A general form for the electroweak corrections to the dark matter relic density

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Introduction

The dark matter (DM) candidate

- **WIMP** (weakly interacting massive particle) with mass $M_\chi \sim \mathcal{O}$(few TeV)
- lightest, electrically neutral member of a non-degenerate **SU(2) multiplet** χ, by which we extend the Standard Model

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{\chi}\left(i\gamma^\mu D_\mu - M_\chi\right)\chi$$

  where $D_\mu = \partial_\mu - igt^a W_\mu^a$
- non-relativistic at the time of freeze-out in the early universe: **cold DM**
- motivated from wino limit of MSSM $\rightarrow$ **R-parity** like multiplicative conserved quantum number

Introduction

For cold, thermally produced DM, including **coannihliations**, the **RELIC DENSITY** is given by

$$\Omega h^2 = \frac{1.07 \cdot 10^9 \,\text{GeV}^{-1} x_f}{g_*^{1/2} M_{\text{Pl}} (a_{\text{eff}} + 3 b_{\text{eff}}/x_f)} \, .$$

with

- the **effective cross section** $\sigma_{\text{eff}} v = \sum_{i,j=1}^{k} r_i r_j \sigma_{ij} v = a_{\text{eff}} + b_{\text{eff}} v^2 + \mathcal{O}(v^4)$ being the sum over the Boltzmann weighted inclusive annihilation cross sections
- freeze-out time $x_f = M_\chi / T_f \approx 20$ (determined iteratively)
- M_{Pl} - Planck mass, and g_* - effective number of relativistic degrees of freedom at the freeze-out

Motivation

Great precision on experimentally determined relic abundance:
$\Omega^{\text{Planck}}_{\text{CDM}} h^2 = 0.1198 \pm 0.0026$ (Planck satellite's recent data)

$\Downarrow$

refine the theoretical calculations to a **comparable level**

- calculation of corrections to inclusive annihilation cross sections σ_{ij}
- EW corrections with infrared (IR) origin are of the form $\alpha v^2 \ln(M_\chi/m_W)$
- cancellation theorems do not apply for non-degenerate multiplets
- potentially large due to $M_\chi \gg m_W$

OUR PROJECT:
Resummation of these logarithms, and the derivation of a general form the resummed inclusive annihilation cross section at LL precision

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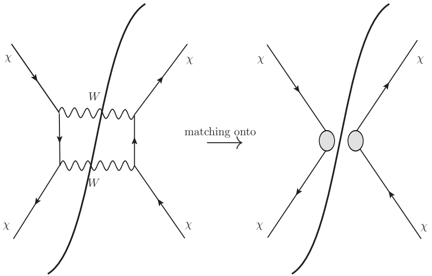
The framework of effective field theories (EFTs)

Heavy particle effective theory (HPET):

- in analogy to HQET, but with particles and antiparticles
- hard gauge bosons are ‘integrated out’
- additionally assume small relative velocity



The inclusive annihilation process $\chi^{a_1}\chi^{a_2} \to X$ is related to the forward scattering amplitude $\chi^{a_1}\chi^{a_2} \to \chi^{a_2}\chi^{a_1}$ via the **OPTICAL THEOREM**:

$$\sigma(\chi^{a_1}\chi^{a_2} \to X) = 2\,\mathrm{Im}\, i\,\mathcal{A}(\chi^{a_1}\chi^{a_2} \to \chi^{a_1}\chi^{a_2}).$$

SU(2) 'color' decomposition

Small relative velocity $\rightarrow$ initial and final states form **two-particle states** with respect to the SU(2) charge
$\rightarrow$ express effective four fermion operators in terms of irreducible representations

For SU(2) triplets (with only vector coupling):

- $3 \otimes 3 = 1 \oplus 3 \oplus 5$.
- The **four-fermion vertices** are described by effective operators in irreducible representation:

$$\mathcal{L}_{\text{4-fermion-op}} = C^i \mathcal{O}^i \,,$$

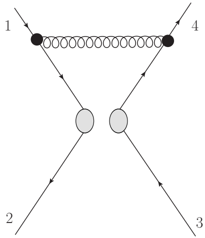
where

$$\mathcal{O}^1 = N_1 \, \bar{h}_{\omega_4} \gamma^\mu f_{\omega_3} \, \bar{f}_{\omega_2} \gamma_\mu h_{\omega_1} \,, \quad \mathcal{O}^3 = N_3 \, \bar{h}_{\omega_4} \gamma^\mu \left(t^a\right) f_{\omega_3} \, \bar{f}_{\omega_2} \gamma_\mu \left(t^a\right) h_{\omega_1} \,,$$
$$\mathcal{O}^5 = N_5 \, \bar{h}_{\omega_4} \gamma^\mu \left(q^A\right) f_{\omega_3} \, \bar{f}_{\omega_2} \gamma_\mu \left(q^A\right) h_{\omega_1} \,,$$

with $h_{\omega_i} = (h^1_{\omega_i}, h^2_{\omega_i}, h^3_{\omega_i})^T$ ($f_{\omega_i} = (f^1_{\omega_i}, f^2_{\omega_i}, f^3_{\omega_i})^T$) describing particles (antiparticles) with momentum p_i

Describing of the IR structure: RGE

- Real emission and virtual exchange of EW gauge bosons from and between annihilating particles is described by one-loop diagrams in HPET
- IR singularities in full theory $\leftrightarrow$ UV divergence of the operator matrix element $\langle \mathcal{O}^i \rangle$ in HPET



$\rightarrow$ calculate divergences in dimensional regularization, and absorb them into multiplicative **Z-factors**

$$\langle \mathcal{O}_i^{\text{bare}} \rangle = Z^{ij}(\mu) \langle \mathcal{O}_j(\mu) \rangle \, .$$

The running with scale μ is described by the anomalous dimension $\gamma = \sum_{n=0} \gamma^{(n)} \left(\alpha/4\pi\right)^{n+1}$, and the **renormalization group equation (RGE)**

$$\frac{d}{d \ln \mu} \langle \mathcal{O}_i^{\text{bare}} \rangle = 0 \;\rightarrow\; \frac{d}{d\, \ln \mu} \langle \vec{\mathcal{O}}(\mu) \rangle = -\underbrace{Z^{-1}(\mu) \frac{d\, Z(\mu)}{d \ln \mu}}_{\gamma} \langle \vec{\mathcal{O}}(\mu) \rangle \, .$$

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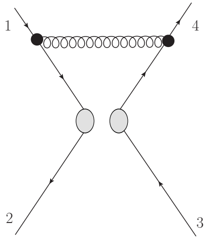

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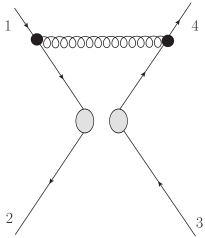

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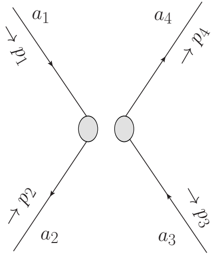
A general form of the anomalous dimension

The general form of the one-loop anomalous dimension matrix $\mathbf{\Gamma}$ in color space

$$\mathbf{\Gamma} = -\sum_{(I,J)} \frac{\mathbf{T}_I \cdot \mathbf{T}_J}{2}\,\gamma_{\mathsf{cusp}}(\beta_{IJ},\alpha) + \sum_I \gamma_I(\alpha)\,,$$

$$\overset{\text{our process}}{=} \frac{8}{3}\,v^2\left(C_I + \mathbf{T}_1\cdot\mathbf{T}_4\right)\frac{\alpha}{4\pi}\,,$$



is projected onto the irreducible 'color' basis via

$$\gamma^{ij} = c^i_{\{a\}}\mathbf{\Gamma}_{\{a\}\{a\prime\}}{c^j}^\dagger_{\{a\prime\}}\,.$$

For the SU(2) triplet: the 'color' basis reads

$$c^1_{\{a\}} = N_1\,\delta_{a_1a_2}\,\delta_{a_3a_4}\,, \qquad c^3_{\{a\}} = N_3\,\left(t^a\right)_{a_2a_1}\left(t^a\right)_{a_4a_3}\,,$$
$$c^5_{\{a\}} = N_5\,\left(q^A\right)_{a_2a_1}\left(q^A\right)_{a_4a_3}\,,$$

Resumming the forward scattering amplitude

The general process $\chi^{a_1}\chi^{a_2} \to \chi^{a_3}\chi^{a_4}$ leads to the scattering amplitude

$$\mathcal{A}_{\{a\}} v = \frac{1}{4} \sum_{\{s\}} C^i \, W^{ij} \, \langle \mathcal{O}^j_{\text{tree}} \rangle^{\{s\}}_{\{a\}} \, ,$$

where interactions were absorbed into the soft function W (by field redefinitions using Wilson lines).
Scale-dependence of $\langle \mathcal{O}^i \rangle$ is transfered onto the soft function $\to$ **RGE**:

$$\frac{d}{d \ln \mu} W^{ij} = -\gamma^{ik} W^{kj} \, .$$

From the RGE's solution, we obtain the running of the soft function:

$$W^{ij}(\mu) = \left[V \left(\left(\frac{\alpha(\mu)}{\alpha(m_W)} \right)^{\frac{\vec{\gamma}_0}{2\beta_0}} \right)_D V^{-1} \right]^{ik} W^{kj}(m_W) \, .$$

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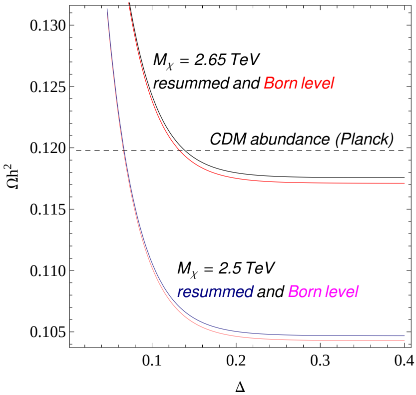
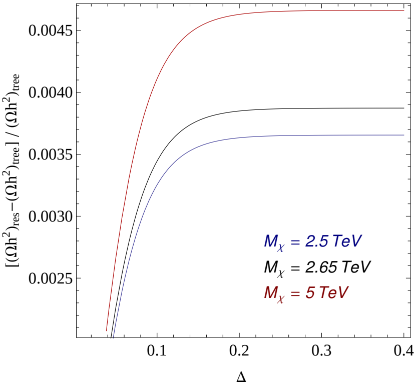
The general form of the resummed cross section

- choose the renormalization scale to be the factorization scale, and also $\mu = M_\chi$
- since $M_\chi > m_t > m_W$:
  $U_{n_f}(M_\chi, m_W) \to U_6(M_\chi, m_t)\, U_5(m_t, m_W) \equiv U(M_\chi, m_t, m_W)$
- We obtain

$$
\begin{aligned}
\sigma^{\text{res}}_{a_1 a_2} v &= \frac{1}{2} \sum_{s_1, s_2} C^i(M_\chi)\, U^{ij}(M_\chi, m_t, m_W)\, \langle \mathcal{O}^j_{\text{tree}} \rangle^{\{s\}}_{\{a\}} \Big|_{\substack{s_{4/3}=s_{1/2} \\ a_{4/3}=a_{1/2}}} \\
&= \sum_{a_3, a_4} \sigma^i_{\text{Born}}\, v\, U^{ij}(M_\chi, m_t, m_W)\, c^j_{\{a\}} \delta_{a_4 a_1} \delta_{a_3 a_2} \\
&\to \sum_{a_3, a_4} \sigma^i_{\text{Born}}\, v \left[\mathbb{1} - \gamma^{(0)} \frac{\alpha(M_\chi)}{4\pi} \ln \left(\frac{M_\chi}{m_W} \right) \right]^{ij} c^j_{\{a\}} \delta_{a_4 a_1} \delta_{a_3 a_2} \, .
\end{aligned}
$$

Numerical analysis: the impact on the relic density





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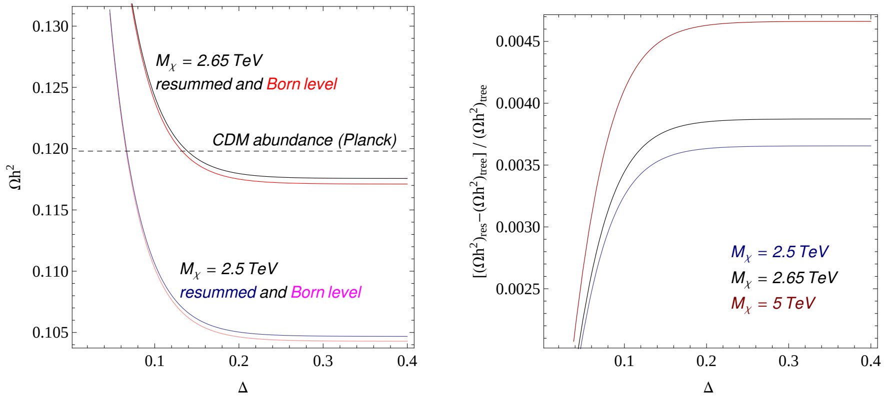

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Summary

- directly accessible, model-independent, general form of the resummed inclusive annihilation cross section at LL precision for heavy dark matter

- Only including gauge bosonic final states $\rightarrow$ the impact of EW logarithmic corrections is at per mil level
  Reason: logarithmic corrections suppressed by αv^2 in comparison to LO Born level contribution

- Combining fermionic and bosonic final states with coupling of same order of magnitude: Born cross section into fermions suppressed by v^2 $\rightarrow$ overshadowed by LO bosonic result

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