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Lecture 3: Neutrinoless double β-decay

Distribute computation of decay rate!

Introduction:

- why lepton number is (most probably) violated in Nature:
  - SM: lepton number is <u>only</u> conserved at perturbative level
    ↳ within the SM, ~~so called~~ non-perturbative processes called "sphalerons" violate B+L but conserve B−L
  - new physics generically violates L
    ↳ Weinberg operator : $\sim (HL)^T (HL) \Rightarrow \Delta L = 2$
- if L is violated, all kinds of LNV processes could potentially exist
  ↳ experimentally most promising one:

$$\boxed{(A,Z) \rightarrow (A,Z+2) + e^- + e^-} \quad \underline{0\nu\beta\beta}$$

Experimental aspects:

- spectrum:

<u>in fact</u>
$2\nu\beta\beta$ in the endpoint region is the biggest background to fight

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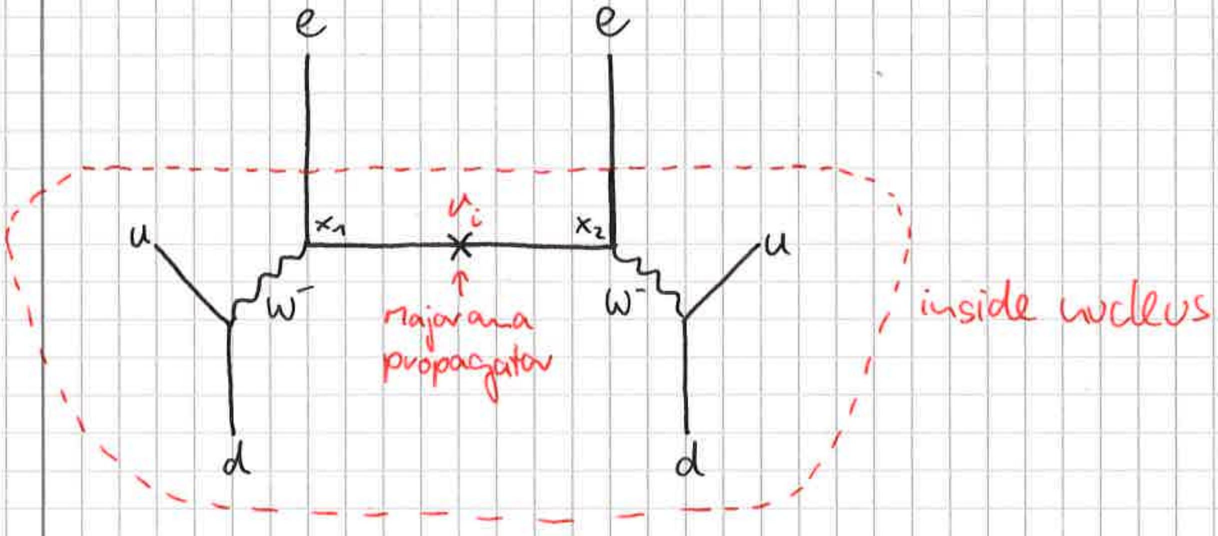
- current limit: $T_{1/2} > 1.9 \cdot 10^{25}$ years (KamLAND-Zen, using Xe-136)
  $\Rightarrow$ process must be quite suppressed to have escaped detection so far

Sketch of the computation:

- Feynman diagram: "2-nucleon mechanism"



- 2nd order perturbation theory in real space:

$$\mathcal{M}_{0\nu\beta\beta} = \langle f | S^{(2)} | i \rangle = \frac{(-i)^2}{2!} \cdot \left(\frac{2\,G_F}{\sqrt{2}}\right)^2 \int d^4x_1\, d^4x_2 \cdot$$

2nd order perturbation

double weak vertex (4-Fermi)

"sum" over all possible space-time points for the interaction

$$\cdot \underbrace{\langle A' | T\{J_\mu(x_1) J_\nu(x_2)\} | A\rangle}_{\text{nuclear physics part}} \cdot \underbrace{\langle f | N\{\bar{e}_L(x_1)\gamma^\mu \overbrace{\nu_{eL}(x_1)\nu_{eL}^T}(x_2)\gamma^{\nu T}\bar{e}_L^T(x_2)\} | i\rangle}_{\text{particle physics part}}$$

- (electron exchange term)

to account for Fermi statistics

- particle physics part:
  - the electron-neutrino is a superposition of (typically) 3 light mass eigenstates:

$$\nu_e = \sum_{i=1}^{3} \tilde{U}_{ei}\, \nu_i$$

"CKM-part" of the full PMNS matrix: $U = \tilde{U}\, \mathrm{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, e^{i\varphi_3/2})$

majorana phases

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⇒ here: $\nu_{eL} = P_L \nu_e = \sum_{i=1}^{3} \tilde{U}_{ei} P_L \nu_i$, $\nu_{eL}^T = \nu_e^T P_L = \sum_{j=1}^{3} \tilde{U}_{ej} \nu_j^T P_L$

- Majorana condition: "fermion = antifermion"

$$\boxed{\nu_i^c = C \bar{\nu}_i^T \overset{!}{=} e^{-i\varphi_i} \nu_i} \Rightarrow \nu_i = e^{i\varphi_i} C \bar{\nu}_i^T \Rightarrow \nu_i^T = -e^{i\varphi_i} \bar{\nu}_i C$$

- fermion propagator: $\overbrace{\psi(x) \bar{\psi}(y)} = i\, S_F(x-y)$

↳ with: $$S_F(x-y) = \int \frac{d^4q}{(2\pi)^4} \frac{\not{q} + m}{q^2 - m^2 + i\varepsilon} e^{-iq(x-y)}$$

⇒ [illegible] thus we can [illegible] compute the neutrino propagator:

$$\overbrace{\nu_{eL}(x_1) \nu_{eL}^T}(x_2) = \sum_{i,j=1}^{3} \tilde{U}_{ei} \tilde{U}_{ej} P_L \overbrace{\nu_i(x_1) \nu_j^T}(x_2) P_L =$$

$$= -\sum_{i,j=1}^{3} \tilde{U}_{ei} \tilde{U}_{ej} P_L e^{i\varphi_i} \underbrace{\overbrace{\nu_i(x_1) \bar{\nu}_j}(x_2)}_{= i S_F^{(i)}(x_1-x_2)\,\delta_{ij} \text{, with } S_F^{(i)} = S_F|_{m \to m_i}} P_L C =$$

$$= -i \sum_{i=1}^{3} \underbrace{(\tilde{U}_{ei} e^{i\varphi_i/2})^2}_{\equiv U_{ei}} \underbrace{P_L S_F^{(i)}(x_1-x_2) P_L}_{= \int \frac{d^4q}{(2\pi)^4} \frac{P_L(\overset{\to 0}{\not{q}} + m_i) P_L}{\underbrace{q^2 - m_i^2 + i\varepsilon}_{\simeq q^2 \text{, since } \langle q^2 \rangle \sim (100\,\text{MeV})^2 \gg m_i^2 \sim 1\,\text{eV}^2}} e^{-iq(x_1-x_2)}} C$$

$$\simeq m_i \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq(x_1-x_2)}}{q^2} P_L$$

⇒ hence: $$\overbrace{\nu_{eL}(x_1) \nu_{eL}^T}(x_2) \simeq -i \underbrace{\left(\sum_{i=1}^{3} m_i U_{ei}^2 \right)}_{\equiv\, m_{ee} \,:\, \text{"effective neutrino mass"}} \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq(x_1-x_2)}}{q^2} \cdot C P_L$$

↳ this is the decisive quantity in what concerns particle physics:

$$\boxed{|m_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta)}|}$$

↳ Majorana phases: $\alpha_{i1} \equiv \varphi_i - \varphi_1$

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- **nuclear physics part:**

- one first inserts a complete set of intermediate nuclear states:

$$\sum_n |n\rangle\langle n| = \mathbb{1}$$

$\Rightarrow$ thus:

$$\langle A'| T\{ J_\mu(x_1) J_\nu(x_2) \} |A\rangle =$$

$$= \Theta(t_1 - t_2) \langle A'| J_\mu(x_1) \sum_n |n\rangle\langle n| J_\nu(x_2) |A\rangle +$$

$$+ \Theta(t_2 - t_1) \langle A'| J_\nu(x_2) \sum_n |n\rangle\langle n| J_\mu(x_1) |A\rangle$$

↳ the Fourier representations of the Θ-functions yield the "energy denominators": e.g. $\sum_n \frac{\langle A'| J_\mu(x_1) |n\rangle\langle n| J_\nu(x_2) |A\rangle}{E_i - E_1 - E_n - |\vec{q}|}$

- then, one applies a set of standard approximations:

1. closure approximation: $E_n \to \langle E_n\rangle \hat{=}$ suitable average energy

   $\Rightarrow$ one can again use $\sum_n |n\rangle\langle n| = \mathbb{1}$

2. non-relativistic nuclear currents:

$$J_\alpha(\vec{x}) \simeq \sum_m \tau_m^+ \Big[\overset{\text{vector coupling}}{g_V} g_{\alpha 0} + \overset{\text{axial vector coupling}}{g_A} g_{\alpha j} \sigma_m^j \Big] \delta^{(3)}(\vec{x} - \vec{r}_m)$$

   sum over all nucleons

   nuclear isospin ladder operator: $\tau_m^+ |n\rangle_m = |p\rangle_m$, $\tau_m^+ |p\rangle_m = 0$

   spin-flip of the n-th nucleon in the j-direction

   position of the m-th nucleon

3. Long wavelength approximation:

$$\left.\begin{array}{l} |\vec{x}| \sim |\vec{r}_m| \sim R \sim (100\,\text{MeV})^{-1} \\ |\vec{p}| \sim Q << 100\,\text{MeV} \end{array}\right\} \Rightarrow |\vec{p}||\vec{x}| << 1 \Rightarrow e^{-i\vec{p}\vec{x}} \sim 1$$

   Q-value: $Q \equiv M_i - M_f \simeq E_i - E_f$, where $E_f \simeq M_f + \frac{\vec{p}^{\,2}}{2M_f} \approx M_f$

- after some tedious computation, one arrives at:

$$\mathcal{M}_{0\nu\beta\beta} = \frac{i G_F^2 m_{ee}}{2^3 \pi^2 \sqrt{E_1 E_2}} \delta(Q - \underset{e^-\text{-energies}}{E_1 - E_2}) \cdot \bar{u}^{s_2}(p_2) P_R v^{s_1}(p_1) \sum_{m,m'} \int d^3q \frac{e^{+i\vec{q}(\vec{r}_m - \vec{r}_{m'})}}{|\vec{q}|} \cdot$$

$$\cdot \left[\frac{1}{|\vec{q}| + (E_1 + \langle E_n\rangle - E_i)} + \frac{1}{|\vec{q}| + (E_2 + \langle E_n\rangle - E_i)} \right] \cdot \left[g_V^2 \langle A'| \tau_m^+ \tau_{m'}^+ |A\rangle - g_A^2 \langle A'| \tau_m^+ \tau_{m'}^+ \vec{\sigma}_m \vec{\sigma}_{m'} |A\rangle \right]$$

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- neutrino potential functions: $H(|\vec{x}|, a) \equiv \frac{1}{2\pi^2} \int \frac{e^{+i\vec{q}\vec{x}}\, d^3q}{|\vec{q}|\,(|\vec{q}|+a)}$

$\Rightarrow$ with $h(|\vec{r}_m - \vec{r}_{m'}|) \equiv \frac{R}{2}\left[H(|\vec{r}_m-\vec{r}_{m'}|, E_1 + \langle E_n\rangle - E_i) + H(|\vec{r}_m-\vec{r}_{m'}|, E_2 + \langle E_n\rangle - E_i)\right]$:

$$\mathcal{M}_{0\nu\beta\beta} = \frac{i G_F^2\, m_{ee}}{2R\sqrt{E_1 E_2}}\, \delta(Q - E_1 - E_2)\, \bar{u}^{s_2}(p_2)\, P_R\, v^{s_1}(p_1)\, \cdot$$

$$\cdot \Big[g_V^2\, \underbrace{\langle A'| \sum_{m,m'} \tau_m^+ \tau_{m'}^+\, h(|\vec{r}_m - \vec{r}_{m'}|)\, |A\rangle}_{\equiv M_F:\ \text{Fermi matrix element}} - g_A^2\, \underbrace{\langle A'| \sum_{m,m'} \tau_m^+ \tau_{m'}^+\, \vec{\sigma}_m \vec{\sigma}_{m'}\, h(|\vec{r}_m - \vec{r}_{m'}|)\, |A\rangle}_{\equiv M_{GT}:\ \text{Gamov-Teller matrix element}} \Big]$$

$\Rightarrow$ total nuclear matrix element (NME):

$$\boxed{M^{0\nu} \equiv M_{GT} - \frac{g_V^2}{g_A^2} M_F}$$

↳ this quantity contains all the nuclear physics:
- highly non-trivial to compute
- currently a big uncertainty

- to arrive at a decay rate:
  1. square to matrix element & sum over electron spins
  2. use Fermi's Golden Rule & integrate over phase space
  3. don't forget a factor of $\frac{1}{2}$ for identical electrons

$\Rightarrow$ decay rate in the limit $m_e \simeq 0$:

$$\boxed{\Gamma_{0\nu\beta\beta} \approx \frac{g_A^4 G_F^4}{(2\pi)^5 R^2} |M^{0\nu}|^2 |m_{ee}|^2 \frac{Q^5}{30}} \qquad \Rightarrow \quad T_{1/2} = \frac{\ln 2}{\Gamma_{0\nu\beta\beta}}$$

$\Rightarrow$ characteristic Q^5-dependence $\Rightarrow$ preference for large Q

~~[illegible] ⇒ slight preference for light nuclei~~

The effective mass:

- formula: $|m_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta)}|$

- one can use the smallest neutrino mass m as parameter:
  - NO: $m_1 \equiv m$, $m_2 = \sqrt{m^2 + \Delta m_\odot^2}$, $m_3 = \sqrt{m^2 + \Delta m_A^2}$
  - IO: $m_3 \equiv m$, $m_1 = \sqrt{m^2 + \Delta m_A^2}$, $m_2 = \sqrt{m^2 + \Delta m_A^2 + \Delta m_\odot^2}$

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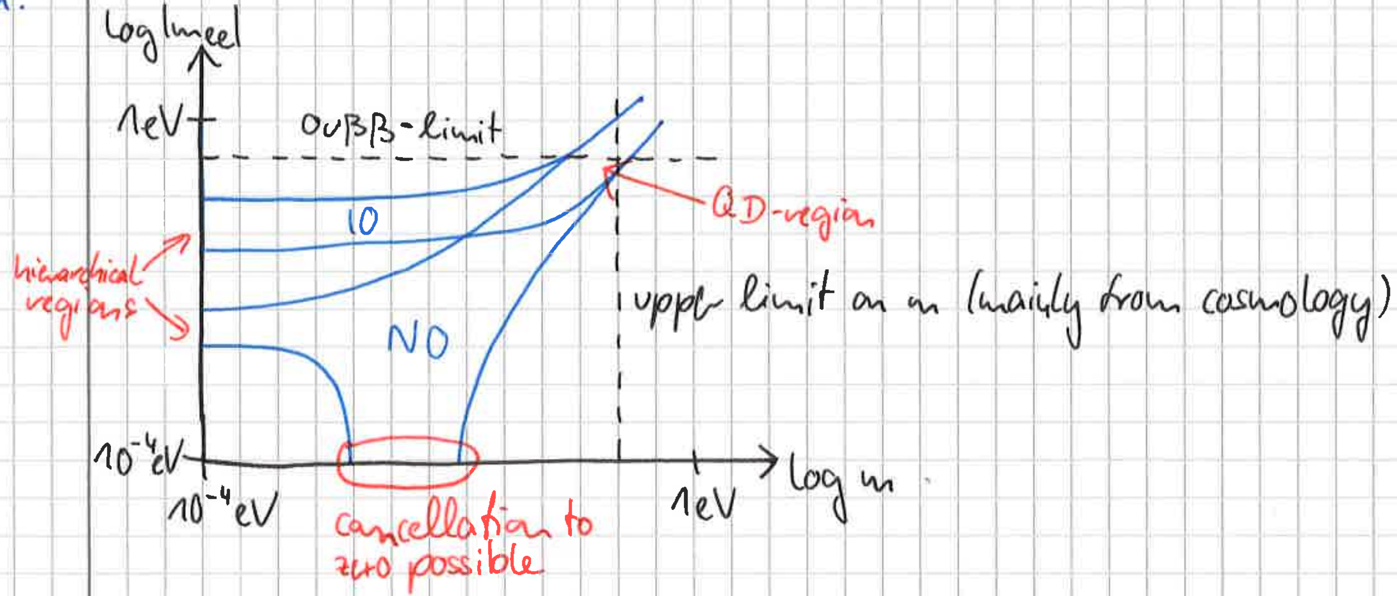
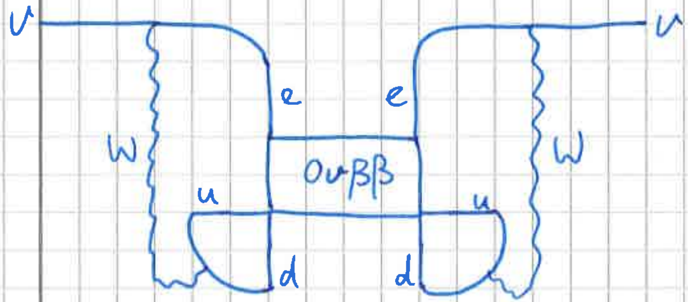
- sketch:



⇒ if we are unlucky, we could have NO with Majorana-ν's, but we would <u>never</u> see it

Implications of $0\nu\beta\beta$-detection:

- if we observe $0\nu\beta\beta$...
  ... we have discovered LNV: $\Delta L = 2$
  .... but what about the ν-mass?
- <u>Schechter-Valle theorem:</u> <u>any</u> $0\nu\beta\beta$-operator yields a Majorana ν-mass



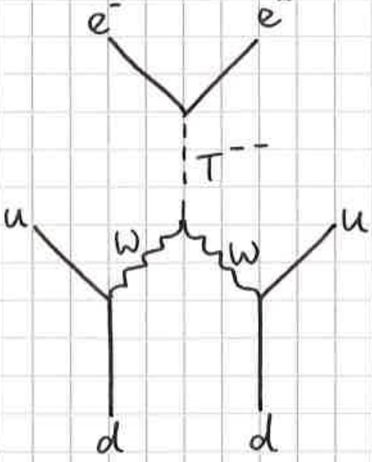
"Butterfly diagram"

↳ BUT: Dürr, Lindner, Merle '11
<u>This mass contribution can be zero for some operators!</u>
⇒ one might have to draw a more complicated diagram

Alternative mechanisms for $0\nu\beta\beta$:

- other mechanisms could transmit $0\nu\beta\beta$
  ⇒ probably hard to disentangle

- e.g. triplet Higgs $T \equiv (T^{++}, T^{+}, T^{0})$:



- e.g. heavy neutrinos N: