Gauge Symmetries in F-theory

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Work done with Andres Collinucci

Université Libre de Bruxelles

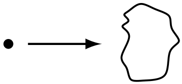
IMPRS Workshop, Munich

March 16, 2015

Introduction

F-theory $\subset$ String theory

Basic idea: particles are in fact tiny strings



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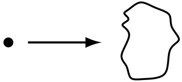
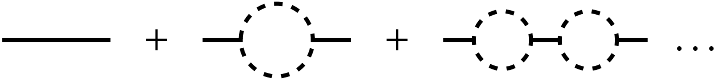
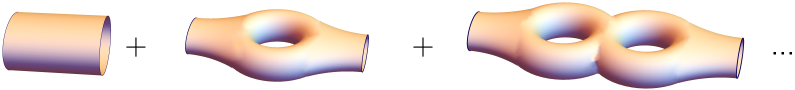


QFT loop expansion



replaced by expansion in surfaces with holes



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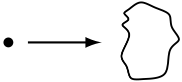

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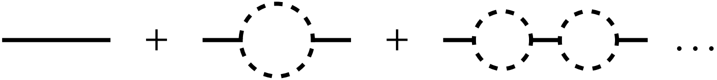

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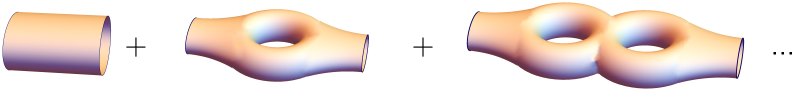
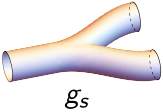


$\Longrightarrow$ Defined perturbatively,
parameter: **string coupling g_s**



String coupling

g_s determined by the value of the **dilaton** field ϕ

$$g_s \sim e^{\phi}$$

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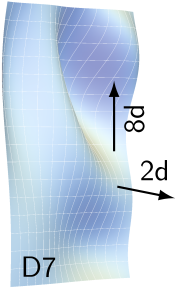
Inevitable in presence of **D7-branes**

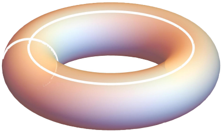
$\hookrightarrow$ Magnetic potential: **axion** field $\boldsymbol{C_0}$

$\Longrightarrow$ 2 scalars fields varying along the space

$$\phi \quad \text{and} \quad C_0$$



Geometric formulation



General idea of F-theory :
2 scalars $\leftrightarrow$ Kaluza-Klein reduction on a torus

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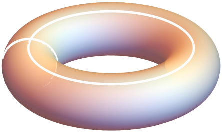

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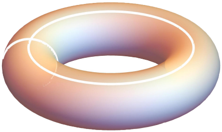

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Type IIB effective Lagrangian

$$\mathcal{L} \sim -\frac{\mathrm{d}\,\tau \wedge \star\, \mathrm{d}\,\bar{\tau}}{2\,(\mathrm{Im}\,\tau)^2} + \ldots$$

Invariant under

$$\tau \to \frac{a\,\tau + b}{c\,\tau + d}, \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

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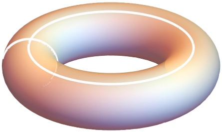

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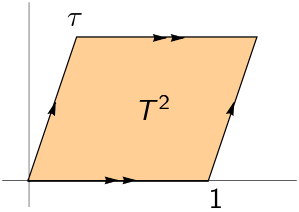
$\Rightarrow$ $\tau \sim$ modular parameter of a torus

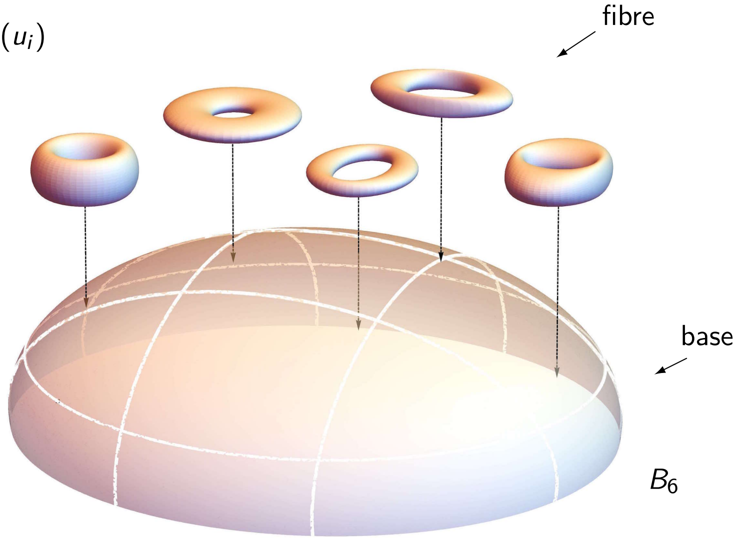
[Vafa '96]

$$\tau = \tau(u_i)$$



Elliptic fibration

$\tau = \tau(u_i)$



Algebraic description of a torus

Weierstrass equation

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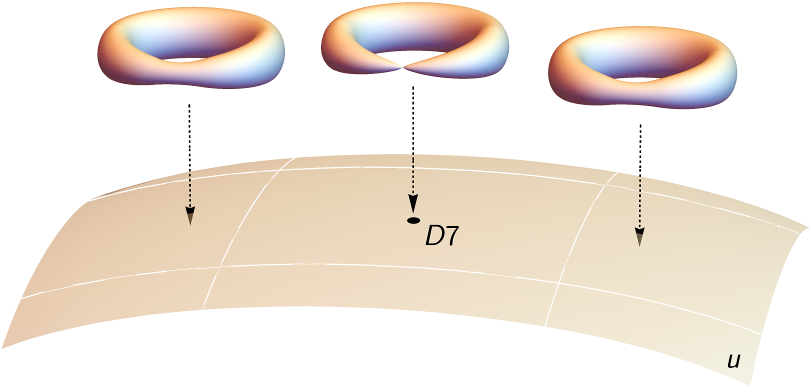
Geometric picture for D7-branes

$\tau \to i\infty$ at the position of a D7 $\Leftrightarrow$ a cycle of the torus collapses



$$\Delta = 4f^3 + 27g^2 = 0$$

Weierstrass equation for D7-branes

# D7	Singular fibre
1	$y^2 = x^3 + x^2 + u$

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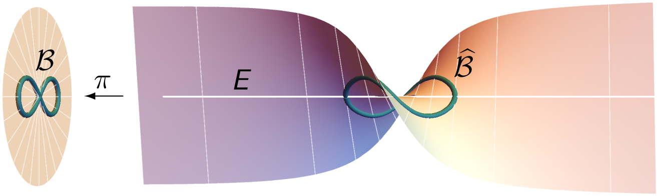
How to treat these singularities ?

Blowing-up

Example of a blow-up



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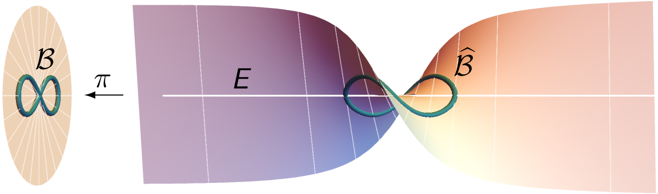
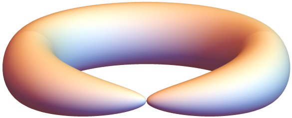


For the A_1 singularity



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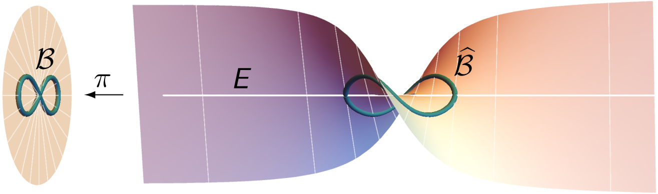

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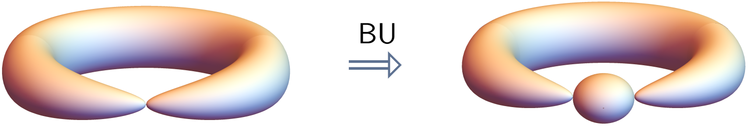

A $\mathbb{P}^1$ appears

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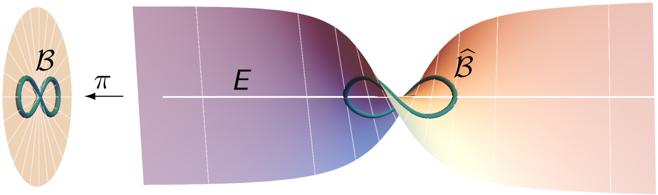
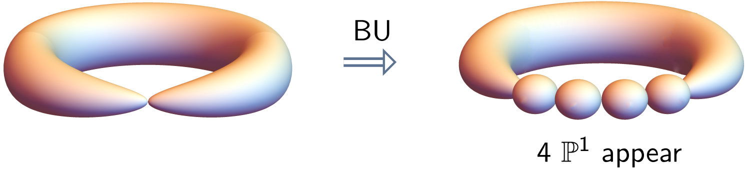


For the A_4 singularity



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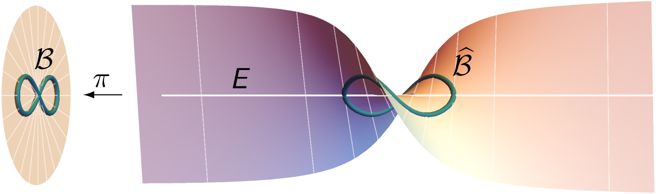

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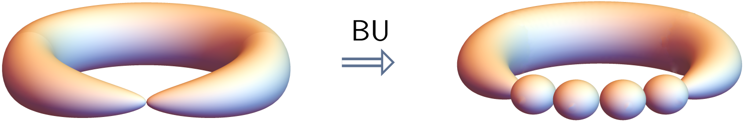

4 $\mathbb{P}^1$ appear

Every time: (extended) **Dynkin diagram** $\Gamma_i \cap \Gamma_j \sim -C_{ij}$

Gauge theory

These new cycles yield to a **gauge theory**

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- Cartan subalgebra: $\Gamma_i \leftrightarrow \omega_i$ (harmonic 2-form) by Poincaré duality.

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# D7	Singularity	Gauge group
2	$x\,y = u^2$	SU(2)
n	$x\,y = u^n$	SU(n)
	$y^2 = x^3 + u^5$	E_8

Abelian Gauge symmetries

U(1)'s have no Dynkin diagram !

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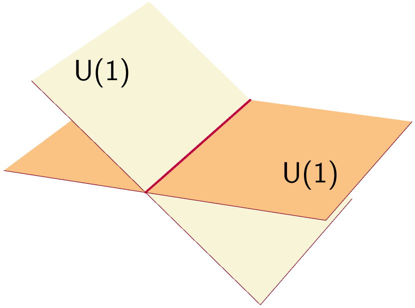

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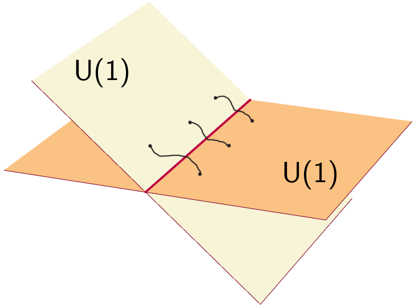
- In F-theory: the space is smooth.

$\Rightarrow$ trick : 2 branes intersecting at angle.

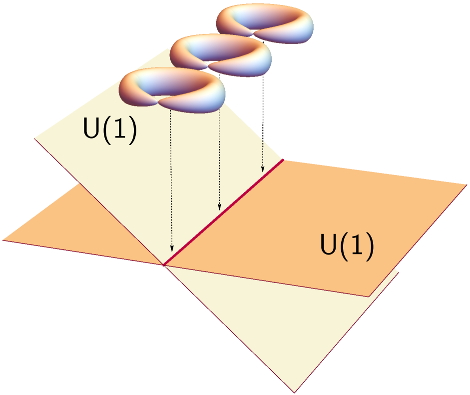
$\hookrightarrow$ charged matter under the 2 U(1)'s.



F-theory lift

Higher codimension singularity, of **conifold** type

$$x\,y = u\,v$$



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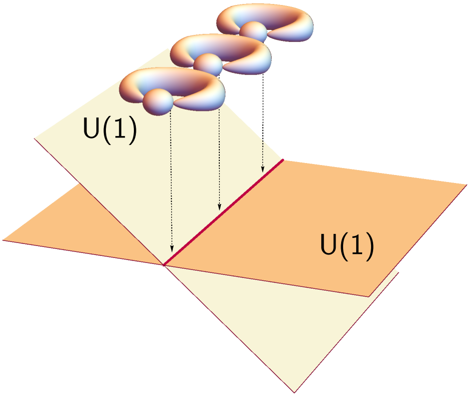

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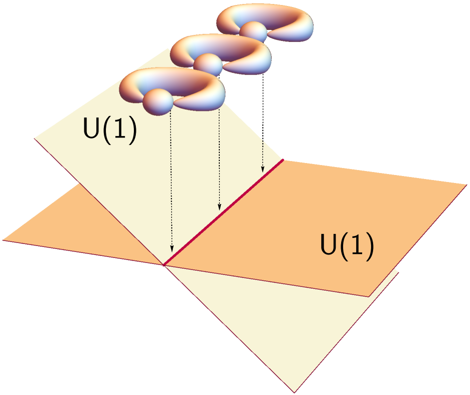
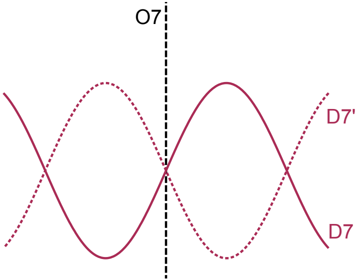
Wrapped by M2-branes $\rightsquigarrow$ charged matter

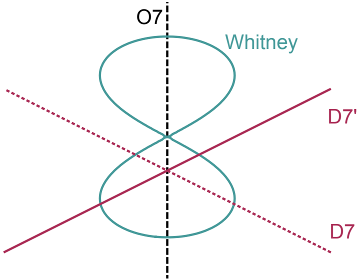


Global Models

Studied in [Braun, Collinucci, Valandro '14]

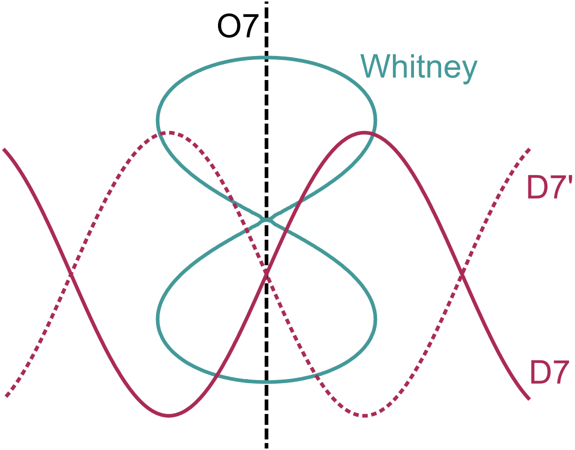


$$(\eta_1^2 - \xi^2 \psi_1^2) = 0$$



$$(\eta_1^2 - \xi^2)(\eta_2^2 - \xi^2 \chi_2) = 0.$$

Combining the two



$$(\eta_1^2 - \xi^2\psi_1^2)\,(\eta_2^2 - \xi^2\chi_2) = 0.$$

Results

Weierstrass equation:

$$y^2 = x^3 + b_2\,x^2 + 2\,\eta_1\eta_2\,x + \eta_2^2\psi_1^2 + \eta_1^2\chi_2 - b_2\psi_1^2\chi_2.$$

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Going to a Zariski open $\psi_1 \neq 0$, and neglecting the x^3 term, we manage to bring it into a conifold form

$$\begin{aligned} \left(y + \eta_2 \psi_1 + \frac{\eta_1}{\psi_1}\, x \right) \left(y - \eta_2 \psi_1 - \frac{\eta_1}{\psi_1}\, x \right) \\ = \left(b_2 - \frac{\eta_1^2}{\psi_1^2} \right) \left(x^2 - \chi_2 \psi_1^2 \right). \end{aligned}$$

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New weak coupling limit of [Clingher, Donagi, Wijnholt '12]
$\rightsquigarrow$ we have detected massless U(1)'s at **weak coupling**.

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New weak coupling limit of [Clingher, Donagi, Wijnholt '12]
⇝ we have detected massless U(1)'s at **weak coupling**.

We checked this was still the case at **strong coupling** (as it was expected since $[D7] = [D7']$)

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Mathematical tool to treat singularities [Eisenbud '80] introduced in F-theory in [Collinucci, Savelli '14]

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Singularity : $P = 0 \quad \longrightarrow$ Matrix factorisation :

$$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} = P \cdot \mathbb{1}$$

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Conifold : $x\,y - u\,v = 0 \quad \longrightarrow$ Matrix factorisation :

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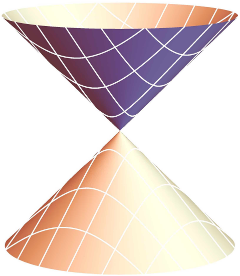

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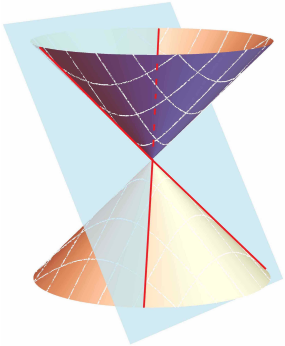
Reveals the structure of the singularity

$$\begin{pmatrix} x & u \\ v & y \end{pmatrix} \begin{pmatrix} s \\ t \end{pmatrix} = 0$$

$\hookrightarrow$ non-Cartier divisor $\;\leftrightarrow\;$ U(1).



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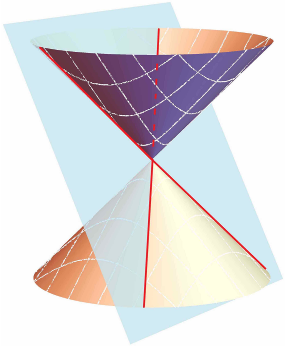
$$\begin{pmatrix} x & u \\ v & y \end{pmatrix} \begin{pmatrix} y & -u \\ -v & x \end{pmatrix} = (x\,y - u\,v) \cdot \mathbb{1}$$

Reveals the structure of the singularity



$$\begin{pmatrix} x & u \\ v & y \end{pmatrix} \begin{pmatrix} s \\ t \end{pmatrix} = 0$$

$\hookrightarrow$ non-Cartier divisor $\;\leftrightarrow\;$ U(1).

Hope: no need to go to a Zariski open

Thank you

Weak coupling limit

Weierstrass equation can be rewritten

$$y^2 = x^3 + b_2 x^2 + 2b_4 x + b_6$$

Weak coupling $\epsilon \to 0$

$$b_4 \to \epsilon\, b_4 - \frac{1}{2}\xi_2 \psi_1^2 \epsilon^2$$

$$b_6 \to \epsilon\, b_6$$

$$\widehat{W_5}: \qquad y^2 = x^3\, v + b_2 x^2 + (2b_4\, t - \xi_2 \psi_1^2\, t^2 v)\, x + b_6 t^2$$

$$\left(y + \eta_2 \psi_1\, t + \frac{\eta_1}{\psi_1}\, x\right)\left(y - \eta_2 \psi_1\, t - \frac{\eta_1}{\psi_1}\, x\right) = \left(b_2 - \frac{\eta_1^2}{\psi_1^2}\right)\left(x^2 - \chi_2 \psi_1^2\, t^2\right).$$

(at weak coupling)

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$$\left(y + \eta_2 \psi_1\, \epsilon + \frac{\eta_1}{\psi_1}\, x\right)\left(y - \eta_2 \psi_1\, \epsilon - \frac{\eta_1}{\psi_1}\, x\right) = \left(x + b_2 - \frac{\eta_1^2}{\psi_1^2}\right)\left(x^2 - \chi_2 \psi_1^2\, \epsilon^2\right).$$

(at strong coupling)