Generalized Metric Formulation of DFT_{WZW}

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based on : [1502.02428]
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SUGRA Action [1]

- Reformulation of SUGRA action for massless string excitations:

$$S_{NS} = \int d^D x \sqrt{-g} e^{-2\phi} \Big[R + 4(\partial\phi)^2 - \frac{1}{12} H_{ijk} H^{ijk} \Big]$$

- Geometrization of SUGRA action in terms of generalized Ricci scalar?
- DFT action:

$$S_{DFT} = \int d^{2D} X \; e^{-2d} \mathcal{R}(\mathcal{H}, d)$$

- Generalized curvature scalar:

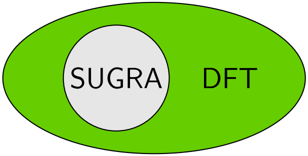
$$\begin{aligned}\mathcal{R} \equiv \; & 4\mathcal{H}^{MN} \partial_M d \, \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4\mathcal{H}^{MN} \partial_M d \, \partial_N d \\ & + 4\partial_M \mathcal{H}^{MN} \partial_N d + \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_N \mathcal{H}^{KL} \partial_L \mathcal{H}_{MK}\end{aligned}$$

DFT Action [2]

- Doubled coordinates and derivative $X^M = \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$ resp. $\partial_M = \begin{pmatrix} \tilde{\partial}^i \\ \partial_i \end{pmatrix}$, along with generalized metric

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}B_{kj} \\ B_{ik}g^{kj} & g_{ij} - B_{ik}g^{kj}B_{lj} \end{pmatrix} \in O(D,D) \to \text{T-Duality}$$

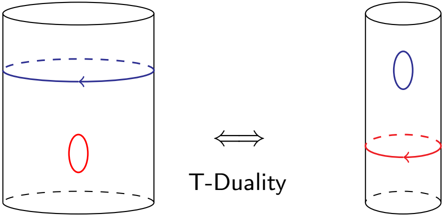
- Additionally, identify $e^{-2d} = e^{-2\phi}\sqrt{-g}$ (dilaton density)
- How to retrieve SUGRA action from DFT?



$$S_{DFT} \xrightarrow{\tilde{\partial}^i = 0} S_{NS}$$

T-Duality

- Closed strings can wrap non-contractable cycles around compact dimensions:



momenta p_i $\Longleftrightarrow$ winding modes $\tilde{p}^i$

$\updownarrow$ T-Duality $\updownarrow$

coordinates x^i $\Longleftrightarrow$ winding coordinates $\tilde{x}_i$

- Connects different background topologies
- Manifest symmetry of DFT action

Generalized Diffeomorphisms [2]

- Generalized metric:

$$\hat{\mathcal{L}}_\xi \mathcal{H}^{MN} = \xi^P \partial_P \mathcal{H}^{MN} + \left(\partial^M \xi_P - \partial_P \xi^M\right) \mathcal{H}^{PN} + \left(\partial^N \xi_P - \partial_P \xi^N\right) \mathcal{H}^{MP}$$

- Dilaton:

$$\hat{\mathcal{L}}_\xi d = -\frac{1}{2} \partial_M \xi^M + \xi^M \partial_M d \,, \quad \text{and} \quad \hat{\mathcal{L}}_\xi e^{-2d} = \partial_M (\xi^M e^{-2d})$$

- $O(D,D)$ metric:

$$\hat{\mathcal{L}}_\xi \eta^{MN} = 0$$

$\Longrightarrow$ S_{DFT} invariant when strong constraint imposed

DFT gauge algebra [2]

- Closure:

$$\left[\hat{\mathcal{L}}_{\xi_1}, \hat{\mathcal{L}}_{\xi_2}\right] = \hat{\mathcal{L}}_{[\xi_1,\xi_2]_C} \text{ modulo } \underline{\text{strong constraint}}$$

with C-bracket:

$$\left[\xi_1, \xi_2\right]_C^M = \xi_1^N \partial_N \xi_2^M - \xi_2^N \partial_N \xi_1^M - \frac{1}{2}\xi_{1N}\partial^M \xi_2^N + \frac{1}{2}\xi_{2N}\partial^M \xi_1^N$$

- Strong constraint:

$$\partial^M \partial_M (A \cdot B) = 0 \quad \forall \text{ fields } A\,, B$$

e.g. solved by $\begin{cases} \partial_i & \text{if } M = i \\ 0 & \text{else} \end{cases}$

DFT on Group Manifolds [3]

- Use group manifold instead of torus to derive DFT
- Representation for semisimple Lie algebras

$$D_a = e_a{}^i \partial_i \,, \quad \text{and commutation relation} \quad [D_a, D_b] = F_{ab}{}^c D_c$$

$\Rightarrow$ same goes for the anti-chiral flat derivative $D_{\bar{a}}$ ...
$\Rightarrow$ unimodularity of the Lie algebra allows for integration by parts!

- Perform CSFT calculations to obtain action and gauge transformations up to cubic order ... (lengthy formulas) (in terms of $\epsilon^{a\bar{b}}$, $\tilde{d}$, D_a, $D_{\bar{a}}$, F_{abc}, $F_{\bar{a}\bar{b}\bar{c}}$)

Generalized Diffeomorphisms [3, 4]

- Introduce doubled flat derivative : $D_A = \begin{pmatrix} D_a \\ D_{\bar{a}} \end{pmatrix}$
- We can rewrite the gauge transformations as:

$$\mathcal{L}_\xi \mathcal{H}^{AB} = \xi^C \nabla_C \mathcal{H}^{AB} + (\nabla^A \xi_C - \nabla_C \xi^A)\mathcal{H}^{CB} + (\nabla^B \xi_C - \nabla_C \xi^B)\mathcal{H}^{AC}$$

with

$$\nabla_A V^B = D_A V^B + \tfrac{1}{3} F^B{}_{AC} V^C\,, \quad \text{and} \quad F_{AB}{}^C = \begin{cases} F_{ab}{}^c \\ F_{\bar{a}\bar{b}}{}^{\bar{c}} \\ 0 \quad \text{otherwise} \end{cases}$$

$$\delta_\xi \tilde{d} = \mathcal{L}_\xi \tilde{d} = \xi^A D_A \tilde{d} - \tfrac{1}{2} D_A \xi^A\,, \quad \text{while} \quad \nabla_A d = D_A \tilde{d}$$

$$\delta_\xi \eta^{AB} = \mathcal{L}_\xi \eta^{AB} = 0\,, \text{ however } \delta_\xi S^{AB} = 0 \text{ but } \mathcal{L}_\xi S^{AB} \neq 0$$

DFT gauge algebra [2]

- Closure:

$$\left[\mathcal{L}_{\xi_1}, \mathcal{L}_{\xi_2}\right] = \mathcal{L}_{[\xi_1,\xi_2]_C} \text{ modulo } \underline{\text{strong constraint}}$$

with C-bracket:

$$\left[\xi_1, \xi_2\right]_C^A = \xi_1^B \partial_B \xi_2^A - \xi_2^B \partial_B \xi_1^A - \frac{1}{2}\xi_{1B}\partial^A \xi_2^B + \frac{1}{2}\xi_{2B}\partial^A \xi_1^B$$

- Strong constraint:

$$\partial^A \partial_A (f \cdot g) = 0 \quad \forall \text{ fields } f,\ g$$

DFT_{WZW} gauge algebra [3]

- Closure:

$$\left[\mathcal{L}_{\xi_1}, \mathcal{L}_{\xi_2}\right] = \mathcal{L}_{[\xi_1,\xi_2]_C} \text{ modulo } \underline{\text{strong + closure constraint}}$$

with C-bracket:

$$\left[\xi_1, \xi_2\right]_C^A = \xi_1^B \nabla_B \xi_2^A - \xi_2^B \nabla_B \xi_1^A - \frac{1}{2}\xi_{1B}\nabla^A \xi_2^B + \frac{1}{2}\xi_{2B}\nabla^A \xi_1^B$$

- Strong constraint:

$$D^A D_A (f \cdot g) = 0 \quad \forall \text{ fluctuations } f,\, g$$

- Closure Constraint:

$$F_{E[AB}F^E{}_{C]D} = 0 \quad \text{(background fields)}$$

DFT action [2]

- Rewrite action?

$$S_{DFT} = \int d^{2D}X \; e^{-2d}\mathcal{R}(\mathcal{H}, d)$$

- Generalized curvature scalar:

$$\mathcal{R} \equiv \; 4\mathcal{H}^{AB}\partial_A d\,\partial_B d - \partial_A\partial_B\mathcal{H}^{AB} - 4\mathcal{H}^{AB}\partial_A d\,\partial_B d + 4\partial_A\mathcal{H}^{AB}\partial_B d + \frac{1}{8}\mathcal{H}^{AB}\partial_A\mathcal{H}^{CD}\partial_B\mathcal{H}_{CD} - \frac{1}{2}\mathcal{H}^{AB}\partial_B\mathcal{H}^{CD}\partial_D\mathcal{H}_{AC}$$

?

DFT_{WZW} action [4]

- Rewrite action?

$$S_{DFT_{\text{WZW}}} = \int d^{2D}X \; e^{-2d}\mathcal{R}(\mathcal{H}, d)$$

- Generalized curvature scalar:

$$\mathcal{R} \equiv 4\mathcal{H}^{AB}\nabla_A d \,\nabla_B d - \nabla_A\nabla_B\mathcal{H}^{AB} - 4\mathcal{H}^{AB}\nabla_A d \,\nabla_B d + 4\nabla_A\mathcal{H}^{AB}\nabla_B d + \frac{1}{8}\mathcal{H}^{AB}\nabla_A\mathcal{H}^{CD}\nabla_B\mathcal{H}_{CD} - \frac{1}{2}\mathcal{H}^{AB}\nabla_B\mathcal{H}^{CD}\nabla_D\mathcal{H}_{AC}$$

?

DFT$_{\mathrm{WZW}}$ action [4]

- Rewrite action?

$$S_{DFT_{\mathrm{WZW}}} = \int d^{2D}X \, e^{-2d} \mathcal{R}(\mathcal{H}, d)$$

- Generalized curvature scalar:

$$\begin{aligned}\mathcal{R} \equiv \; & 4\mathcal{H}^{AB}\nabla_A d \, \nabla_B d - \nabla_A \nabla_B \mathcal{H}^{AB} - 4\mathcal{H}^{AB}\nabla_A d \, \nabla_B d \\ & + 4\nabla_A \mathcal{H}^{AB} \nabla_B d + \frac{1}{8}\mathcal{H}^{AB}\nabla_A \mathcal{H}^{CD}\nabla_B \mathcal{H}_{CD} - \frac{1}{2}\mathcal{H}^{AB}\nabla_B \mathcal{H}^{CD}\nabla_D \mathcal{H}_{AC} \\ & + \frac{1}{6} F_{ACE} F_{BDF} \mathcal{H}^{AB} S^{CD} S^{EF}\end{aligned}$$

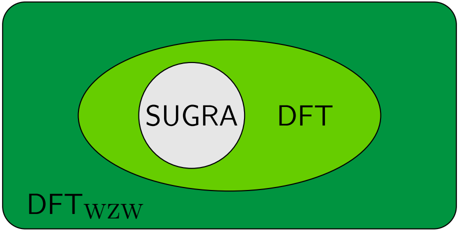
$\Longrightarrow$ invariant under generalized diffeomorphisms when s.c. + c.c. imposed

$\Longrightarrow$ additional $2D$-diffeomorphism invariance

Transition to toroidal DFT? [4]

- Relationship between both DFT formulations?
- Extended strong constraint:

$$D^A D_A (f \cdot b) = 0 \quad \forall \text{ fluctuations } f \text{ , background fields } b$$
$$\Longrightarrow \mathcal{L}_\xi V^M = \hat{\mathcal{L}}_{DFT,\xi} V^M$$
$$\Longrightarrow S_{DFT_{\mathrm{WZW}}} = S_{DFT}$$



- Found generalized metric formulation of $\mathrm{DFT}_{\mathrm{WZW}}$:
  $\Rightarrow$ theory invariant under generalized and $2D$-diffeomorphisms
- $\mathrm{DFT}_{\mathrm{WZW}}$ 'generalizes' original DFT description
- Truly non-geometric backgrounds with new physical information?
  $\Rightarrow$ Well-defined?
- Extension of $\mathrm{DFT}_{\mathrm{WZW}}$ to arbitrary background geometries possible?
- Tool to analyze non-associativity, non-commutativity of non-geometric backgrounds?

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