Precision cosmology (dark matter and neutrino mass limits)

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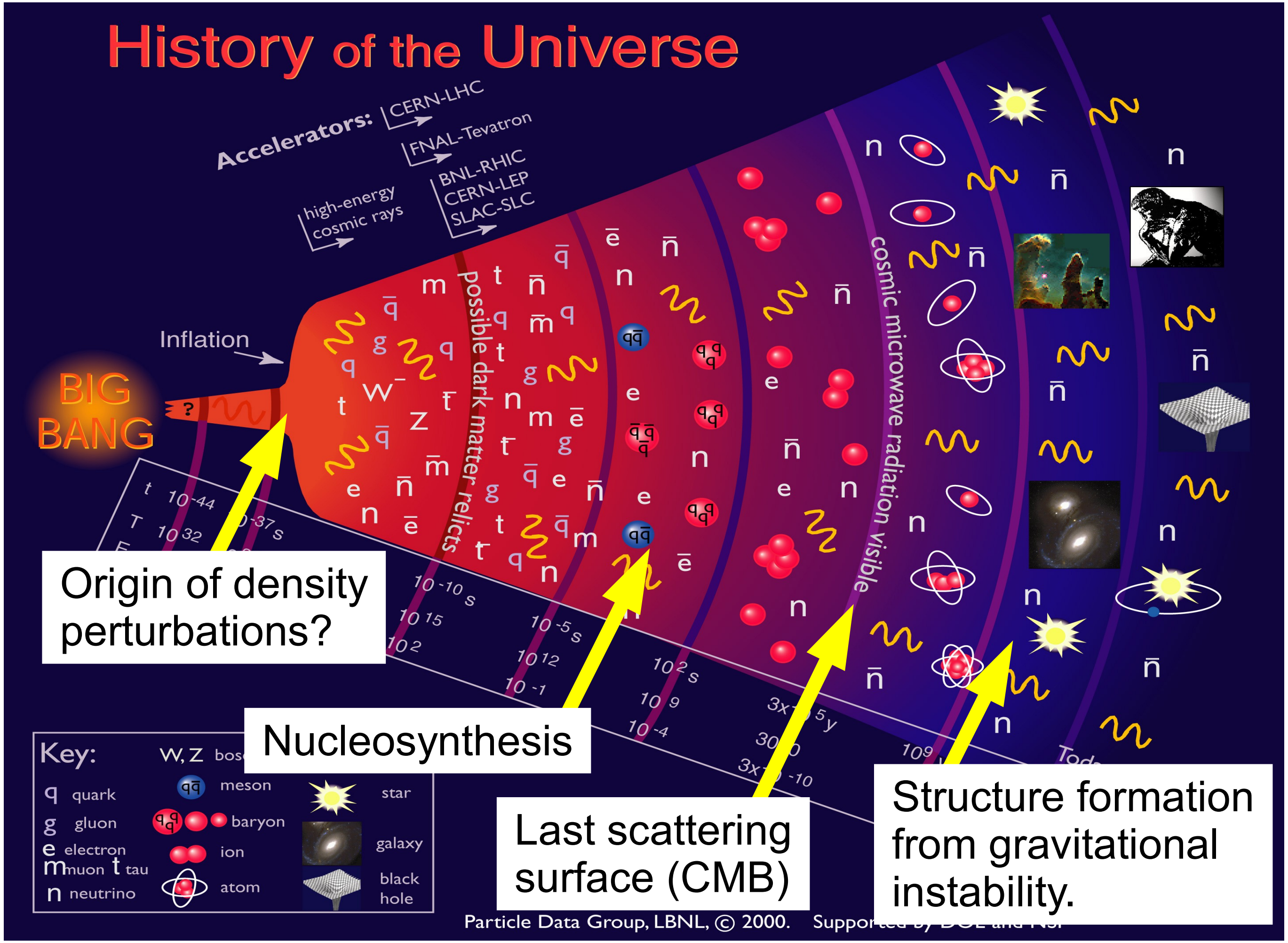
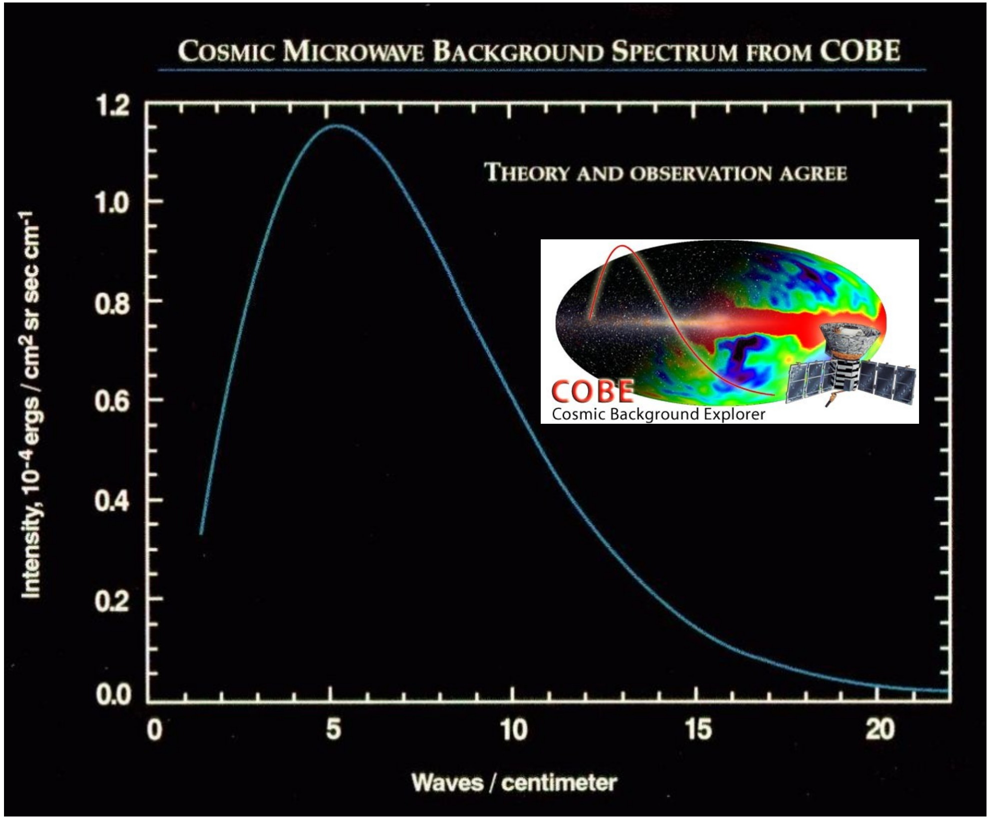


What are the components and how much? What is the geometry?
The initial conditions? The eventual fate? ... → Precision cosmology

Tool 1: Cosmic microwave background...



Mather et al., 1994

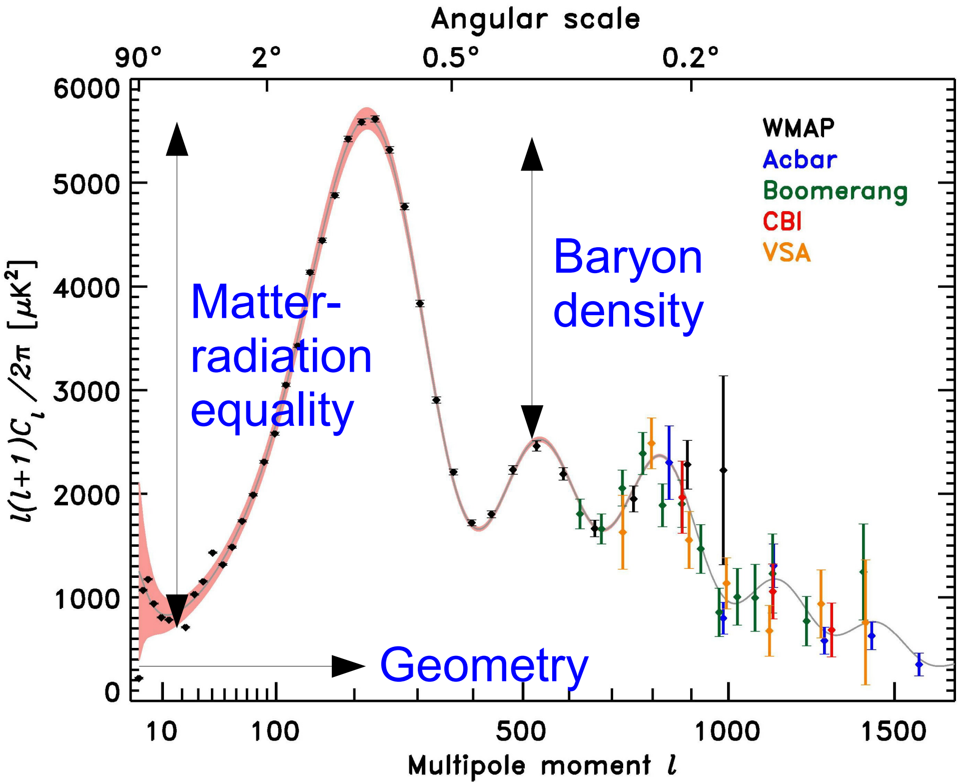
- FIRAS on COBE measured **Planck spectrum**, with temperature:

$$T_{\mathrm{CMB}} = 2.725 \pm 0.001\,\mathrm{K}$$

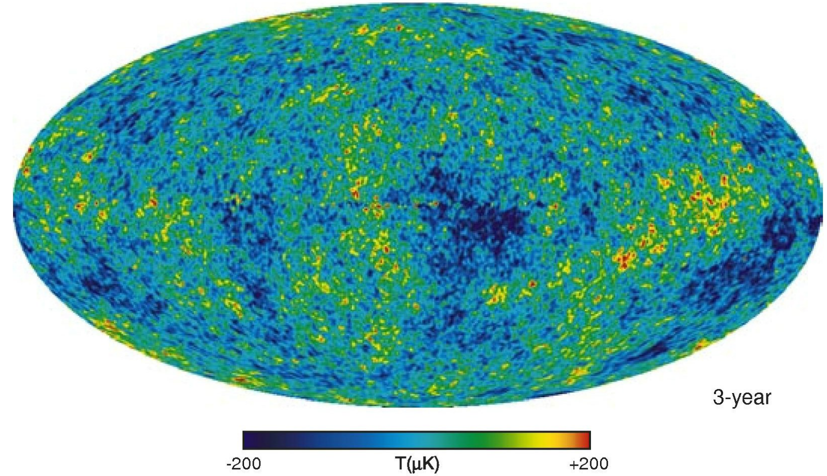
- T_{CMB} fixes:
  - Photon energy density.
  - Relic neutrino number density per flavour:

$$n_\nu = 112\,\mathrm{cm}^{-3}$$

... and its anisotropies...

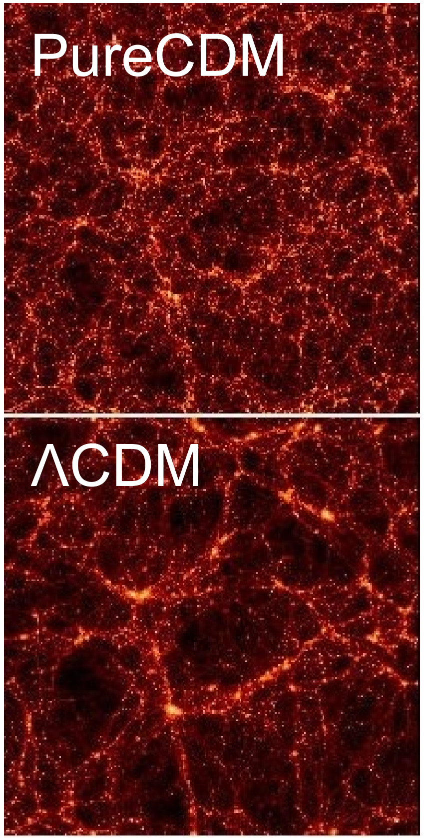


WMAP, 2006



- Temperature fluctuations from **acoustic oscillations** of the photon-baryon fluid frozen on the last scattering surface.

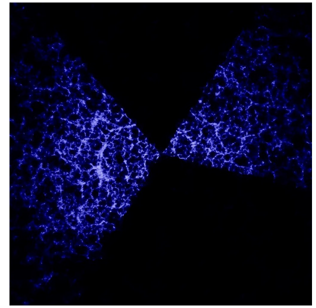
Tool 2: Large-scale structure...



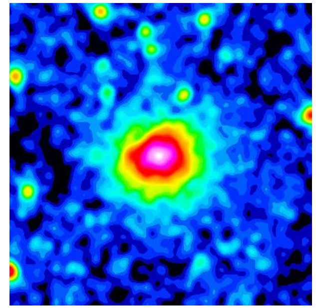
Matter distribution
(luminous and dark)

Virgo collaboration, 1996

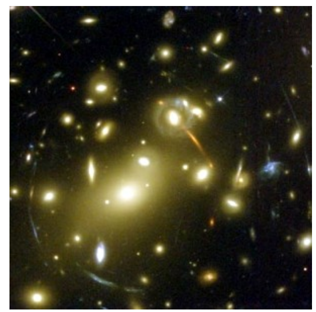
Probed by



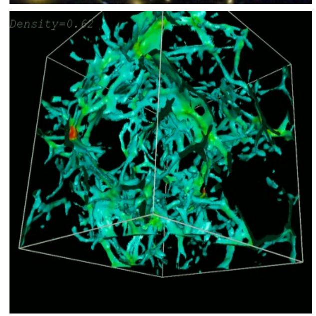
Galaxy clustering



Cluster abundance



Gravitational lensing

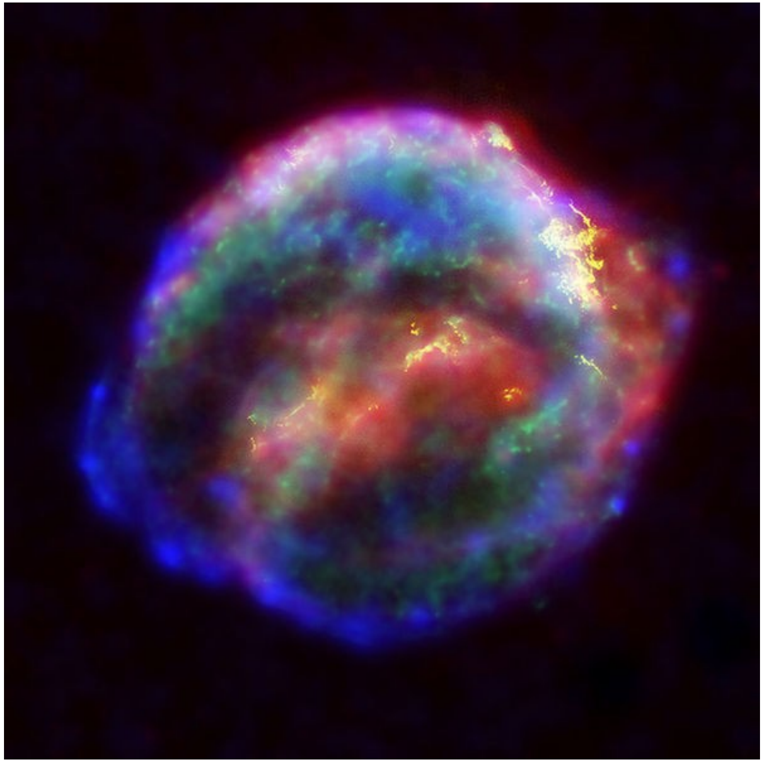


Intergalactic hydrogen clumps; Lyman-α

$300\ h^{-1}$ Mpc

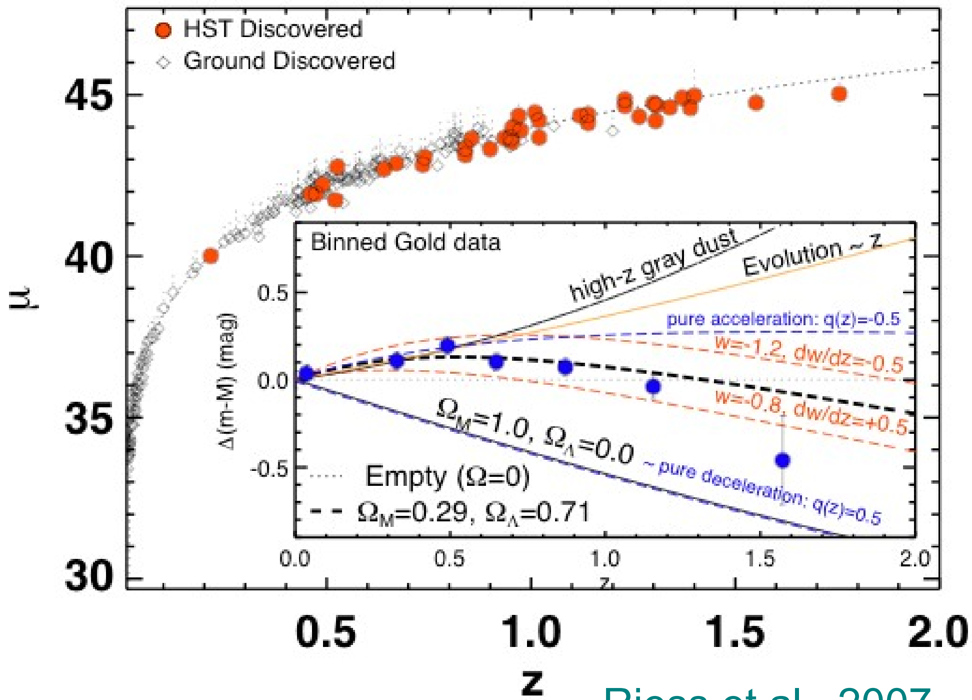
$1\ h^{-1}$ Mpc

Tool 3: Standard candles...



Type Ia supernova (SNIa).

- Hubble diagram of **SNIa** provided the first evidence for a negative pressure fluid, the "**dark energy**".



Riess et al., 2007

The concordance model...

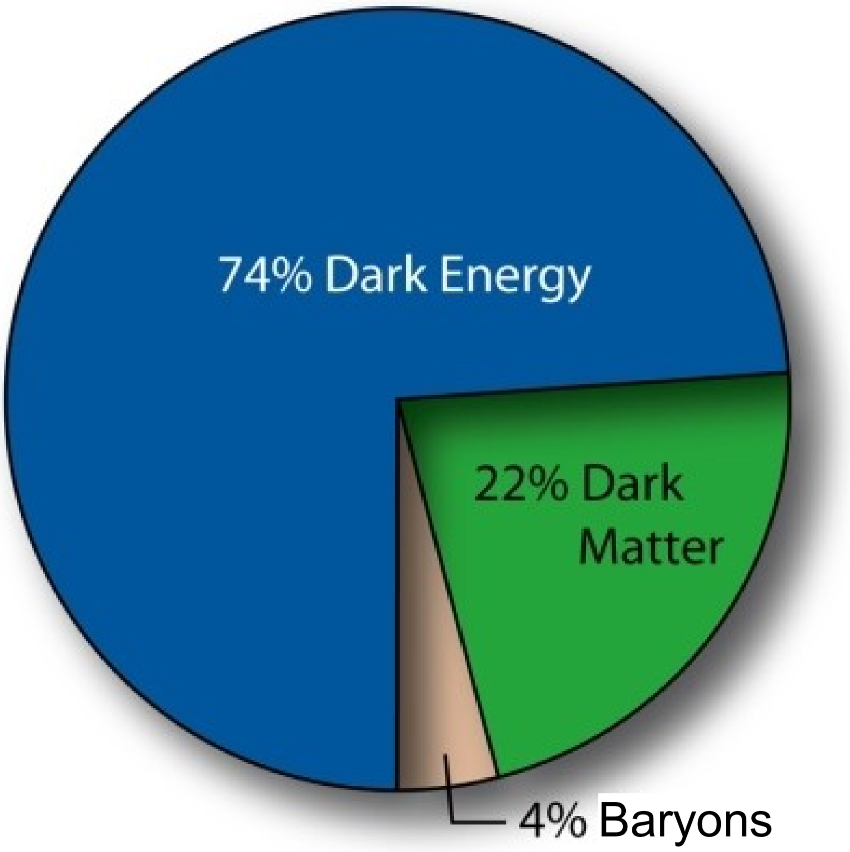
- The simplest model consistent with all **present data**:

→ Flat geometry.

→ 74% Negative pressure fluid.

→ Dark matter is cold.

→ Initial conditions consistent with predictions of inflation.



- Six-parameter vanilla model:

Optical depth to reionisation, CMB only

$$P_{\text{vanilla}} = (\Omega_c h^2, \Omega_b h^2, \Omega_\Lambda, n_s, A_s, \tau)$$

Energy content

Initial conditions

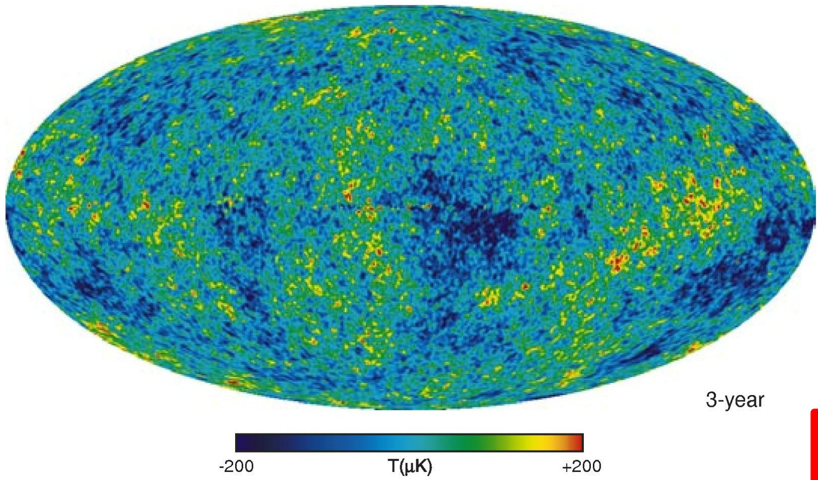
Flat geometry

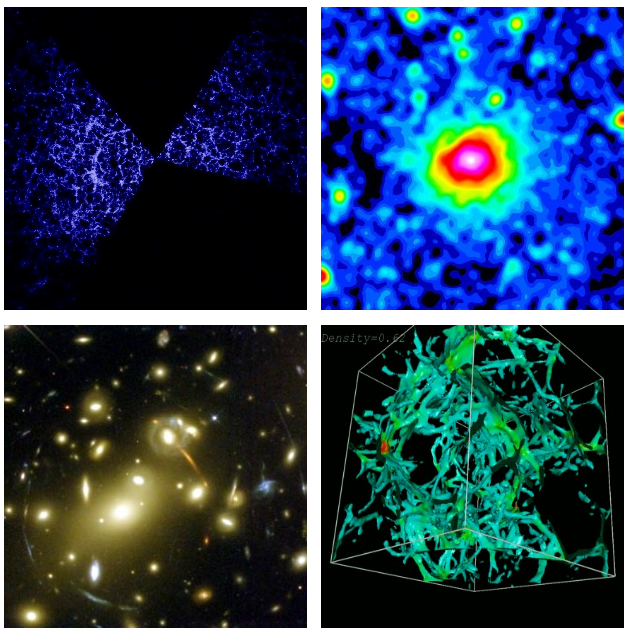
$$\sum_i \Omega_i = 1$$

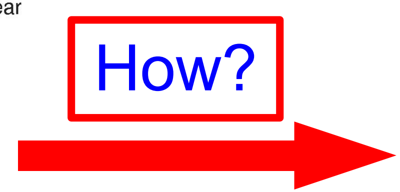
Table 5: ΛCDM Model: Joint Likelihoods These values are calculated using the $N_{side} = 8$ likelihood code with $A_{PS} = 0.017$

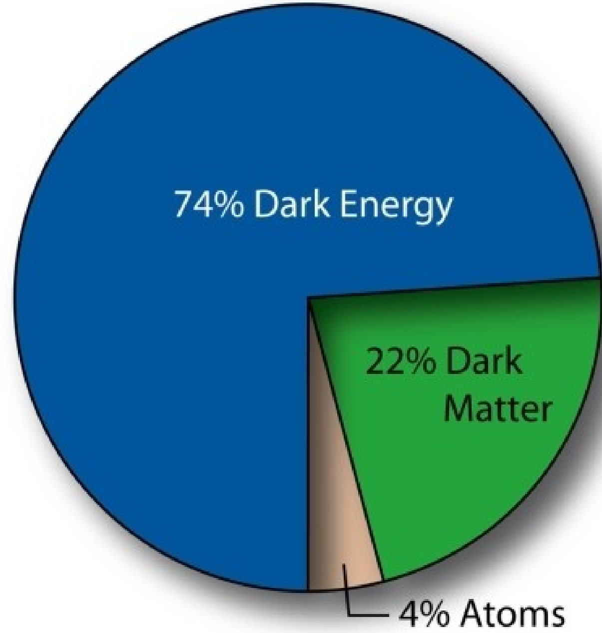
	WMAP Only	WMAP +CBI+VSA	WMAP+ACBAR +BOOMERanG	WMAP + 2dFGRS
$100\Omega_b h^2$	$2.230^{+0.075}_{-0.073}$	2.208 ± 0.071	2.232 ± 0.074	$2.223^{+0.069}_{-0.068}$
$\Omega_m h^2$	$0.1265^{+0.0081}_{-0.0080}$	$0.1233^{+0.0075}_{-0.0074}$	0.1260 ± 0.0081	0.1261 ± 0.0050
h	0.735 ± 0.032	0.742 ± 0.031	$0.739^{+0.033}_{-0.032}$	$0.733^{+0.020}_{-0.021}$
τ	$0.088^{+0.029}_{-0.030}$	0.087 ± 0.029	$0.088^{+0.031}_{-0.032}$	0.083 ± 0.028
n_s	0.951 ± 0.016	0.947 ± 0.015	0.951 ± 0.016	0.948 ± 0.015
σ_8	0.742 ± 0.051	$0.721^{+0.047}_{-0.046}$	$0.739^{+0.050}_{-0.051}$	0.737 ± 0.036
Ω_m	0.237 ± 0.034	0.226 ± 0.031	$0.233^{+0.033}_{-0.034}$	0.236 ± 0.020

Spergel et al. (WMAP), 2006









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Plan...

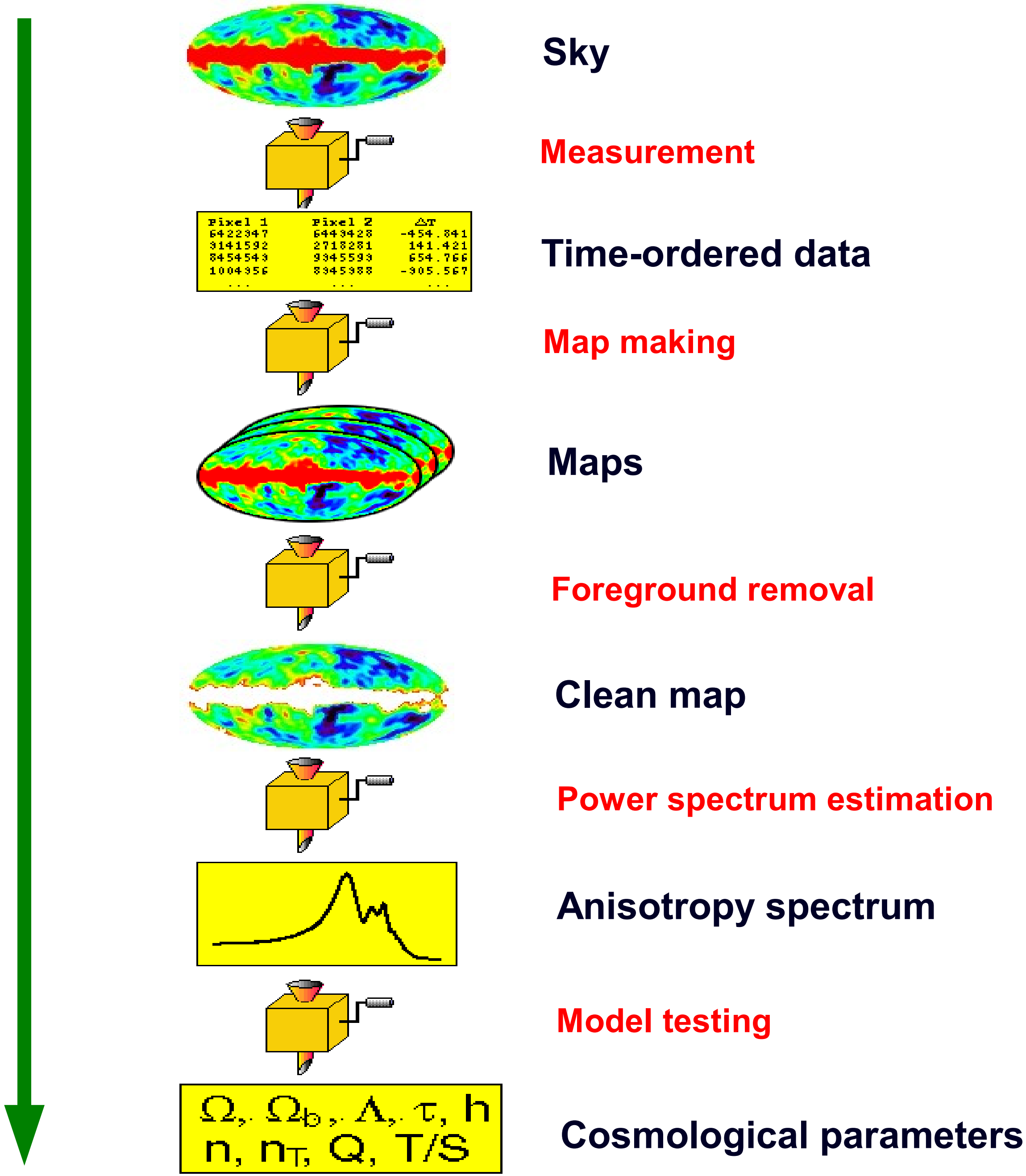
- From observations to parameter constraints.

- Neutrino masses.

- (Future probes.)

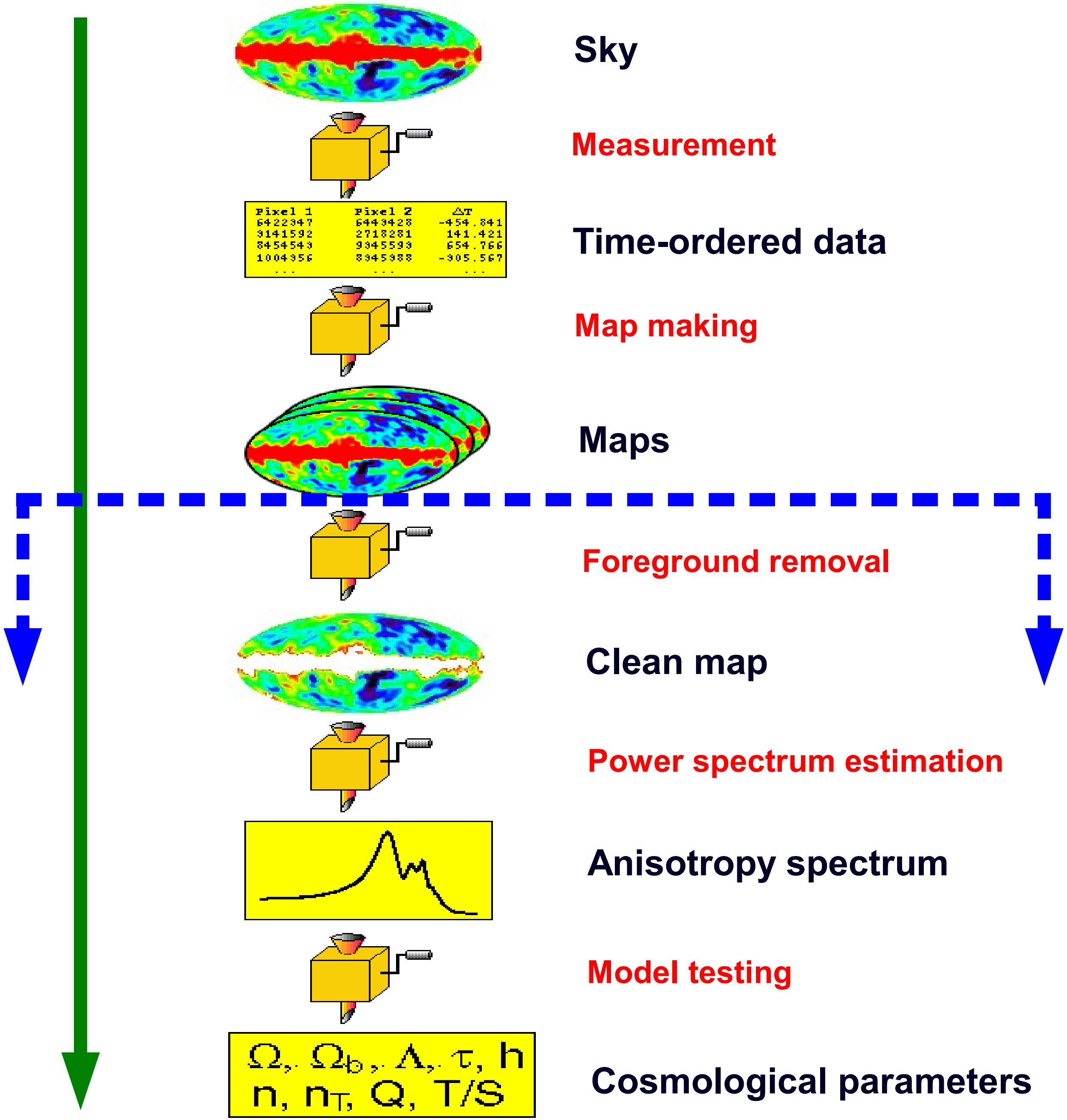
1. From observations to parameter constraints…

The analysis pipeline



http://space.mit.edu/~tegmark

The analysis pipeline



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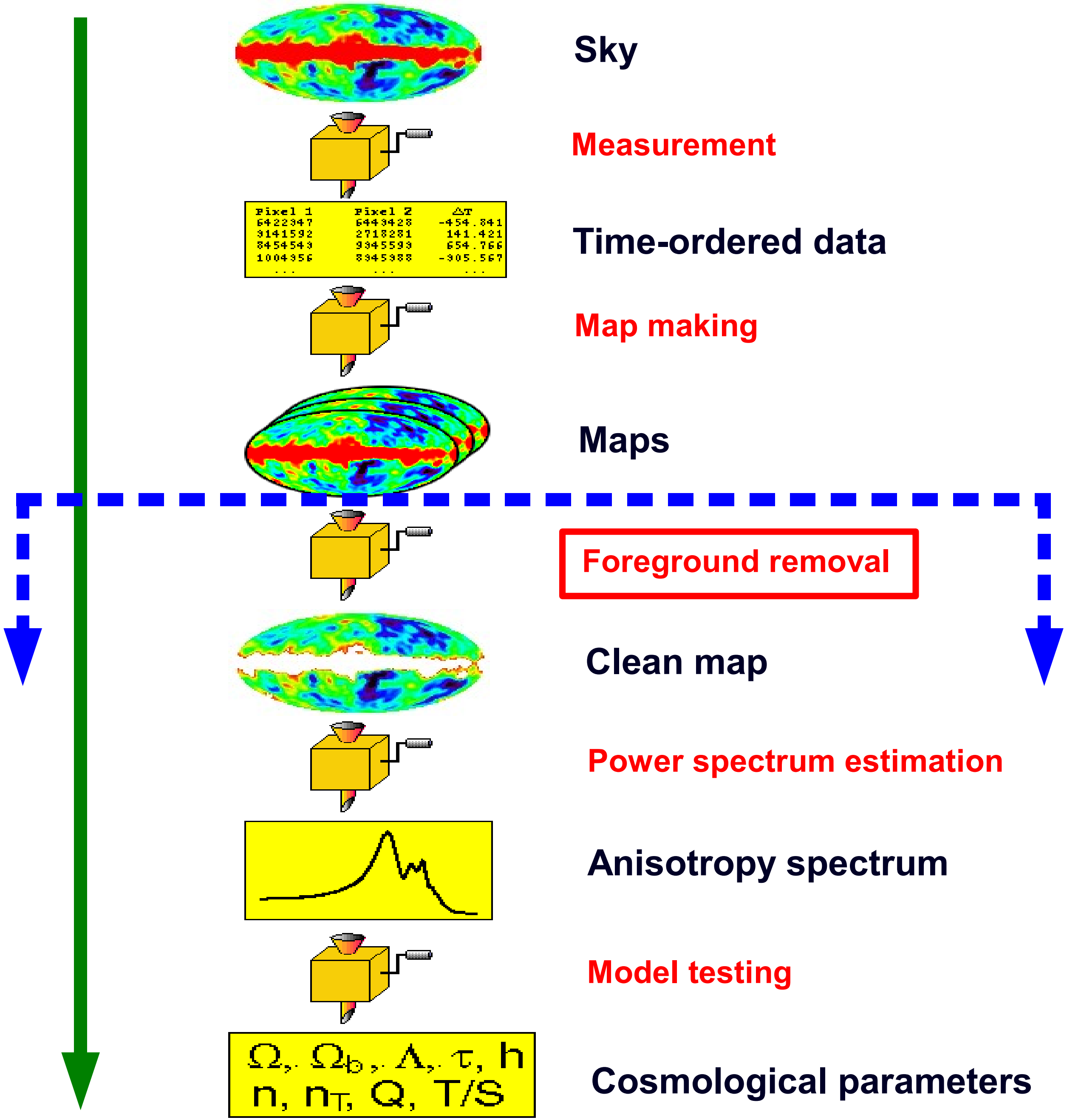

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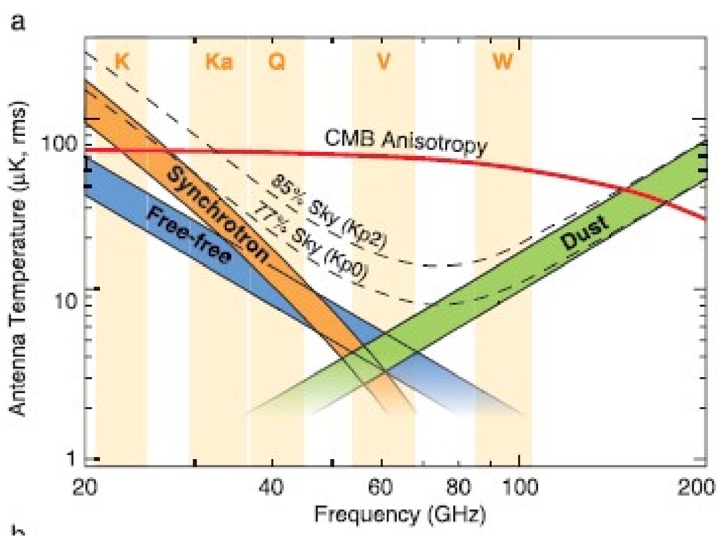
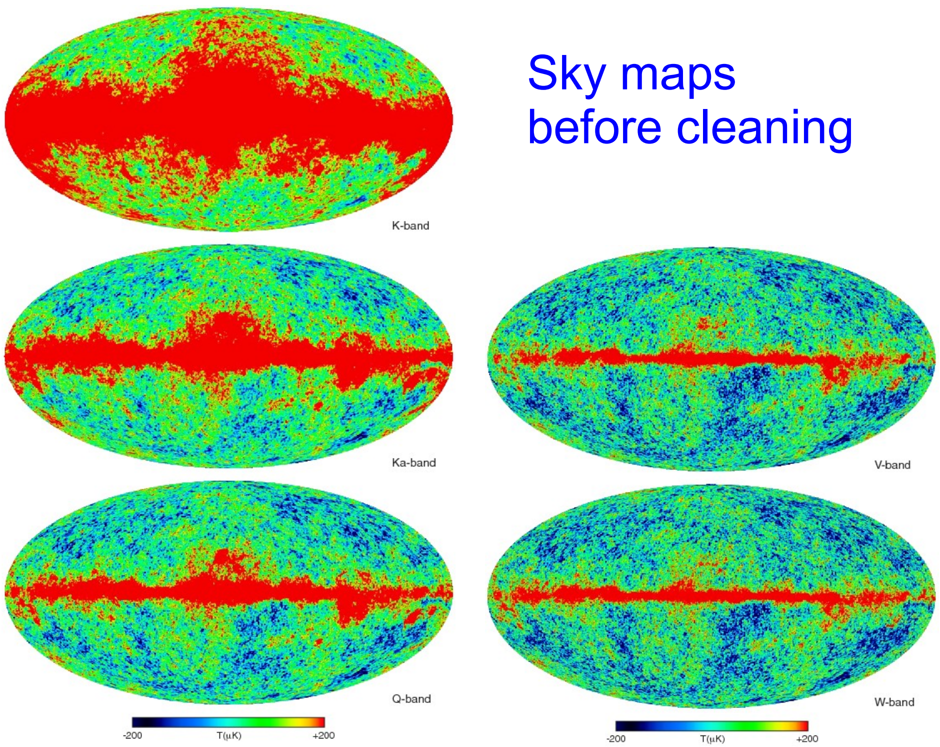
Foreground removal...

- Subtract diffuse galactic emission.
- Remove known point sources.
- **Discard** regions of high contamination.





Sky maps before cleaning

Hinshaw et al. (WMAP3), 2006

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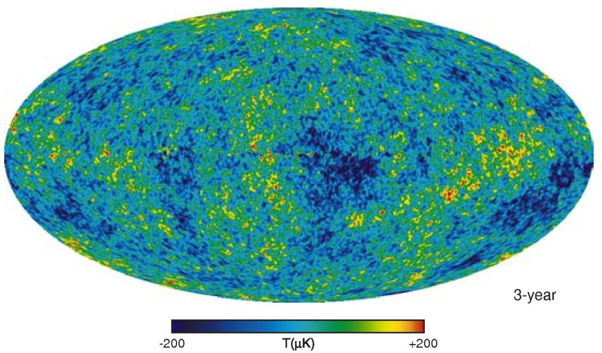
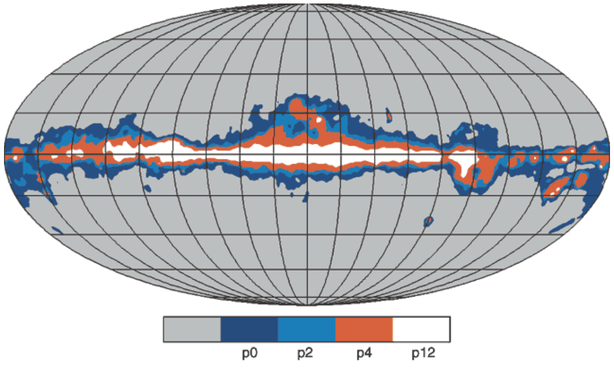
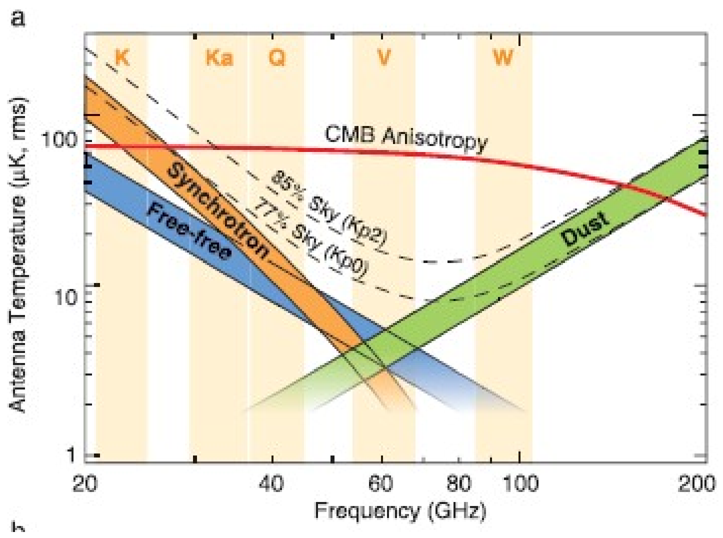


The cleaned, “publicity” map.



Masks: regions not used in cosmological analyses.



Hinshaw et al. (WMAP3), 2006

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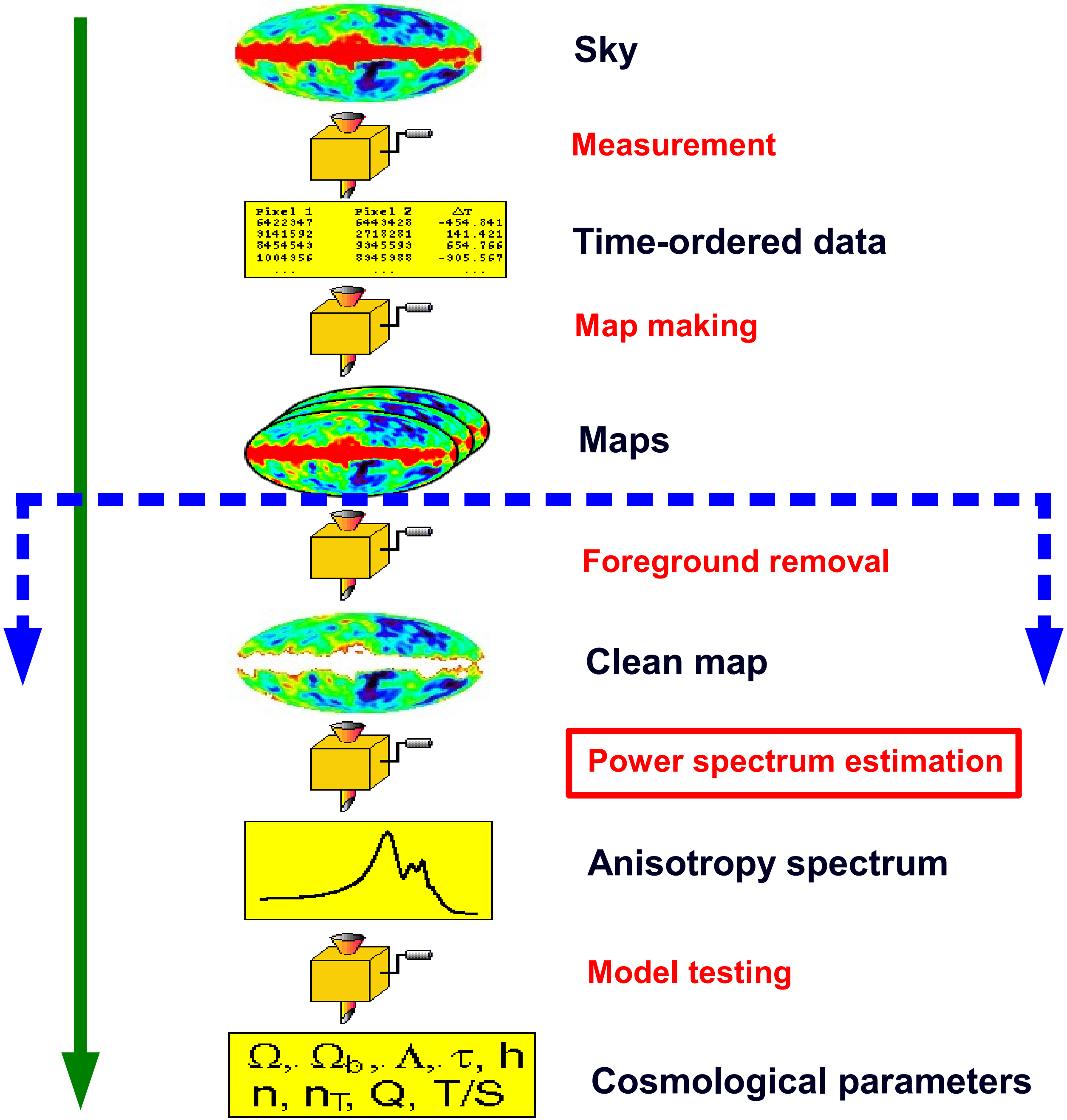

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Power spectrum estimation...

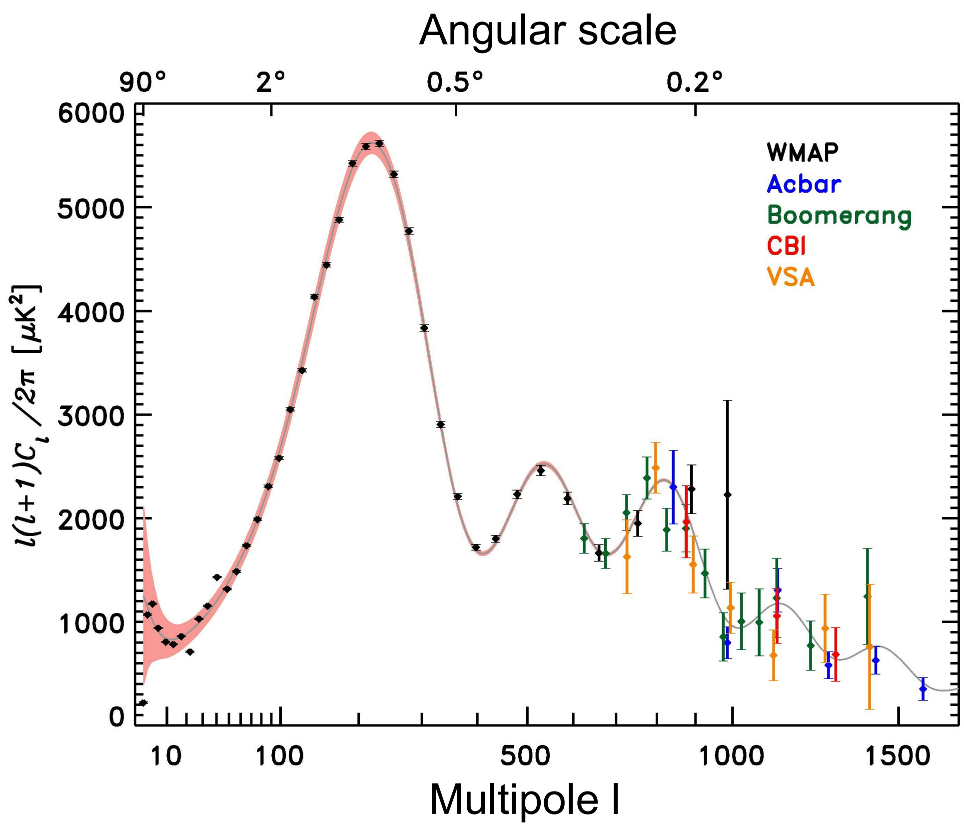
Temperature fluctuation

Spherical harmonics

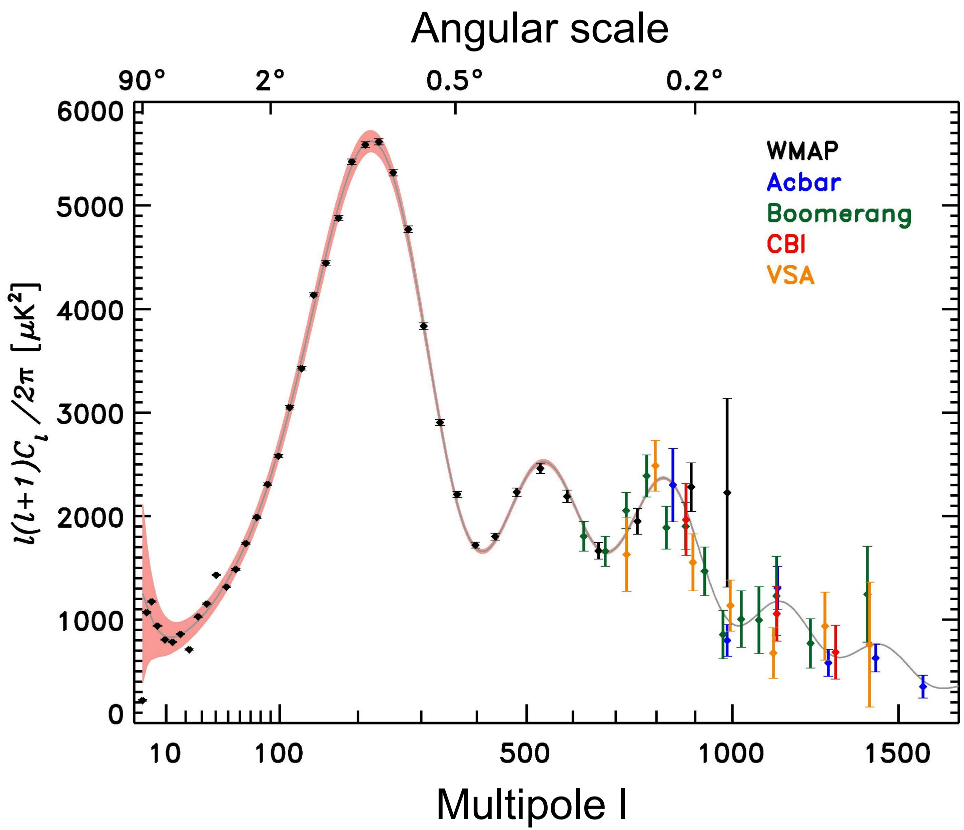
$$\frac{\Delta T}{T}(\hat{n}) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{n})$$

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} a_{lm} a_{lm}^{*}$$

C_l = Power spectrum



WMAP, 2006



$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} a_{lm} a_{lm}^{\star}$$

- Easy to compare with theory:
  - Calcs in Fourier space.
  - C_l completely describe Gaussian random fluctuations.
- Black error bars = instrumental noise.
- Red band = cosmic variance (sampling error):

$$\frac{\Delta C_l}{C_l} = \sqrt{\frac{2}{2l+1}}$$

Power spectrum from theory...

Initial conditions
$P_{ini}(k)$ = Initial perturbation power spectrum

Projection
$j_l(x)$ = spherical Bessel function

$$C_l \propto \int \frac{dk}{k} P_{\mathrm{ini}}(k) \left[\int d\eta \, T_\gamma(k, \eta) \, j_l[k(\eta_0 - \eta)] \right]^2$$

Transfer function, T(k)
Depends on energy content, geometry, interactions, velocity dispersions.

k = Wavenumber
η = Time

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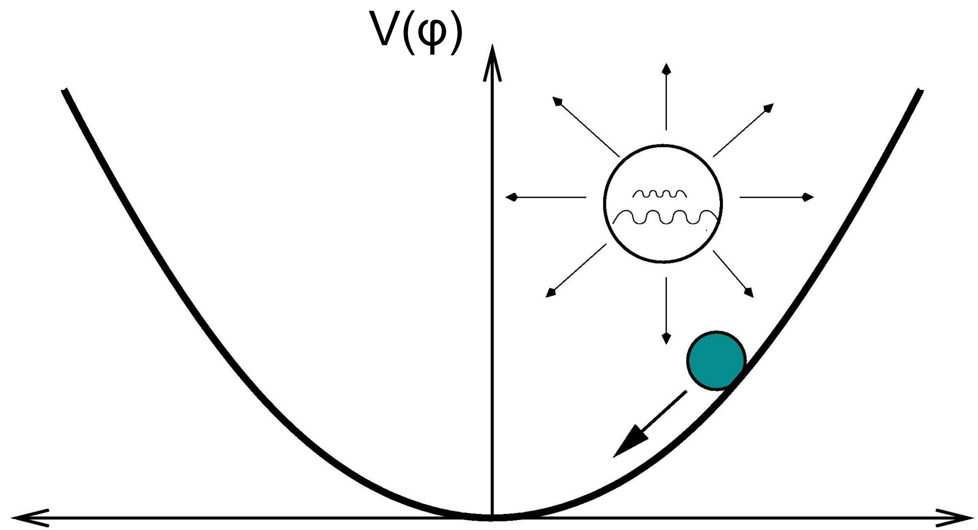
Initial perturbations...

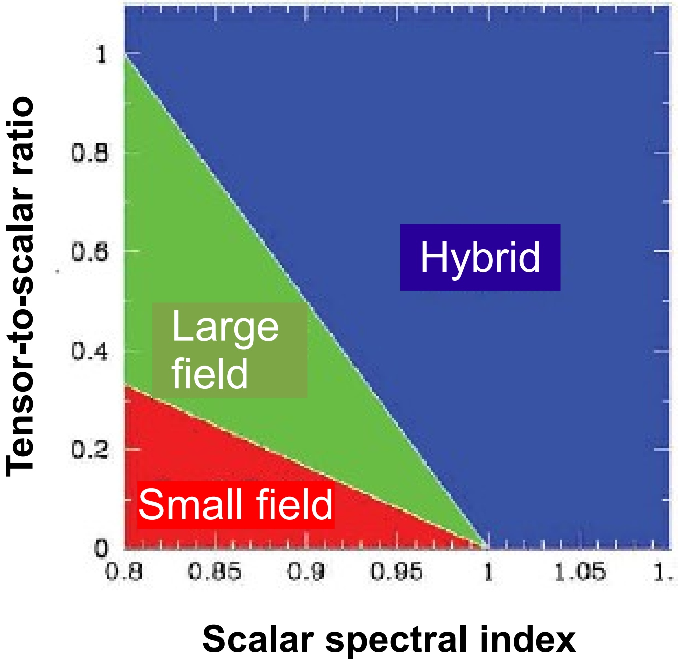
- Leading theory = **Inflation**.
- Scalar field φ driven exponential expansion:

$$a(t) \sim \exp(H\,t)$$

- Quantum fluctuations of φ seed perturbations.



- Inflation generates two types of perturbations on the space-time metric:
  - Scalar ↔ matter density perturbations.
  - Tensor ↔ primordial gravity waves (can affect CMB).



$r = A_T/A_S$.
Depends on V(φ).

Perturbation power spectra

$$P_S(k) = A_s k^{n_s - 1}, \qquad P_T(k) = A_T k^{n_T}$$

n_S, n_T = Spectral indices.
Depend on V(φ).

- Are the perturbations:
  - Adiabatic? $$\frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}=\frac{1}{4}\frac{\delta\rho_\nu}{\rho_\nu}=\frac{1}{3}\frac{\delta\rho_{\mathrm{dm}}}{\rho_{\mathrm{dm}}}=\frac{1}{3}\frac{\delta\rho_b}{\rho_b}$$ Generic prediction of single-field inflation.
  - Isocurvature? $\sum_i \delta\rho_i = 0$ **Ruled out.**
  - Mixed ? $$\frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}\neq\frac{1}{4}\frac{\delta\rho_\nu}{\rho_\nu}\neq\frac{1}{3}\frac{\delta\rho_{\mathrm{dm}}}{\rho_{\mathrm{dm}}}\neq\frac{1}{3}\frac{\delta\rho_b}{\rho_b}, \quad \sum_i \delta\rho_i \neq 0$$ Can be realised in multi-field inflation models, cosmic strings, etc.

No evidence for mixed perturbations, but up to ~10% isocurvature is allowed.

Power spectrum from theory...

Initial conditions
$P_{ini}(k)$ = Initial perturbation power spectrum

Projection
$j_l(x)$ = spherical Bessel function

$$C_l \propto \int \frac{dk}{k} P_{\mathrm{ini}}(k) \left[\int d\eta \, T_\gamma(k,\eta) \, j_l[k(\eta_0 - \eta)] \right]^2$$

Transfer function, T(k)
Depends on energy content, geometry, interactions, velocity dispersions.

k = Wavenumber
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Transfer function, T(k)
Depends on energy content, geometry, interactions, velocity dispersions.

k = Wavenumber
η = Time

Transfer functions...

- Describe evolution of density perturbations in various components:

$$\frac{\delta\rho_i}{\rho_i}(k,t_2)=T_i(k;t_2,t_1)\frac{\delta\rho_i}{\rho_i}(k,t_1)$$

Density perturbations we see today

Density perturbations from inflation

Transfer function for component i

i = photon, neutrino, cdm, baryon

Boltzmann equation to track density pertrubations

$$\frac{Df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = \left(\frac{\partial f}{\partial \tau}\right)_C ,$$

Friedmann equation of an expanding background

$$\begin{aligned} \left(\frac{\dot a}{a}\right)^2 &= \frac{8\pi}{3} G a^2 \bar\rho - \kappa , \\ \frac{d}{d\tau}\left(\frac{\dot a}{a}\right) &= -\frac{4\pi}{3} G a^2 (\bar\rho + 3\bar P) , \end{aligned}$$

General relativity

Perturbed Friedmann-Robertson-Walker metric

$$ds^2 = a^2(\tau)\left\{-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i\right\}$$

1st order perturbed Einstein equations

$$\begin{aligned} k^2\phi + 3\frac{\dot a}{a}\left(\dot\phi + \frac{\dot a}{a}\psi\right) &= 4\pi G a^2 \delta T^0{}_0(\mathrm{Con}) , \\ k^2\left(\dot\phi + \frac{\dot a}{a}\psi\right) &= 4\pi G a^2 (\bar\rho + \bar P)\theta(\mathrm{Con}) , \\ \ddot\phi + \frac{\dot a}{a}(\dot\psi + 2\dot\phi) + \left(2\frac{\ddot a}{a} - \frac{\dot a^2}{a^2}\right)\psi + \frac{k^2}{3}(\phi - \psi) &= \frac{4\pi}{3} G a^2 \delta T^i{}_i(\mathrm{Con}) , \\ k^2(\phi - \psi) &= 12\pi G a^2 (\bar\rho + \bar P)\sigma(\mathrm{Con}) , \end{aligned}$$

Perturbed energy-momentum tensor

$$\begin{aligned} T^0{}_0 &= -(\bar\rho + \delta\rho) , \\ T^0{}_i &= (\bar\rho + \bar P)v_i = -T^i{}_0 , \\ T^i{}_j &= (\bar P + \delta P)\delta^i{}_j + \Sigma^i{}_j , \qquad \Sigma^i{}_i = 0 , \end{aligned}$$

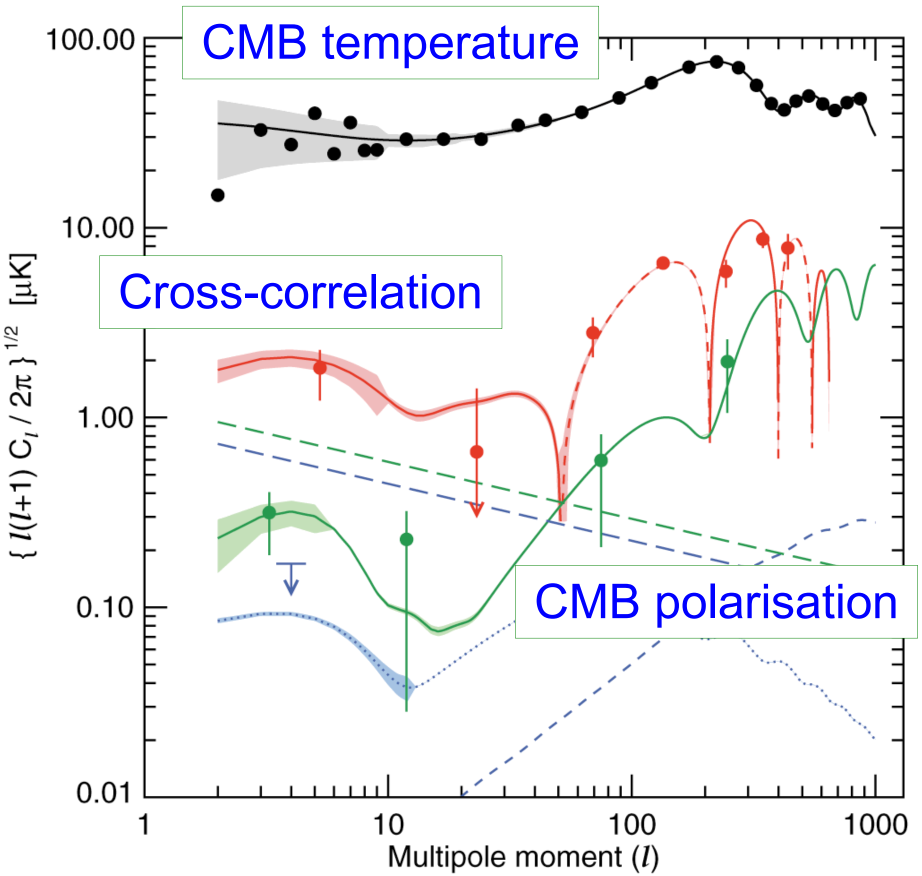
e.g., Ma & Bertschinger, 1995

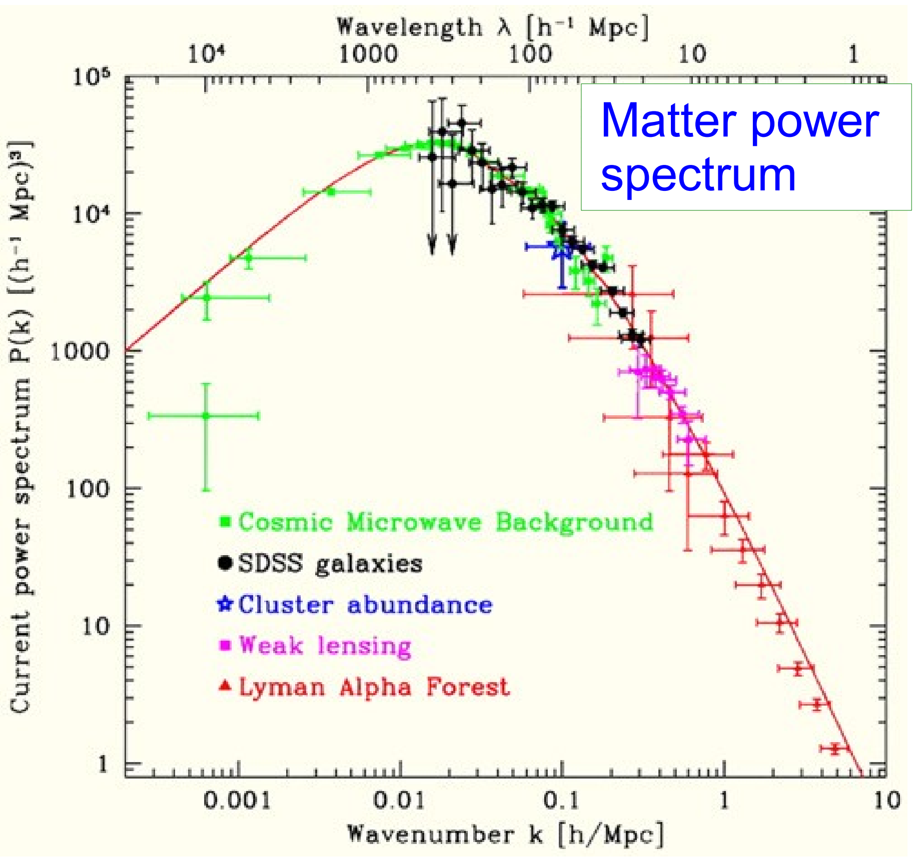
- Evolution of perturbations depends on
  - **Background evolution**.
    - Radiation, matter, or dark energy domination?
  - **Velocity dispersion**.
    - Cold versus hot dark matter.
  - **Interactions**.
    - Baryons versus dark matter.

The observables...

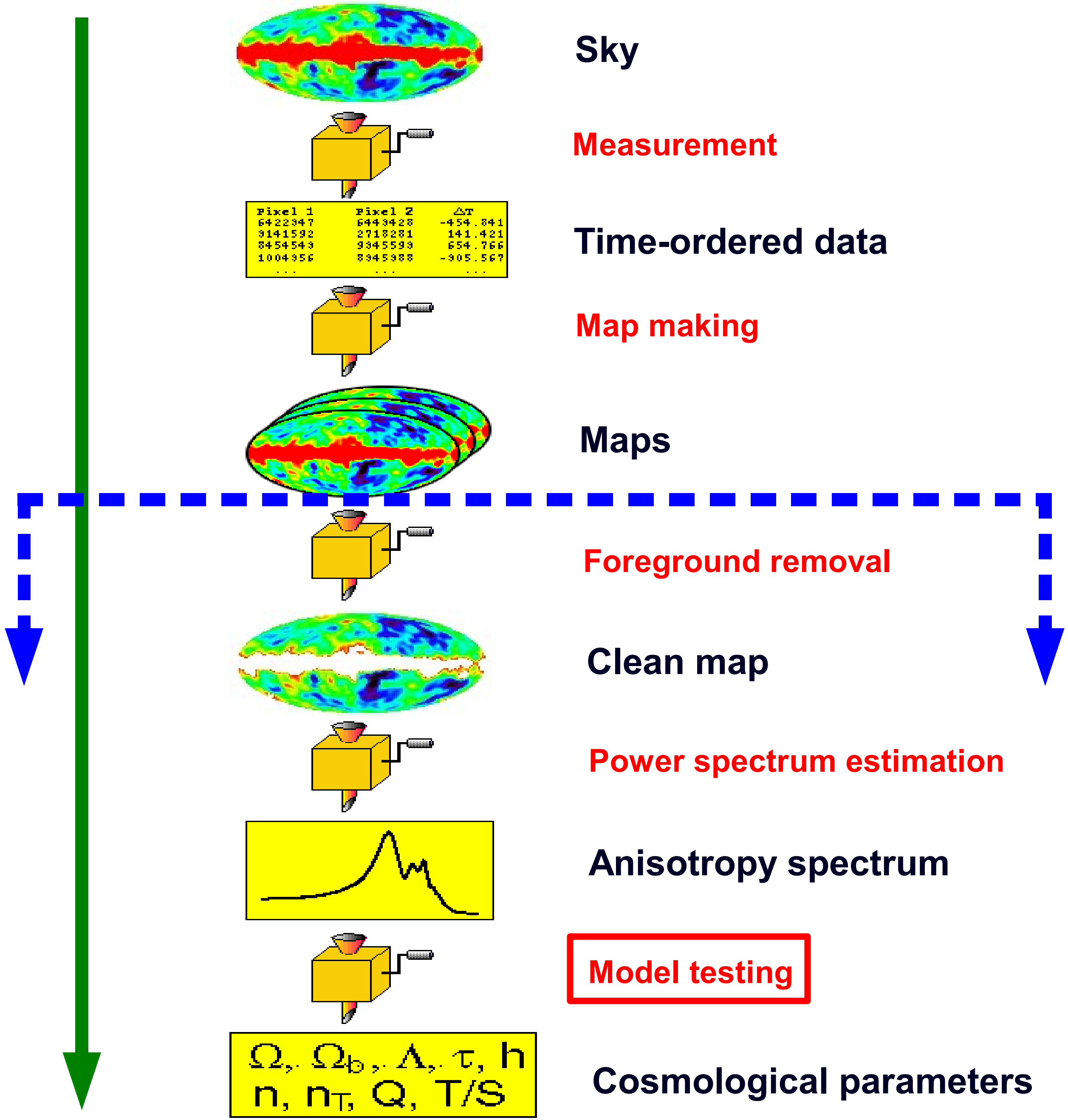
$$C_l \propto \int \frac{dk}{k} P_{\rm ini}(k) \left[\int d\eta\, T_\gamma(k,\eta)\, j_l[k(\eta_0-\eta)] \right]^2$$

$$P_m(k,t_0) = T_m^2(k,t_0)\, P_{\rm ini}(k)$$





The analysis pipeline



http://space.mit.edu/~tegmark

Model testing and parameter inference...

- The simplest model that fits all **present** data:

Dark matter density

Cosmological constant

Optical depth to reionisation, CMB only

$$P_{\text{vanilla}} = (\Omega_c h^2, \Omega_b h^2, \Omega_\Lambda, n_s, A_s, \tau)$$

Baryon density

Perturbation spectral index

Perturbation amplitude

Flat geometry

$$\sum_i \Omega_i = 1$$

$$P_{\text{vanilla}} = (\Omega_c h^2, \Omega_b h^2, \Omega_\Lambda, n_s, A_s, \tau) \qquad \sum_i \Omega_i = 1$$

Table 3: Goodness of Fit, $\Delta\chi^2_{eff} \equiv -2\ln\mathcal{L}$, for WMAP data only relative to a Power-Law ΛCDM model. $\Delta\chi^2_{eff} > 0$ is a worse fit to the data.

	Model	$-\Delta(2\ln\mathcal{L})$	N_{par}
M1	Scale Invariant Fluctuations ($n_s = 1$)	6	5
M2	No Reionization ($\tau = 0$)	7.4	5
M3	No Dark Matter ($\Omega_c = 0, \Omega_\Lambda \neq 0$)	248	6
M4	No Cosmological Constant ($\Omega_c \neq 0, \Omega_\Lambda = 0$)	0	6
M5	**Power Law ΛCDM**	0	6
M6	Quintessence ($w \neq -1$)	0	7
M7	Massive Neutrino ($m_\nu > 0$)	-1	7
M8	Tensor Modes ($r > 0$)	0	7
M9	Running Spectral Index ($dn_s/d\ln k \neq 0$)	-4	7
M10	Non-flat Universe ($\Omega_k \neq 0$)	-2	7
M11	Running Spectral Index & Tensor Modes	-4	8
M12	Sharp cutoff	-1	7
M13	Binned $\Delta^2_{\mathcal{R}}(k)$	-22	20

"Vanilla"

\- = better fit
\+ = worse fit

Spergel et al. (WMAP), 2006

$$P_{\text{vanilla}} = (\Omega_c h^2, \Omega_b h^2, \Omega_\Lambda, n_s, A_s, \tau) \qquad \sum_i \Omega_i = 1$$

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The simplest inflation models predict $n_s < 1$.

"Vanilla"

- = better fit
+ = worse fit

Spergel et al. (WMAP), 2006

$$P_{vanilla} = (\Omega_c h^2, \Omega_b h^2, \Omega_\Lambda, n_s, A_s, \tau) \qquad \sum_i \Omega_i = 1$$

Table 3: Goodness of Fit, $\Delta\chi^2_{eff} \equiv -2\ln\mathcal{L}$, for WMAP data only relative to a Power-Law ΛCDM model. $\Delta\chi^2_{eff} > 0$ is a worse fit to the data.

	Model	$-\Delta(2\ln\mathcal{L})$	N_{par}
M1	Scale Invariant Fluctuations ($n_s = 1$)	6	5
M2	No Reionization ($\tau = 0$)	7.4	5
M3	No Dark Matter ($\Omega_c = 0, \Omega_\Lambda \neq 0$)	248	6
M4	No Cosmological Constant ($\Omega_c \neq 0, \Omega_\Lambda = 0$)	0	6
M5	**Power Law ΛCDM**	0	6
M6	Quintessence ($w \neq -1$)	0	7
M7	Massive Neutrino ($m_\nu > 0$)	-1	7
M8	Tensor Modes ($r > 0$)	0	7
M9	Running Spectral Index ($dn_s/d\ln k \neq 0$)	−4	7
M10	Non-flat Universe ($\Omega_k \neq 0$)	−2	7
M11	Running Spectral Index & Tensor Modes	−4	8
M12	Sharp cutoff	−1	7
M13	Binned $\Delta^2_\mathcal{R}(k)$	−22	20

Astrophysics...

"Vanilla"

- = better fit
+ = worse fit

Spergel et al. (WMAP), 2006

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Absolutely necessary!!!

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CMB only. **Incompatible** with other probes.

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	Massive N		
	Tensor		
	g Spectr		
	Non-flat		
	g Spectra		
M12	Sl		
M13	Bi		

CMB only. **Incompatible** with other probes.

"Vanilla"

Non-flat universe

$$\Omega_c+\Omega_b=1.3$$

$$H_0=30\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

More complicated initial conditions

$$P_1(k)=A_1 k^{n_1-1}, \quad P_2(k)=A_2 k^{n_2-1}$$

$$\Omega_c+\Omega_b=0.88$$

$$\Omega_\nu=0.12$$

$$H_0=46\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Blanchard, Douspis, Rowan-Robinson & Sarkar, 2003

\- = better fit
\+ = worse fit

WMAP), 2006

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	Binned $\Delta^2_{\mathcal{R}}(k)$	-22	20

"Vanilla"

\- = better fit
\+ = worse fit

\+ number of neutrinos, helium fraction, dark energy clustering, isocurvature mode, etc.

Spergel et al. (WMAP), 2006

Uncertainties...

	1σ errors (all data)
• **Baryon density**, $\Omega_b h^2$=0.022	3%
• **Dark matter density**, $\Omega_c h^2$=0.105	3%
• **Dark energy density**, Ω_Λ=0.76	2%
• **Scalar spectral index**, n_s=0.95	2%

Vanilla fit
Spergel et al. , 2006
Tegmark et al., 2006

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Provided that:
- Dark energy is exactly a cosmological constant.
- Neutrino mass is exactly zero.
- The Helium-4 fraction is exactly 0.24.
- Tensor contribution is exactly zero.
- There are exactly 3 thermalised neutrino species.
- Space is exactly flat.
- Initial conditions are exactly adiabatic.
- etc.

Vanilla fit
Spergel et al. , 2006
Tegmark et al., 2006

Uncertainties...

	1σ errors (all data)	
• **Baryon density**, $\Omega_b h^2=0.022$	3%	
• **Dark matter density**, $\Omega_c h^2=0.105$	3%	[10%]
• **Dark energy density**, $\Omega_\Lambda=0.76$	2%	
• **Scalar spectral index**, $n_s=0.95$	2%	[8%]

Provided that:
- ~~Dark energy is exactly a cosmological constant~~.
- ~~Neutrino mass is exactly zero~~.
- The Helium-4 fraction is exactly 0.24.
- ~~Tensor contribution is exactly zero~~.
- There are exactly 3 thermalised neutrino species.
- ~~Space is exactly flat~~.
- Initial conditions are exactly adiabatic.
- etc.

Vanilla fit
Spergel et al. , 2006
Tegmark et al., 2006

Conservative fit
Hamann, Hannestad,
Sloth & Y^3W, 2006

The bottom line...

- It's pretty difficult to get away from the concordance ΛCDM framework, **but**...

- ... some strong physical assumptions are required to push errors down to the percent level.

2. Neutrino masses

Neutrino dark matter...

- Massive neutrinos (m_ν > 1 meV) are **dark matter**:

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$$ ← Energy density

- From lab experiments:

$$\min \sum m_\nu \simeq 0.05\,\mathrm{eV}$$ (Neutrino oscillations)

$$\max \sum m_\nu \simeq 6\,\mathrm{eV}$$ (Tritium β decay)

→ Small but not negligible!

$$\Omega_\nu \sim 0.1\,\% \rightarrow 12\,\%$$

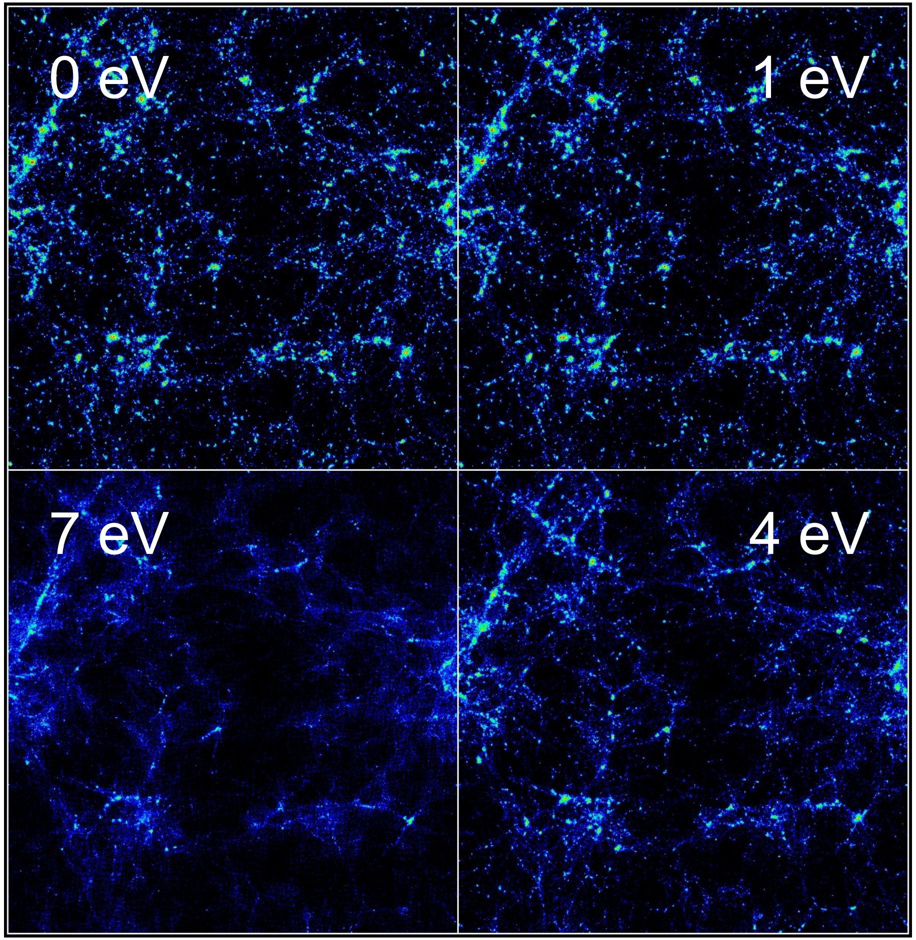
Neutrino DM is hot:

→ **Distinctive imprint** on the large-scale structure

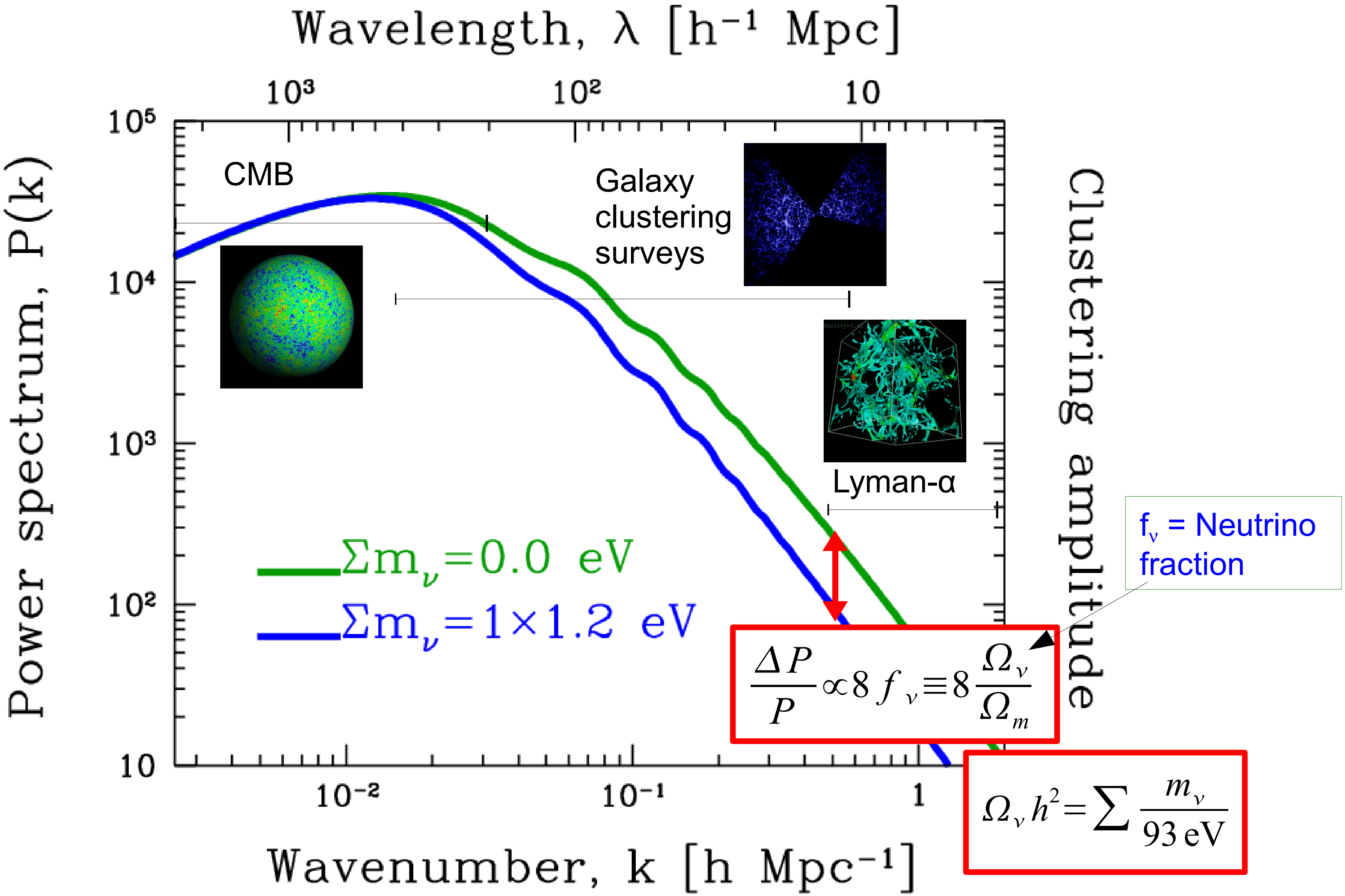
→ Detecting neutrino DM = measuring the neutrino mass

The idea...

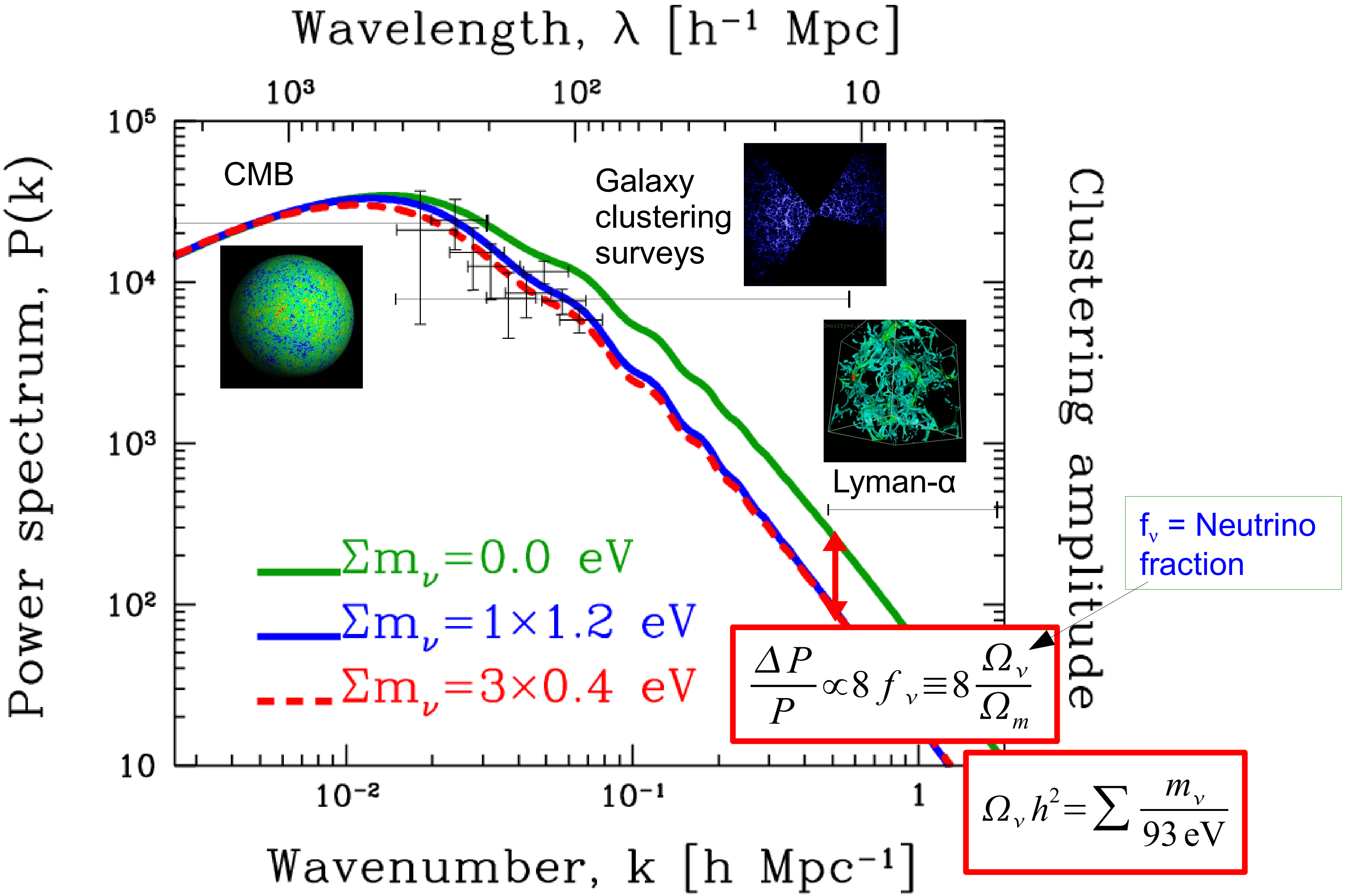
- Massive neutrinos are **hot dark matter**.
  - 1 eV neutrino becomes nonrelativistic at z_{nr} ~ 2000.
  - Structure formation begins at z_{eq} ~ 3000.
- Free-streaming from z_{eq} to z_{nr} suppresses formation of structures on small scales.



N-body simulation, Ma 1996

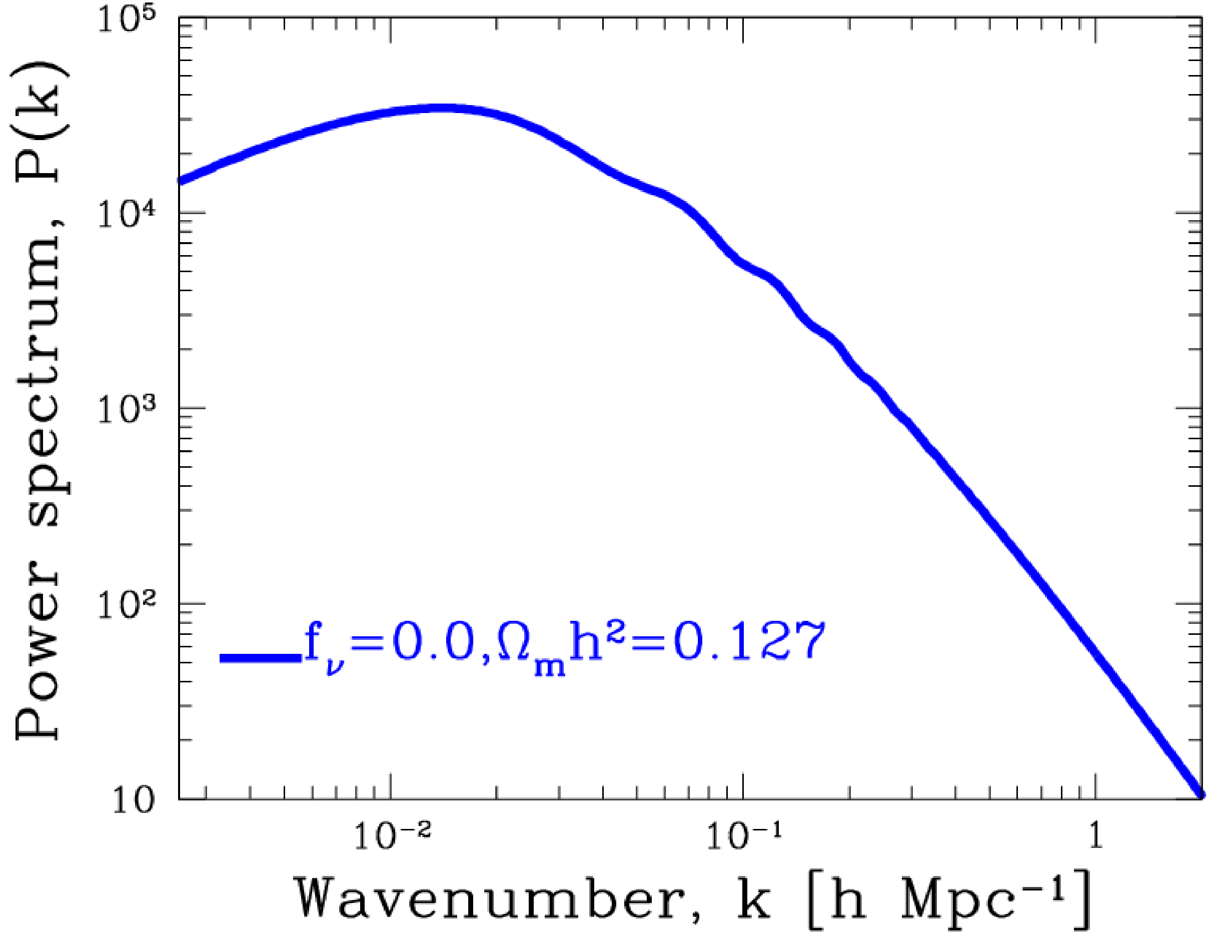


Wavelength, λ [h^{-1} Mpc]
10^3
10^2
10
10^5
10^4
10^3
10^2
10
CMB
Galaxy clustering surveys
Lyman-α
$\Sigma m_\nu = 0.0$ eV
$\Sigma m_\nu = 1 \times 1.2$ eV
Power spectrum, P(k)
Clustering amplitude
f_ν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$
10^{-2}
10^{-1}
1
Wavenumber, k [h Mpc^{-1}]



Wavelength, λ [h⁻¹ Mpc]
10³
10²
10
Power spectrum, P(k)
10⁵
10⁴
10³
10²
10
CMB
Galaxy clustering surveys
Lyman-α
Clustering amplitude
Σm_ν=0.0 eV
Σm_ν=1×1.2 eV
Σm_ν=3×0.4 eV
f_ν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$
10⁻²
10⁻¹
1
Wavenumber, k [h Mpc⁻¹]

Limitations...



Limitations...

Cosmic variance

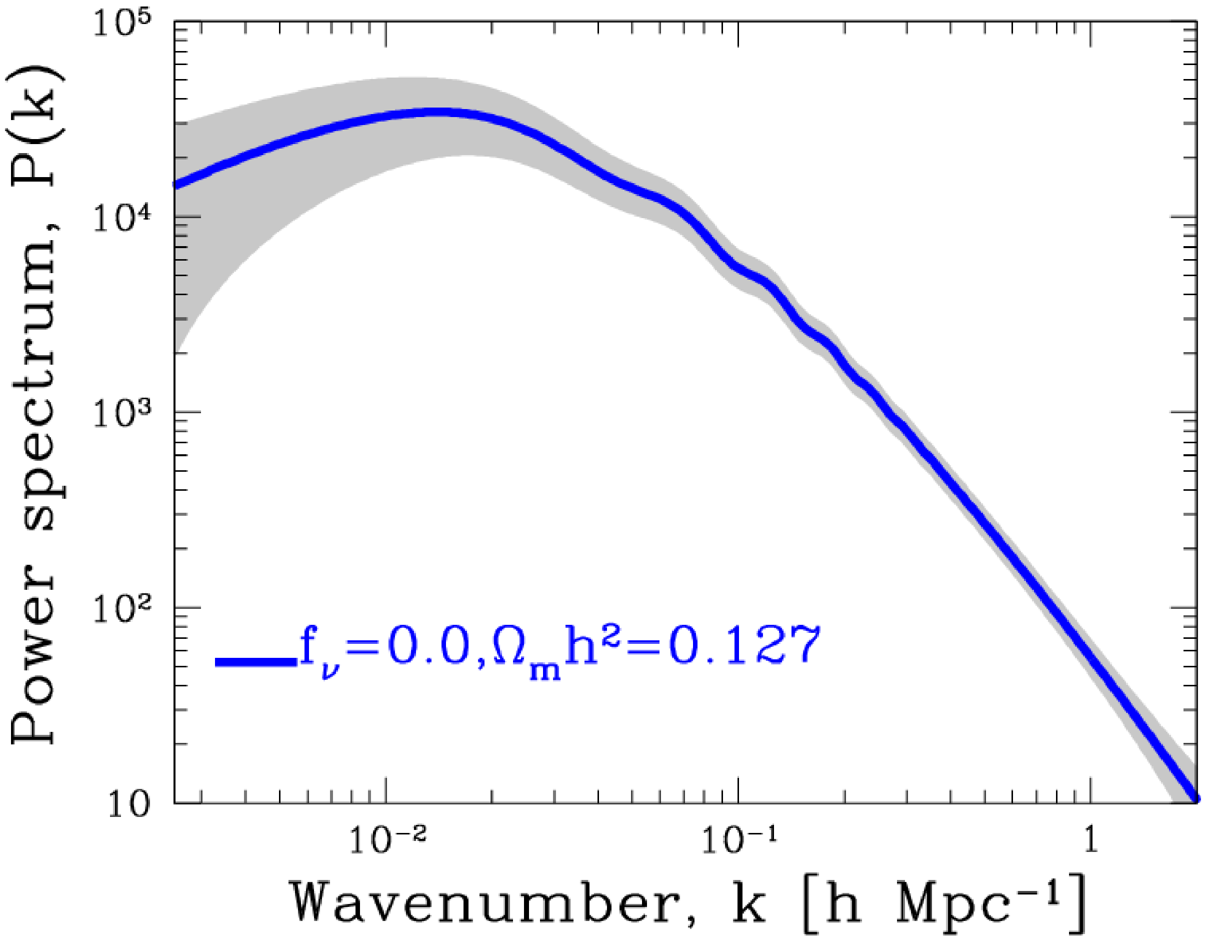
No. of modes in (k, k+dk):

$$V\frac{d^3k}{(2\pi)^3}=V\frac{k^2dk}{2\pi^2}$$

→ **Error bars**:

$$\frac{\delta P}{P}\propto\frac{1}{\sqrt{Vk^2\delta k}}$$

Survey volume



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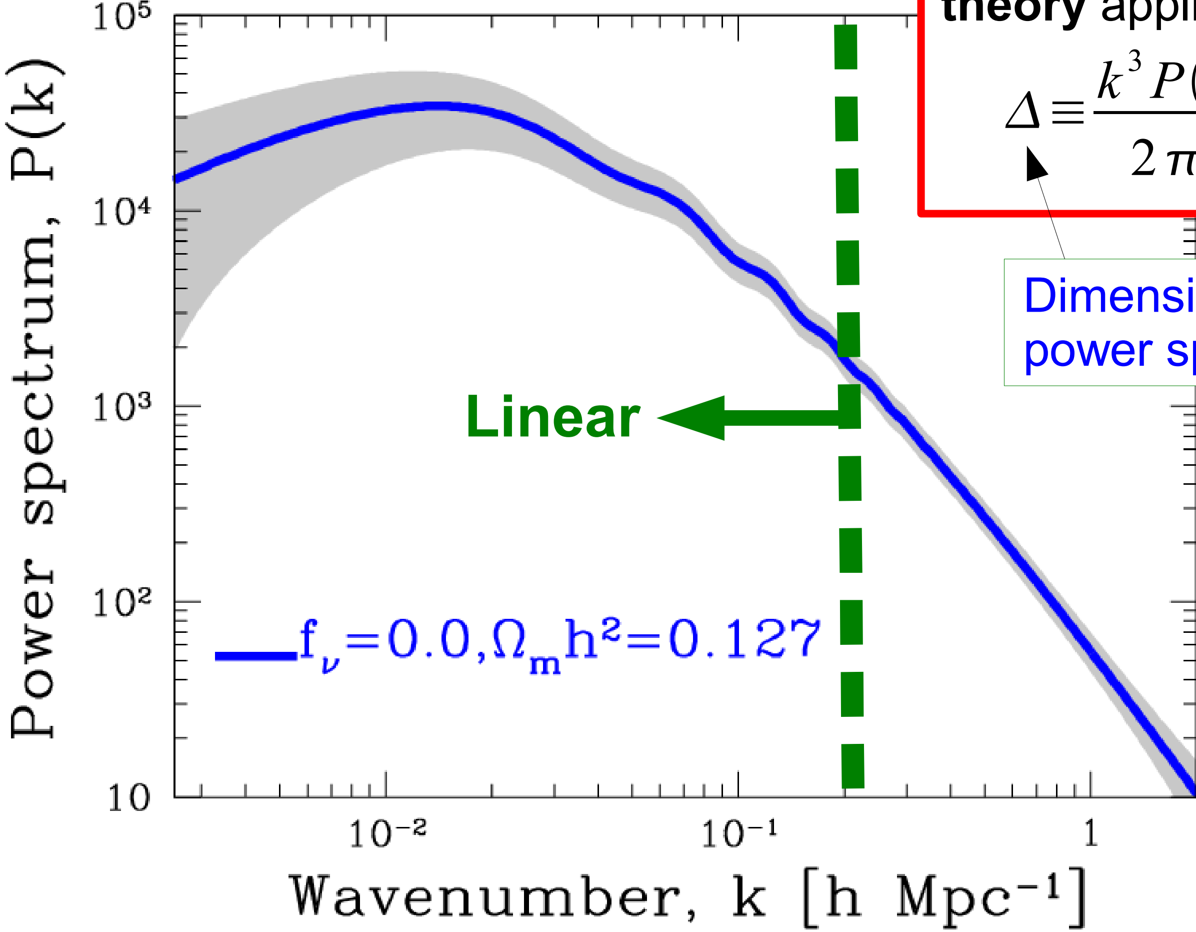
Survey volume



Nonlinearity

Linear perturbation theory applicable when:

$$\Delta \equiv \frac{k^3 P(k)}{2\pi^2} \ll 1$$

Dimensionless power spectrum

Limitations...

Cosmic variance

No. of modes in (k, k+dk):

$$V\frac{d^3k}{(2\pi)^3} = V\frac{k^2\,dk}{2\pi^2}$$

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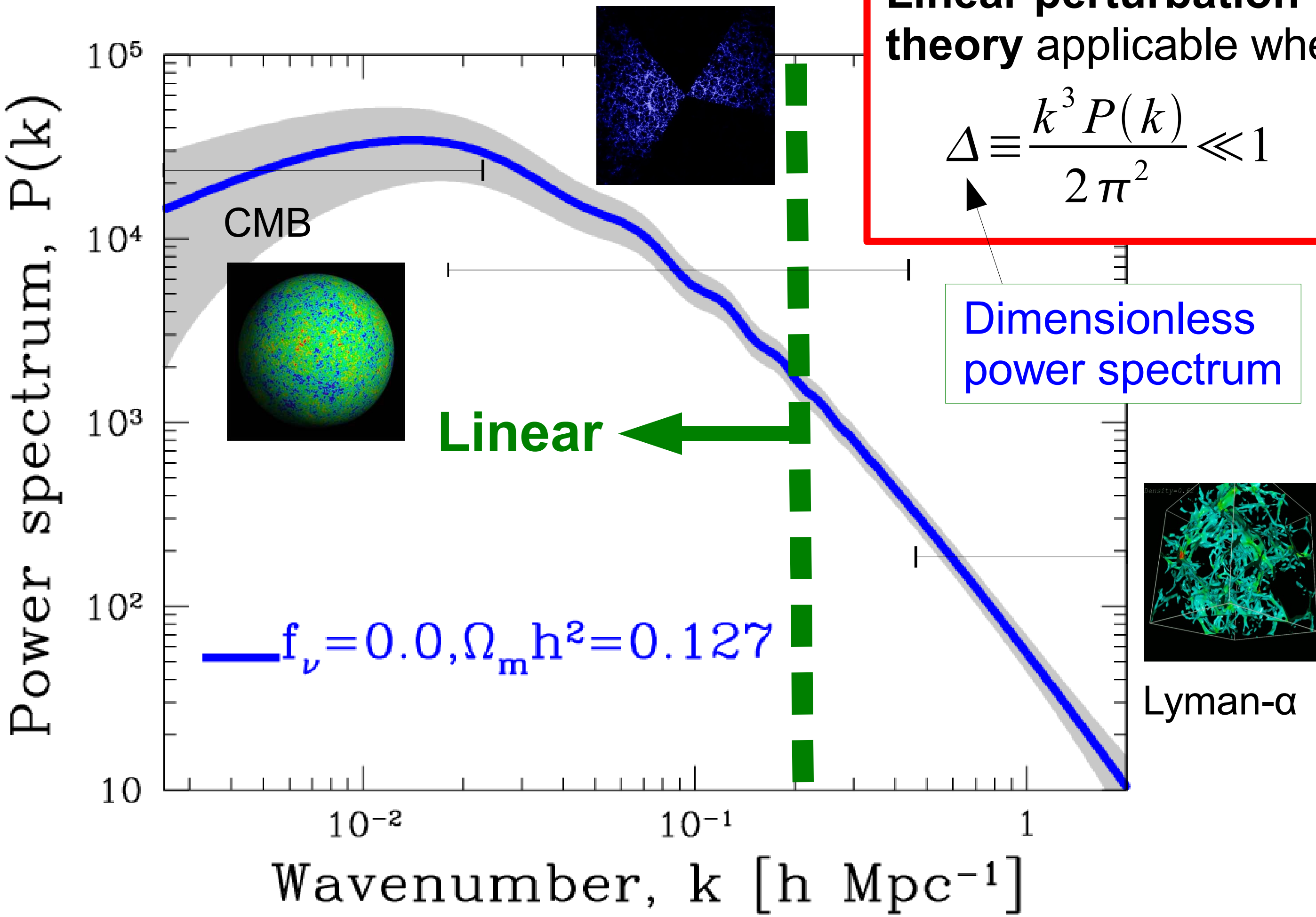
Survey volume



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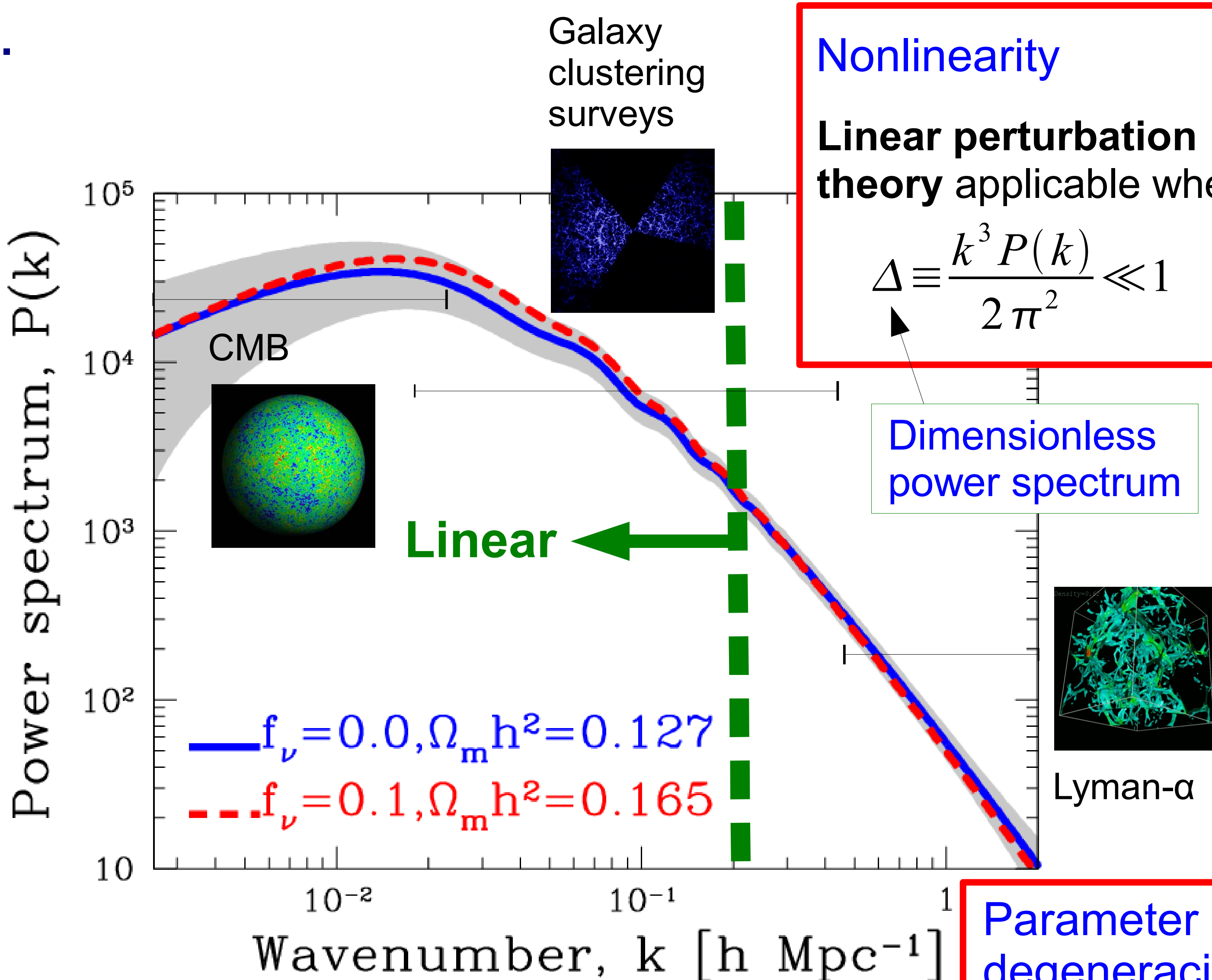
Survey volume



Nonlinearity

Linear perturbation theory applicable when:

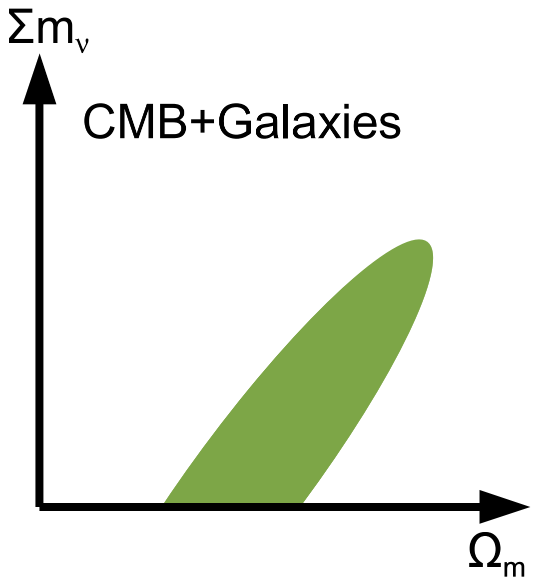
$$\Delta \equiv \frac{k^3 P(k)}{2\pi^2} \ll 1$$

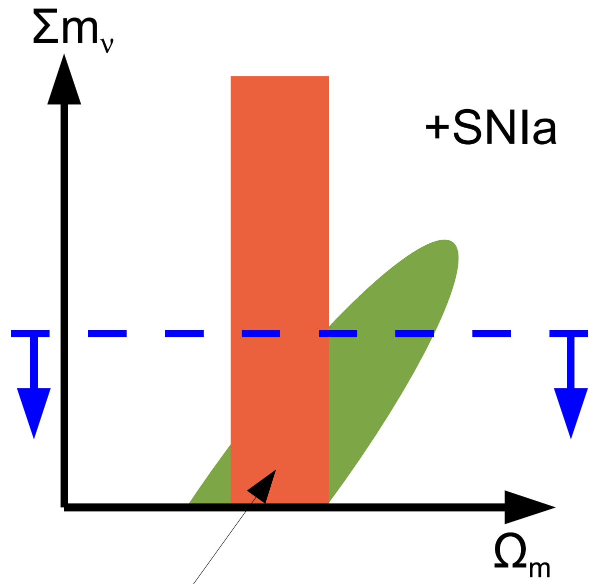
Dimensionless power spectrum

Parameter degeneracies

- Add SNIa...

Schematic only!
Not to scale!





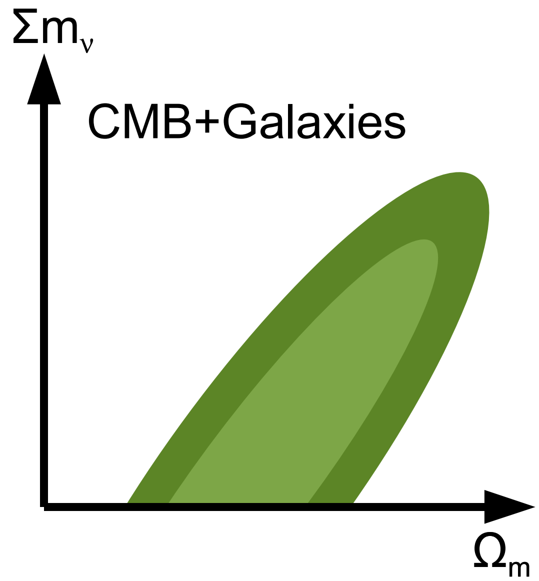
Flat geometry, DE=Cosmological constant

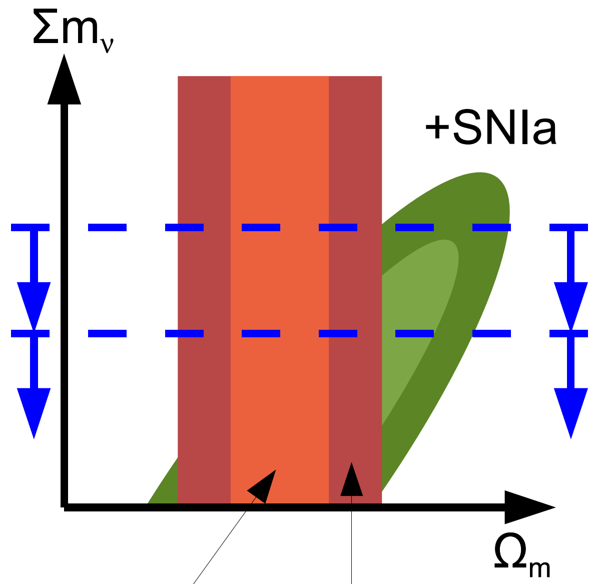
$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + (1-\Omega_m)}$$

H(z) = Expansion rate

- Add SNIa...

Schematic only!
Not to scale!





Flat geometry, DE=Cosmological constant

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + (1-\Omega_m)}$$

Flat geometry, but DE=something else?

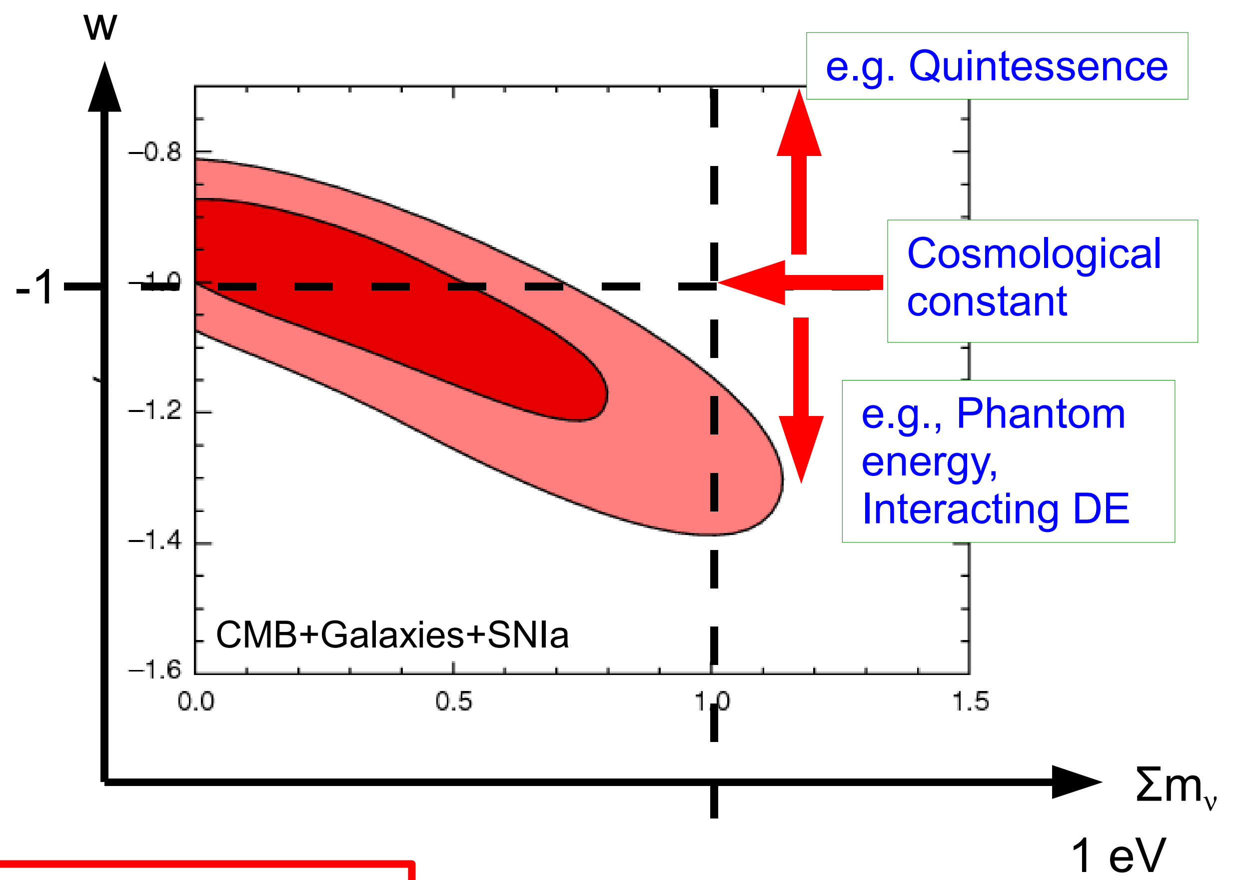
$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_{DE} (1+z)^{3(1+w)}}$$

H(z) = Expansion rate

- Net effect for the neutrino mass measurement...

Dark energy equation of state parameter

$$w_{\rm DE} \equiv \frac{P_{\rm DE}}{\rho_{\rm DE}}$$



$$\sum m_\nu < 0.6\,\mathrm{eV}\,(95\,\%\,\mathrm{C.L.}),\quad w = -1$$
$$\sum m_\nu < 1.0\,\mathrm{eV}\,(95\,\%\,\mathrm{C.L.}),\quad \text{free } w$$

Hannestad, 2005; WMAP, 2006

Present status...

It's pretty difficult to get away from $\Sigma m_\nu < 1 - 2$ eV when all available data have been considered.

Reference	Σm_ν (95% C.L.)	Model	Data
Ichikawa et al. 2006	< 2.0 eV	m	CMB only
Tegmark et al. 2006	< 0.94 eV	m	CMB, Galaxies
Seljak et al. 2006	< 0.17 eV	m	CMB, Galaxies, Lyman-α, SNIa, HST
Zunckel & Ferreira 2007	< 2.2 eV	x	CMB, Galaxies
Spergel et al. 2006	< 0.7 eV	m	CMB, Galaxies, SNIa, HST
Goobar et al. 2006	< 0.6 eV	x	CMB, Galaxies, SNIa, BAO, HST
Kristiansen et al. 2006	< 1.43 eV	m	CMB, Cluster mass function
Hannestad et al. 2007	< 0.65 eV	x	CMB, Galaxies, SNIa, BAO

... and many more.

m=minimal; x=extended

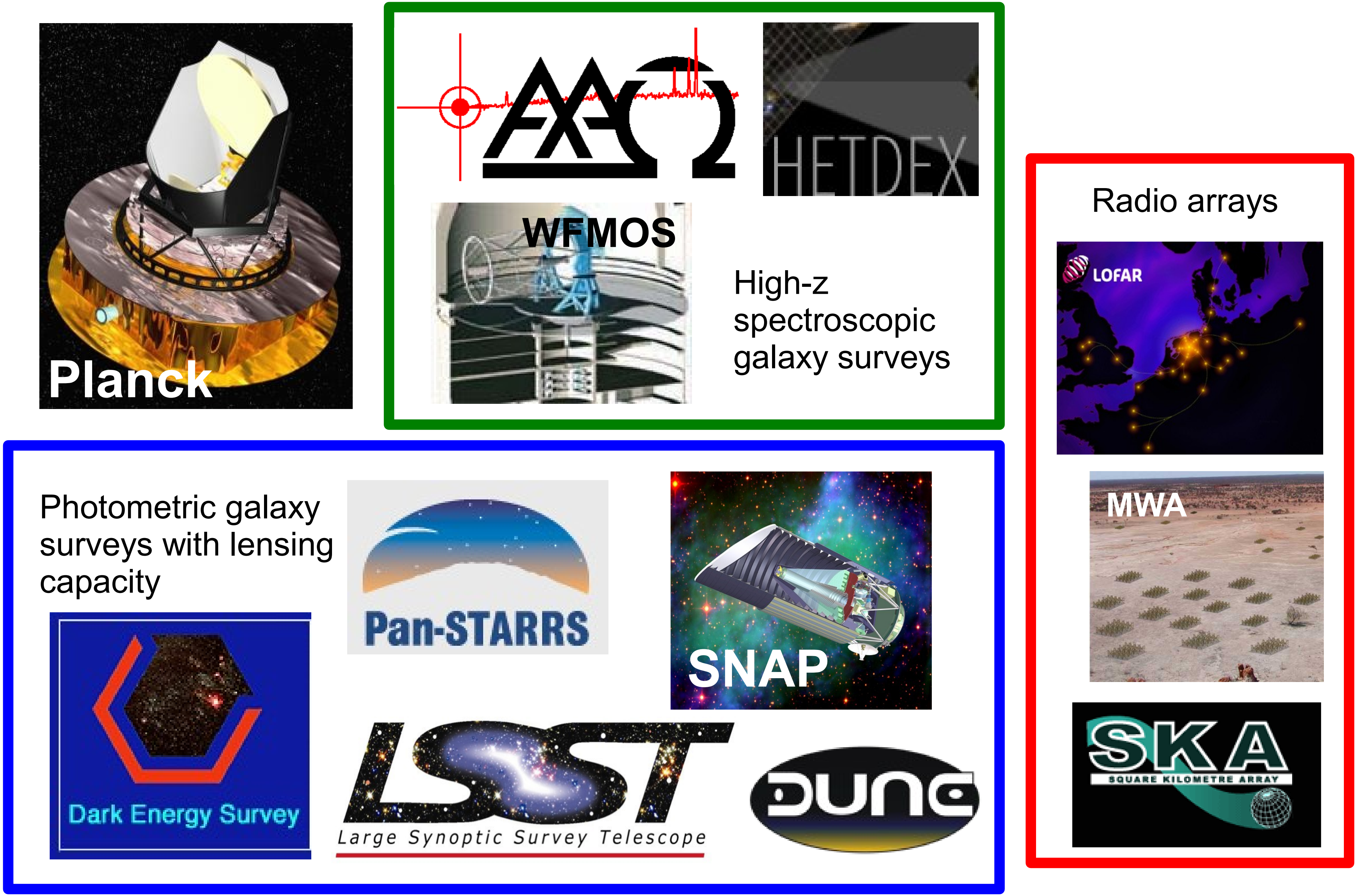
Towards improvement (detection?!?)...

- **Larger survey volume** for better statistics.

- More **reliable** probes and predictions at **large wavenumbers k**.

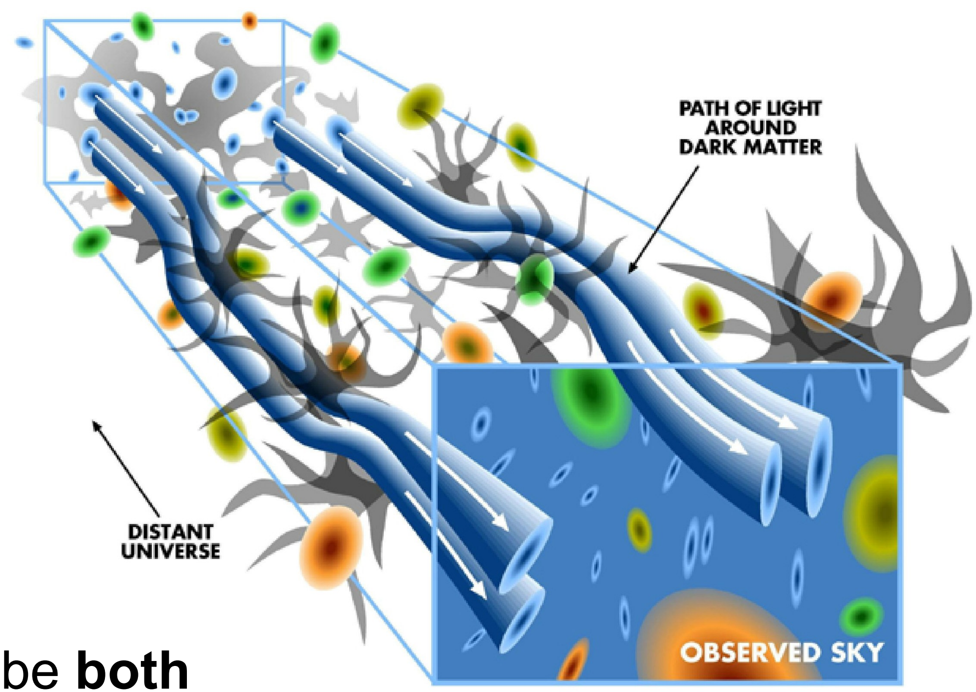
- **New methods** to probe large-scale structure to **break degeneracies**.

3. Future probes…

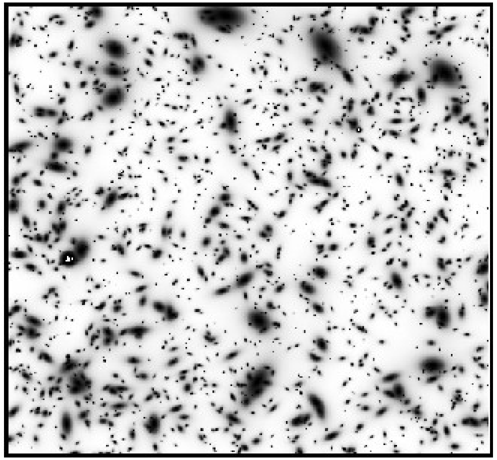

Planck
AAΩ
HETDEX
WFMOS
High-z
spectroscopic
galaxy surveys
Radio arrays
LOFAR
MWA
SKA
SQUARE KILOMETRE ARRAY
Photometric galaxy
surveys with lensing
capacity
Pan-STARRS
SNAP
Dark Energy Survey
LSST
Large Synoptic Survey Telescope
DUNE

Weak lensing of galaxies/Cosmic shear...

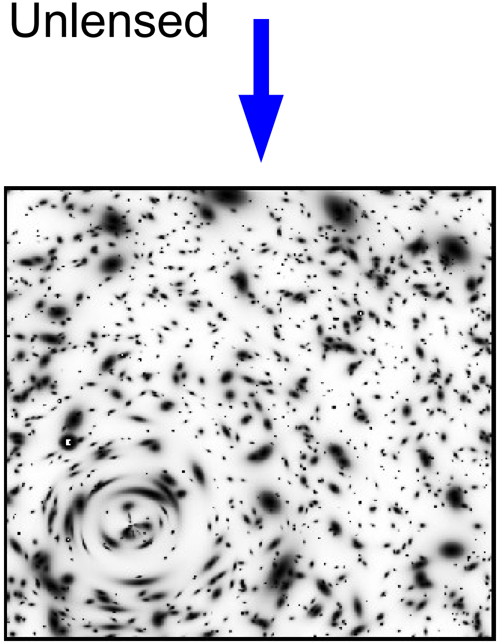
- Distortion (magnification or stretching) of distant galaxy images by **foreground matter**.



Distortions probe **both** luminous and dark matter.



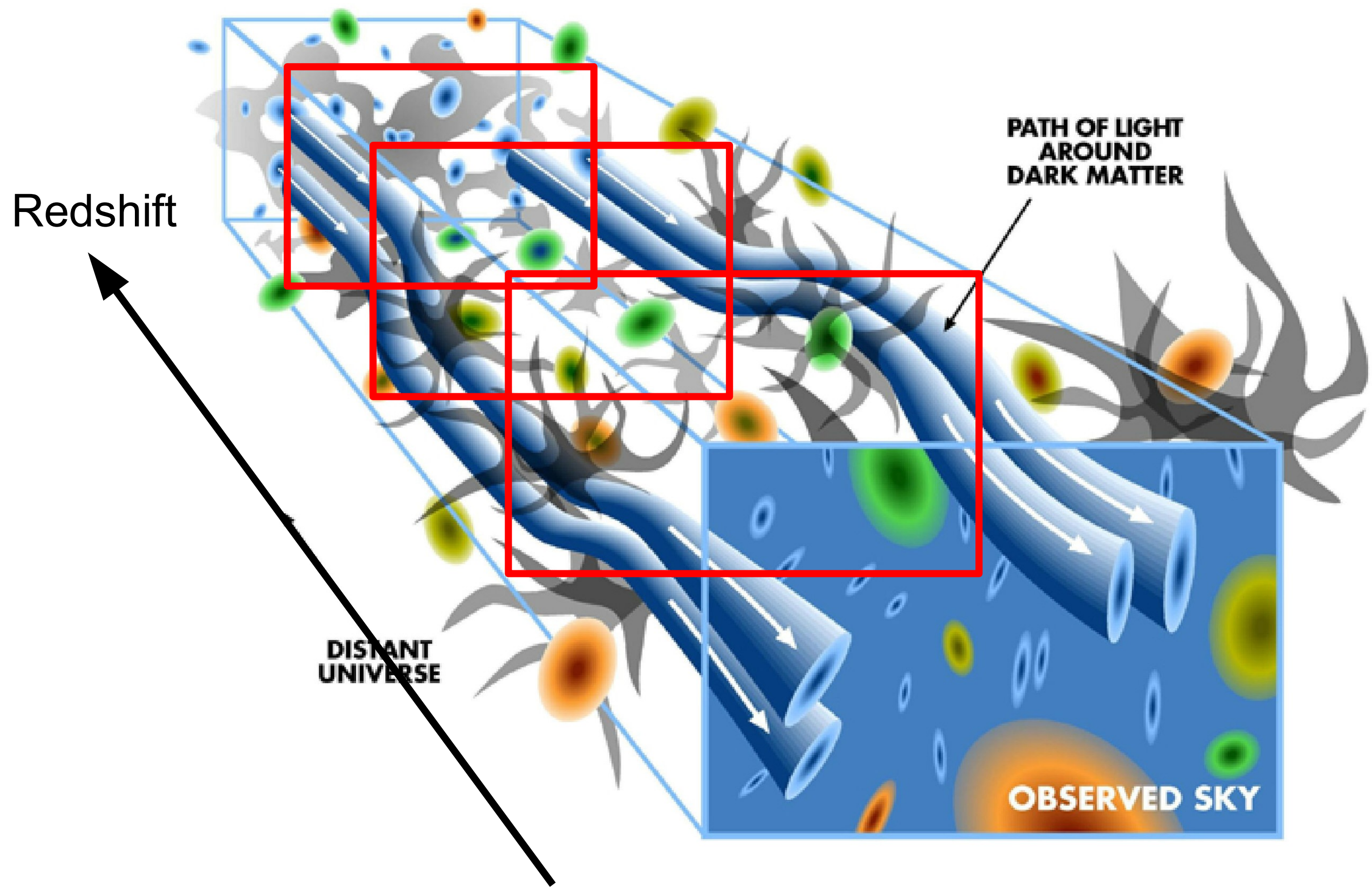
Unlensed



Lensed

- **Tomography** = bin galaxies by redshift

  → Probe evolution of density perturbations.



- Past: Weak lensing first detected in 2000.

- Present: There are some ongoing surveys (e.g., CFHTLS).

- Future:

  - **Dark Energy Survey** (**DES**) to start in 2012.

  - **Large Synoptic Survey Telescope (LSST)** to start in 2014.

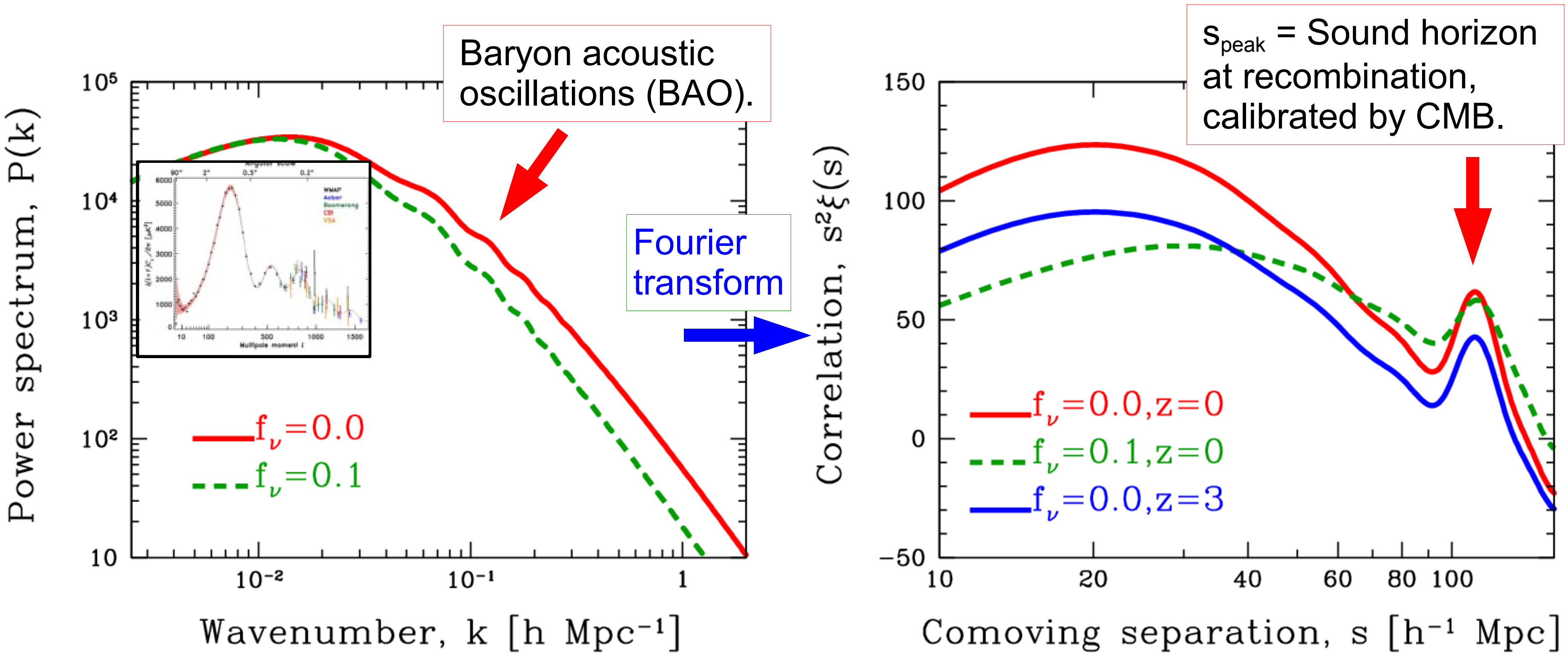
- Planck+LSST sensitivities (based on a 11-parameter model):

	1σ sensitivities
– **Neutrino mass**, Σm_ν	0.05 eV
– **Dark energy density**, Ω_{de}	1%
– **Dark matter density**, $\Omega_c h^2$	1%
– **Baryon density**, $\Omega_b h^2$	0.6%
– **DE equation of state**, w	3%
– **Optical depth to reionisation**, τ	8%
– **Scalar spectral index**, n_s	1%
– **Number of neutrino species**, N_ν	2%

Hannestad, Tu & Y^3W, 2006

Baryon wiggles...

- **Acoustic oscillations** of coupled photon-baryon fluid at recombination (cf CMB anisotropies).

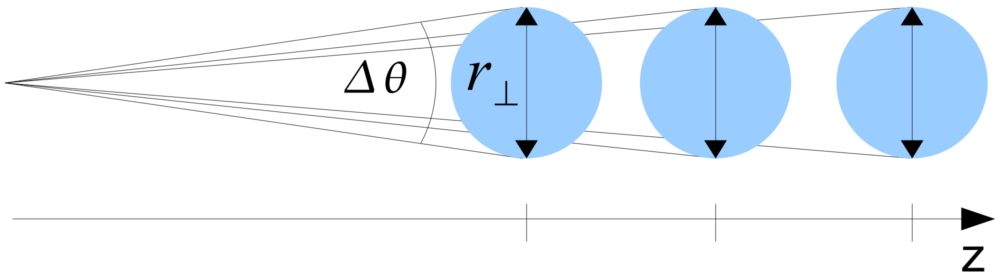


- BAO as a **standard ruler**:

$$r_\perp = r_\parallel = s_{\rm peak} \sim 150\ h^{-1}\,{\rm Mpc}$$

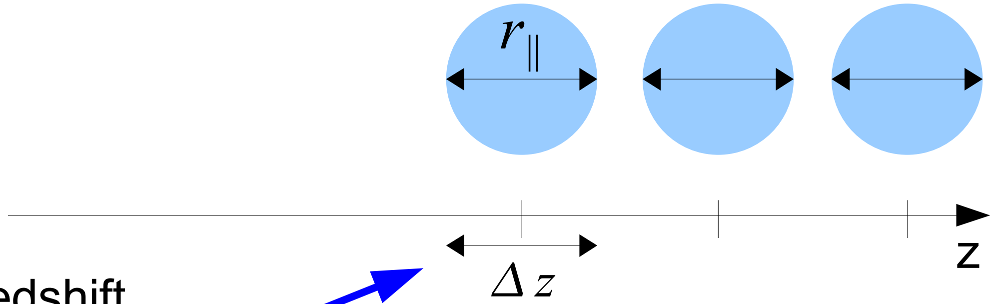
Correlation in transverse direction
→ Angular diameter distance

$$D_A(z) = \int_0^z \frac{dz'}{H(z')} = \frac{r_\perp}{\Delta\theta}$$



Correlation in radial direction
→ Hubble expansion rate

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_{\rm DE}(1+z)^{3(1+w)}}$$
$$= \frac{\Delta z}{r_\parallel}$$

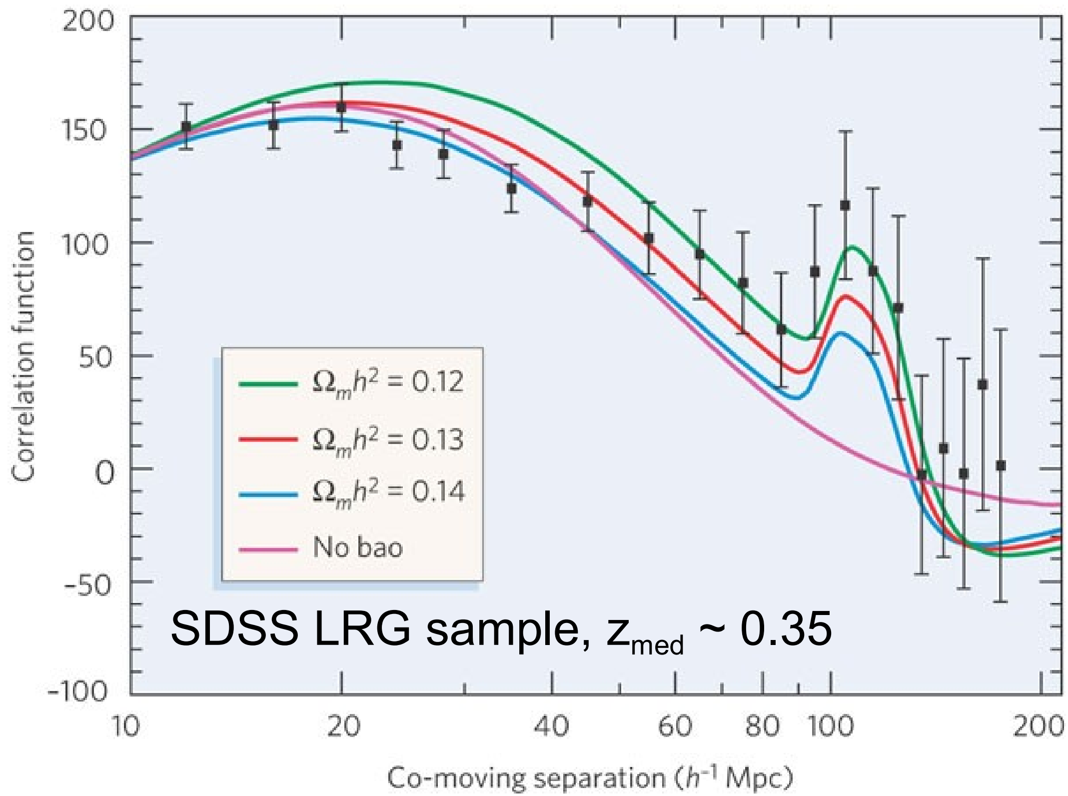


Requires good redshift resolution → spectroscopic galaxy surveys only

- BAO has been **detected** @ z ~ 0.35.

- Planned/proposed spectroscopic surveys, (WFMOS, HETDEX, etc.) will observe @ 2 < z < 4.

- Complementary to SNIa for probing dark energy.



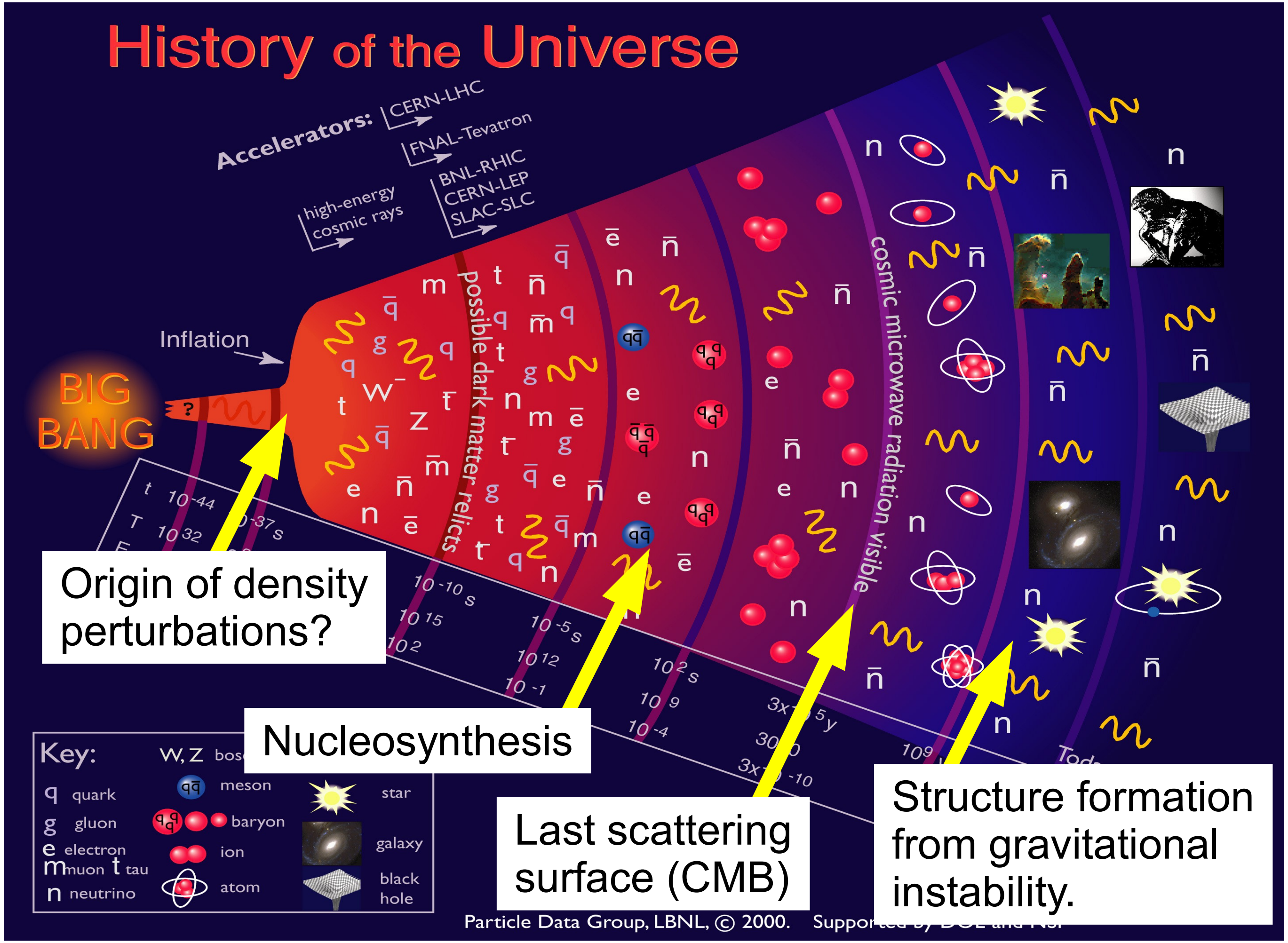
Eisenstein et al. (SDSS), 2005

Even further down the road...

- 21 cm emission lines from Hydrogen spin flip.

- *Lensing* of 21 cm.

- ...



What are the components and how much? What is the geometry?
The initial conditions? The eventual fate? ... → Precision cosmology