
AdS/CFT
T [MeV]

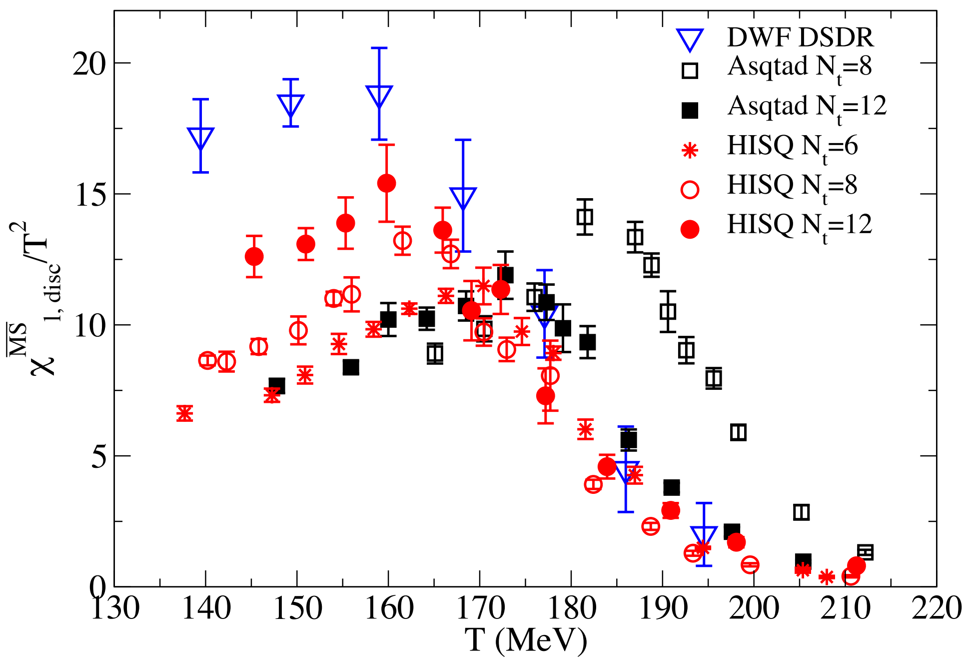
Domain wall Fermions and $U_A(1)$ symmetry restoration

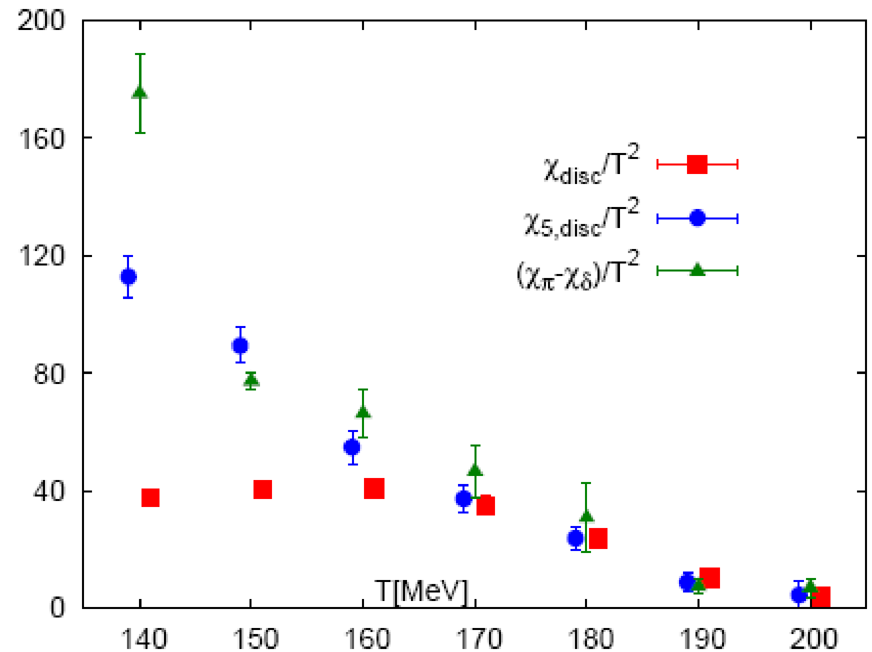
Domain Wall Fermions, Bazavov et al (HotQCD), PRD86 (2012) 094503

$$\chi_i = \int d^4x G_i(x)$$



chiral:	axial:
$\chi_\pi = \chi_\delta + \chi_{\rm disc}$	$\chi_\pi = \chi_\delta$
$\chi_\delta = \chi_\pi - \chi_{5,{\rm disc}}$	$\chi_\delta + \chi_{\rm disc} = \chi_\pi - \chi_{5,{\rm disc}}$
$\chi_{\rm disc} = \chi_{5,{\rm disc}}$	$\chi_{\rm disc} = -\chi_{5,{\rm disc}}$





Peak position roughly agrees with previous staggered results

axial symmetry is effectively restored *T*>200 MeV !

Improved staggered calculations at finite temperature

low T region

$T<200$ MeV

$\mathcal{O}(\alpha_s^n (a\Lambda_{QCD})^2)$ errors

$a>0.125fm$

hadronic degrees of freedom

improvement of the flavor symmetry is important $\rightarrow$ fat links

cutoff effects are different in :

$a = 1/(TN_\tau)$

$N_\tau = 8$

for #flavors < 4 rooting trick

$detD \rightarrow (detD)^{\frac{n_f}{4}}$

high-T region

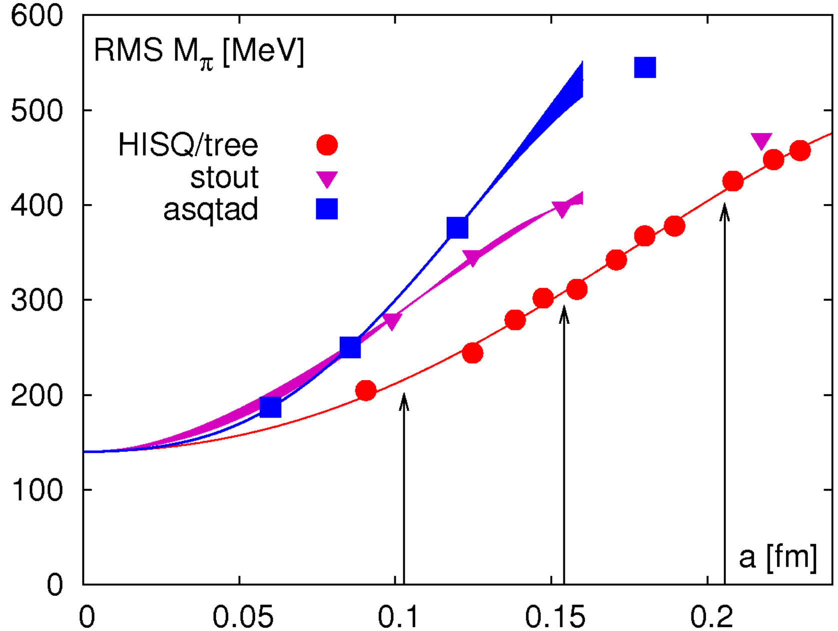
$T>200$MeV

$\mathcal{O}((aT)^2)$ errors

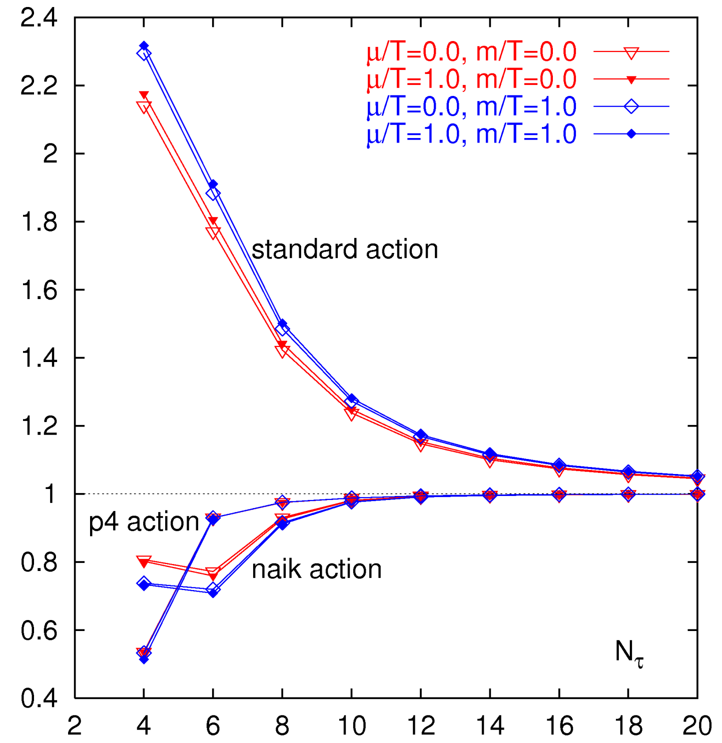
$a<0.125fm$

quark degrees of freedom

quark dispersion relation



p4, asqtad, HISQ, stout



The Highly Improved Staggered Quark (HISQ) Action

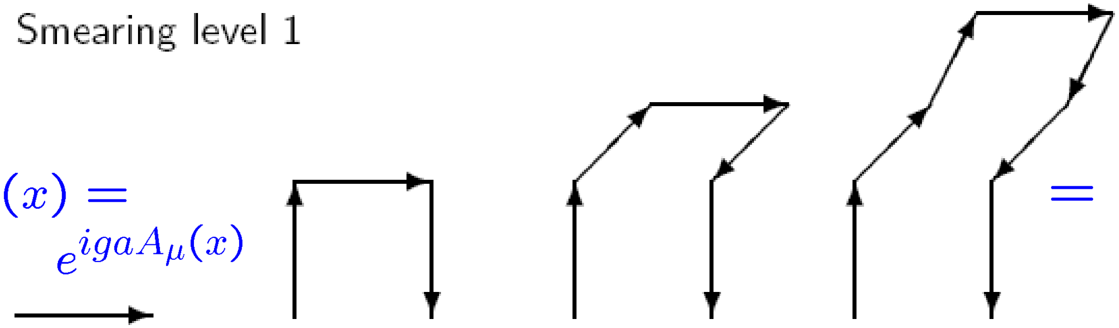
HISQ action

two levels of gauge field smearing with re-unitarization

Follana et al, PRD75 (07) 054502

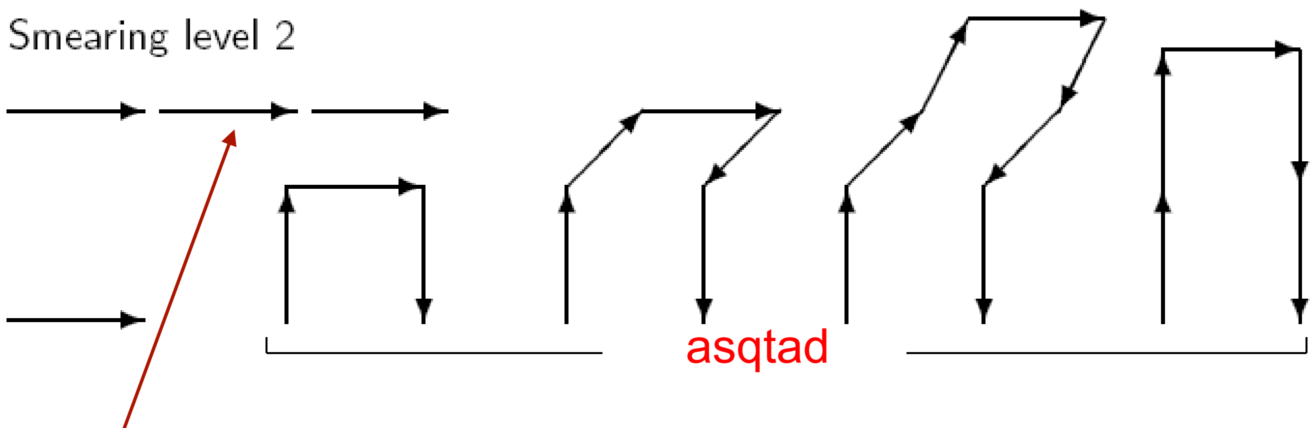
Smearing level 1

$U_\mu(x) = e^{igaA_\mu(x)}$



$= U_\mu^{fat7} \to \tilde{U}_\mu = \frac{U_\mu^{fat7}}{\sqrt{U_\mu^{fat7} U_\mu^{fat7\dagger}}}$

Smearing level 2



projection onto U(3) improves flavor symmetry

Hasenfratz, arXiv:hep-lat/0211007

3-link (Naik) term to improve the quark dispersion relation + asqtad smearing