

## Particle production beyond the thermal model

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**Abstract.** The sources of particle production in relativistic heavy-ion collisions are investigated from RHIC to LHC energies. Whereas charged-hadron production in the fragmentation sources follows a  $\ln(s_{NN}/s_0)$  law, particle production in the mid-rapidity low- $x$  gluon-gluon source exhibits a much stronger dependence  $\propto \ln^3(s_{NN}/s_0)$ , and becomes dominant between RHIC and LHC energies. The equilibration of the three sources is investigated in a relativistic diffusion model (RDM). It agrees with the thermal model only for  $t \rightarrow \infty$ .

### 1 Introduction

The statistical hadronization or thermal model [1] has consistently provided good descriptions of relative or absolute particle production yields in  $e^+e^-$ ,  $pp$  and relativistic heavy-ion collisions, e.g. [2, 3]. However, a necessary and sufficient condition for statistical equilibrium in the systems under investigation is provided by the distribution functions of the relevant observables rather than the particle yields. An example for a purely thermal distribution may be found in the cosmic microwave background radiation. It has a blackbody spectrum with a temperature of 2.735 K at redshift zero [4], although there are temperature fluctuations on the level of less than 1 part in  $10^5$  which give rise to structure formation, and have meanwhile been measured with excellent accuracy by the WMAP [5] and Planck [6] collaborations.

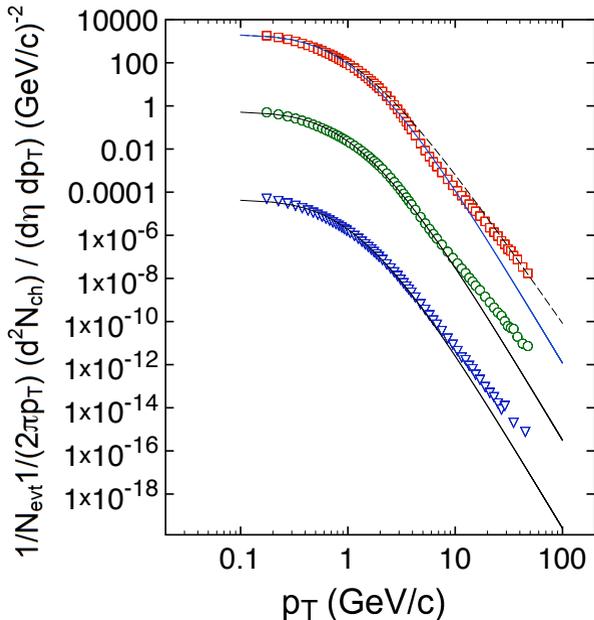
In relativistic heavy-ion collisions, the distributions of both transverse momentum  $p_T$  (Fig. 1) as well as rapidity  $y$  (or pseudorapidity  $\eta$ ) of produced charged hadrons (Figs. 2, 4) clearly deviate from thermal distributions. At RHIC and LHC energies, the deviations in a  $p_T$ -region of  $0.5 < p_T < 8$  GeV/ $c$  and the ensuing transition from exponential to power-law  $p_T$ -distributions are usually attributed to collective expansion. This transition is conveniently parametrized by the distribution functions shown in Fig 1. Above 8 GeV/ $c$ , however, hard processes become visible which require a pQCD treatment. When integrated over  $p_T$  to obtain particle yields, their contribution is almost negligible, but decisive as an indicator for nonequilibrium processes.

Indications for nonequilibrium behaviour can also be found in (pseudo)rapidity distributions of produced charged hadrons as measured by ALICE in PbPb [7], and both ATLAS and ALICE in  $p$ Pb collisions [8, 9]. Here the distribution functions have important contributions from the fragmentation regions that are clearly visible in net-proton rapidity distributions at SPS and RHIC energies [10],

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increase in particle content with  $\log(s_{NN})$  for produced particles (Fig. 3), and are not in equilibrium with particles produced in the midrapidity source.



**Figure 1.** Transverse momentum distributions of produced charged hadrons in 2.76 TeV PbPb collisions as measured by ALICE [3] for 0-5%, 30-40% and 70-80% centralities compared with distribution functions  $\propto [1 + (q - 1)m_T/T]^{-q/(q-1)}$  for  $q = 1.12$  that implicitly account for thermal emission and collective expansion, but not for hard (pQCD) processes at  $p_T \geq 8$  GeV/c. The dashed curve is for  $q = 1.14$ . From GW, unpublished.

## 2 Relativistic diffusion model

In the Relativistic Diffusion Model (RDM) [11, 12], the (pseudo-)rapidity distribution of produced particles emerges from an incoherent superposition of the beam-like fragmentation components at larger rapidities  $y$  arising from valence quark-gluon interactions, and a component centered at (or near) midrapidity due to gluon-gluon collisions. All three time-dependent distribution functions (sources)  $R_{1,2,gg}(y, t)$  are broadened in rapidity space as a consequence of diffusion-like processes governed by a Fokker-Planck equation (FPE). The fragmentation sources tend to shift towards midrapidity due to the drift term. Whereas this drift is sizeable at lower (AGS, SPS) energies, it is not pronounced at LHC energies where the interaction times are very short.

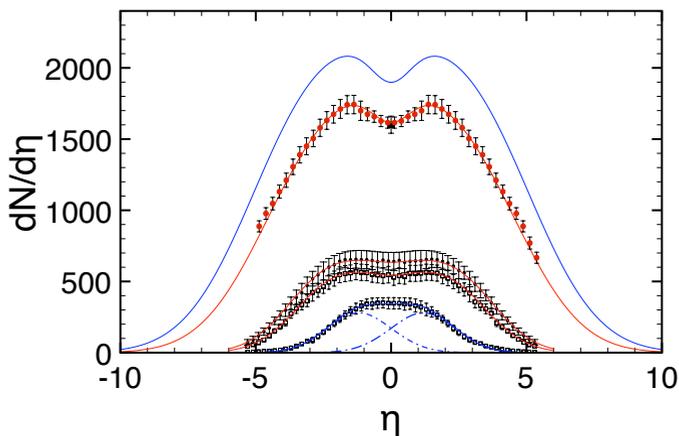
Since the FP-equation is linear, a superposition of the distribution functions using the initial conditions  $R_{1,2}(y, t = 0) = \delta(y \mp y_{max})$  with the absolute value of the beam rapidities  $y_{max} = \ln(\sqrt{s_{NN}}/m_p)$  and  $R_{gg}(y, t = 0) = \delta(y - y_{eq})$  yields the solution. The charged-particle content of the respective sources is  $N_{ch}^{qq,1}$  (projectile-like),  $N_{ch}^{qq,2}$  (target-like) and  $N_{ch}^{gg}$  for the midrapidity low- $x$  gluon-gluon source. In symmetric collisions the equilibrium value is  $y_{eq} = 0$ , whereas it is calculated from energy and momentum conservation at each centrality for asymmetric systems like  $p$ Pb. The total rapidity distribution of produced charged particles becomes

$$\frac{dN_{ch}^{tot}(y, t = \tau_{int})}{dy} = N_{ch}^{qq,1} R_1(y, \tau_{int}) + N_{ch}^{qq,2} R_2(y, \tau_{int}) + N_{ch}^{gg} R_{gg}(y, \tau_{int})$$

with the rapidity  $y = 0.5 \cdot \ln((E + p)/(E - p))$ , and the interaction time  $\tau_{int}$ . The latter corresponds to the total integration time of the underlying partial differential equation, which is a linear partial differential equation of the Fokker-Planck type, as described in [16].

In the solution, the mean values and variances are obtained analytically from the moments equations. Relaxation time and diffusion coefficient are related through a dissipation-fluctuation theorem (Einstein relation). Due to the collective expansion of the system, however, the effective diffusion coefficient is substantially larger. Hence the partial widths  $\Gamma_k$  (FWHM) are treated here as independent variables, which are related to the standard deviations through  $\Gamma_k = \sqrt{8 \ln 2} \cdot \sigma_k$ . For symmetric systems, the RDM then has four parameters (five including the total number of produced charged particles). This nonequilibrium-statistical model goes beyond the thermal model: Only for  $t \rightarrow \infty$  a thermal distribution would be attained. At RHIC and LHC energies, comparison of the model results with data clearly shows that equilibrium among the three sources is not reached.

This finding is indirectly related to the current intense theoretical investigations of local equilibration within the gluonic source, cf. [13, 14] and references therein. These works concern the microscopic equilibration mechanisms and eventually aim at a fully QCD-based nonperturbative description. A direct connection to the current macroscopic investigation of equilibration among the three sources is difficult to perform conceptually and mathematically.



**Figure 2.** The RDM pseudorapidity distribution functions for charged hadrons in central AuAu (RHIC) and PbPb (LHC) collisions at c.m. energies of 19.6 GeV, 130 GeV, 200 GeV and 2.76 TeV shown here are optimized in  $\chi^2$ -fits with respect to the PHOBOS [15] (bottom) and ALICE [7] (top) data, with parameters from [16]. The upper distribution function is an extrapolation to the LHC design energy of 5.52 TeV. At the lowest energy, only the fragmentation sources contribute (dash-dotted curves). From [16].

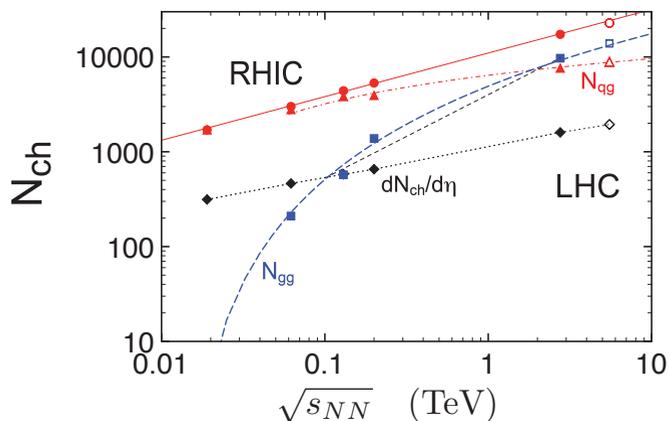
### 3 Pseudorapidity distributions

If particle identification is not available, one has to convert the results to pseudorapidity,  $\eta = -\ln[\tan(\theta/2)]$  with the scattering angle  $\theta$ . The Jacobian used for the conversion is

$$\frac{dN}{d\eta} = \frac{dN}{dy} \frac{dy}{d\eta} = J(\eta, m/p_T) \frac{dN}{dy},$$

$$J(\eta, m/p_T) = \cosh(\eta) \cdot [1 + (m/p_T)^2 + \sinh^2(\eta)]^{-1/2}$$

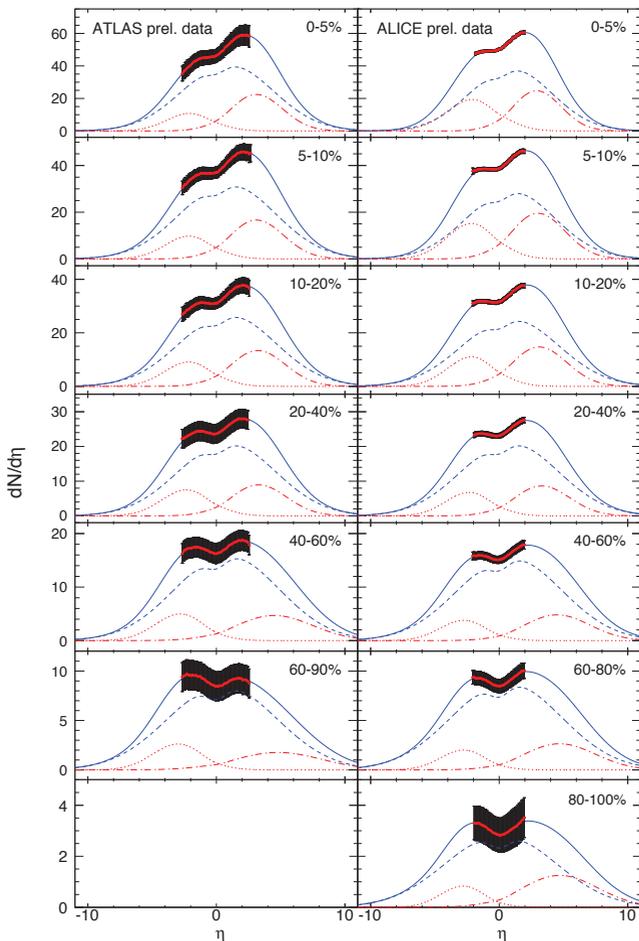
with the mass  $m$  fixed at the pion mass  $m_\pi$ , and a corresponding effective mean transverse momentum [17]  $\langle p_{T,\text{eff}} \rangle = m_\pi J_{y=0} / (1 - J_{y=0}^2)^{1/2}$ . Here the Jacobian  $J_{y=0}$  at midrapidity is taken from experiment (for pions, kaons and protons) as discussed in [17]. The effective transverse momenta are smaller than the mean transverse momenta determined from the  $p_T$ -distributions, and the corresponding effect of the Jacobian is therefore larger than that estimated with  $\langle p_T \rangle$  taken from the transverse momentum distributions for each particle species. At high RHIC and LHC energies the effect of the Jacobian transformation remains, however, essentially confined to the midrapidity source.



**Figure 3.** The total charged-hadron production in central AuAu and PbPb collision in the energy region 19.6 GeV to 5.52 TeV is following a power law  $N_{\text{tot}} \propto (s_{NN}/s_0)^{0.23}$  (solid line), whereas the particle content in the fragmentation sources is  $N_{qg} \propto \ln(s_{NN}/s_0)$ , dash-dotted curve. The particle content in the mid-rapidity source obeys  $N_{gg} \propto \ln^3(s_{NN}/s_0)$ , dashed curve, not too far from a power law (short-dashed line) only in the intermediate energy range 0.1–2.76 TeV. The energy dependence of the mid-rapidity yield is shown as a dotted line, with PHOBOS data [15] at RHIC energies, and ALICE data [18] at 2.76 TeV. From [19].

With parameter values for central collisions given in [16], the 3-sources RDM results for PbPb at 2.76 TeV energy are shown in Fig. 2 [19]. Corresponding centrality-dependent results for  $p$ Pb collisions at 5.02 TeV are shown in Fig. 4 [20]. Here the asymmetric shapes of the distributions are much more sensitive to the details of the model, enhancing the credibility of the nonequilibrium 3-sources approach. The model had previously also been compared with dAu data at RHIC energies in [12].

The total hadron-production yield integrated over  $\eta$  follows  $\ln(s_{NN}/s_0)$  for the fragmentation sources in both  $p$ Pb and PbPb collisions at LHC energies, whereas the midrapidity gluonic source has a cubic log dependence  $\propto \ln^3(s_{NN}/s_0)$ , see Fig. 3 and the discussion in [19] with theoretical arguments for such an energy dependence. For experimental confirmations at RHIC energies based on STAR results cf. [21] and references therein.



**Figure 4.** The centrality-dependent RDM pseudorapidity distribution functions for charged hadrons in  $p\text{Pb}$  collisions at LHC c.m. energy of 5.02 TeV are adjusted in the mid-rapidity region through  $\chi^2$ -minimizations to the preliminary ATLAS (left) [9] and ALICE (right) [8] data (systematic error bars only for ALICE). The underlying distributions in the three-sources RDM are also shown, with the dashed curves arising from gluon-gluon collisions, the dash-dotted curves from valence quark-gluon events in the Pb-like region, and the dotted curves in the proton-like region (fragmentation sources). From [20].

## 4 Conclusion

Deviations from thermal distribution functions for produced particles in relativistic heavy-ion collisions are sensitive indicators for nonequilibrium processes. These are discussed for both transverse momentum and pseudorapidity distributions of produced charged hadrons.

A three-sources nonequilibrium-statistical relativistic diffusion model (RDM) is used for the description and prediction of pseudorapidity distributions of produced charged hadrons. Two fragmentation sources and a midrapidity gluonic source evolve as functions of time according to a Fokker-Planck equation, but they remain far from the thermal equilibrium distribution due to the short interaction times of about  $10^{-23}$  s at RHIC and LHC energies.

The particle content integrated over pseudorapidity is proportional to  $\ln(s_{NN}/s_0)$  only in the fragmentation sources. In the midrapidity source that arises essentially from gluon-gluon collisions, a dependence  $\propto \ln^3(s_{NN}/s_0)$  is found and discussed in [19].

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