Precision calculations in Higgs physics: Standard Model and Beyond

*Phänomenologie Seminar*, Max Planck Institut für Physik München

Julien Baglio | 16. 02. 2015
Outline

1. Introduction
2. SM Higgs pair production at the LHC
3. NLO corrections in the NMSSM Higgs sector
4. Outlook
Once upon a time...

4/7/2012: CERN presents the discovery of a bosonic particle
Its properties are compatible with those of the Higgs boson

\[ M_H \simeq 125.5 \text{ GeV} \]

2013 analyses have confirmed the discovery of a Higgs boson: spin 0, couplings to fermions and bosons as a function of their masses ⇒ 2013 Nobel Prize awarded to Englert and Higgs

Key question: what is the exact nature of the observed Higgs boson? Standard Model (SM)-like or more exotics?
Coining the scalar boson

Introduction

J. Baglio – SM and BSM (N)NLO calculations in Higgs physics

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Coining the scalar boson

**ATLAS** Preliminary

- Data
- Spin 2
- H → ZZ^* → 4l
- \( \tilde{t} = 7 \) TeV: \( \mathcal{L} = 4.6 \) fb \(^{-1}\)
- \( \mathcal{L} = 8 \) TeV: \( \mathcal{L} = 20.7 \) fb \(^{-1}\)
- \( H \rightarrow \gamma \gamma \)
- \( J^{P}_H = 0^+ \)
- \( J^{P}_H = 2^+ \)

**CMS** Preliminary

- Observed
- GB, VH
- \( gg \rightarrow X(2_m) \rightarrow ZZ + WW + \gamma \gamma \)
- \( \tilde{t} = 8 \) TeV: \( \mathcal{L} = 20.7 \) fb \(^{-1}\)
- \( \mathcal{L} = 7 \) TeV: \( \mathcal{L} = 5.1 \) fb \(^{-1}\)

Pseudoexperiments

- Observed
- 0^*
- 2_m


Introduction

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Coining the scalar boson
Coining the scalar boson

CP-even spin 0 hypothesis strongly preferred, no significant deviations from SM couplings: data up to now points toward a SM Higgs boson...

The SM ultimate test: probing the scalar potential

- From the scalar potential before EWSB:

\[ V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4 \]

- To \( V(\phi) \) after EWSB, with \( M_H^2 = 2m^2, v^2 = m^2/\lambda \):

\[
\phi = \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow V(H) = \frac{1}{2} M_H^2 H^2 + \frac{1}{2} \frac{M_H^2}{v} H^3 + \frac{1}{8} \frac{M_H^2}{v^2} H^4 + \text{constant}
\]
The SM ultimate test: probing the scalar potential

- From the scalar potential before EWSB:

$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$

- To $V(\phi)$ after EWSB, with $M_H^2 = 2m^2$, $v^2 = m^2/\lambda$:

$$\phi = \left( \frac{0}{v + H(x)} \right) \Rightarrow V(H) = \frac{1}{2} M_H^2 H^2 + \frac{1}{2} \frac{M_H^2}{v} H^3 + \frac{1}{8} \frac{M_H^2}{v^2} H^4 + \text{constant}$$

$\times (-i)$
Precision calculation is crucial at the LHC...

The case of the Higgs discovery in $gg \rightarrow H \rightarrow \gamma\gamma$ channel

QCD corrections can be large, uncertainties need to be under control

⇒ higher orders are compulsory!
Why going beyond the SM?

**Standard Model cannot be the final answer:**
- **Unification** of fundamental interactions: QCD and electroweak interaction (EW) fully unified is not possible in the SM (not to mention gravity)
- What about dark matter, neutrino masses?
- **Naturalness problem:** the quantum corrections to the Higgs boson mass are quadratically divergent, leading to a cumbersome adjustment of the parameters to cancel them; how to avoid this problem?
- etc.

Supersymmetry (SUSY) gives some solutions to these fundamental questions

SUSY = fermion ↔ boson symmetry
SM Higgs pair production at the LHC

[J.B., Djouadi, Gröber, Mühlleitner, Quevillon, Spira, JHEP 1304 (2013) 151]

[J.B. et al, VBFNLO 2.7.0 release note, arXiv:1404.3940]
The main production channels

- **gluon fusion**

  \[ g + g \rightarrow H + g + g + H \]  

  \[ H + g + g \rightarrow t + \bar{t} + H + H \]  

  \[ t + \bar{t} + H + H \rightarrow q + \bar{q} + g + g \]  

  \[ M_H = 125 \text{ GeV} \]  

  \[ \sigma (pp \rightarrow HH + X) \] \[ [\text{fb}] \]  

  \[ \sqrt{s} \text{ [TeV]} \]  

- **vector boson fusion**

  \[ W, Z \rightarrow q + \bar{q} + H + H \]  

  \[ q + \bar{q} + H + H \rightarrow gg + gg \]  

  \[ gg + gg \rightarrow t + \bar{t} + HH \]  

  \[ \sqrt{s} \text{ [TeV]} \]  

- **double Higgs–strahlung**

  \[ W, Z \rightarrow H + g + g \]  

  \[ H + g + g \rightarrow q + \bar{q} + q + \bar{q} \]  

  \[ q + \bar{q} + q + \bar{q} \rightarrow W + Z + H + H \]  

  \[ W, Z \rightarrow H + g + g \]  

  \[ H + g + g \rightarrow q + \bar{q} + q + \bar{q} \]  

  \[ q + \bar{q} + q + \bar{q} \rightarrow W + Z + H + H \]  

  \[ \sqrt{s} \text{ [TeV]} \]  

- **associated production with top quark**

  \[ t + \bar{t} + HH \rightarrow q + g + g \]  

  \[ q + \bar{q} + H + H \rightarrow gg + gg \]  

  \[ gg + gg \rightarrow t + \bar{t} + HH \]  

  \[ t + \bar{t} + HH \rightarrow q + \bar{q} + q + \bar{q} \]  

  \[ q + \bar{q} + q + \bar{q} \rightarrow W + Z + H + H \]  

  \[ \sqrt{s} \text{ [TeV]} \]  

\( \sim 1000 \text{ times smaller than } \sigma (pp \rightarrow H + X) \)
Gluon fusion: the largest cross section


\[ K-\text{factor} \simeq 2 \]

<table>
<thead>
<tr>
<th>(\sqrt{s} \text{ [TeV]})</th>
<th>(\sigma^{\text{NLO}} \text{ [fb]})</th>
<th>(\sigma^{\text{NNLO}} \text{ [fb]})</th>
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NNLL resummation: \(\simeq +20 - 30\%\) on top of NLO cross section, scale dependence stabilized [Shao, C.S. Li, H.T. Li, Wang, JHEP 1307 (2013) 169]
Gluon fusion: theoretical uncertainties

\( gg \rightarrow HH \) affected by sizable uncertainties:

- **Scale uncertainty:** calculated at NLO with \( \frac{1}{2} \mu_0 \leq \mu_R, \mu_F \leq 2 \mu_0, \mu_0 = M_{HH} \):
  \[ \Delta_{\text{scale}} \simeq +20\% (+12\%) / -17\% (-10\%) \] at \( \sqrt{s} = 8(100) \text{ TeV} \)

- **PDF uncertainty:** gluon PDF at high-\( x \) less constrained, \( \alpha_s(M_Z^2) \) uncertainty
  \[ \Rightarrow \text{large discrepancy between PDFs predictions} \]
  \[ \Delta_{\text{PDF} + \alpha_s} \simeq \pm 9\% (\simeq \pm 6\% \text{ at } 100 \text{ TeV}) \]

- **EFT approximation:** NLO correction only known in a top mass expansion
  \[ \Rightarrow \text{estimate of } \pm 10\% \text{ uncertainty} \] (confirmed in [Grigo, Hoff, Melnikov, Steinhauser, Nucl.Phys. B875 (2013) 1])

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**Total uncertainty:** \( \simeq \pm 40\% \) \( \simeq \pm 30\% \) at 100 TeV

\(^1\) With NNLO corrections, \( \simeq +20\% \) on total rate, scale uncertainty reduced to \( \pm 9\% (\pm 6\%) \) at 8(100) TeV [De Florian, Mazzitelli, Phys.Lett. B724 (2013) 306-309; PRL 111 (2013) 201801]
Current issues of the gluon fusion channel

Big issue in gluon fusion: the effective theory approach

- **Single Higgs production:** works pretty well provided a rescaling to the full Born result
- **Higgs pair production:** approximation worse than for $\sigma(gg \rightarrow H)$ as $m_t \gg \sqrt{s} \geq 4M_H^2$
  never fullfilled (remember the EFT $\pm 10\%$ uncertainty)
- **Critical:** fails for the $HH$ NLO differential distributions


The goal: having the full 2-loop calculation including the yet unknown box contributions

SM Higgs pair production at the LHC
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Vector boson fusion at NLO

$pp \rightarrow qq \rightarrow qq\,WW/ZZ \rightarrow qqHH$: the second production channel at the LHC


$\simeq +7\%$ correction
(similar to single Higgs case)

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<th>$\sqrt{s}$ [TeV]</th>
<th>$\sigma^{\text{NLO}}$ [fb]</th>
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Interlude: handling infrared singularities in VBFNLO

How to handle infrared singularities? Soft and collinear singularities may arise, notably cumbersome as arising in different phase–spaces ⇒ substraction method to handle them!

\[ \sigma^{\text{NLO}} = \int_{\phi_n} d\sigma^{\text{Born}} + \int_{\phi_n} d\sigma^{\text{virt}} + \int_{\phi_{n+1}} d\sigma^{\text{real}} \]

with each contribution divergent ⇒ cancel soft & collinear singularities before Monte-Carlo integration:

\[ \sigma^{\text{NLO}} = \int_{\phi_{n+1}} \left( d\sigma^{\text{real}} |_{\varepsilon=0} - d\sigma^{A} |_{\varepsilon=0} \right) + \int_{\phi_n} \left( d\sigma^{\text{Born}} + d\sigma^{\text{virt}} + \int_{\phi_1} d\sigma^{A} \right) |_{\varepsilon=0} \]

where \( d\sigma^{A} \) a substraction term with the following properties:

- \( d\sigma^{A} \) cancels soft & collinear divergences of \( d\sigma^{\text{real}} \)
- \( \int_{\phi_1} d\sigma^{A} \) done (partially) analytically in \( d \) dimensions ⇒ \( I, P, K \) operators, left–over collinear singularities absorbed into PDFs

The calculation in VBFNLO has been done with Catani-Seymour dipoles

Vector boson fusion: theoretical uncertainties

$qq \rightarrow HHqq$ is a clean process:

- Scale uncertainty: calculated at NLO with $\frac{1}{2} \mu_0 \leq \mu_R, \mu_F \leq 2 \mu_0, \mu_0 = Q_W/Z$; 
  $\Delta_{\text{scale}} \simeq +3\% (+2\%) / -2\% (-1\%)$ at $\sqrt{s} = 8(33) \text{ TeV}$
  Good precision compared to LO $\Delta_{\text{scale}} \simeq \pm 10\%$

- PDF uncertainty: total $\Delta_{\text{PDF}+\alpha_s}^{90\% \text{CL}} \simeq +7\% / -4\% \left( \simeq +5\% / -4\% \text{ at } 33 \text{ TeV} \right)$

Total uncertainty: $\simeq +8\% / -5\% \left( 14 \text{ TeV} \right)$

NNLO QCD corrections in the structure function approach: $+0.5\%$ on top of the NLO result, scale uncertainty at the percent level $[L. Liu-Sheng \ et \ al, \ PRD \ 89 \ (2014) \ 073001]$
Vector boson fusion: differential distributions

Example of NLO differential distributions with VBFNLO in VBF → $H(\rightarrow b\bar{b})H(\rightarrow \tau\tau)jj$:

$pp \rightarrow HHjj \rightarrow b\bar{b}\tau\tau jj$

$\sqrt{s} = 14$ TeV

$1/\sigma \, d\sigma/dM_{HH}$

$M_{HH}$ [GeV]

$1/\sigma \, d\sigma/d\eta_H$

$\eta_H$

SM Higgs pair production at the LHC

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Associated $W/Z +$ Higgs pair production

$$pp \rightarrow Z^*/W^* \rightarrow Z/W + HH:$$ clean but very small rates

- **NLO QCD corrections:** Drell-Yan $\sigma(pp \rightarrow V^*)$ corrections $\simeq +20\%$
  

- **NNLO QCD corrections:** Drell-Yan $\simeq +4\%$ \[\text{[J.B. et al, JHEP 1304 (2013) 151]}\]

- **NNLO QCD corrections (II):** specific $gg \rightarrow ZHH$ channel $\Rightarrow \simeq +20 - 30\%$, sharp contrast with $\simeq +5\%$ in $ZH$ production \[\text{[J.B. et al, JHEP 1304 (2013) 151]}\]

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<th>$\sqrt{s}$ [TeV]</th>
<th>$\sigma_{WHH}^{\text{NNLO}}$ [fb]</th>
<th>$\sigma_{ZHH}^{\text{NNLO}}$ [fb]</th>
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<td>1.68</td>
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<tr>
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<td>8.00</td>
<td>8.27</td>
</tr>
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Theoretical uncertainties in double Higgs–strahlung

\( pp \rightarrow VHH \) is also a very clean process:

- **Scale uncertainty**: calculated at NNLO with \( \frac{1}{2} \mu_0 \leq \mu_R, \mu_F \leq 2 \mu_0, \mu_0 = M_{VHH}; \)
  \( \Delta_{\text{scale}}^{\text{WHH}} \lesssim 1\% \) in \( WHH \) channel
  In \( ZHH \) channel, worse due to \( gg \rightarrow ZHH \): \( \Delta_{\text{scale}}^{\text{ZHH}} \simeq \pm 3\% \)

- **PDF uncertainty**: total \( \Delta_{90\% \text{CL}}^{\text{PDF} + \alpha_s} \simeq \pm 4\% \) (\( \simeq \pm 3\% \) at 100 TeV)

\[ \sigma(q\bar{q}' \rightarrow VHH) \text{ [fb]} \]

NNLO QCD, \( M_H = 125 \text{ GeV} \)

**Total uncertainty**: 
\( \Delta_{\text{tot}}^{\text{WHH}} \simeq \pm 4\%, \Delta_{\text{tot}}^{\text{ZHH}} \simeq \pm 7\% \)
Main search channels

Where to look for $HH$ production? production cross section small
⇒ use min. one $H \to b\bar{b}$ decay channel to retain some signal, $\mathcal{L} = 3000 \text{ fb}^{-1}$

3 interesting final states $a priori$:

- $b\bar{b}W(\to \ell\nu)W(\to 2j)$: difficult but might be interesting with MVA techniques
  [Papaefstathiou, Lin Yang, Zurita, PRD 87 (2013) 011301]
- $b\bar{b}\tau\tau$: rates small, but quite promising

Even 4$b$ final state has been investigated!
[Ferreira de Lima, Papaefstathiou, Spannowsky, JHEP 1408 (2014) 030]

Remark: most of the analyses use the $gg \to HH$ production channel; $HH + 2j$ (using also VBF process) analyses have started to be analysed
[Dolan, Englert, Greiner, Spannowsky, PRL 112 (2014) 101802] as well as $pp \to t\bar{t}HH$ process
Main search channels

**Where to look for HH production?** production cross section small
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3 interesting final states *a priori*:

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How sensitive are the three main channels to HHH coupling?

- VBF mode is the most sensitive channel
- Identical shape when increasing the center–of–mass energy but reduced sensibility

\[
\frac{\sigma(pp \rightarrow HH + X)}{\sigma_{SM}} \quad \text{with} \quad \sqrt{s} = 8 \text{ TeV, } M_H = 125 \text{ GeV}
\]

Triple Higgs coupling sensitivity in the production channels

How sensitive are the three main channels to HHH coupling?

- VBF mode is the most sensitive channel
- Identical shape when increasing the center–of–mass energy but reduced sensibility

\[
\frac{\sigma(pp \rightarrow HH + X)}{\sigma^{SM}}
\]

\[\sqrt{s} = 14 \text{ TeV}, M_H = 125 \text{ GeV}\]

\[
\frac{\lambda_{HHH}}{\lambda_{HHH}^{SM}}
\]

Triple Higgs coupling sensitivity in the production channels

**How sensitive are the three main channels to HHH coupling?**

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\[
\sigma(pp \rightarrow HH + X)/\sigma^{SM}
\]

\[\sqrt{s} = 33 \text{ TeV}, M_H = 125 \text{ GeV}\]

\[
\lambda_{HHH}/\lambda_{HHH}^{SM}
\]

Precision calculations in the Higgs sector of the NMSSM

[J.B., Gröber, Mühlleitner, Nhung, Rzehak, Spira, Streicher, Walz, CPC 185 (2014) 12]

[J.B., Krauss, Mühlleitner, Walz, to appear (2015)]
From the SM to the (N)MSSM

- **SM Hierarchy problem:** Higgs mass corrections quadratically divergent in the SM ⇒ Higgs mass stabilized by SUSY, MSSM minimal SM extension, 2 Higgs doublets

- **But μ–problem:** $\mu \hat{H}_u \cdot \hat{H}_d$ term in the superpotential, $\mu$ SUSY-invariant but of the order of EW scale ⇒ naturalness problem!

- **Solution to the μ–problem:** effective $\mu$–term generated dynamically ⇔ NMSSM

**NMSSM Superpotential**

$$\mathcal{W} = \hat{u}_R^* y_u \left( \hat{Q} \cdot \hat{H}_u \right) - \hat{d}_R^* y_d \left( \hat{Q} \cdot \hat{H}_d \right) - \hat{e}_R^* y_e \left( \hat{L} \cdot \hat{H}_d \right) + \lambda \hat{S} \left( \hat{H}_u \cdot \hat{H}_d \right) + \frac{1}{3} \kappa \hat{S}^3$$

- 3+2 neutral Higgs bosons, 2 charged Higgs bosons, 4+1 neutralinos
- Singlet field $S$ gets a vacuum expectation value ⇒ $\mu_{\text{eff}} = \lambda v_s / \sqrt{2}$
- Higgs sector less restricted in NMSSM compared to MSSM:
  - upper bound on tree-level in the MSSM: $M_H \leq M_Z$
  - ⇒ large quantum corrections needed to get $M_H \geq 114.7$ GeV
  - Fine-tuning less important in the NMSSM, higher tree-level bound
    $$M_H^2 \leq M_Z^2 \left( \cos(2\beta)^2 + \frac{\lambda^2}{g^2} \sin(2\beta)^2 \right), \quad g^2 = \frac{g_1^2 + g_2^2}{2}$$

[see Ellwanger, Hugonie, Teixeira (2010)]
From the real NMSSM to the complex NMSSM

- **CP violation**: CKM matrix can explain $CP$ violation in $K - \bar{K}$ system but not the baryon asymmetry in the Universe:
  
  \[ SM \, CP \text{ phases are too small} \]

  One solution
  
  $CP$ violation enhanced with complex parameters

- **Richer Higgs spectrum**: complex parameters in the NMSSM
  
  $\Rightarrow$ $CP$ violation at tree–level, all the Higgs bosons mix:

  \[
  H_i = \sum_k S_{ik} \phi_k, \quad \phi = (h_d, h_u, s, a, a_s)
  \]

  $(S_{ik})$ $5 \times 5$ matrix
The scalar potential in the NMSSM

- From the superpotential to the scalar potential:

\[
V = |\lambda|^2 |S|^2 \left( H_u^\dagger H_u + H_d^\dagger H_d \right) + |\lambda (H_u^\dagger \epsilon H_d) + \kappa S^2|^2 + \frac{1}{2} g_2^2 |H_u^\dagger H_d|^2 \\
+ \frac{1}{8} (g_1^2 + g_2^2) \left( H_u^\dagger H_u - H_d^\dagger H_d \right)^2 + m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_S^2 |S|^2 \\
+ \left( \lambda A_{\lambda} (H_u^\dagger \epsilon H_d) S + \frac{1}{3} \kappa A_{\kappa} S^3 + c.c \right)
\]

- Higgs fields with phases:

\[
H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d + h_d + ia_d \\ \sqrt{2} H_d^- \end{pmatrix}, \quad H_u = \frac{e^{i \phi_u}}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H_u^+ \\ v_u + h_u + ia_u \end{pmatrix}, \quad S = \frac{e^{i \phi_s}}{\sqrt{2}} (v_s + h_s + ia_s) \\
\tan \beta = \frac{v_u}{v_d}
\]
1-loop corrections to Higgs masses in the real NMSSM

- **LO Higgs masses**: diagonalizing the $3 \times 3$ CP-even $M_S$ and the $3 \times 3$ CP-odd (Goldstone) $M_A$ mass matrices give the LO masses $m_{H_1} < m_{H_2} < m_{H_3}$ and $m_{A_1} < m_{A_2}$.

  sizable higher order corrections compulsory to obtain one SM-like $H_i$ boson

- **1-loop corrections**: define input parameters, calculate renormalized self-energies in a mixed renormalization scheme \cite{Graf, Ender, Graf, Mühlleitner, Rzehak, PRD 85 (2012) 075024}

  - 7 parameters renormalized on-shell: $e, M_W, M_Z, M_{H^\pm}, t_{h_u}, t_{h_d}, t_s$
  - 5 parameters renormalized $\overline{\text{DR}}$: $\tan \beta, \lambda, \kappa, A_k, \nu_s$

  $\overline{\text{DR}}$ field renormalization constants for the Higgs fields

  \[
  H_d = \left(1 + \frac{1}{2} \delta Z_d\right) \hat{H}_d \quad H_u = \left(1 + \frac{1}{2} \delta Z_u\right) \hat{H}_u \quad S = \left(1 + \frac{1}{2} \delta Z_s\right) \hat{S}
  \]

  - $\delta M_V^2 = \text{Re}\Sigma^T_{VV}(M_V^2)$, $\delta M_{H^\pm}^2 = \text{Re}\Sigma_{H^+H^+}(M_{H^\pm}^2)$

  - $\delta Z_e$ at the scale $M_Z$ to absorb the dependence on light quark masses

  Tadpole condition: $\hat{T}_{h_{u/d/s}} = 0 \Rightarrow \delta t_{h_{u/d/s}} = T_{h_{u/d/s}}$

  with $T_{h_i} = R_{ji} T_{H_j}$, $i = u, d, s$ and $j = 1, 2, 3$; $(R_{ij})$ tree-level mixing matrix
1-loop corrections to Higgs masses in the real NMSSM

- **1-loop corrections**: with the renormalized self-energies calculate the zeros of the 1-loop 2-point vertex matrix

\[
\begin{pmatrix}
 p^2 - m_{H_1}^2 + \hat{\Sigma}_{H_1 H_1}^{(1)}(p^2) & \hat{\Sigma}_{H_1 H_2}^{(1)}(p^2) & \hat{\Sigma}_{H_1 H_3}^{(1)}(p^2) \\
 \hat{\Sigma}_{H_2 H_1}^{(1)}(p^2) & p^2 - m_{H_2}^2 + \hat{\Sigma}_{H_2 H_2}^{(1)}(p^2) & \hat{\Sigma}_{H_2 H_3}^{(1)}(p^2) \\
 \hat{\Sigma}_{H_3 H_1}^{(1)}(p^2) & \hat{\Sigma}_{H_3 H_2}^{(1)}(p^2) & p^2 - m_{H_3}^2 + \hat{\Sigma}_{H_3 H_3}^{(1)}(p^2)
\end{pmatrix}
\]

\[
\hat{\Sigma}_{H_i H_j}^{(1)}(p^2) = \Sigma_{H_i H_j}^{(1)}(p^2) + \frac{1}{2} p^2 (\delta Z_{H_i H_i} + \delta Z_{H_i H_j}) - \frac{1}{2} (\delta Z_{H_i H_i} m_{H_i}^2 - m_{H_i}^2 \delta Z_{H_i H_j}) - (R \delta M S R^\top)_{H_i H_j}
\]

Algorithm to calculate the masses

1. Set \( p^2 = m_{H_1}^2 \) in the self-energies
2. Solve \( \det \hat{\Gamma}(p^2) = 0 \)
3. Extract the smallest root obtained \( m_{\text{tmp}}^2 \)
4. Set \( p^2 = m_{\text{tmp}}^2 \) in the self-energies and back to 2 as long as \(|m_{\text{tmp},n}^2 - m_{\text{tmp},n-1}^2| > \epsilon\), \( n \) is the \( n^{\text{th}} \) iteration, \( \epsilon \) the desired accuracy \( \Rightarrow \) get the 1-loop mass \( M_{H_1} \)
5. Start again at 1 with the 2\(^{\text{nd}}\) Higgs mass, extract the next-to-smallest root, and so on
Complex parameters and renormalization

- Many new phases in the complex NMSSM: $M_1$, $M_2$, $A_t$, etc, all complex parameters with associated phases

In the Higgs sector: 6 phases $\phi_u$, $\phi_S$, $\phi_\lambda$, $\phi_\kappa$, $\phi_{A_\lambda}$ and $\phi_{A_\kappa}$

At tree–level, only one relevant phase: only 3 combinations in the mass matrix

$$\Psi = \phi_\lambda - \phi_\kappa + \phi_u - 2\phi_s$$
$$\Psi_\lambda = \phi_\lambda + \phi_{A_\lambda} + \phi_u + \phi_s$$
$$\Psi_\kappa = \phi_\kappa + \phi_{A_\kappa} + 3\phi_s$$

$\Psi_\lambda$, $\Psi_\kappa$ fixed by two tadpole conditions

$$\frac{1}{\sqrt{2}} |A_\lambda| \sin \Psi_\lambda = - \frac{|\kappa| v_s}{2} \sin \Psi,$$
$$\frac{1}{\sqrt{2}} |A_\kappa| \sin \Psi_\kappa = - \frac{3|\lambda| v_d v_u}{2v_s} \sin \Psi$$

- Renormalization in the Higgs sector:
  - on–shell renormalization for $e$, $M_W$, $M_Z$, $M_{H^\pm}$
  - DR renormalization for $\tan \beta$, $v_s$, $|\lambda|$, $|\kappa|$, $|A_\kappa|$ and the phases
  - vanishing phases counter–terms but for two tadpole conditions $t_d$ and $t_{d_s}$
Higgs masses at the one loop order

Complex phases have sizable effects [Graf, Gröber, Mühlleitner, Rzehak, Walz, JHEP 1210 (2012) 122]:

- **Tree–level CP violation ($\psi \neq 0$):**

![Graph showing Higgs masses at one loop order](image)

- $|\lambda| = 0.72, \quad |\kappa| = 0.2, \quad \tan \beta = 3, \quad v_s = 389 \text{ GeV}, \quad |A_\kappa| = 27 \text{ GeV}$,
- $|A_\lambda| = 928 \text{ GeV}, \quad A_t = -875 \text{ GeV}, \quad A_b = A_I = -963 \text{ GeV}, \quad M_{\text{SUSY}} = 1 \text{ TeV}$,
- $M_1 = 145 \text{ GeV}, \quad M_2 = 200 \text{ GeV}, \quad M_3 = 600 \text{ GeV}, \quad Q = 300 \text{ GeV}$
Complex phases have sizable effects [Graf, Gröber, Mühlleitner, Rzehak, Walz, JHEP 1210 (2012) 122]:

- **No tree–level CP violation** \( (\Psi = 0), \phi_\lambda = \phi_\kappa \neq 0\):

  - One loop corrections have a *sizable impact* (restore some scenarios)

  - No tree-level *CP violation* scenario: still slight impact of \( \phi_\kappa \) over \( H_3 \) through its coupling to stops
NMSSMCALC: calculate Higgs masses and decay widths in the NMSSM

2-loop Higgs boson masses in the NMSSM, allowing for complex parameters
+ new implementation to calculate the Higgs boson decay widths in the NMSSM based on the latest version of HDECAY


- Real or complex parameters
- $\mathcal{O}(\alpha_t\alpha_s)$ 2-loop masses
- Decay widths and branching ratios including higher order corrections
- Off-shell decays into $VV^*, H_i V^*, tt^*, bt^*$
- Dominant SUSY corrections in the decays into quarks and leptons: implemented with two effective couplings $\Delta_b, \Delta_1$ calculated in the NMSSM

\[
\mathcal{L} = -\lambda_b \bar{b}_R (1 + \Delta_1) \left[ H_d^0 + \frac{\lambda^* e^{i\phi_u}}{\mu^*_\text{eff}} \frac{\Delta_b}{\tan \beta} S^* H_u^0 \right] b_L + h.c.
\]

\[
\Delta_1 = -\frac{C_F \alpha_s}{2 \pi} M_3^* A_b I(m_{b_1}^2, m_{b_2}^2, m_g^2), \quad \Delta_b = \frac{1}{1 + \Delta_1} (\Delta_b^{QCD(1)} + \Delta_b^{\text{elw}(1)})
\]
**NMSSMCALC**: calculate Higgs masses and decay widths in the NMSSM

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- New implementation to calculate the Higgs boson decay widths in the NMSSM based on the latest version of HDECAY


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**NMSSMCALC**

- **Input/Output**
  - inp.dat
  - bhdecay.in
  - slha.in
  - slha_decay.out

- **CalcMasses(2loop).F**
  - Calculates Higgs boson masses at 1(2)-loop order
  - slha.in

- **bhdecay(_c).f**
  - Calculates Higgs decay widths and branching fractions

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NLO corrections in the NMSSM Higgs sector

J. Baglio – SM and BSM (N)NLO calculations in Higgs physics

MPI München

16. 02. 2015
Status of the NMSSM Higgs mass calculation

Current status in NMSSMCALC: approximate 2-loop order for the calculation of the Higgs spectrum with $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the effective potential approach using $\hat{\Sigma}_{ij}(p^2 = 0)$


To download NMSSMCALC: http://www.itp.kit.edu/~maggie/NMSSMCALC

NLO corrections in the NMSSM Higgs sector

J. Baglio – SM and BSM (N)NLO calculations in Higgs physics

MPI München 16. 02. 2015 26/33
Focus on one decay process: full NLO corrections to $A_k \rightarrow \tilde{t}_1\tilde{t}_2$ in the real NMSSM

**CP-odd Higgs decay into stops:**
  ⇒ what about them in the NMSSM?
- Possible detection channel for stops given the exp limits on their mass or a discovery channel for the $A_k$ bosons

**Calculation strategy:** complete NLO QCD and EW corrections:
- **virtual corrections:** vertex corrections, wave-function renormalization, counterterms diagrams, $A-Z$ mixing diagrams ⇒ ultraviolet finite 1-loop amplitude
- **real corrections:** 1-loop amplitude infrared-divergent ⇒ regularization with the gluon and photon real corrections in the **the mass regularization scheme**
Renormalization procedure

Main framework for the calculation: the mixed renormalization scheme in NMSSMCalc

LO amplitude is the $A_k \tilde{t}_1 \tilde{t}_2^*$ coupling, with $(P)_{ij}$ the CP-odd mixing matrix:

$$G_{A_i}^{12} = - \frac{e m_t}{2} \frac{1}{M_W^2} - \frac{1}{M_Z^2} \left[ \left( \frac{A_t}{\tan \beta} + \mu_{\text{eff}} \right) P_{i1} + \frac{\lambda v}{\sqrt{2}} \frac{1}{\tan \beta} P_{i2} \right]$$

- on-shell parameters: $M_Z$, $M_W$, $e$ + new parameters $m_t$, $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_t$ (stop mixing angle)
- DR parameters: $\tan \beta$, $\lambda$, $v_s$

- **CP-odd mixing matrix for the EW corrections**: use finite $Z$–factors to dress the LO mixing matrix [see e.g. Frank et al, JHEP 0702 (2007) 047; Nhung, Mühlleitner, Streicher, Walz, JHEP 1311 (2013) 181]

$$P_{i,j}^{(1)} = Z_{ik} P_{kl}^{(0)}, Z_{ik} = \sqrt{\hat{Z}_i \hat{Z}_{ik}}$$

with the factors $\hat{Z}_i, \hat{Z}_{ik}$ related to the effective self-energies

⇒ full momentum dependence of the self-energies kept

- **Renormalization of the stop sector**: on-shell field renormalization and

$$\delta \theta_t = \frac{1}{2} \frac{\text{Re} \left( \Sigma_{12}(m_{\tilde{t}_2}^2) + \Sigma_{21}(m_{\tilde{t}_1}^2) \right)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}$$
Scenarios for selected examples

ICHEP 2014 ATLAS results for the direct stop search:

- Select regions in the gap $m_{t_1} \in [250 - 300]$ GeV
- Check against current Higgs results with HiggsBounds and HiggsSignal

Two following scenarios extracted from a scan:

1. $m_{t_1} = 333.7$ GeV, $m_{t_2} = 691.2$ GeV, $M_{A_1} = 716.1$ GeV (singlet-like), $M_{A_2} = 1109$ GeV, $M_{H_1} = 125.7$ GeV (SM-like), $m_{\chi_1^0} = 169.7$ GeV

2. $m_{t_1} = 378.5$ GeV, $m_{t_2} = 660.4$ GeV, $M_{A_1} = 382.8$ GeV (singlet-like), $M_{A_2} = 1076$ GeV, $M_{H_1} = 126.1$ GeV (SM-like), $m_{\chi_1^0} = 172.1$ GeV
Numerical results for $\Gamma(A_2 \rightarrow \tilde{t}_1\tilde{t}_2)$

SUSY-EW corrections compulsory for meaningful results, NLO corrections can be sizable

- Pink line: original scenarios 1 with $\text{BR}^{\text{NLO}} = 25\%$ (up) and 2 with $\text{BR}^{\text{NLO}} = 29\%$ (down)
- Kinks from the threshold $m_{\tilde{t}_1} = m_t + m_{\chi_1^0}$ in the $\tilde{t}_1$ field renormalization

NLO corrections in the NMSSM Higgs sector

J. Baglio – SM and BSM (N)NLO calculations in Higgs physics

MPI München 16. 02. 2015 30/33
Reversed decay $\tilde{t}_2 \rightarrow \tilde{t}_1 A_k$

Same setup, reverse the kinematical relations: access to the stop decays into $A_k$

\[ \Gamma(\tilde{t}_2 \rightarrow \tilde{t}_1 A_2) \text{ [GeV]} \]

\[ A_t \text{ [GeV]} \]

(MSSM cross-check)
What about SUSY at the LHC?

We always say that SUSY is around the corner...
What about SUSY at the LHC?

We always say that SUSY is around the corner...
What about SUSY at the LHC?

We always say that SUSY is around the corner...

We are still waiting! Perhaps our corner will be the 13/14 TeV LHC?
Conclusion and outlook

**Precision calculation in the Higgs sector: an essential tool for discoveries!**

- **Major news since 2012:** a Higgs boson has been observed, now it is time to solve the next big question: *is it standard or a first window on BSM physics?*

- **The SM Higgs frontier at the high luminosity LHC:** the measure of the triple Higgs coupling to probe directly the scalar potential

- **$HH$ production status:** all main inclusive production channels known at least NLO, even NNLO for double Higgsstrahlung and gluon fusion

- **The big issue in $HH$ gluon fusion:** (N)NLO corrections only in the effective approach, still sizable uncertainties ⇒ complete 2-loop calculation for $gg \to HH$ is awaited

- **NMSSM, one very attractive SUSY framework for BSM Higgs physics:** with NMSSMCALC it is possible to obtain 2-loop Higgs spectrum and decay widths/branching fractions with relevant higher order corrections, complex parameters included

- **$\Gamma(A_k \to \tilde{t}_1\tilde{t}_2)$ known fully at NLO in the NMSSM:** a possible discovery channel for the NMSSM $A_2$ boson OR a discovery channel for the heavier stop, sizable NLO corrections
Das war’s für heute, vielen Dank!