(2.1)

Lecture 2: Neutrino phenomenology

Neutrino mass:

- experiments $\Rightarrow m_\nu \lesssim O(0.1 - 1\,eV) << m_e = 511\,keV$
  ↳ BUT: we don't know the scale yet
- Why so difficult?
  - $m_\nu <<$ typical Q-values $\Rightarrow m_\nu \neq 0$ has hardly any effect
    ↑ energy release in a reaction
  - neutrinos interact only weakly $\Rightarrow$ problem to sufficient # events
- experimental probes:
  - kinematical measurements } now
  - cosmology } now
  - $0\nu\beta\beta \Rightarrow$ lecture 3

Single β-decay (kinematics):

- ν's are produced in many decays: $\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$, $n^0 \to p^+ + e^- + \bar{\nu}_e, \dots$
  ↳ which decay is best?
  - low Q-value $\Rightarrow$ bigger effect of $m_\nu \neq 0$ ✓
    $\Rightarrow$ small rates ↯
  - large enough decay rate to accumulate statistics
- experimentally suitable: tritium

$$^3H \to {}^3He + e^- + \bar{\nu}_e \qquad (Q = 18.6\,keV)$$

if the e^- carries away all the energy, the neutrino is at rest

- β-spectrum:

$$N(T_e) = K \cdot F(E_e, Z) \cdot p_e E_e (Q - T_e) \sqrt{(Q-T_e)^2 - m_\nu^2}$$

electron kinetic energy

Fermi function (influence of Coulomb potential)

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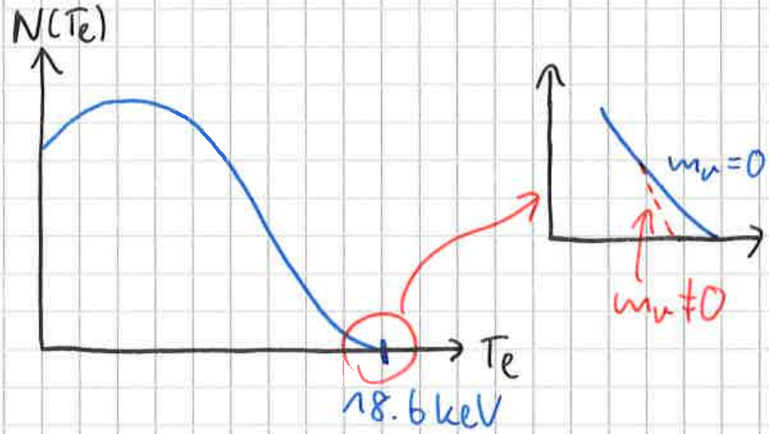
↳ for 3 generations of ν's: $\sqrt{(Q-T_e)^2 - m_\nu^2} \rightarrow \sum_{i=1}^{3} |U_{ei}|^2 \sqrt{(Q-T_e)^2 - m_i^2}$

$\Rightarrow m_i^2 << (Q-T_e)^2$: $\sim (Q-T_e) \underbrace{\sum_{i=1}^{3} |U_{ei}|^2}_{=1 \text{ (unitarity)}} - \frac{1}{2(Q-T_e)} \sum_{i=1}^{3} |U_{ei}|^2 m_i^2$

$\sum_{i=1}^{3} |U_{ei}|^2 m_i^2 \equiv m_\beta^2$ ("effective kinematic ν_e-mass")

$\Rightarrow$ kinematical experiments are sensitive to m_β^2

↳ effect on spectrum:



- limits: MAINZ/TROITSK (current) $\Rightarrow m_\beta^2 \lesssim 2.1\,eV^2$
  KATRIN (future) $\Rightarrow m_\beta^2 < 0.2\,eV^2$

↳ BUT: $\boxed{m_\beta^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2}$

$\Rightarrow$ conclusion on mass scale depends on mixing angles

Neutrino mass from cosmology:

- early Universe: neutrinos "in thermal equilibrium"
  $\Rightarrow$ constantly produced & destroyed
  $\Rightarrow$ follow Fermi-Dirac distribution:

$$f_\nu(T_\nu) = \frac{1}{e^{p_\nu/T_\nu} + 1}$$

- later: reaction rate falls below expansion rate H at $T \sim 2\,MeV$
  $\Rightarrow$ decoupling from equilibrium ("freeze-out")

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↳ at $T \sim 1\,\mathrm{MeV}$, also e^+e^- freeze-out ⇒ "reheating" of photons but not of neutrinos:

$$T_\nu = \sqrt[3]{\frac{4}{11}}\, T_\gamma \simeq 0.7\, T_\gamma \quad \text{(from entropy conservation)}$$

⇒ number density of neutrinos today:

$$n_{\nu 0} = g_\nu \int \frac{d^3p}{(2\pi)^3} f_\nu(p, T_{\nu 0}) = \frac{g_\nu}{2\pi^2} \underbrace{T_{\nu 0}^3}_{=\frac{4}{11} T_{\gamma 0}^3} \underbrace{\int_0^\infty dx \frac{x^2}{e^x+1}}_{=\frac{3}{2}\zeta(3)}$$

$$= \frac{3}{4} \frac{g_\nu}{g_\gamma} \cdot \underbrace{n_{\gamma 0}}_{=411\,\mathrm{cm}^{-3}\text{ (photon density from CMB)}} \cdot \frac{4}{11}$$

⇒ with $g_\gamma = 2$ (polarisations) & $g_\nu = 6$ ($\nu_{e,\mu,\tau}$ & $\bar{\nu}_{e,\mu,\tau}$):

$$\boxed{n_{\nu 0} = 336\,\mathrm{cm}^{-3} \Rightarrow 112\,\mathrm{cm}^{-3}/\text{flavour}}$$

- energy density today:

$$\Omega_{\nu 0} = \frac{\rho_{\nu 0}}{\rho_c} = \frac{m_\nu \cdot 336\,\mathrm{cm}^{-3}}{1.05 \cdot 10^{-5} h^2\,\mathrm{GeV\,cm}^{-3}}$$

- easy estimate: Gerstein-Zeldovich limit

$$\Omega_{\nu 0} < \Omega_{\text{non-rel.}} = \frac{0.142}{h^2} \Rightarrow m_\nu < 4.4\,\mathrm{eV} \Rightarrow \text{already good}$$

- more refined: structure formation data, CMB, ...

⇒ Planck '15: $\boxed{\sum_i m_i < 0.194\,\mathrm{eV}}$ ⇒ good limit, but systematics...

Neutrino oscillations:

- process: periodic change of flavour with time

⇒ a neutrino produced as, e.g., ν_μ could after some propagation be detected as ν_e

(see next page)

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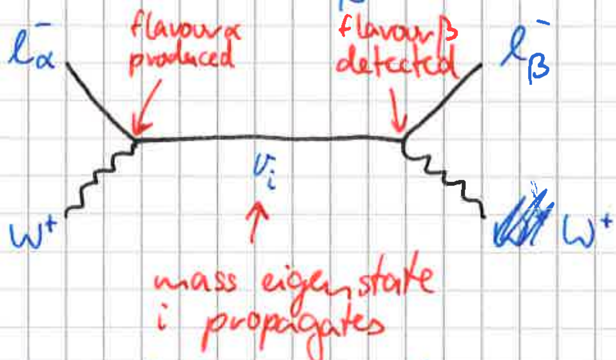
- wrong (!) derivation:

• Feynman diagram: neutrino produced as flavour ν_α propagates and is detected as ν_β



• crucial: due to their different energies E_i, the mass eigenstates ν_i will receive different phases during propagation

$$|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i\rangle$$

• assumption (wrong!): momenta are all equal

$$\Rightarrow E_i = \sqrt{p^2+m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq p + \frac{m_i^2}{2E}$$

⇒ thus, the neutrino produced as ν_α evolves as:

$$|\nu(t)\rangle = \sum_{i=1}^{3} U_{\alpha i}|\nu_i(t)\rangle = \sum_{i=1}^{3} U_{\alpha i}\, e^{-iE_i t}|\nu_i(0)\rangle \cancel{= e^{-ipt}}$$

$$\cancel{= e^{-ipt}\sum}$$

↳ this state has non-zero overlap with flavour $|\nu_\beta\rangle = \sum_{j=1}^{3} U_{\beta j}|\nu_j(0)\rangle$:

$$A_{\alpha\to\beta}(t) = \langle \nu_\beta|\nu_\alpha(t)\rangle = \underbrace{e^{-ipt}}_{|\dots|=1} \cdot \sum_{i,j=1}^{3} U_{\alpha i}\, e^{-i\frac{m_i^2}{2E}t}\, U_{\beta j}^* \underbrace{\langle \nu_i(0)|\nu_j(0)\rangle}_{=\delta_{ij}}$$

⇒ oscillation probability:

$$P_{\alpha\to\beta}(t) = |A_{\alpha\to\beta}(t)|^2 = \sum_{i,j=1}^{3} U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}\, e^{-\frac{it}{2E}\underbrace{(m_i^2-m_j^2)}_{\Delta m_{ij}^2}} =$$

$$= \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 + \left(\sum_{i<j} + \sum_{i>j}\right) U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}\, e^{-\frac{it}{2E}\Delta m_{ij}^2} =$$

$$= \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2\,\mathrm{Re}\left[\sum_{i<j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}\, e^{-\frac{it}{2E}\Delta m_{ij}^2}\right] =$$

rename indices for 2nd term

~~Average probability~~ ~~actual oscillation~~

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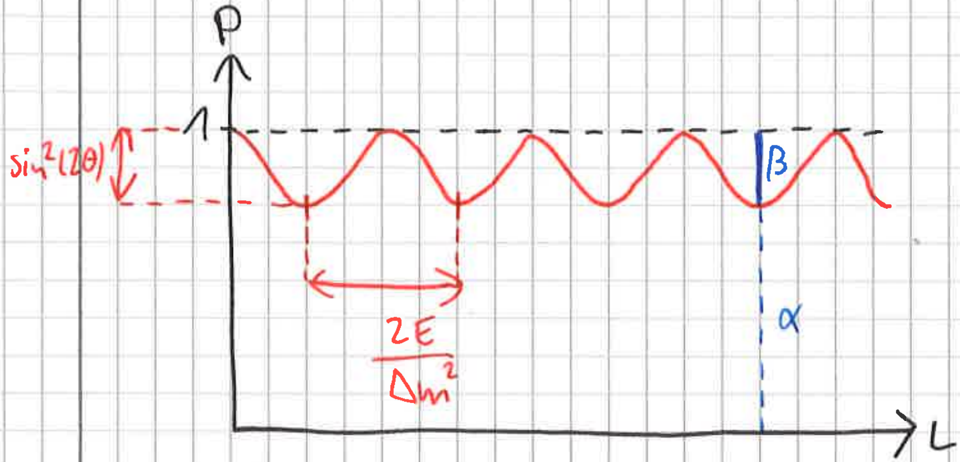
$$= \delta_{\alpha\beta} - 2\,\mathrm{Re}\left[\sum_{i<j} U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j} \left(1 - e^{-\frac{it}{2E}\Delta m^2_{ij}}\right)\right]$$

unitarity

- for 2 flavours only: $\boxed{P_{\alpha\to\beta}(L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{2E}\right)}$



$\Rightarrow$ 2 types of experiments: × disappearance: flavour α detected
× appearance: flavour $\beta \neq \alpha$ detected

- issue with the derivation: in general, not all p_i are equal
  ↳ can be resolved by ~~in~~ taking into account the production process or by using another formalism such as wave packets
  → BUT: same result!

Neutrino oscillations in matter:

- Schrödinger-like equation (yields correct result):

$$i\frac{d}{dt}\begin{pmatrix} \nu_1(t) \\ \nu_2(t)\end{pmatrix} = \frac{1}{2E}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2\end{pmatrix}\begin{pmatrix} \nu_1(t) \\ \nu_2(t)\end{pmatrix}$$

- steps:
  - transform to flavour basis: $\begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$
  - add an "interaction" to the Hamiltonian: from coherent forward scattering

$$H \to H + \begin{pmatrix} A & 0 \\ 0 & 0\end{pmatrix}, \quad A = 2\sqrt{2}\,G_F\, n_e\, p$$

electron number density (only those exist in matter)

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- diagonalise the Hamiltonian $\Rightarrow$ effective mixing angle and mass-square difference in matter:

$$\boxed{\begin{aligned} \sin^2(2\theta_m) &= \frac{\sin^2(2\theta)}{\sqrt{\sin^2(2\theta) + \left(\frac{A}{\Delta m^2} - \cos(2\theta)\right)^2}} \\ \Delta m_m^2 &= \frac{\sin(2\theta)}{\sin(2\theta_m)} \Delta m^2 \end{aligned}}$$

$(= \sin^2(2\theta)$ for $A=0)$

$(= \Delta m^2$ for $A=0)$

- MSW effect:
  - observation: the mixing angle angle in matter can be maximal for a suitable matter density

$$\sin^2(2\theta_m) \overset{!}{=} 1 \Rightarrow \boxed{A \overset{!}{=} \Delta m^2 \cos(2\theta) \overset{!}{=} 2\sqrt{2}\, G_F n_e p}$$ resonance condition

↳ furthermore: antineutrinos would have $A \to -A$

$$\Rightarrow A + \Delta m^2 \cos(2\theta) \neq 0$$

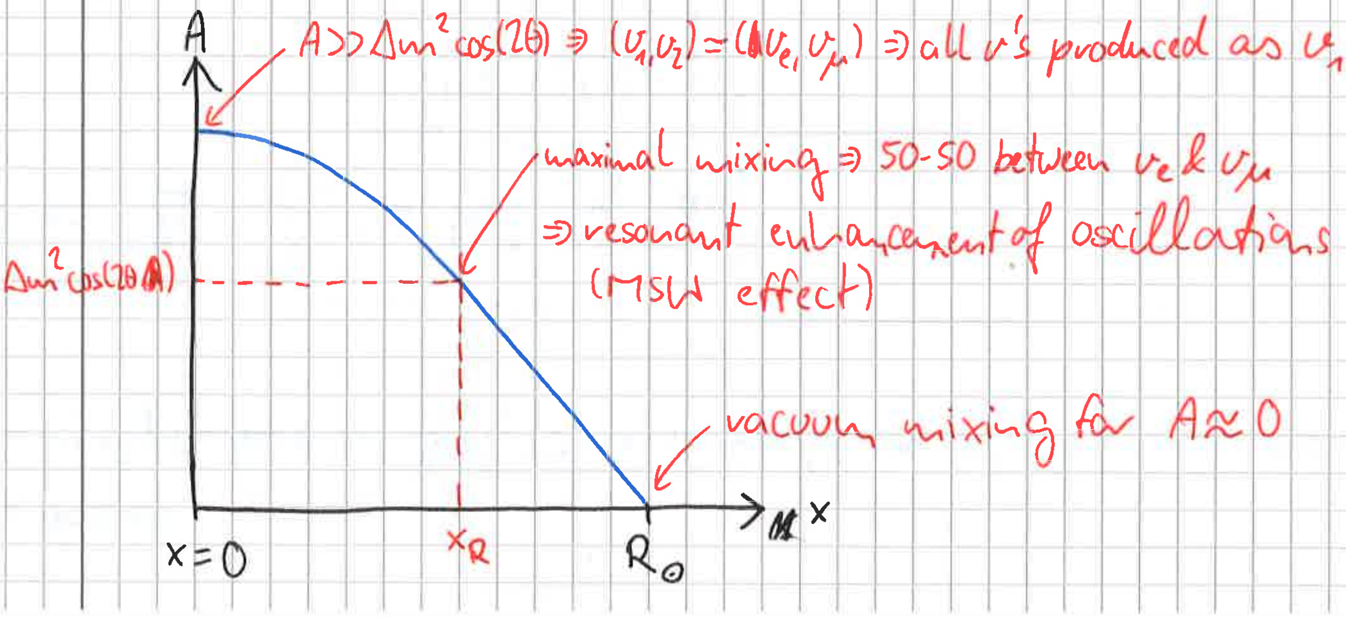
$\Rightarrow$ enhancement does NOT appear for antineutrinos!

↳ same if Δm^2 is negative for neutrinos

$\Rightarrow$ enhancement is sensitive to the sign of Δm^2

  - What if we start with very high density that ~~falls of~~ gradually falls off?

↳ just like inside the Sun:



(2.7)

↳ keep in mind: ~~[illegible]~~ $A \sim \rho \simeq E$

$\Rightarrow$ resonance enhancement also depends on the energy of the neutrinos

$\Rightarrow$ the MSW effect manifests itself in the neutrino spectrum, and this is what is measured

$\Rightarrow$ this is why we know: $\boxed{\Delta m_{21}^2 \equiv m_2^2 - m_1^2 > 0}$

$\Rightarrow$ thus, only two mass orderings are possible