INTRODUCTION TO NUCLEAR MODELS

Daniel Kollar – Friday Physics Session

NUCLEUS BASICS

$^{A}_{Z}X_{N}$ with A=Z+N

Z – proton number
N – neutron number
A – atomic/weight number

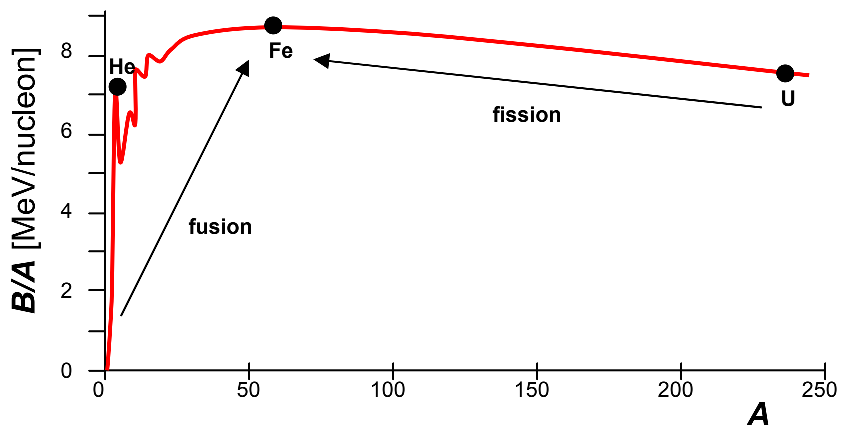
$^{76}_{32}Ge_{44}$

size: $R \cong R_0 A^{\frac{1}{3}}$ $R_0 \cong 1.2\ \text{fm}$ $R \cong 5.1\ \text{fm}$

mass: $m(X) < Zm_p + Nm_n$ (that's why it holds together)

binding energy: $B(Z,N) = \left[Zm_p + Nm_n - m(Z,N)\right] \cdot c^2$

$$B(^{76}\text{Ge}) = \left[36m_p + 44m_n - m(^{76}\text{Ge})\right] \cdot c^2 = 661.6\ \text{MeV}$$



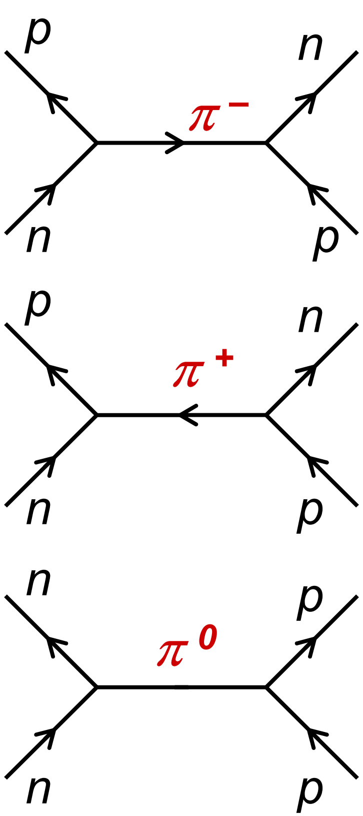
NUCLEAR FORCES

short range, spin-orbital character

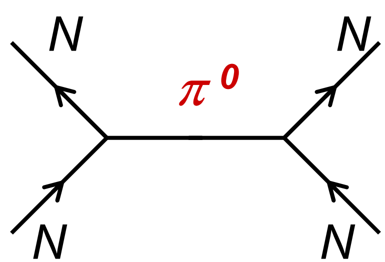
YUKAWA THEORY OF NUCLEAR FORCES

based on exchange of π^0, π^+, and π^-

$p + n \rightarrow p + n$

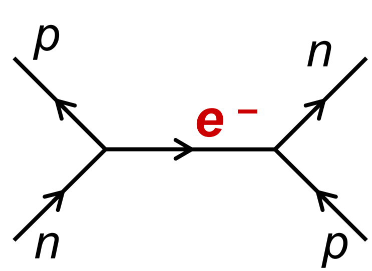


$N + N \rightarrow N + N$



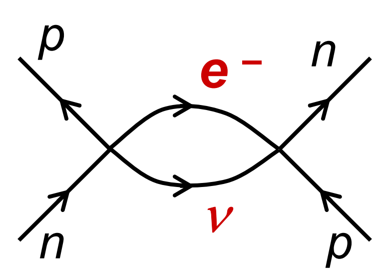
consider range of 1.4 fm
$\Rightarrow$ from uncertainty principle
$m\,c\,r \approx \hbar$
$\rightarrow$ $m_\pi \approx$ **140 MeV**

Heisenberg



violates angular momentum conservation

Fermi



too weak

NUCLEAR MODELS

- total *wavefunction* of the nucleus is far too complicated to be useful even if it was possible to calculate it (only possible for the lightest nuclei)

$\Rightarrow$ **we make use of models and use simple analogies**

Types of nuclear models

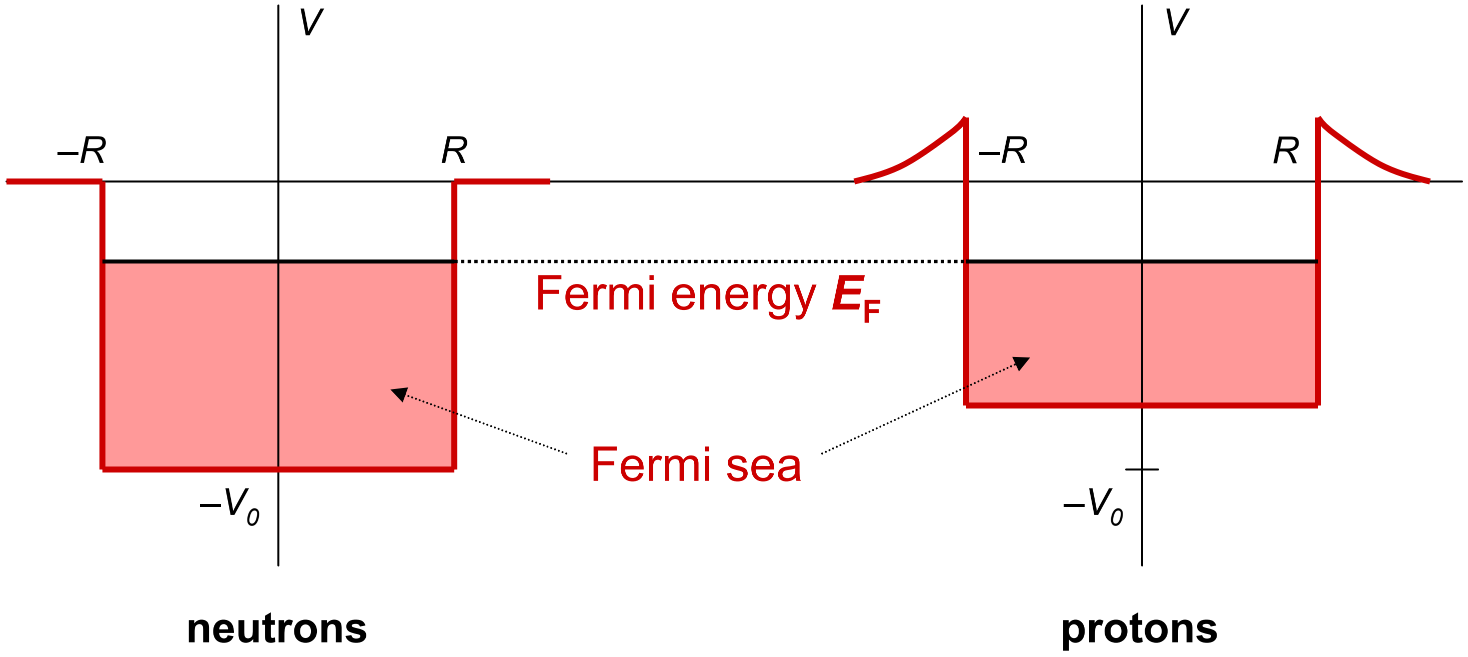
	Semiclassical	Quantum mechanical
Independent particle	Fermi gas	Shell
Collective	Liquid drop	Rotational Vibrational

FERMI GAS MODEL

Built on analogy between nucleus and ideal gas

- particles don't interact
- particles move independently in the mean field of the nucleus

Ground state → particles occupy lowest energy states allowed by the Pauli principle



FERMI MODEL

Distribution of nucleon momentum states: $dn = \frac{2Vd^3p}{(2\pi\hbar)^3}$

- total number of states up to $\boldsymbol{E_F}$: $n = \int_0^{E_F} dn$
  - momentum → energy
  - $n_p = Z\,;\; n_n = A - Z$
  - volume ⇒ $V = 4/3\,\pi r_0^{\,3} A$
- Fermi energy:

$$E_F^p = \frac{\hbar^2}{2mr_0^{\,2}}\left(\frac{9\pi}{4}\cdot\frac{Z}{A}\right)^{\frac{2}{3}} \qquad E_F^n = \frac{\hbar^2}{2mr_0^{\,2}}\left(\frac{9\pi}{4}\cdot\frac{A-Z}{A}\right)^{\frac{2}{3}}$$

→ for $Z = A - Z = A/2$

$$E_F = \text{const}\cdot\frac{1}{m}$$

⇒ **equal for all nuclei**

$$E_F \approx 30\text{ MeV} \quad (V_0 \approx 40\text{ MeV})$$

NUCLEAR LIQUID DROP MODEL

Weizsäcker formula for the binding energy ($A \geq 30$)

$$B(A,Z) = a_V \cdot A$$

condensation energy $\propto V$
holding nucleus together

$$- a_S \cdot A^{\frac{2}{3}}$$

surface tension $\propto S$
near-surface nucleons are bound less

$$- a_C \cdot Z(Z-1) \cdot A^{-\frac{1}{3}}$$

Coulomb potential

$$- a_A \cdot (A-2Z)^2 \cdot A^{-1}$$

asymmetry

$$- \Delta$$

$$\Delta = \begin{cases} -\delta & \text{even-even} \\ 0 & \text{even-odd} \\ +\delta & \text{odd-odd} \end{cases}$$

pairing energy

$$\delta \propto A^{-\frac{1}{2}}$$

$$R \propto A^{\frac{1}{3}}$$

NUCLEAR LIQUID DROP MODEL

Weizsäcker formula for the mass of the nucleus

$$m(A,Z) = Zm_p + (A-Z)m_n - B(A,Z)$$

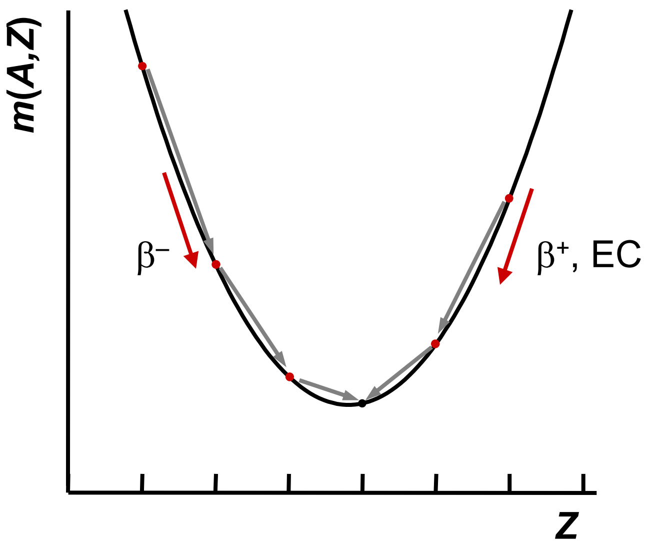
$$m(A,Z) = Zm_p + (A-Z)m_n - a_V A + a_S A^{\frac{2}{3}} + a_C Z(Z-1)A^{-\frac{1}{3}} + a_A (A-2Z)^2 A^{-1} + \Delta$$

$$\Delta = \begin{cases} -\delta & \text{ee} \\ 0 & \text{oe} \\ +\delta & \text{oo} \end{cases}$$

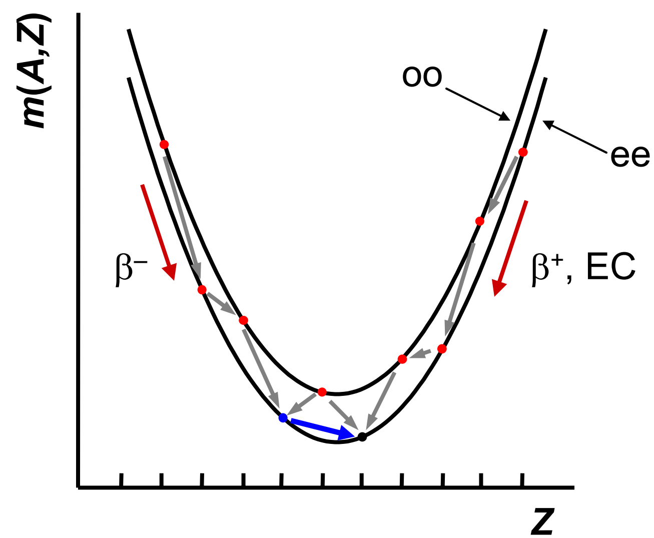
$$\delta \propto A^{-\frac{1}{2}}$$

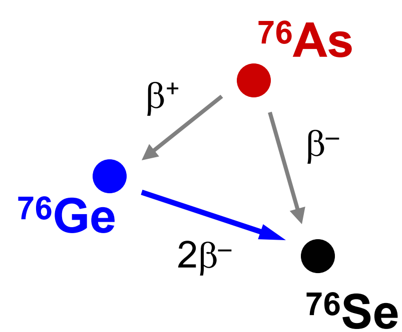
for constant ***A*** $\Rightarrow$ ***m(A,Z)*** is quadratic in ***Z***

odd ***A***



even ***A***



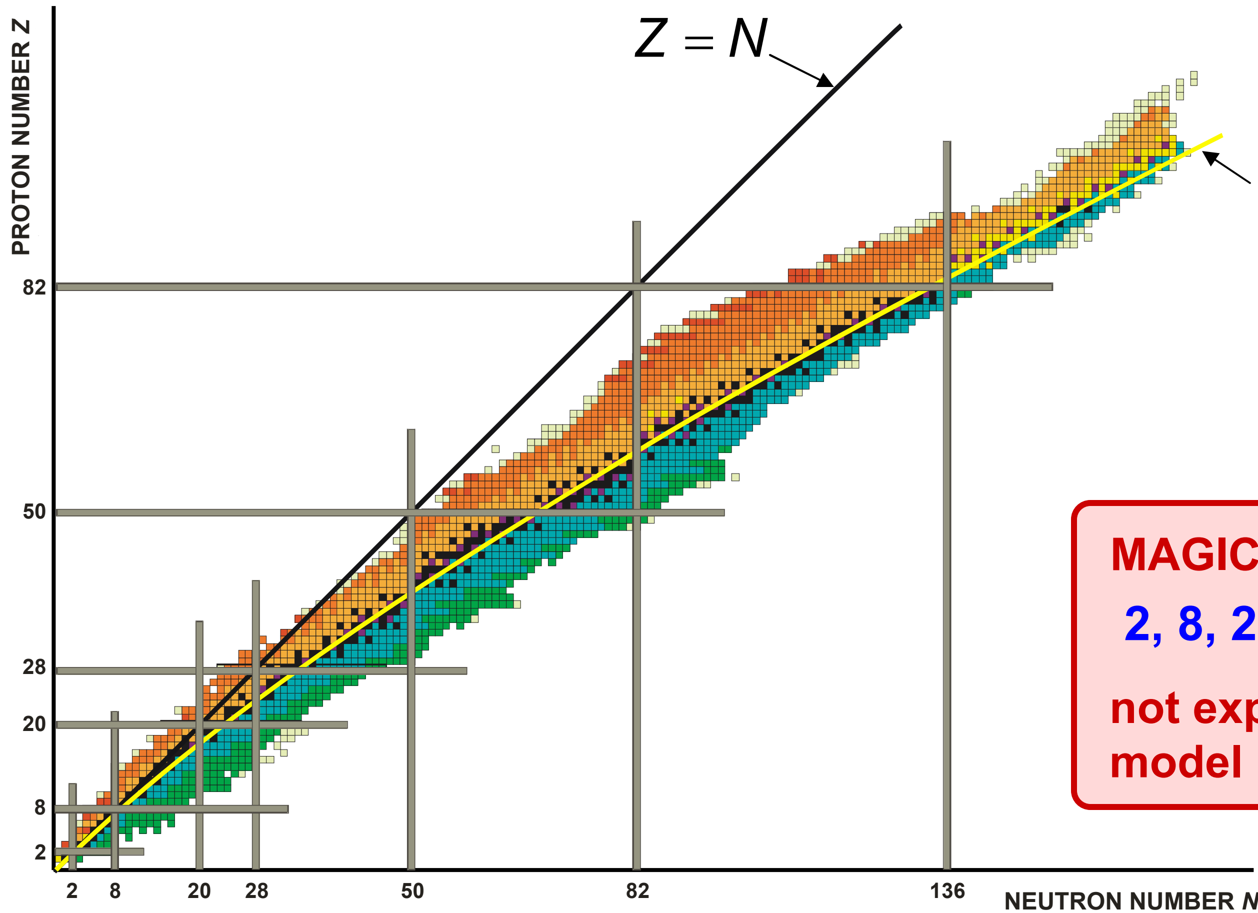


NUCLEAR LIQUID DROP MODEL

Valley of stability

$$m(A,Z) = Zm_p + (A-Z)m_n - a_V A + a_S A^{\frac{2}{3}} + a_C Z(Z-1)A^{-\frac{1}{3}} + a_A (A-2Z)^2 A^{-1} + \Delta$$

For fixed ***A*** the most stable ***Z*** is obtained by differentiating **m(*A,Z*)**



$$Z \cong \frac{A}{2} \cdot \frac{1}{1 + 0.0075 A^{\frac{2}{3}}}$$

MAGIC NUMBERS:

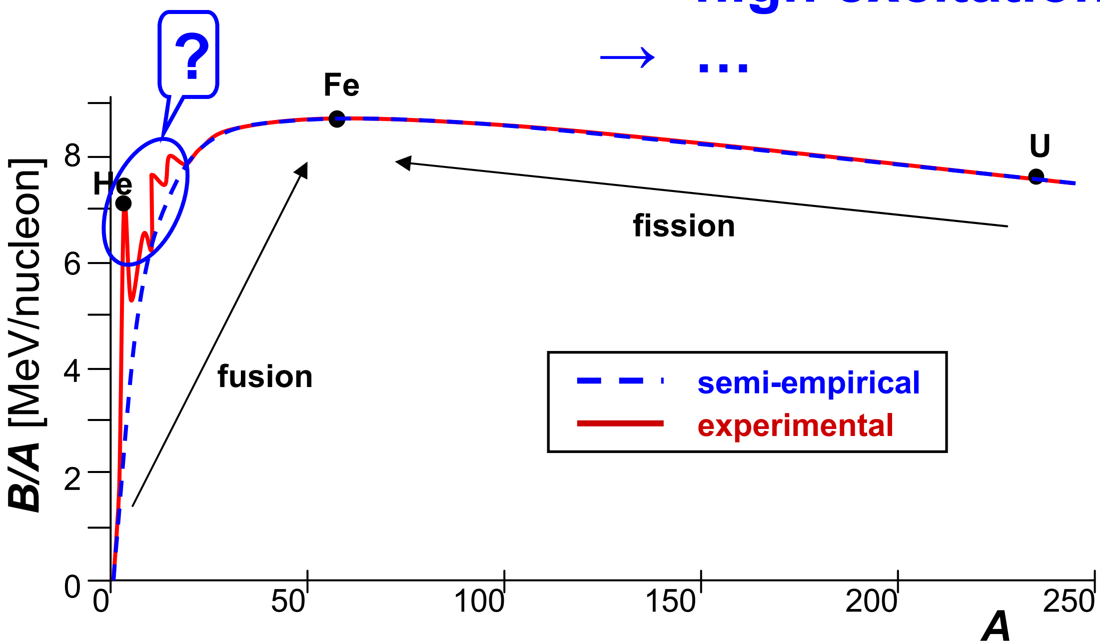
2, 8, 20, 28, 50, 82, 126

not explained by Fermi gas model nor liquid drop model

NUCLEAR SHELL MODEL – WHY?

New model needed to explain discontinuities of several nuclear properties → **binding energy**
→ **high relative abundances**
→ **low n-capture cross section**
→ **high excitation energies**
→ **...**

MAGIC NUMBERS



Magic number = closed shell

Magic numbers indicate similarity of nucleus to electron shells of atom, **BUT** still different from “Atomic magic numbers” (2, 10, 18, 36, 54, 86)

NUCLEAR SHELL MODEL

AIM	→	Explain the magic numbers
ASSUMPTION	→	Interactions between nucleons are neglected
	→	Each nucleon can move independently in the nuclear potential
STEPS	→	Find the potential well that resembles the nuclear density
	→	Consider the spin-orbit coupling

NUCLEAR SHELL MODEL

Potential well

Hamiltonian of a nucleus:

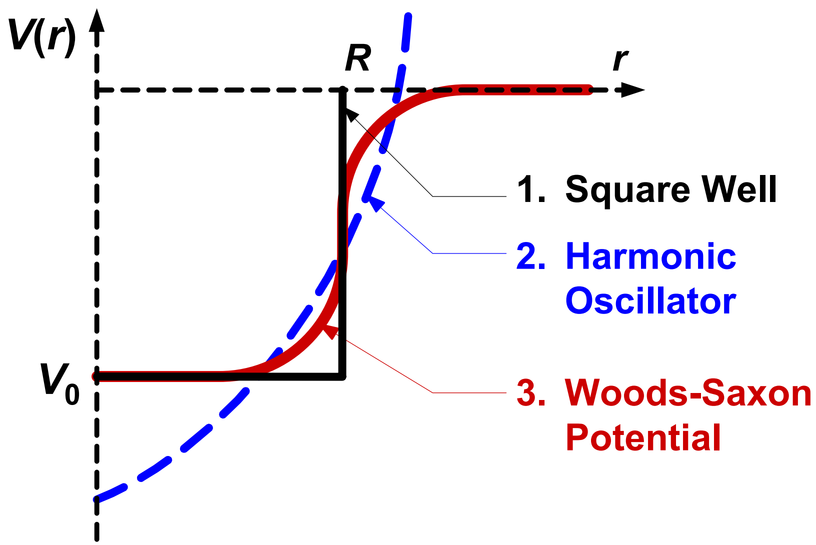
$$H = \sum_i \left(T_i + V(r_i)\right) + \lambda \left[\sum_{i,j,i\neq j} v(r_{ij}) - \sum_i V(r_i) \right]$$

central potential

residual potential

Central potential $\gg$ Residual potential $\Rightarrow \lambda \to 0$

Potential well candidates



Solve Schrödinger equation

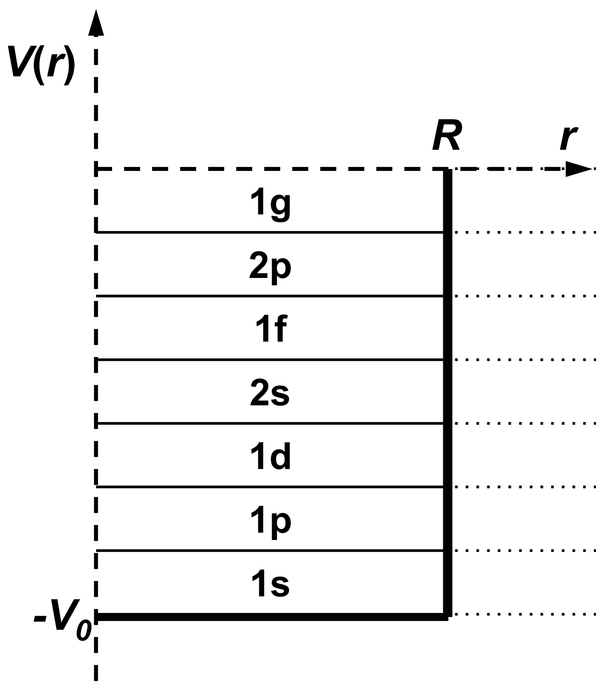
$$\frac{d^2}{dr^2} R_{nl} + \frac{2M}{r\hbar^2}\left(E_{nl} - V(r) - \frac{l(l-1)\hbar^2}{2Mr^2} \right) R_{nl} = 0$$

NUCLEAR SHELL MODEL

Square well potential

$$V_0(r) = \begin{cases} -V_0 & r \le R \\ 0 & r > R \end{cases}$$

$\Rightarrow$ **no analytical solution**



Occupation	Total
18	58
6	40
14	34
2	20
10	18
6	8
2	2

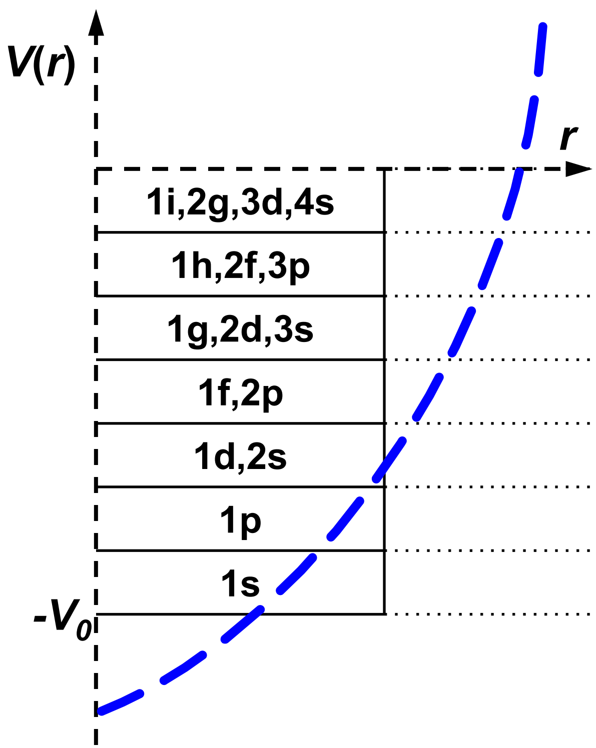
Closed Shell
$\neq$
Magic Number

NUCLEAR SHELL MODEL

Harmonic potential

$$V_0(r) = -V_0 + \frac{1}{2}M\omega^2 r^2$$

$\Rightarrow$ **analytical solution possible**



	Occupation	Total
1i,2g,3d,4s	56	168
1h,2f,3p	42	112
1g,2d,3s	30	70
1f,2p	20	40
1d,2s	12	20
1p	6	8
1s	2	2

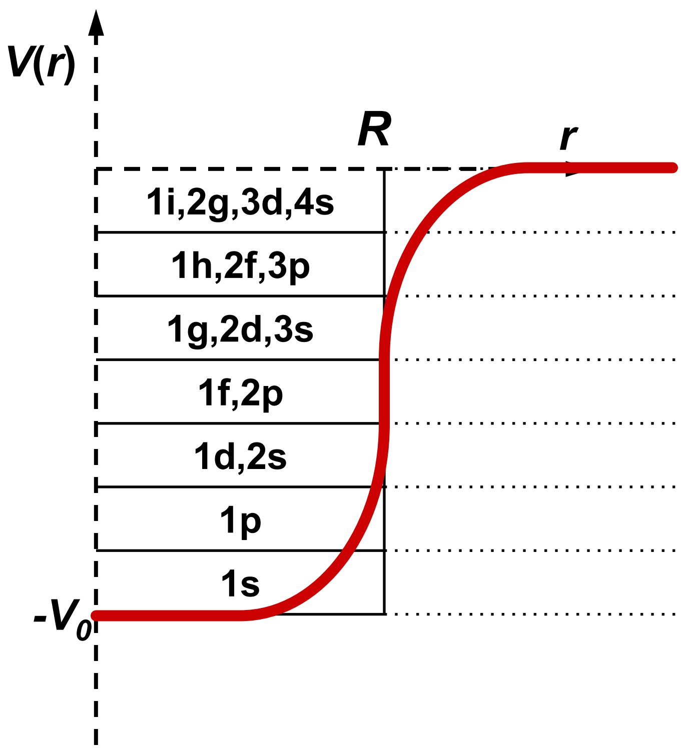
Closed Shell
$\neq$
Magic Number

NUCLEAR SHELL MODEL

Woods-Saxon potential ⇒ resembles the nuclear density from scattering measurements

$$V_0(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

⇒ **no analytical solution**



Shell	Occupation	Total
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1s	2	2

Closed Shell ≠ Magic Number

NUCLEAR SHELL MODEL

Spin-orbit coupling contribution

Maria Mayer (*Physical Review 78 (1950), 16*) suggested:

1. **There should be a non-central component**
2. **It should have a magnitude which depends on S & L**

$$V(r) = V_0(r) + V_s(r)\,\mathbf{L}\cdot\mathbf{s}$$

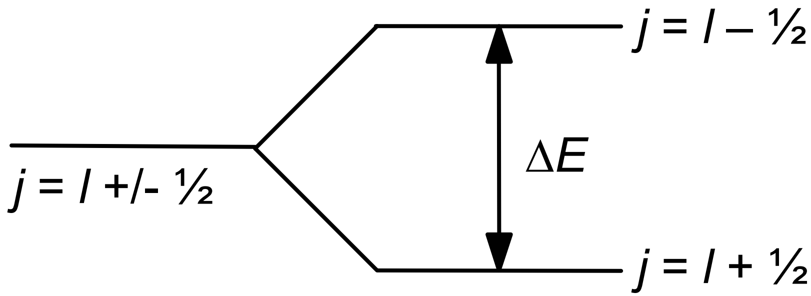
non-central potential

with $$V_s(r) = V_{0s}\frac{1}{r}\frac{d}{dr}f(r)$$

Woods-Saxon shape

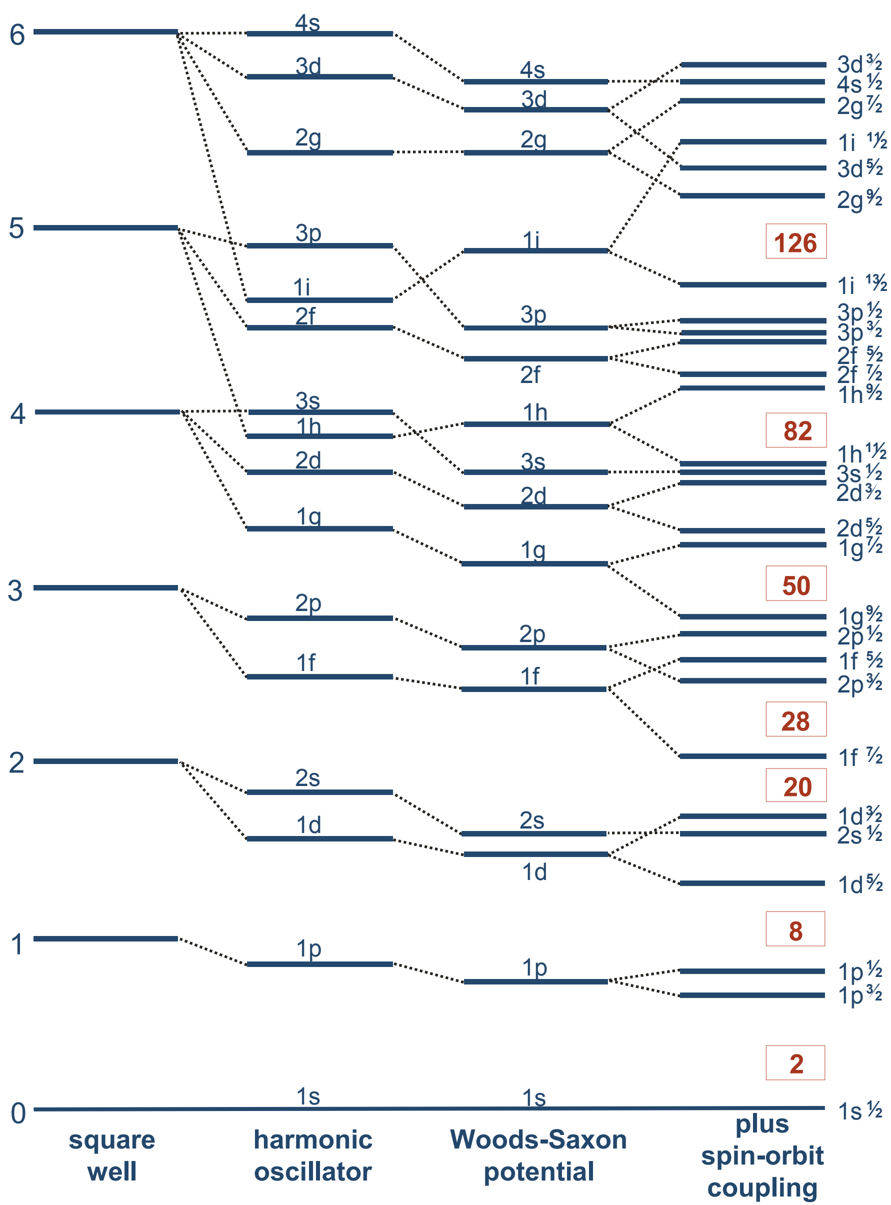
Results in **energy splitting** of individual levels
for given ***J*** (angular momentum)

for $l > 0$



e.g. $\Rightarrow$ 1d → 1d 3/2, 1d 5/2

NUCLEAR SHELL MODEL



Level splitting

REMARKS

The essential features are given by any potential of the form

$$V(r) = V_0(r) + V_s(r)\mathbf{L}\cdot\boldsymbol{\sigma}$$

Energies of levels are parameter dependent

Shell model fails when dealing with deformed nuclei, i.e., nuclei far from magic numbers

Collective models: rotational, vibrational

Other models

Close to **CLOSED-SHELL** nuclei well described by shell model

However, most of the nuclear properties are indeed determined by nucleons outside the closed shells

Collective models → treating the closed shells as inert and only dealing with the rest

Models not mentioned (but used):

1. **rotational model** → **rotations of permanently deformed nuclei**
2. **vibrational model** → **excitations within shell – multipole account**
3. **Nilsson model** → **shell model with deformed potential**
4. **α-particle model** → **α-particle clusters inside the nucleus**
5. **interacting boson model** → **considering pairs of nucleons as bosons**