The τ-model of Bose-Einstein Correlations: Some recent results

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Abstract. Bose-Einstein correlations of pairs of identical charged pions produced in hadronic Z decays and in 7 TeV pp minimum bias interactions are investigated within the framework of the τ-model.

1 Introduction

After a brief review of relevant previous results, new *preliminary* results are presented on the dependence of the Bose-Einstein correlation function on track and jet multiplicity, pair transverse momentum and pair rapidity, using a parametrization which has been found [1] to describe well Bose-Einstein correlations in hadronic Z decay, namely that of the τ-model [2, 3].

1.1 Parametrization

The Bose-Einstein correlation function, R_2, is often parametrized as

$$R_2 = \gamma \left[1 + \lambda \exp \left(- (rQ)^2 \right) \right] (1 + \epsilon Q) , \tag{1}$$

and is measured by $R_2(Q) = \rho(Q)/\rho_0(Q)$, where $\rho(Q)$ is the density of identical boson pairs with invariant four-momentum difference $Q = \sqrt{-(p_1 - p_2)^2}$ and $\rho_0(Q)$ is the similar density in an artificially constructed reference sample, which should differ from the data only in that it does not contain the effects of Bose symmetrization of identical bosons.

However, the "classic" parametrization of Eq. (1) is found to be inadequate, even when it is generalized to allow for a Lévy distribution of the source:

$$R_2 = \gamma \left[1 + \lambda \exp \left(- (rQ)^\alpha \right) \right] (1 + \epsilon Q) , \quad 0 < \alpha \leq 2 \tag{2}$$

This was not realized for a long time because the correlation function was only plotted up to $Q = 2$ GeV or less. In Ref. [1] Q was plotted to 4 GeV, and it became apparent that there is a region of anti-correlation ($R_2 < 1$) extending from about $Q = 0.5$ to 1.5 GeV. This anti-correlation, which one might term Bose-Einstein Anti-Correlations (BEAC), as well as the Bose-Einstein correlations (BEC) are well described by the τ-model.

In the τ-model R_2 is found to depend not only on Q, but also on quantities a_1 and a_2. For two-jet events $a = 1/m_t$, where $m_t = \sqrt{m^2 + p_t^2}$ is the transverse mass of a particle). Parameters of the model

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are the parameters of the Lévy distribution which describes the proper time of particle emission: α, the index of stability of the Lévy distribution; a width parameter $\Delta\tau$; and the proper time τ_0 at which particle production begins.

We shall use a simplified parametrization [1] where τ_0 is assumed to be zero and a_1 and a_2 are combined with $\Delta\tau$ to form an effective radius R:

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right](1 + \epsilon Q), \tag{3a}$$

$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}. \tag{3b}$$

Note that the difference between the parametrizations of Eqs. (2) and (3) is the presence of the cosine term, which provides the description of the anti-correlation. The parameter R describes the BEC peak, and R_a describes the anti-correlation region. While one might have had the insight to add, *ad hoc*, a cos term to Eq. (2), it is the τ-model which provides a physical reason for it and which predicts a relationship, Eq. (3b), between R and R_a.

A fit of Eq. (3) to L3 two-jet events is shown in Fig. 1, from which it is seen that the τ-model describes both the BEC peak and the BEAC dip quite well. Also the three-jet data is well described [1], which is perhaps surprizing since the τ-model is inspired by a picture of fragmentation of a single string.

It must also be pointed out that the τ-model has its short-comings: The τ-model predicts that R_2 depends on the two-particle momentum difference only through Q, not through components of Q. However, this is found not to be the case [1]. Nevertheless, regardless of the validity of the τ-model, Eq. (3) provides a good description of the data. Accordingly, we shall use it in most of the following.

Figure 1. The Bose-Einstein correlation function R_2 for two-jet events. The curve corresponds to the fit of Eq. (3). Also plotted is Δ, the difference between the fit and the data. The dashed line represents the long-range part of the fit, *i.e.*, $\gamma(1 + \epsilon Q)$. The figure is taken from Ref. [1].

1.2 Data

The e^+e^- data were collected by the L3 detector at a center-of-mass energy of $\sqrt{s} \simeq 91.2\,\text{GeV}$. Approximately 36 million like-sign pairs of well-measured charged tracks from about 0.8 million hadronic Z decays are used. This data sample is identical to that of Ref. [1]. The same event mixing technique is used to construct ρ_0 as in Ref. [1].

The minimum-bias pp data were collected by the ATLAS detector at a center-of-mass energy of $\sqrt{s} = 7\,\text{TeV}$. The sample contains approximately 1.8×10^9 like-sign charged track pairs from approximately 10^7 events. The acceptance in pseudorapidity is $|\eta| < 2.5$. This data sample is identical to the 7 TeV sample used in the recent ATLAS paper on BEC [4]. However the new results reported here, which are taken from a Ph.D. thesis [5], use, unless otherwise stated, a reference sample constructed by the opposite-hemisphere (OHP) method where the three-momentum, $\vec{p}$ of one of the particles of the pair is replaced by $-\vec{p}$. The recent ATLAS paper [4] used unlike-sign pairs to construct ρ_0, which is unsuitable for studying the region of anti-correlation, since the ρ^0 meson is a dominant feature there.

The Durham and JADE algorithms are used to classify e^+e^- events according to the number of jets. The number of jets in a particular event depends on the jet resolution parameter of the algorithm, y_{cut}. We define y_{23}^D (y_{23}^J) as the value of y_{cut} at which the number of jets in the event changes from two to three using the respective algorithms. Small y_{23} corresponds to narrow two-jet events, large y_{23} to events with three (or more) well-separated jets. The two algorithms lead to similar results.
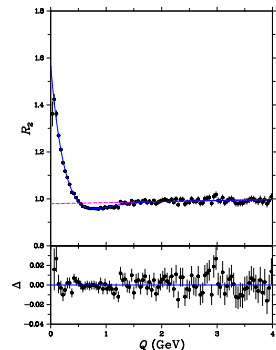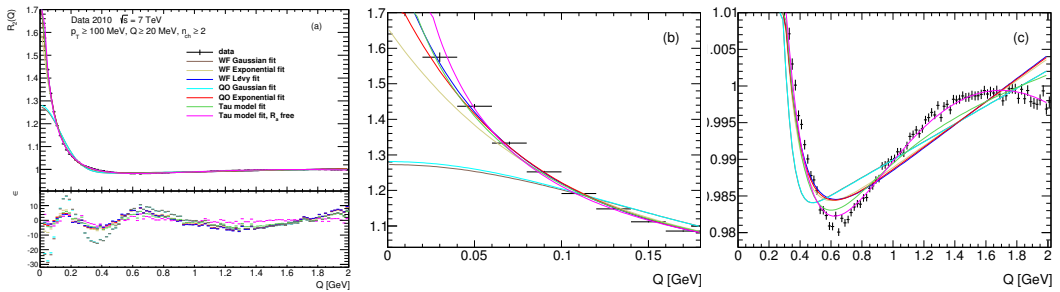
Figure 2. R_2 for 7 TeV pp minimum bias interactions with the results of fits of several parametrizations [5].

2 New *Preliminary* Results

As in e^+e^- annihilation [1], the pp minimum bias data are found [5] to be best described by Eq. (3a). While other parametrizations can describe the BEC peak, only this parametrization (with R_a a free parameter) comes close to describing not only the BEC peak but also the BEAC region, as is seen in Fig. 2. Previously, CMS had also observed the BEAC and fit it with the τ-model parametrization [6].

As has frequently been pointed out (see, *e.g.*, Ref. [7]) the values of the parameters of BEC depend strongly on the choice of reference sample. This is seen again in Fig. 3, where the dependence of R on the charged track multiplicity, N, and on the average pair transverse momentum, $k_t = (\vec{p_{t1}} + \vec{p_{t2}})/2$, is shown for the exponential parametrization of R_2, *i.e.*, Eq. 2 with $\alpha = 1$, which is the parametrization used in the ATLAS paper [4]. The dependences with unlike-sign (ULS) reference sample are markedly different from those with the other samples. Whereas R increases somewhat less than linearly with N for the other reference samples, it appears to saturate for the ULS sample. And, whereas R decreases approximately linearly with k_t for the ULS sample, for the rotated reference sample it decreases only up to $k_t \approx 450$ MeV and then increases, and for the other reference samples it is first roughly constant with k_t and then increases.

An unfortunate property of the τ-model parameterization is that the estimates of α, R, and R_a from the fits tend to be highly correlated. Therefore, when studying the dependence of R on quantities such as N or k_t, in order to stabilize the fits α is fixed to the value obtained in a fit to all events. In examining the de-
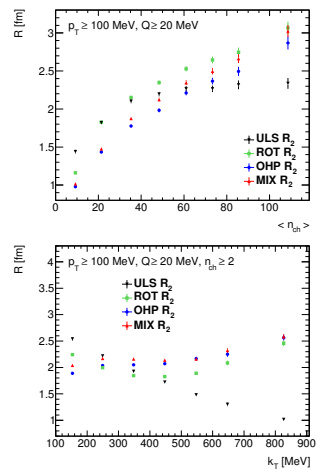

Figure 3. The dependence of R (top) on N and (bottom) on k_t with respect to the beam axis for different reference samples in 7 TeV pp minimum bias events. The exponential parametrization is used [5].
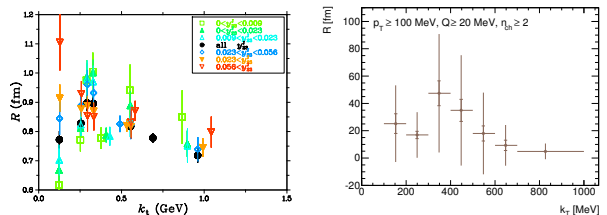

Figure 4. The dependence of R on k_t for e^+e^- (top) and pp (bottom). The e^+e^- results are shown for various selections on y_{23}^J. The τ-model parametrization is used with R_a a free parameter.

pendence of R on charged track multiplicity and on k_t, the parametrization of Eq. (3a) is used (with R_a given by Eq. (3b) for e^+e^- and a free parameter for pp).

The dependence of R on k_t with respect to the thrust axis for e^+e^- and with respect to the beam axis for pp is shown in Fig. 4 for the τ-model parametrization with R_a a free parameter is shown in Fig. 4. The behavior of two-jet e^+e^- seems similar to pp, although the error bars are large.

For both e^+e^- and pp, R is found [5, 7] to increase with multiplicity (not shown). Further, for e^+e^- R is found [7] to increase with 'jettiness', as measured by y_{23}.

In e^+e^-, R is also found to depend on the rapidity of the pair, as shown in Fig. 5. Here the rapidity, y_E, is defined with respect to the thrust axis with the positive thrust axis chosen to be in the same hemisphere as the most energetic jet. Then $y_E > 1$ selects almost always tracks from the most energetic quark jet, and $y_E < -1$ selects mostly tracks from the other quark jet with the contribution of tracks from the gluon jet increasing as y_{23} increases, *i.e.*, as the events become more three-jetlike. The

Figure 5. R obtained in fits of Eq. (3) for various rapidity and y_{23}^D intervals.

intermediate y_E region contains tracks from the gluon jet and low-energy tracks from both quark jets. One observes that R is roughly independent of y_{23} for $y_E > 1$. This value of R is also found for $y_E < -1$ in the case of two-jet events. These are the situations of 'pure' quark jets. As y_{23} increases R for $y_E < -1$ also increases, reflecting the increasing contribution of tracks from the gluon. The region $-1 < y_E < 1$ has a larger value of R for two-jet events, and this value increases with y_{23}. For three-jet events, where the gluon contributes to both $-1 < y_E < 1$ and $y_E < -1$, the values of R are approximately equal.

As mentioned above, the τ-model is known to break down. In the τ-model R_2 depends on Q, and not on components of Q. This was tested in Ref. [1]. In Eq. (3) $R^2 Q^2$ is replaced by $R_L^2 Q_L^2 + R_{side}^2 Q_{side}^2 + R_{out}^2 Q_{out}^2$ and fits for two-jet events performed in the LCMS. Different values were found for R_L, R_{out}, and R_{side}. Here results are shown in Fig. 6 for R_L and R_{side} for various selections of y_{23}^D. One sees that R_L is independent of y_{23}, while R_{side} increases with 'jettiness'. Thus the increase in R occurs mainly in the transverse plane. This is consistent with the increase in R being due to the increasing hardness of the gluon, which was seen in the rapidity dependence above.

A large source of systematic uncertainty on the parameter values is the range of Q over which the fit is performed. A higher upper limit, Q_U, of the fit range gives more data to determine the baseline, which in turn affects the values of the parameters describing BEC and BEAC. Clearly, Q_U should be chosen well beyond the anti-correlation region. Fig. 7 and Ta-

Figure 6. R_L and R_{side}/R_L obtained in fits of Eq. (3) for various rapidity and y_{23}^D intervals.

ble 1 show this effect. The values of the parameters change drastically when the upper limit, Q_U, is increased from 2 to 3 GeV. As Q_U is further increased the change is less, but still large. Clearly the parametrization does not fully describe the data. Nevertheless, it is much better than all the other parametrizations that were tried. It is also worth noting that the value of α is quite different from the value $\alpha = 0.41 \pm 0.02^{+0.04}_{-0.06}$ found for two-jet e^+e^- events [1].
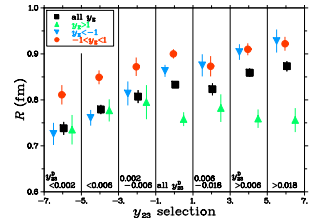
Figure 7. Fits of Eq. (3a), with R_{a} a free parameter, to 7 TeV minimum bias data for various choices of Q_{U} [5].

Table 1. Results of fits of Eq. (3a), with R_{a} a free parameter, for various choices of Q_{U} [5].

Q_{U}	2 GeV	3 GeV	4 GeV	5 GeV
α	0.108 ± 0.001	0.186 ± 0.005	0.235 ± 0.003	0.261 ± 0.003
R (fm)	17.8 ± 0.7	6.7 ± 0.5	4.1 ± 0.2	3.3 ± 0.1
R_{a} (fm)	43.4 ± 1.2	3.0 ± 0.2	1.80 ± 0.04	1.52 ± 0.02
λ	3.08 ± 0.05	1.91 ± 0.10	1.36 ± 0.05	1.15 ± 0.03

3 BEAC in more detail

In the τ-model BEAC arises through the corre-
lation of coordinate space and momentum space.
Recently, another explanation has been proposed,
namely the non-zero size of the pion [8, 9]. A
detailed investigation of the BEAC region seems
therefore warranted, in the hope of distinguishing
between these explanations.

Since fits of Eq. (3a), with R_{a} a free parame-
ter, provide a reasonable description of the anti-
correlation region, they are used to show how
BEAC depend on 'jettiness' in e^+e^- and to com-
pare the dependence of BEAC on multiplicity and
k_{t} in two-jet e^+e^- and pp minimum bias events.

Figure 8. The result of fits of Eq. (3a) to (left)
e^+e^- two-jet ($y_{23}^{\mathrm{J}} < 0.023$) and (right) pp min-
imum bias data for various intervals of multi-
plicity, observed in the case of e^+e^- and cor-
rected in the case of pp.

Fig. 8 shows the dependence of the BEAC on track multiplicity. The anti-correlation dip is deeper
and at somewhat higher Q for two-jet e^+e^- than for pp. With increasing N the minimum moves to
lower Q; this effect is larger in pp than in e^+e^-. The dip also becomes less deep as N increases, an
effect also noticed by cms [6].

Fig. 9 shows the dependence of the BEAC on y_{23}^{J}. It is seen that the anti-correlation dip becomes
deeper and its minimum moves slighlty lower in Q as the 'jettiness' increases, i.e., as one moves from
two- to three-jet events.

Fig. 10 shows the dependence of the BEAC on k_{t}. Little dependence is seen for two-jet e^+e^- data.
For pp minimum bias data, the depth of the dip decreases with k_{t}, but the position of the minimum is
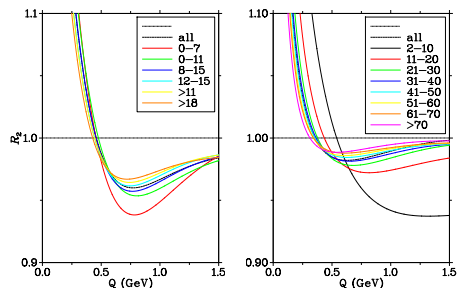approximately constant.

4 Conclusions

The τ-model, which is closely related to the string picture, provides a reasonable explanation of both the BEC peak and the BEAC dip in both e^+e^- and minimum bias pp interactions. Of all the parametrizations tried, only that of the τ-model survives as a candidate to explain the data. Another possible explanation is that the anti-correlation arises because of the non-zero size of the pion. To discover which of these explanations (or what combination of them) is the best explanation requires detailed investigation of both BEC and BEAC.

R and R_a are found to depend on track multiplicity and transverse momentum in both pp and e^+e^- and on jets and rapidity in e^+e^-. What more does it depend on? If the BEAC are due to the correlation between coordinate space and momentum space, as in the τ-model, it is reason-

Figure 9. The result of fits of Eq. (3) to e^+e^- for various y_{23}^J selections.

able that the dependence on jet structure, as seen in e^+e^-, occurs. Then one would also expect similar dependences in pp. This possibility makes the study of BEC (and BEAC) in jet events interesting. One might also expect a rapidity dependence in minimum bias events, as the number of strings involved or the presence of color reconnection may vary with rapidity. On the other hand, it is not clear (at least to me) why the effect of the pion size should depend on either rapidity or the presence of a jet.

In studying these correlations, the reference sample used plays a crucial role. The use of an unlike-sign reference sample is not possible in studying BEAC. Further, the upper limit of the fit range must be well above the BEAC region. It is very difficult, if not impossible, to compare quantitatively the results using different reference samples. It would therefore be useful if the LHC experiments could agree on a standard method of constructing the reference sample, and give results using this standard method as well as any other method deemed superior. This is especially important to investigate, *e.g.*, dependence on rapidity,

Figure 10. The result of fits of Eq. (3a) to (left) e^+e^- two-jet ($y_{23}^J < 0.023$) and (right) pp minimum bias data for various intervals of k_t.

where the acceptances of the experiments are very different, *e.g.*, LHCb and ATLAS, CMS, ALICE.

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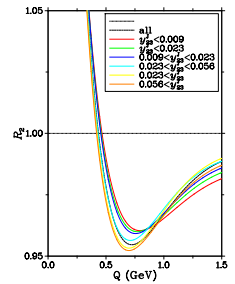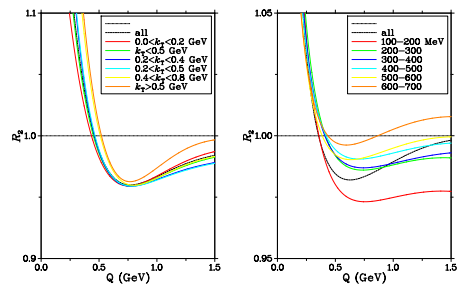[9] A. Białas, W. Florkowski, K. Zalewski, Phys. Lett. **B748**, 9 (2015), 1503.02807