Transport phenomena in a plasma of confining gluons

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- RHIC and LHC HIC data suggest that QGP is a strongly-interacting dissipative fluid

  $\Rightarrow$ relativistic dissipative fluid dynamics (equation of state?, transport coefficients?)

- equation of state (at high T)
  - lattice QCD
    (M. Cheng et al., PRD 77, 014511 (2008); S. Borsanyi et al., JHEP 11, 077 (2010);S. Borsanyi et al., JHEP 07, 056 (2012))
  - re-summed perturbation theory
    (J.-P. Blaizot, E. Iancu, Phys. Rept. 359, 355 (2002), ;J. O. Andersen, M. Strickland, Annals Phys. 317, 281 (2005); G. D. Moore, O. Saremi, JHEP 09, 015 (2008); J. Hong, D. Teaney, PRC 82, 044908 (2010); N. Su, Commun. Theor. Phys. 57, 409 (2012);)

- transport coefficients
  (W. Israel, J. M. Stewart, Ann. Phys. (NY) 118, 341 (1979); A. Muronga, PRC 69, 034903 (2004); A. El et al., PRC 81, 041901(R) (2010); G. S. Denicol et al., PRL 105, 162501 (2010); A. Jaiswal, PRC 87, 051901(R) (2013))

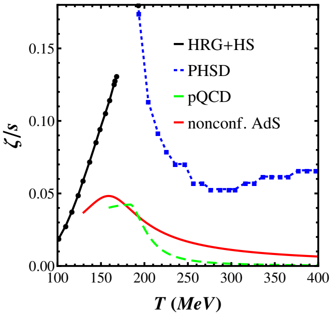
$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\frac{\zeta}{\tau_\Pi}\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

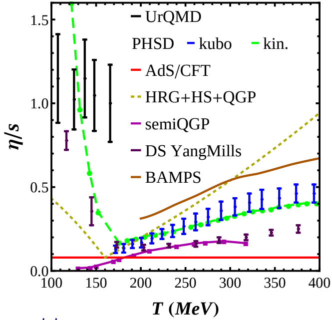
$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\frac{\eta}{\tau_\pi}\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$

$\zeta \to$ bulk viscosity $\qquad \eta \to$ shear viscosity

- by force of the Kubo formulas, they are sensitive to the long-distance dynamics of the underlying microscopic theory

- transport coefficients (cont'd)
  - pQCD (P. B. Arnold et al., JHEP 11, 001 (2000); P. B. Arnold et al., JHEP 05, 051 (2003); G. D. Moore, O. Saremi, JHEP 09, 015 (2008); P. B. Arnold et al., PRD 74, 085021 (2006);)
  - functional renormalization techniques (M. Haas et al., PRD90, 091501 (2014); N. Christiansen et al., (2014), arXiv:1411.7986 (hep-ph))
  - low energy theorems (D. Kharzeev, K. Tuchin, JHEP 09, 093 (2008); F. Karsch et al., PLB 663, 217 (2008);)
  - *l*QCD (H. B. Meyer, PRL 100, 162001 (2008); H. B. Meyer, PRD 76, 101701 (2007);)
  - $\mathcal{N} = 4$ supersymmetric YM plasma with broken conformal symmetry (G. Policastro et al., PRL 87, 081601 (2001); P. Benincasa et al., NPB 733, 160 (2006); A. Buchel, PRD 72, 106002 (2005); S. I. Finazzo et al., JHEP 02, 051 (2015)))
  - HRG+HS (G.P. Kadam, H. Mishra, NPA 934 (2014) 133-147))
  - parton-hadron-string dynamics transport approach (V. Ozvenchuk, O. Linnyk, M. I. Gorenstein, E. L. Bratkovskaya and W. Cassing, PRC 87, no. 6, 064903 (2013)))
  - . . .





around T_C ζ and η are comparable

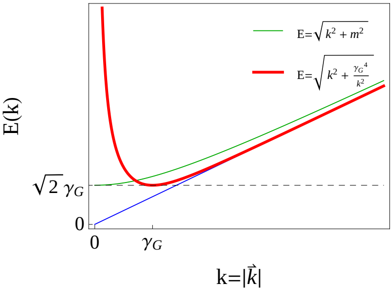
- Gribov quantization of YM theory - fixing the infrared (IR) residual gauge transformations remaining after Faddeev-Popov procedure

$\Downarrow$

a new scale γ_G that leads to an IR-improved dispersion relation for gluons (Coulomb gauge)
(V. Gribov, NPB 139, 1 (1978);
D. Zwanziger, NPB 323, 513 (1989))

$$E(\mathbf{k}) = |\mathbf{k}| \qquad \rightarrow \qquad E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{\mathbf{k}^2}}$$

- attracted a lot of attention recently
(G. Burgio, M. Quandt, H. Reinhardt, PRL 102, 032002 (2009);
N. Su, K. Tywoniuk, PRL. 114, 161601 (2015);
D. E. Kharzeev, E. M. Levin, PRL 114, 242001 (2015);
. . .)



- reduction of the physical phase space due to the large energy cost of the excitation of soft gluons

$\Downarrow$

essential feature of the confinement
(V. Gribov, NPB 139, 1 (1978);
R.P. Feynman, NPB 188, 479 (1981);
D. Zwanziger, NPB 485, 185 (1997);)

Covariant setup

local rest frame → comoving frame

$$E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{\mathbf{k}^2}} \quad \rightarrow \quad E(k\cdot u) = \sqrt{(k\cdot u)^2 + \frac{\gamma_G^4}{(k\cdot u)^2}}$$

- thermodynamics of the system of confining gluons

(D. Zwanziger, PRL 94, 182301 (2005);) (W. F., R.Ryblewski, N. Su, K. Tywoniuk, arXiv:1504.03176;)

$$\varepsilon = g_0 \int \frac{d^3k}{(2\pi)^3} E(\mathbf{k})\, f(\mathbf{k}) \quad \rightarrow \quad \varepsilon = \int dK\, E(k\cdot u)\, f(x,k)$$

$$P = \frac{g_0}{3} \int \frac{d^3k}{(2\pi)^3} |\mathbf{k}| \frac{\partial E(\mathbf{k})}{\partial |\mathbf{k}|}\, f(\mathbf{k}) \quad \rightarrow \quad P = \frac{1}{3} \int dK\, \frac{(k\cdot u)^2}{E(k\cdot u)} \left(1 - \frac{\gamma_G^4}{(k\cdot u)^4}\right) f(x,k)$$

$$\int dK\, (\dots) = g_0 \int \frac{d^3k}{(2\pi)^3 k^0} (k\cdot u)\, (\dots)$$

$$g_0 = 2(N_c^2 - 1) \qquad (SU(N_c))$$

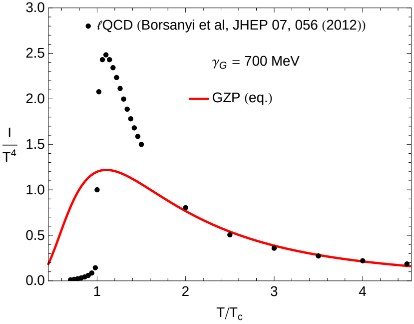
in high-T limit $\gamma_G \to \frac{d}{d+1} \frac{N_c}{4\sqrt{2}\pi} g^2 T$

D. Zwanziger, PRD 76, 125014 (2007)

$T \approx (2-4)\, T_c \qquad \Rightarrow \qquad \gamma_G \approx \text{const.}$

$\gamma_G(T)$ derived from the gap equation with running coupling from lQCD

J. O. Andersen et al., PRL 104, 122003 (2010);
K. Fukushima, N. Su, PRD 88, 076008 (2013);
W. F., R.Ryblewski, N. Su, K. Tywoniuk, arXiv:1504.03176



(0+1)D Bjorken symmetry implementation

- consider the case of a transversely homogeneous boost-invariant system
- assume the Bjorken flow of matter in longitudinal direction (boost-invariance)
  (J. D. Bjorken, PRD 27, 140 (1983);)

$$u^\mu = (t/\tau, 0, 0, z/\tau)$$

- introduce convenient boost-invariant variables
  (A. Bialas et al., NPB 296, 611 (1988);)

$$v = k^0 t - k_\parallel z$$
$$w = k_\parallel t - k^0 z$$

- EOM follow from the conservation of $T^{\mu\nu}$

$$T^{\mu\nu} = \int dK\, k^\mu k^\nu f(x,k)$$

- $f = f(\tau, w, k_\perp)$

- within the assumed symmetries the $T^{\mu\nu}$ has the spherically anisotropic form
  (W. F., R. Ryblewski, PRC 85, 044902 (2012);
  M. Martinez et al., PRC 85, 064913 (2012);)

$$T^{\mu\nu} = (\varepsilon + P_\perp) u^\mu u^\nu - P_\perp g^{\mu\nu} + (P_\parallel - P_\perp) z^\mu z^\nu$$
$$z^\mu = (z/\tau, 0, 0, t/\tau)$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad \Rightarrow \qquad \frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_\parallel}{\tau} = 0$$

Non-equilibrium fluid dynamics of GZ plasma

(A. Muronga, PRC 69, 034903 (2004);
R. Baier et al., PRC 73, 064903 (2006);)

$$\frac{d\varepsilon}{d\tau} + \frac{1}{\tau}\left(\varepsilon + P_{\rm GZ} + \Pi - \pi\right) = 0 \qquad \Leftrightarrow \qquad \frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_\parallel}{\tau} = 0$$

- dissipative fluxes

$$\pi = \frac{4}{3}\frac{\eta}{\tau} \qquad \pi = \frac{2}{3}\left(P_\parallel - P_\perp\right)$$

$$\Pi = -\frac{\zeta}{\tau} \qquad \Pi = P - P_{\rm GZ} = \frac{1}{3}(P_\parallel + 2P_\perp) - P_{\rm GZ}$$

(Navier-Stokes dissipative fluid dynamics)

- what is the form of ζ and η for GZ plasma?, close to equilibrium one can use the linear response approximation $f \approx f_{\rm GZ} + \delta f + \cdots$

$$\zeta(T,\gamma_{\rm G}) = \frac{g_0 \gamma_{\rm G}^5}{3\pi^2}\frac{\tau_{\rm rel}}{T}\int_0^\infty dy\left[c_s^2 - \frac{1}{3}\frac{y^4-1}{y^4+1}\right] f_{\rm GZ}(1+f_{\rm GZ})$$

$$\eta(T,\gamma_{\rm G}) = \frac{1}{10}\frac{g_0 \gamma_{\rm G}^5}{3\pi^2}\frac{\tau_{\rm rel}}{T}\int_0^\infty dy\,\frac{\left(y^4-1\right)^2}{y^4+1} f_{\rm GZ}(1+f_{\rm GZ})$$

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_{\parallel}}{\tau} = \int dK\, E(\tau, w, k_\perp)\, \frac{\partial f(\tau, w, k_\perp)}{\partial \tau}$$

- (0+1)D kinetic equation in RTA
  P. L. Bhatnagar et al., Phys. Rev. 94, 511 (1954);
  G. Baym, PLB 138, 18 (1984);
  G. Baym, NPA 418, 525C (1984);)

$$\frac{\partial f(\tau, w, k_\perp)}{\partial \tau} = \frac{f_{GZ}(\tau, w, k_\perp) - f(\tau, w, k_\perp)}{\tau_{rel}(\tau)}$$

- satisfied as long as Landau matching condition is satisfied $\varepsilon_{GZ} = \varepsilon$
- formal solution
  NPA 916, 249 (2013);
  PRC 88, 024903 (2013);
  PRC 89, 054908 (2014);)

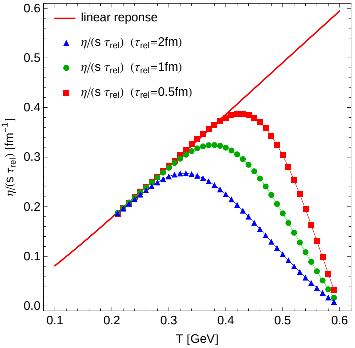
$$f(\tau, w, k_\perp) = f_0(w, k_\perp) D(\tau, \tau_0) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{rel}(\tau')} D(\tau, \tau') f_{GZ}(\tau', w, k_\perp)$$

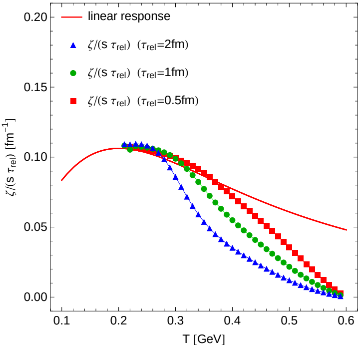
$$\varepsilon = \int dK\, E(\tau, w, k_\perp)\, f(\tau, w, k_\perp)$$

$$P_{\parallel} = \int dK \frac{w^2}{\tau^2 E(\tau, w, k_\perp)} \left[1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_\perp^2)^2}\right] f$$

$$P_{\perp} = \int dK \frac{k_\perp^2}{2\, E(\tau, w, k_\perp)} \left[1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_\perp^2)^2}\right] f$$

$$D(\tau_2, \tau_1) = \exp\left[-\int_{\tau_1}^{\tau_2} \frac{d\tau}{\tau_{rel}(\tau)}\right]$$


linear reponse
η/(s τrel) (τrel=2fm)
η/(s τrel) (τrel=1fm)
η/(s τrel) (τrel=0.5fm)
η/(s τrel) [fm−1]
T [GeV]


linear response
ζ/(s τrel) (τrel=2fm)
ζ/(s τrel) (τrel=1fm)
ζ/(s τrel) (τrel=0.5fm)
ζ/(s τrel) [fm−1]
T [GeV]

$$\frac{\zeta}{\eta} = 15\left(\frac{1}{3} - c_s^2\right)^2$$

- photon gas coupled to hot matter
  (S. Weinberg, Astrophys. J. 168, 175 (1971);)

  scalar theory
  (A. Hosoya et al., Ann. Phys. (N.Y.) 154, 229 (1984); R. Horsley, W. Schoenmaker, NPB 280, 716 (1987);)

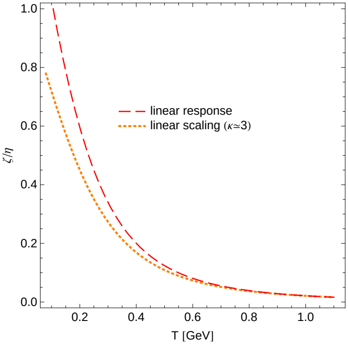
  weakly-coupled QCD (large-T limit)
  (P. B. Arnold et al., JHEP 0011 (2000) 001; P. B. Arnold et al., JHEP 0305 (2003) 051; P. B. Arnold et al., PRD 74 (2006) 085021;)

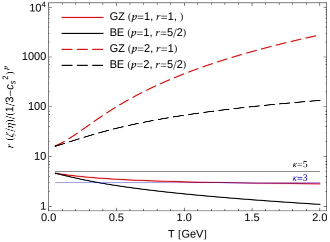
$$\frac{\zeta}{\eta} = \kappa\left(\frac{1}{3} - c_s^2\right)$$

- strongly-coupled nearly-conformal gauge theory plasma using gauge theory–gravity duality (large-T limit) ($\kappa = 2$, $\kappa = 4.558 - 4.935$)
  (P. Benincasa et al., NPB 733, 160 (2006); A. Buchel, PRD 72, 106002 (2005);)

- **Gribov's plasma of confining gluons** ($\kappa = 3$)
  (W. Florkowski, R.R., N. Su, K. Tywoniuk arXiv:1504.03176; W. Florkowski, R.R., N. Su, K. Tywoniuk, forthcoming;)

  different from massive BE gas ! →
  (C. Sasaki, K. Redlich, PRC 79, 055207 (2009);)





Summary

- a dynamic and non-equilibrium description of a plasma consisting of confining gluons (obtained from the Gribov quantization of SU(3) YM theory) introduced for the first time
- the expressions for the shear and bulk viscosities of the Gribov-Zwanziger plasma were derived
- ζ/η T-scaling which is in line with the strong-coupling methods results was found

Outlook

- include the running $\gamma_{\rm G}(T)$
- use our formula in the hydrodynamic symulations