# Warping the effective field theory of string flux compactifications

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### Objective

To study the *warp factor contribution to the Kähler potential* of the effective theory obtained from a IIB flux compactification

# Contents

# What is a compactification?

$$\boxed{\text{STRING THEORY } 10\text{d} \leftrightarrow \text{OBSERVATION } 4\text{d}}$$

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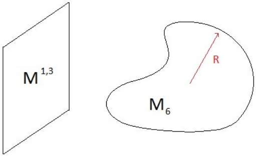
2. *How to get a 4d theory?*

**1 Compactification**

$$M_{10} = M^{1,3} \times M_6$$

$M_6$ compact manifold of radius $R$



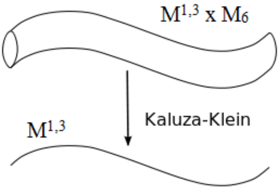

In order to probe extra dimensions we need energies $E \sim \frac{1}{R}$

$R$ "small enough" ➡ extra dimensions are *hidden*

(e.g. $R \simeq 10^{-33} cm \div 10^{-17} cm$)

## 2 Dimensional Reduction (Kaluza–Klein)

Reducing $M_6$ in the limit $R \ll \frac{1}{E_{Phys}}$




10d massless field

⇓

infinite number of 4d massive modes

$m_n \sim \frac{n}{R} \gg E_{Phys}$

$\Downarrow$

truncation to massless modes

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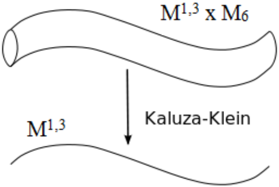


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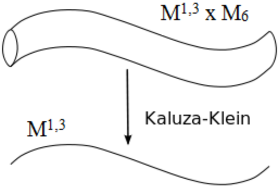


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4d effective theory (low energy) ... presenting *moduli* $u^A(x)$ !?

How to "stabilize" moduli ? → *flux compactifications...*

# IIB Supergravity

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Closed string sector (bosonic action):

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& - \frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g_{\mathrm{s}}}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right) \\
& + \frac{1}{4\kappa_{10}^2}\int C_4\wedge H_3\wedge F_3 \qquad \text{with} \quad H_3, F_1, F_3, F_5, \quad \text{“}\textit{fluxes}\text{”}
\end{aligned}
$$

NS–NS sector: $g, B_2, \phi$
R–R sector: $C_0, C_2, C_4$

closed string

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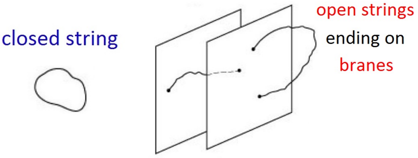
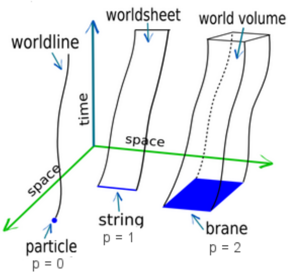


## Open string sector $\to$ D$p$-branes




- $p$ spatial dimensions
- *dynamical*
- gauge theories (on their own surfaces)
  $\hookrightarrow$ *charged matter*
- charged under $C_p$ $\to$ sources of fluxes

$$S_{\mathrm{D}p-\mathrm{brane}} = S_{\mathrm{DBI}} + S_{\mathrm{CS}}$$

$$S_{\mathrm{DBI}} = -T_p \int_W d^{p+1}\sigma\ e^{-\phi} \sqrt{-\det(P[g - B_2] + \lambda F)} \quad ,$$

$$S_{\mathrm{CS}} = \mu_p \int_W \left[ P\left( \sum_{\mathrm{P}} C_p e^{-B_2} \right) e^{\lambda F} \right]_{p+1} \wedge \hat{A}(R)$$

# Purely geometric IIB compactifications

## Most general starting Ansatz $M^{1,3} \times M_6$

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Purely geometric compactifications are characterised by a background in which the only field turned on is the metric:

$$ds^2_{10} = ds^2_4(x) + ds^2_6(y)$$

with

$$ds^2_6 \text{ Ricci–flat } (R_{mn} = 0,\ M_6 \text{ is Calabi–Yau})$$

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Dimensional reduction $\to$ 4d sugra $\mathcal{N} = 2$, not so realistic:

- no chiral matter
- no moduli stabilisation/no ~~SUSY~~ mechanism ($\to \mathcal{N} = 0$)

# Compattificazioni IIB con flussi (GKP)

Solution: *enriching background:* $H_3, F_1, F_3, F_5 \neq 0$

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Among all fluxed backgrounds, we chose a particular class:

**Background GKP [Giddings, Kachru, Polchinski 2002]**

$$ds^2_{10} = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} d\tilde{s}^2_6(y)$$

$d\tilde{s}^2_6$ is Ricci–flat (Calabi–Yau)

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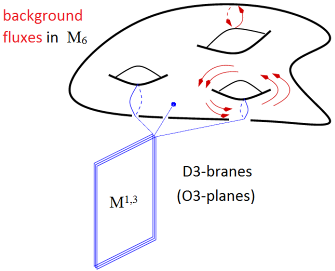
Consistency $\rightarrow$ sources of fluxes:

- D$p$-branes
- O$p$-planes (not dynamical)

# IIB warped compactifications: pros

Dimensional reduction → *richer* 4d theory




10d ⇒ 4d

1. D*p*-branes ⇒ gauge theories and chiral matter

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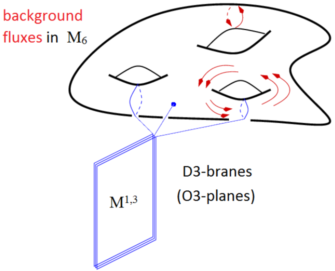


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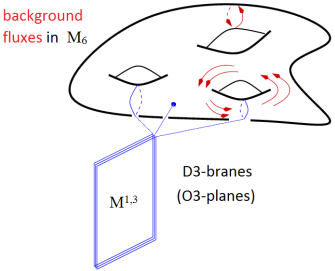


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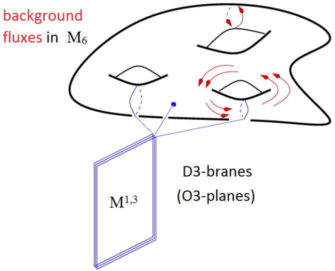


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BACK-REACTION

4. warping $e^{2A(y)}$ $\rightarrow$ hierarchies [RS,KKLT,...]

# 4d effective theory

$$e^{-1}\mathcal{L}_{\rm 4d}^{\mathcal{N}=1} = \frac{M_{\rm P}^2}{2} R - M_{\rm P}^2\; g_{i\bar{\jmath}}\, \partial_\mu \varphi^i \partial^\mu \bar{\varphi}^{\bar{\jmath}} - V_F + \ldots$$

$$\varphi^i(u(x)) \quad \text{chiral fields}$$

$K(\varphi, \bar{\varphi})$ Kähler potential $\quad \rightarrow \quad$ $g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K$ Kähler metric

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$$e^{2A(y)} \approx const$$

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How does it contribute to $K(\varphi,\bar{\varphi})$ ? HOW TO PROCEED ??

# Background GKP symmetry...

An alternative strategy to standard reduction [Douglas, Torroba, Frey,...] comes from the following

## Observation

The dynamical generalisation of the starting Ansatz

$$ds_{10}^2 = e^{2A(y,u)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2A(y,u)} g_{mn}(y,u) dy^m dy^n + \ldots$$

possesses a Weyl symmetry (gauge symmetry)

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4d effective theory is a superconformal supergravity

# … and the Kähler potential

- gauge fixing → 4d $\mathcal{N} = 1$ sugra with

$$K(u) = -3 \log \left( 4\pi \int_{M_6} d^6 y \sqrt{g_6(y;u)}\; e^{-4A(y;u)} \right)$$

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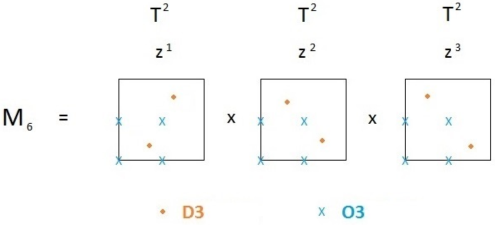
The inversion is generally not *explicitly* known

# Results for a $T^6/\mathbb{Z}_2$ compactification

Background:

- 64 O3-planes
- $N$ D3-branes
- const. $F_3$, $H_3$:
  (particular $\rightarrow$ SUSY!!)



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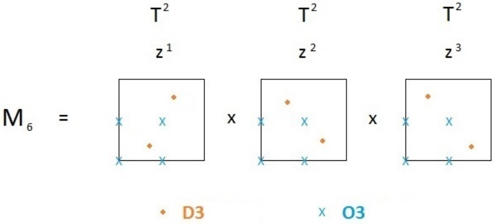
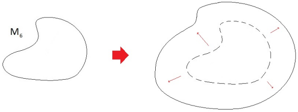



moduli $u$:

- Kähler moduli $v^a$
  (*size deformations*)
- $N$ D3-brane positions $Z_I^j$
  ($I = 1 \ldots N$)



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- Following the procedure described above:

1. chiral fields $\varphi^i$: $Z_I^j$ e $\rho^a(v, Z, \bar{Z})$
2. the explicit Kähler potential is:

$$K(\mathrm{Re}\,\rho, Z, \bar{Z}) = -\log\left(t_1 t_2 t_3 + 2 t_4 t_5 t_6 - t_1 t_4^2 - t_2 t_5^2 - t_3 t_6^2\right) - \log(16\pi^3 \mathrm{V}_0)$$

$$t_a = \mathrm{Re}\,\rho^a - \boxed{\frac{1}{2}\sum_I k_a(Z_I, \bar{Z}_I)}$$ WARPING!

Agreement with reduction of $S_{IIB} + S_{D3}$ (*unwarped limit*) [Grimm,...]

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↪ The strategy is to be generalised


THANK YOU

# Some details

**4D SUPERCONFORMAL SUGRA**
$\mathcal{N}(\Phi,\bar{\Phi}), \mathcal{W}(\Phi), f_{\alpha\beta}(\Phi)$

Super-Poincaré symmetry,
Weyl symmetry, $U(1)$ chiral symmetry,
$S$-supersymmetry, sp. conf. symmetry

$$e^{-1}\mathcal{L} = \frac{1}{2}\mathcal{N}R + 3\mathcal{N}_{I\bar{J}}\partial_\mu\Phi^I\partial^\mu\bar{\Phi}^J + \dots$$

$\xrightarrow[\text{FIXING}]{\text{GAUGE}}$

**4D SUGRA**
$K(\varphi,\bar{\varphi}), W(\varphi), f_{\alpha\beta}(\varphi)$

Super-Poincaré symmetry

$$\frac{M_P^2}{2}R - M_P^2 g_{i\bar{j}}\partial_\mu\varphi^i\partial^\mu\bar{\varphi}^j + \dots$$

$K \leftrightarrow \mathcal{N}$

GKP background (10D) $\xrightarrow{\text{reduction}}$ 4D Superconformal sugra ($S$-susy, sp. conf. symm. fixed)

$$S_{IIB}^{(Ef)} = \frac{2\pi}{\ell_s^8}\int_{M^{1,3}} d^4x\sqrt{-g_4}R_4 \underbrace{\int_{M_6} d^6y\sqrt{g_6(y;u)}e^{-4A(y;u)}}_{\hookrightarrow \mathcal{N} \Rightarrow K} + \dots$$

$$\longrightarrow \quad \text{4D sugra:} \quad K(u) = -3\log\left(4\pi \int_{M_6} d^6y \sqrt{g_6(y;u)}\; e^{-4A(y;u)}\right)$$

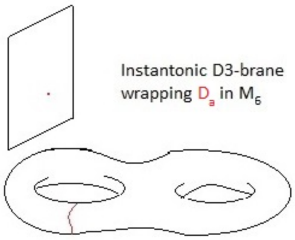
($e^{-4A(y;u)}$ is determined by the "warping equation" – eom/BI $\tilde{F}_5$)

NOW: How to find chiral field $\varphi^i(u)$ ??

## Susy instantonic D3-branes (probes)

$$W_{np} \sim e^{S^{E3}}, \qquad S^{E3} = S_{DBI} + iS_{CS}$$

SUSY: $W_{np}(\varphi) \quad \Longrightarrow \quad S_{DBI} \sim \mathrm{Re}\,\varphi^i$




Ex: $T^6/\mathbb{Z}_2$ compactification: $\qquad u^A = (v^a, Z_I^j) \qquad \varphi^i = (\rho^a, Z_I^j)$

$$S_{DBI} = \pi \underbrace{\int_{D_a} e^{-4A} J \wedge J}_{f(v^a, Z_I, \bar{Z}_I)} + const = \mathrm{Re}\,\rho^a(v^a, Z_I, \bar{Z}_I) + \mathrm{Re}\,g^a(Z_I) + const$$

# Details on the example: the flux choice

$$dH_3 = dF_3 = 0 \quad + \quad \text{quantisation} \quad \Longrightarrow \quad H_3, F_3 \in H^3(M_6, \mathbb{Z}):$$

$$\frac{1}{\ell_s^2} F_3 = a^0 \alpha_0 + a^{ij} \alpha_{ij} + b_{ij} \beta^{ij} + b_0 \beta^0$$

$$\frac{1}{\ell_s^2} H_3 = c^0 \alpha_0 +^{ij} \alpha_{ij} + d_{ij} \beta^{ij} + d_0 \beta^0$$

$(\alpha_0, \alpha_{ij}, \beta^{ij}, \beta^0)$ basis of $H^3$

Flux choice: $\quad (a^{ij}, b_{ij}, c^{ij}, d_{ij}) = (a, b, c, d)\delta^{ij}$

$$\text{SUSY} \Leftrightarrow \begin{cases} complex\ struc.\ (\lambda^{ij} = \lambda\delta^{ij}, \lambda), \tau \qquad (z^i = x^i + \lambda^{ij} y^j) \\ \hookrightarrow G_3 = F_3 - \tau H_3 \sim dz^1 \wedge dz^2 \wedge d\bar{z}^3 + cycl. \\ J \wedge G_3 = 0 \quad (J = v^a \omega_a \in H^{1,1}) \Rightarrow 3/9\ v^a \text{ fixed} \end{cases}$$

For a *general* (supersymmetric) flux choice: $J \wedge G_3 = 0$ fixes 6/9 Kähler moduli and one ends up with

$$K = -\sum_{a=1}^{3} \log\left(\mathrm{Re}\,\rho^a - \sum_{I \in D3's} \frac{|Z_I^a|^2}{2\mathrm{Im}\,\lambda}\right) + const \qquad \text{[Ferrara,...]}$$