Relativistic second-order dissipative fluid dynamics at finite chemical potential

Amaresh Jaiswal[1], Bengt Friman[1] and Krzysztof Redlich[2,3]
[1]GSI, Helmholtzzentrum für Schwerionenforschung, Planckstraße 1, D-64291 Darmstadt, Germany
[2]Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wroclaw, Poland
[3]Extreme Matter Institute EMMI, GSI, Planckstraße 1, D-64291 Darmstadt, Germany

FIAS Frankfurt Institute for Advanced Studies

Introduction

Relativistic hydrodynamics has been applied quite successfully to describe the space-time evolution of the QGP formed in high-energy heavy ion collisions and estimate its transport coefficients [1]. Most of the studies have focused on exploring the effects of the shear viscosity on the QGP. While the effects of bulk viscous pressure has been studied in some details, the dissipative charge current has been largely ignored. This may be attributed to the fact that at very high energies, such as at RHIC and LHC, baryon number and its corresponding chemical potential are negligible. However, at lower collision energies such as those probed in the RHIC low-energy scan or at the upcoming experiments at FAIR, baryon number can no longer be ignored and therefore charge diffusion may play an important role.

In this poster, we present the derivation of second-order evolution equations for shear stress tensor and dissipative charge current for a hydrodynamic system of massless quarks and gluons. The transport coefficients are obtained exactly using quantum statistics for the quark and gluon phase-space distribution functions with a non-vanishing quark chemical potential. We show that, up to second-order in the gradient expansion, the evolution equations for the shear stress tensor and the dissipative charge current can be decoupled. We also demonstrate that the limiting behaviour of the ratio of heat conductivity to shear viscosity is similar to that of a strongly coupled conformal fluid.

Dissipative evolution equations

The equation of motion governing the hydrodynamic evolution of a relativistic system is obtained from the local conservation of energy and momentum, $\partial_\mu T^{\mu\nu} = 0$, and particle four-current, $\partial_\mu N^\mu = 0$. The conserved energy-momentum tensor and the net-quark current can be expressed in terms of the single particle phase-space distribution function as [2]

$$T^{\mu\nu} = \int dp\, p^\mu p^\nu \left[g_q(f_q + f_{\bar q}) + g_g f_g\right] = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{1}$$

$$N^\mu = \int dp\, p^\mu \left[g_q(f_q - f_{\bar q})\right] = n u^\mu + n^\mu, \qquad dp \equiv d\mathbf{p}/[(2\pi)^3|\mathbf{p}|], \tag{2}$$

where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$. Various thermodynamic quantities for a system of massless quarks and gluons is given by

$$u_\mu u_\nu T^{\mu\nu}_{(0)} \equiv \epsilon_0 = \frac{(4g_g + 7g_q)\pi^2}{120}T^4 + \frac{g_q}{4}T^2\mu^2 + \frac{g_q}{8\pi^2}\mu^4, \tag{3}$$

$$-\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu}_{(0)} \equiv P_0 = \frac{(4g_g + 7g_q)\pi^2}{360}T^4 + \frac{g_q}{12}T^2\mu^2 + \frac{g_q}{24\pi^2}\mu^4, \tag{4}$$

$$u_\mu N^\mu_{(0)} \equiv n_0 = \frac{g_q}{6}T^2\mu + \frac{g_q}{6\pi^2}\mu^3, \tag{5}$$

$$\frac{\epsilon_0 + P_0 - \mu\, n_0}{T} \equiv s_0 = \frac{(4g_g + 7g_q)\pi^2}{90}T^3 + \frac{g_q}{6}T\mu^2, \tag{6}$$

where $T^{\mu\nu}_{(0)} = T^{\mu\nu}|_{f\to f^{(0)}}$. The equilibrium distribution functions are given by

$$f^{(0)}_q = \frac{1}{\exp(\beta u\cdot p - \alpha) + 1}, \qquad f^{(0)}_{\bar q} = f^{(0)}_q|_{\alpha\to-\alpha}, \qquad f^{(0)}_g = \frac{1}{\exp(\beta u\cdot p) - 1}, \tag{7}$$

where $\beta = 1/T$ and $\alpha = \mu/T$.

The above expressions for ϵ_0, P_0, n_0, and s_0 can also be obtained directly from the partition function of an ideal QGP [2],

$$\ln Z = \frac{V}{T}\left[\frac{(4g_g + 7g_q)\pi^2}{360}T^4 + \frac{g_q}{12}T^2\mu^2 + \frac{g_q}{24\pi^2}\mu^4\right], \tag{8}$$

by employing the thermodynamic relations

$$\epsilon_0 = \frac{T^2}{V}\frac{\partial \ln Z}{\partial T} + \mu n_0, \quad P_0 = \frac{T}{V}\ln Z n_0 = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu}, \quad s_0 = \frac{1}{V}\frac{\partial (T\ln Z)}{\partial T}, \tag{9}$$

where V is the volume of the system.

For a system close to local thermodynamic equilibrium, $f = f^{(0)} + \delta f$, where $\delta f \ll f^{(0)}$, the shear stress tensor $\pi^{\mu\nu}$ and the particle diffusion current n^μ can be expressed in terms of δf as

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta \left[g_q(\delta f_q + \delta f_{\bar q}) + g_g \delta f_g\right], \tag{10}$$

$$n^\mu = \Delta^\mu_\alpha \int dp\, p^\alpha \left[g_q(\delta f_q - \delta f_{\bar q})\right], \tag{11}$$

where $\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}(\Delta^\mu_\alpha\Delta^\nu_\beta + \Delta^\mu_\beta\Delta^\nu_\alpha) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$.

To obtain δf We start from the relativistic Boltzmann equation with the relaxation-time approximation for the collision term

$$p^\mu\partial_\mu f = -(u\cdot p)\frac{\delta f}{\tau_R} \quad\Rightarrow\quad f = f^{(0)} - \frac{\tau_R}{(u\cdot p)}p^\mu\partial_\mu f, \tag{12}$$

where $f = f^{(0)} + \delta f^{(1)} + \delta f^{(2)} + \cdots$ and τ_R is the relaxation time. Solving Eq. (12) iteratively, we obtain [3],

$$\delta f^{(1)} = -\frac{\tau_R}{u\cdot p}p^\mu\partial_\mu f^{(0)}, \qquad \delta f^{(2)} = \frac{\tau_R}{u\cdot p}p^\mu p^\nu\partial_\mu\Big(\frac{\tau_R}{u\cdot p}\partial_\nu f^{(0)}\Big). \tag{13}$$

Substituting $\delta f = \delta f^{(1)} + \delta f^{(2)}$ in Eqs. (10)-(11) and performing the momentum integrations, we obtain [4]

$$\dot\pi^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}, \tag{14}$$

$$\dot n^{\langle\mu\rangle} + \frac{n^\mu}{\tau_n} = \beta_n\nabla^\mu\alpha - n_\nu\omega^{\nu\mu} - n^\mu\theta - \frac{3}{5}n_\nu\sigma^{\nu\mu} - \frac{3\,\beta_n}{\epsilon + P}\pi^{\mu\nu}\nabla_\nu\alpha. \tag{15}$$

where $\omega^{\mu\nu} \equiv (\nabla^\mu u^\nu - \nabla^\nu u^\mu)/2$. The last term in Eq. (15) is equivalent to $-(6/5)n_\nu\sigma^{\nu\mu}$. Evolution equation for dissipative charge current reduce to [4]:

$$\dot n^{\langle\mu\rangle} + \frac{n^\mu}{\tau_n} = \beta_n\nabla^\mu\alpha - n_\nu\omega^{\nu\mu} - n^\mu\theta - \frac{9}{5}n_\nu\sigma^{\nu\mu}. \tag{16}$$

Transport coefficients

The first-order transport coefficients are obtained as [4]

$$\beta_\pi = \frac{\epsilon + P}{5}, \qquad \beta_n = \frac{\pi^2 + 3\alpha^2}{3\alpha\left(\pi^2 + \alpha^2\right)}n - \frac{n^2 T}{\epsilon + P}. \tag{17}$$

The ratio of charge conductivity to shear viscosity, κ_n/η, and heat conductivity to shear viscosity, κ_q/η, are expressed as

$$\frac{\kappa_n}{\eta} = \frac{\beta_n}{\beta_\pi}; \qquad \frac{\kappa_q}{\eta} = \frac{\beta_n}{\beta_\pi}\left(\frac{\epsilon + P}{n\,T}\right)^2. \tag{18}$$

In the limit of both small and large μ/T,

$$\frac{\kappa_q}{\eta} = C\frac{\pi^2 T}{\mu^2}, \qquad C = \begin{cases} 37/27 & \text{for 2 flavor QGP},\ \mu/T \ll 1 \\ 95/81 & \text{for 3 flavor QGP},\ \mu/T \ll 1 \\ 5/3 & \text{for } \mu/T \gg 1 \\ 32, 8, 2 & \text{AdS/CFT},\ \mathrm{d} = 4, 5, 7 \quad [5] \end{cases} \tag{19}$$

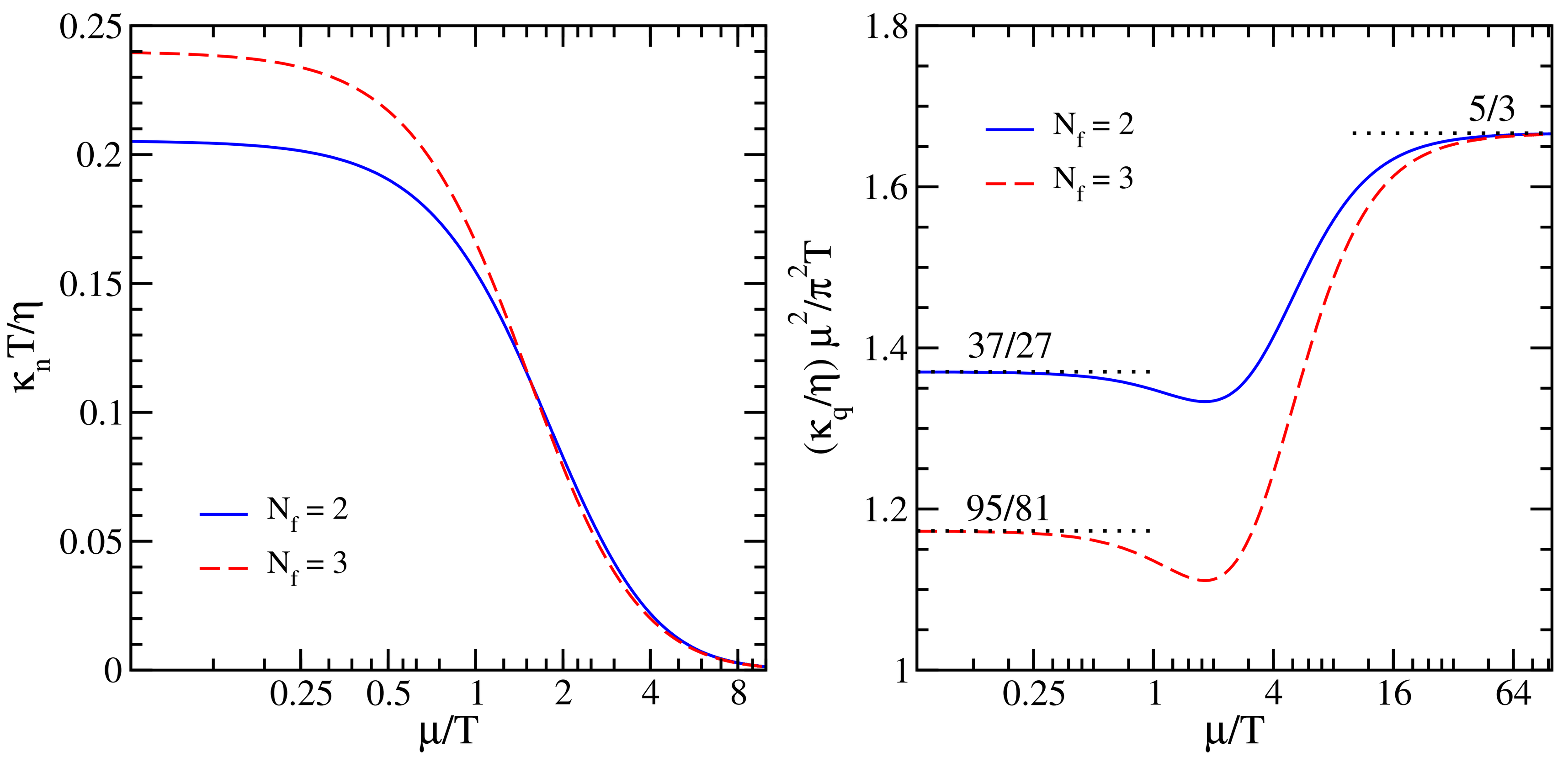

Figure 1: (a): The ratio of charge conductivity to shear viscosity scaled by the temperature, $\kappa_n T/\eta$, and (b): The ratio of thermal conductivity to shear viscosity, κ_q/η, scaled by the factor $\mu^2/\pi^2 T$, for two flavor (solid line) and three flavor (dashed line) massless quarks, plotted against μ/T.

References

[1] U. Heinz and R. Snellings, Ann. Rev. Nucl. Part. Sci. **63**, 123 (2013).
[2] R. Vogt, *Ultrarelativistic Heavy-Ion Collisions*, (North-Holland, 2007).
[3] A. Jaiswal, PRC **87**, 051901 (2013); PRC **88**, 021903 (2013).
[4] A. Jaiswal, B. Friman and K. Redlich, arXiv:1507.02849 [nucl-th].
[5] D. T. Son and A. O. Starinets, JHEP **0603**, 052 (2006); S. Jain, JHEP **1003**, 101 (2010); JHEP **1006**, 023 (2010).