Mean-Field Solutions as Many-Body States in Quantum Field Theory

Artem Averin

advisor Gia Dvali, LMU Munich

March 16, 2015

I Motivation

mean-field approach: the system is described through a classical field

usual intuition: if the system is big, the mean-field approach is a good approximation as quantum effects are negligible

examples: electric field of a capacitor (field strength $F_{\mu\nu}$), gravitational field of planets, black holes (metric $g_{\mu\nu}$), ...

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for black holes this intuition is wrong, mean-field description fails

ignorance of this fact leads exactly to the known paradoxes of black hole physics

macro-quantum behavior caused by quantum criticality

quantum criticality and macro-quantum behavior is already present in theories of non-relativistic, self-interacting Bose fields

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$$H = \int dx \Psi^{\dagger}(-\Delta)\Psi - \frac{g}{2}\int dx \Psi^{\dagger}\Psi^{\dagger}\Psi\Psi$$

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weak-coupling regime $|\gamma| < \frac{1}{2}$: ground-state is a homogeneous condensate

strong-coupling regime $|\gamma| > \frac{1}{2}$: ground-state is a solitonic condensate

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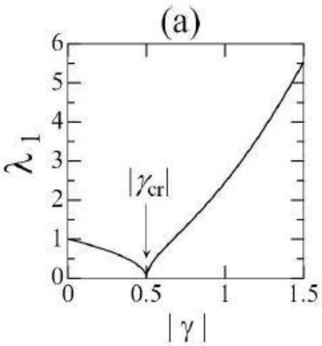

quantum critical point $|\gamma| = \frac{1}{2}$: Quantum Phase Transition from homogeneous to solitonic ground-state

appearance of light excitations

breakdown of mean-field description

III Idea

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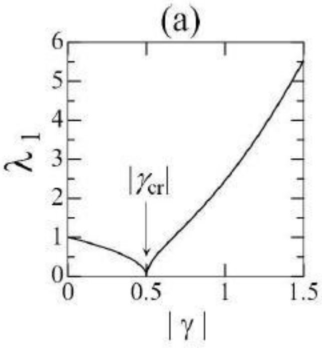

Think of the homogeneous/solitonic condensate as being composed of homogeneous/solitonic corpuscles.

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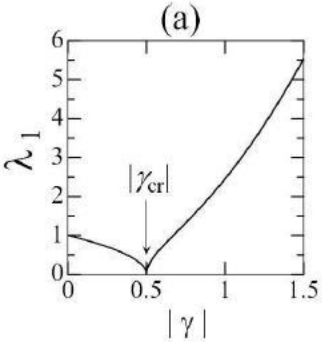

Think of the homogeneous/solitonic condensate as being composed of homogeneous/solitonic corpuscles.

In the weak-coupling regime the homogeneous corpuscles are weakly-coupled whereas their coupling is strong in the strong-coupling regime.

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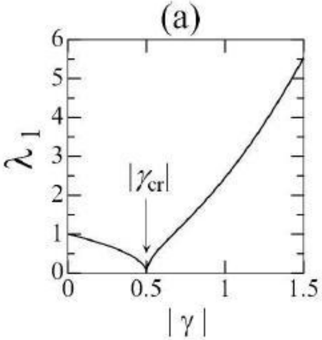

Solitonic corpuscles are weakly-coupled in the strong-coupling regime, while their coupling is strong in the weak-coupling regime.

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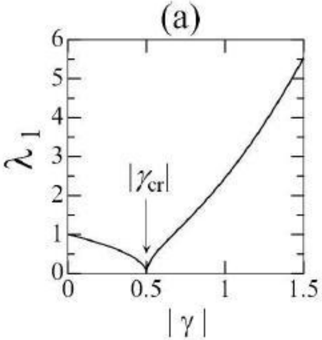

Understand the critical point and the phenomenon of macro-quantumness as the non-existence of weakly-coupled degrees of freedom.

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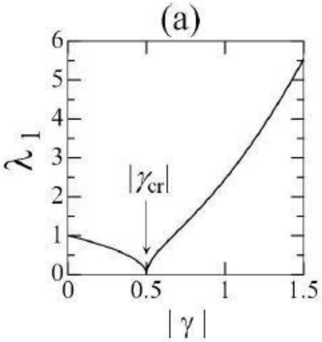

Find a proper framework to formulate and check these proposals!

IV Working out the idea

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$$\Psi = \Phi + \varphi$$

interaction-strength of corpuscles condensed in Φ = coupling of Φ to quantum fluctuations

literature in condensed matter physics: conditions for emergence of strongly-coupled corpuscles

check proposals for homogeneous and solitonic mean-fields for appearance of strongly-coupled corpuscles

verify conditions

homogeneous mean-fields:

strongly-coupled in strong-coupling regime $|\gamma| > \frac{1}{2}$

strong-coupling conditions not fulfilled

solitonic mean-fields:

no stationary solitonic mean-field solutions in the weak-coupling regime $|\gamma| < \frac{1}{2}$

Questions?

Thank you!