# Thermodynamics of the Ernst spacetime

## (Pair production of Black Holes)

Henk Bart

Utrecht University
Supervisor: Stefan Vandoren

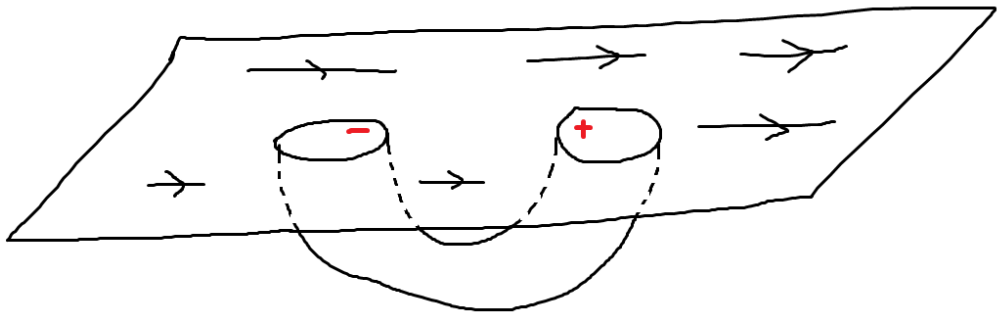
31st IMPRS Workshop
Max Planck Institute, Munich
16-03-2015

Universiteit Utrecht

# Context

## Goal

Verify the first law of thermodynamics in the Ernst spacetime.



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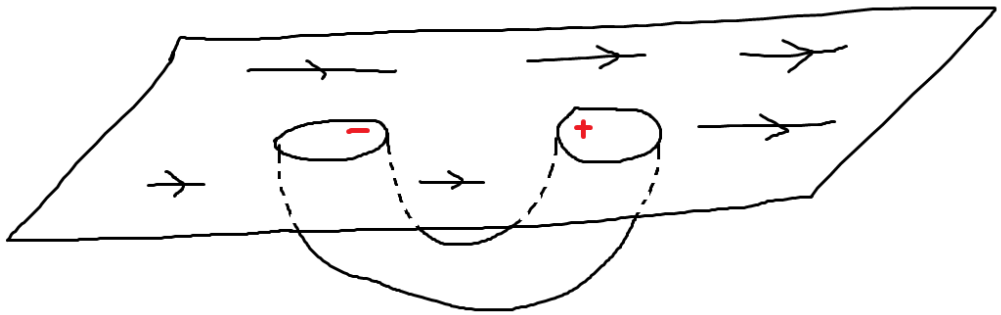


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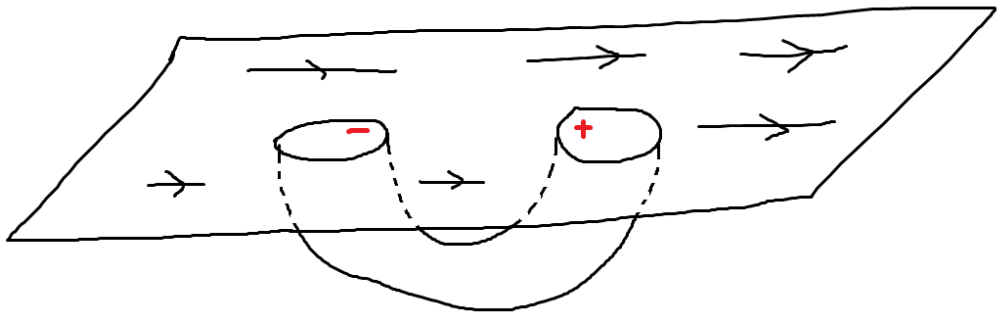


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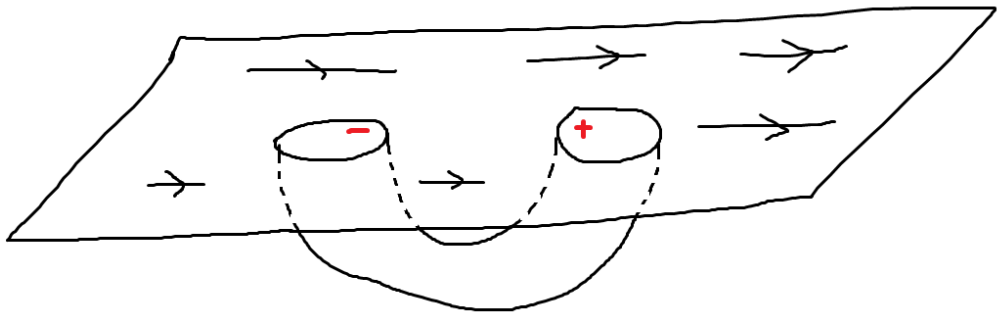


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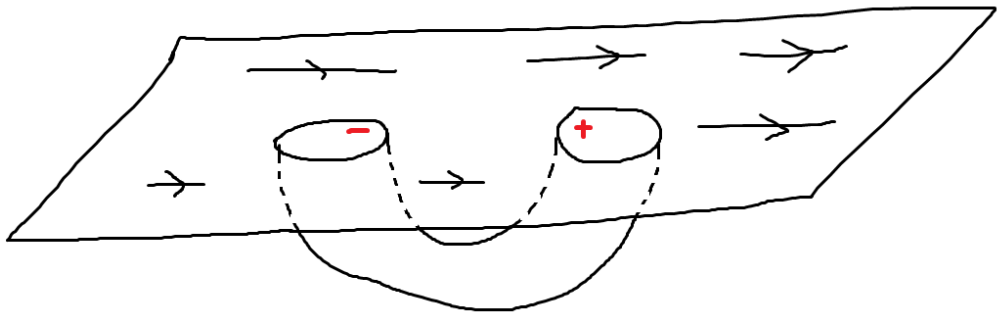


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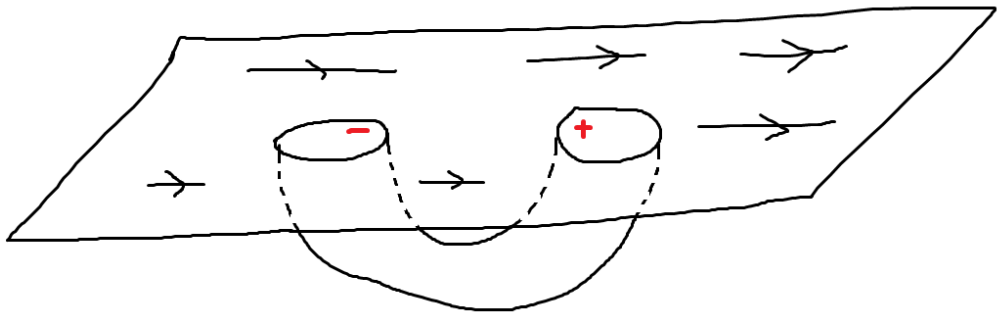


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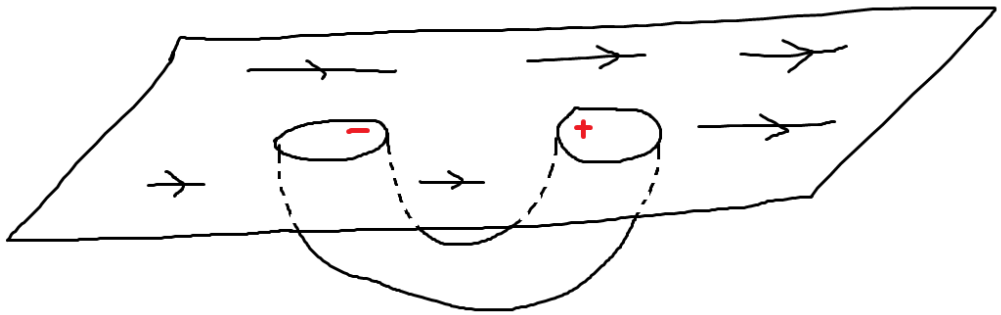


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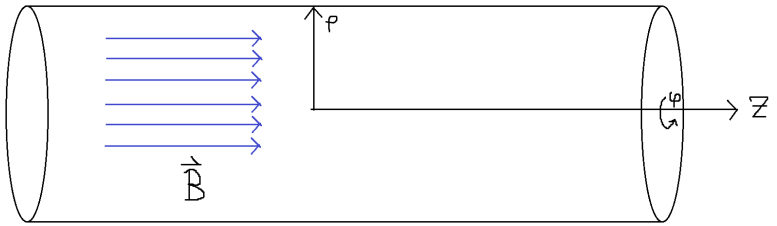
2. **Thermodynamics** of the Ernst spacetime
   - How to define mass/thermodynamic quantities?
   - Does the first law hold?

# Step [1]: The Melvin Universe




Metric:

$$\mathrm{d}s^2 = (1 + \frac{1}{2}B^2\rho^2)^2(-\mathrm{d}t^2 + \mathrm{d}\rho^2 + \mathrm{d}z^2) + \frac{1}{(1 + \frac{1}{2}B^2\rho^2)^2}\,\mathrm{d}\varphi^2$$

Field strength:

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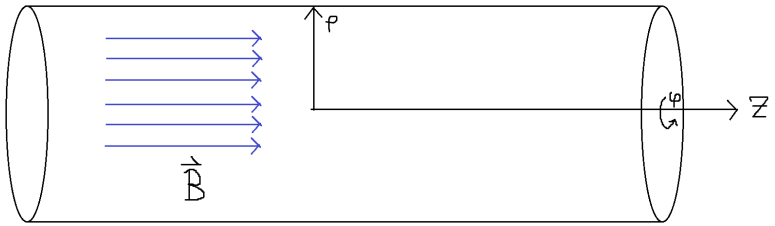


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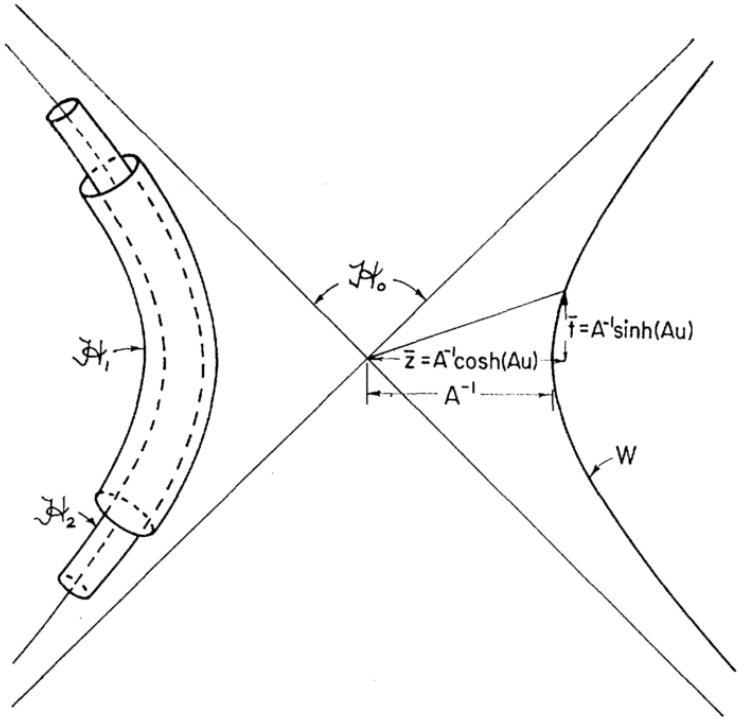
## Fun fact

Flux:

$$\boxed{\Phi = \int F = \frac{4\pi}{B}}$$

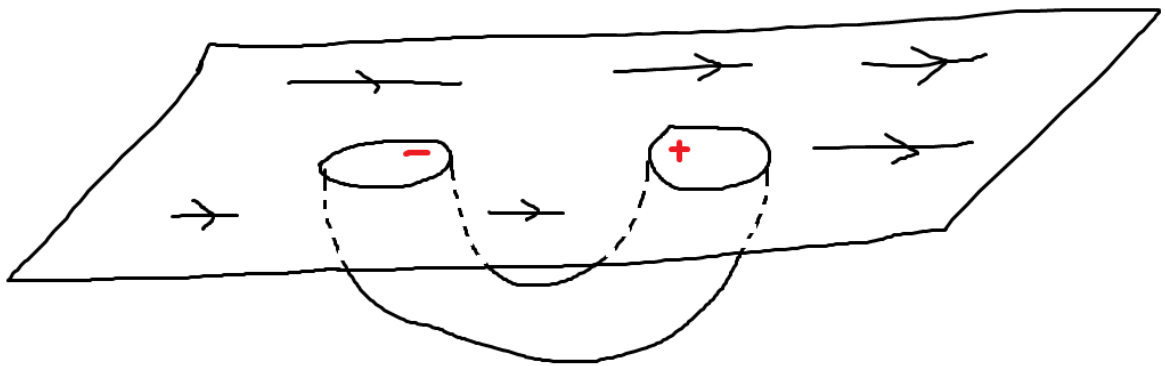
# Step [2]: Adding the black holes




$$ds^2 = \frac{\Lambda^2}{A^2(x-y)^2}\left[G(y)\,dt^2 - G^{-1}(y)\,dy^2 + G^{-1}(x)\,dx^2\right]$$

$$+ \frac{G(x)\,d\tilde{z}^2}{\Lambda^2 A^2(x-y)^2}$$

# A wormhole



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- But the black holes do have a mass!?

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- **Frame dragging**
- For the Ernst metric:

$$\mathrm{d}M = \frac{\kappa}{8\pi}\,\mathrm{d}A_{\mathsf{bh}} + \Phi\,\mathrm{d}Q - \mu\,\mathrm{d}B$$

# What next?

- What about the (generalised) second law (area theorem)?

$$\frac{\mathrm{d}A}{\mathrm{d}t} \geq 0.$$

- Is there an honest way to derive $\Phi$ and $\mu$?

*Thank you.*

# Motivation

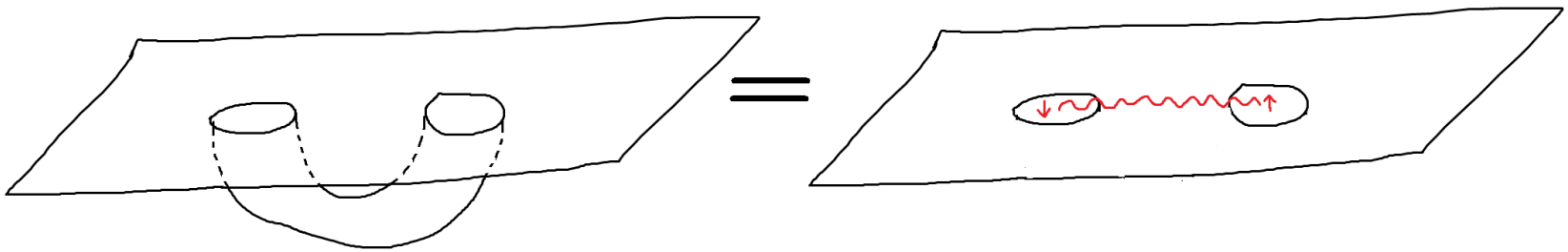
Quantum Gravity.

$$\text{QFT}+\text{GR} \Longrightarrow \text{Firewall paradox}$$

Possible solution:

$$\text{(GR) Wormhole} = \text{Entanglement (QM)}$$

(Maldacena and Susskind)



# Instanton

Wick rotation $u \mapsto iu$.

$$\begin{aligned} \sinh(Au) &\longmapsto i\sin(Au), \\ \cosh(Au) &\longmapsto \cos(Au). \end{aligned} \tag{1}$$