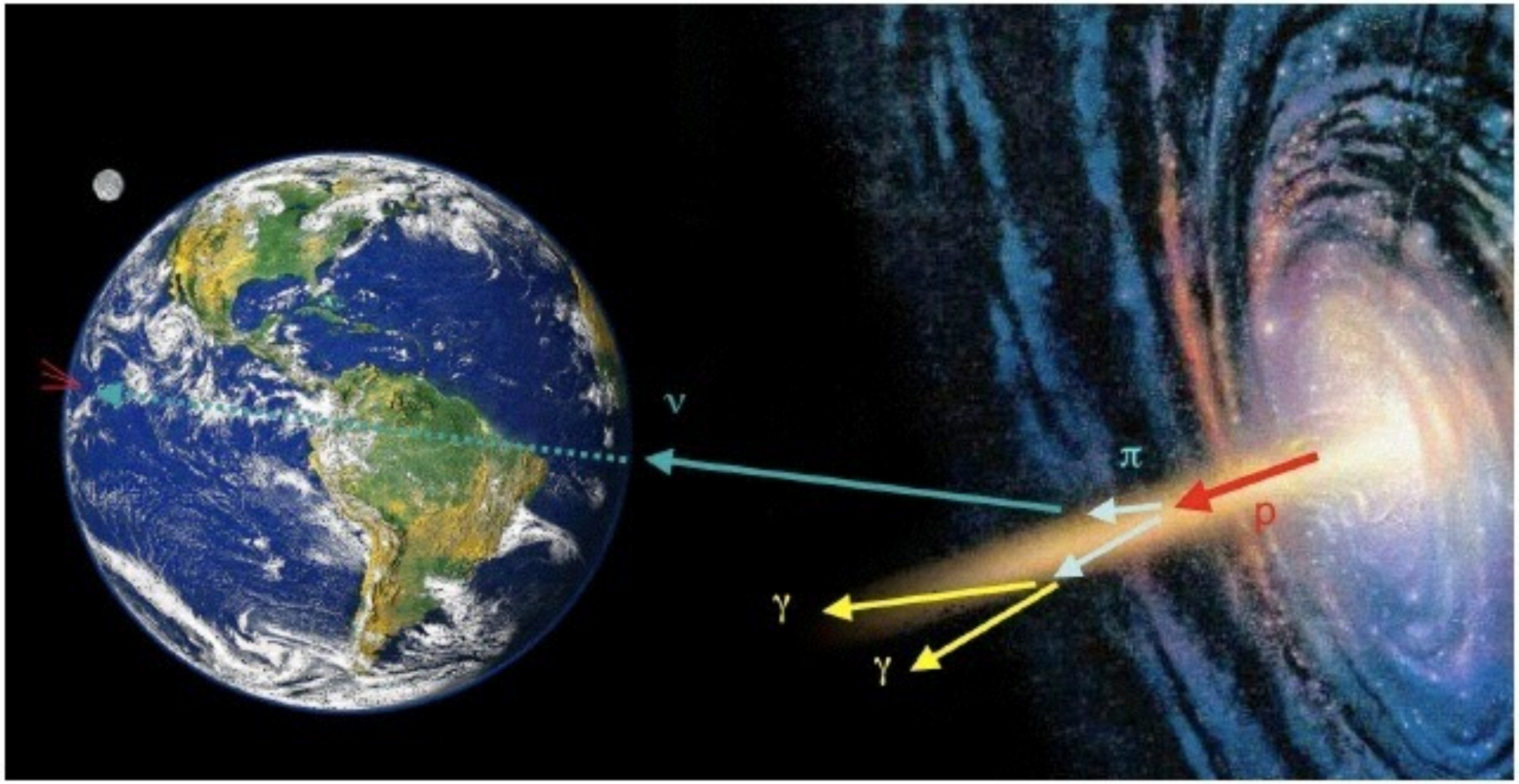
Teilchenphysik mit kosmischen und mit erdgebundenen Beschleunigern



06. Cosmic Accelerators

01.06.2015

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Prof. Dr. Siegfried Bethke
Dr. Frank Simon

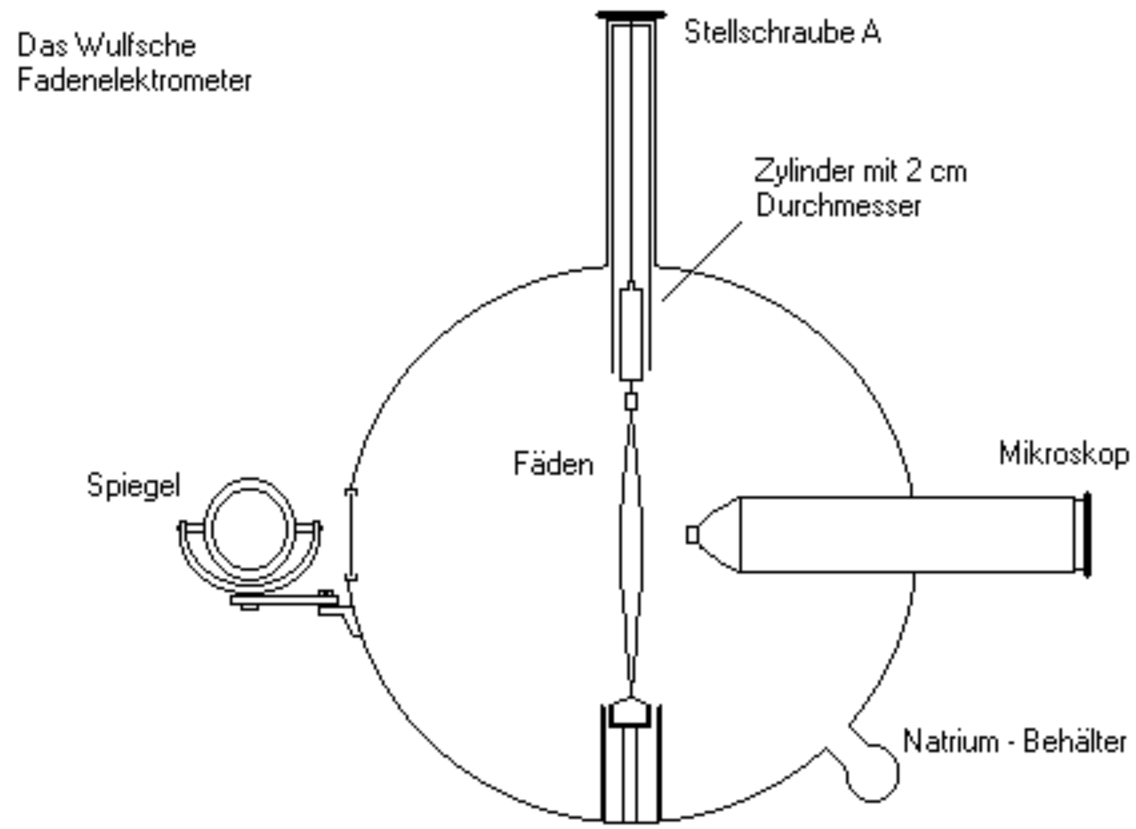
Cosmic Rays: Discovery

- Discovered by Victor Hess 1912
- Nobel Prize in physics 1936

- Observation on balloon flights with electroscopes:
  - Rate of discharge reduces with increasing altitude, up to an altitude of 1000 m
  - Above this a strong increase of the discharge rate is observed, at 5000 m it is several times higher than the rate at ground level

Cosmic Rays: Discovery

- The experimental method:
  - Electrometer, the distance of (electrostatically charged) strings gives the amount of charge on the strings
  - Discharge via ionising radiation



G. Federmann, Diplomarbeit, U. Wien, 2002

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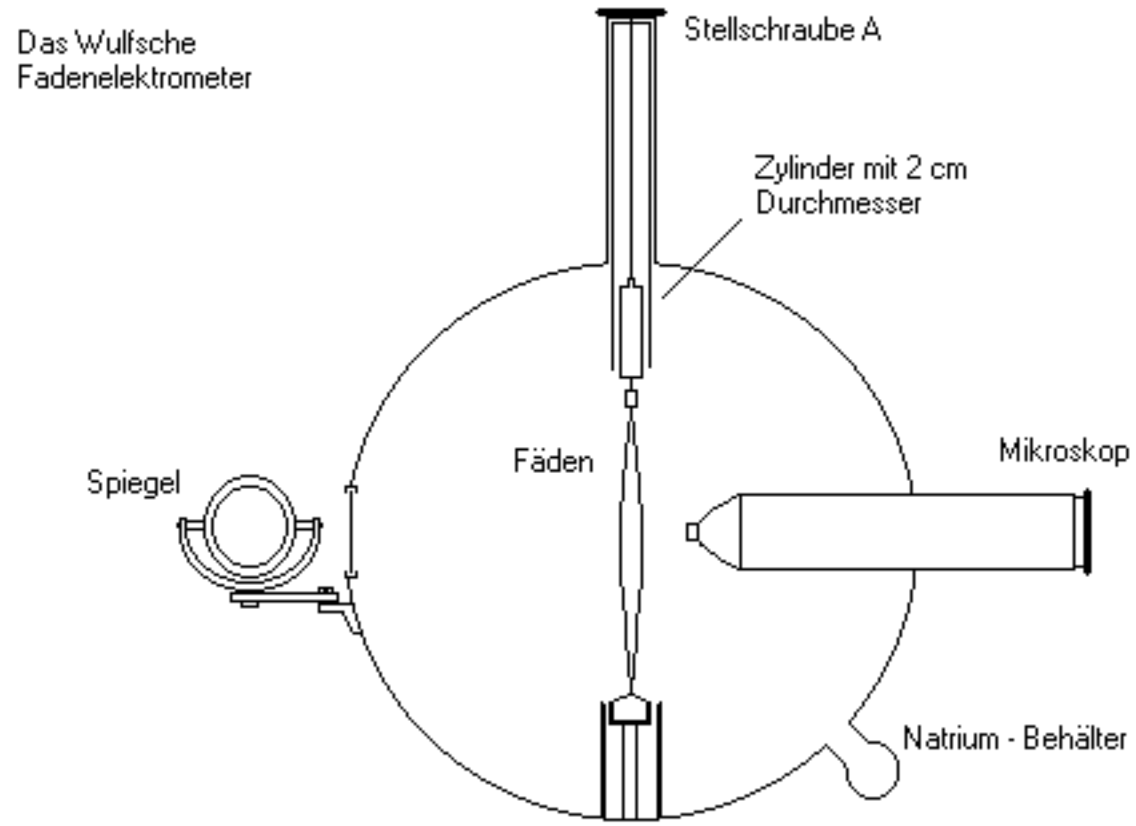

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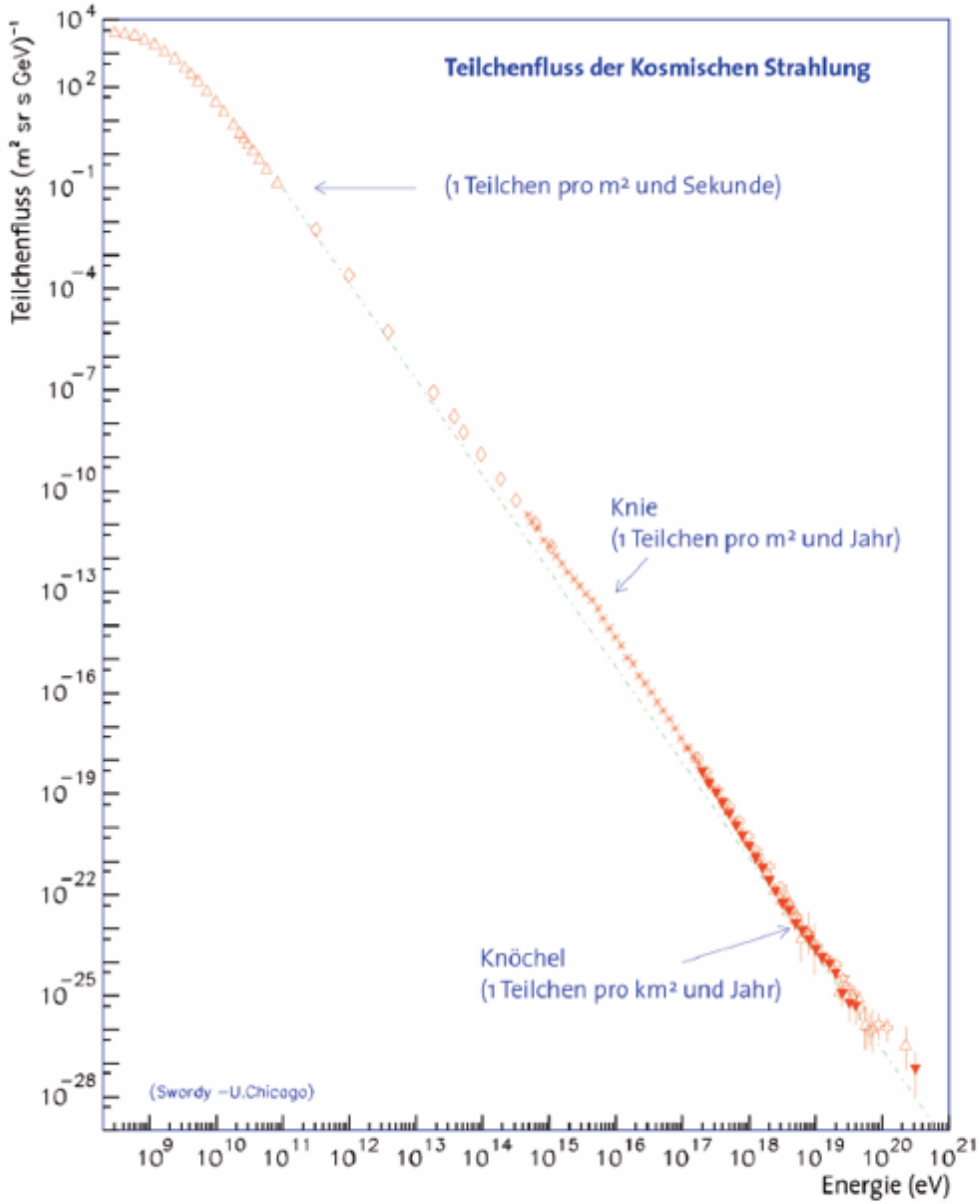
- Interpretation of the observation
  - Reduction of ambient radioactivity with increasing altitude (less radio nuclides, such as Radon)
  - The increase of radiation at high altitudes has to be due to extraterrestrial sources ➪ **"Höhenstrahlung"**

Cosmic Rays: Spectrum



- Extends over many orders of magnitude in energy and flux:
  - GeV (10^9 eV) - ZeV (10^{21})
  - >1 $cm^{-2}s^{-1}$ - < 1 km^{-2} per century
- Follows a power law:

$$\frac{dN}{dE} \propto E^{-\gamma}$$

- $\gamma \sim 2.7$ $E < 10^{15}$ eV
- $\gamma \sim 3.0$ 10^{15} eV < $E < 10^{18}$ eV
- $\gamma \sim 2.7$ 10^{18} eV < E

Energy Density of Cosmic Rays

- Differential flux on earth
  (parametrisation valid from ~ GeV to ~100 TeV):

$$\frac{dN}{dE} \approx \left(\frac{E}{\mathrm{GeV}}\right)^{-2.7} \frac{\mathrm{particles}}{m^2\, sr\, s\, \mathrm{GeV}}$$

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- Energy density:

$$\rho_E = \frac{4\pi}{c} \int E \frac{dN}{dE} \approx 1 \frac{eV}{cm^3}$$

Cosmic Rays: Power

- Rough estimate of the energy of cosmic rays within the Milky Way

$$V = \pi R^2 d \;\sim\; \pi (15\,kpc)^2\,(200\,pc) \sim\; 4 \times 10^{60}\,m^3$$

$$\Rightarrow\; E_{Strahlung} \;\sim\; 4 \times 10^{66} eV \sim\; 6.4 \times 10^{47}\,J$$

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As comparison:
therm. power of the sun
~4×10^{26} W,
Milky Way ~2×10^{11} stars

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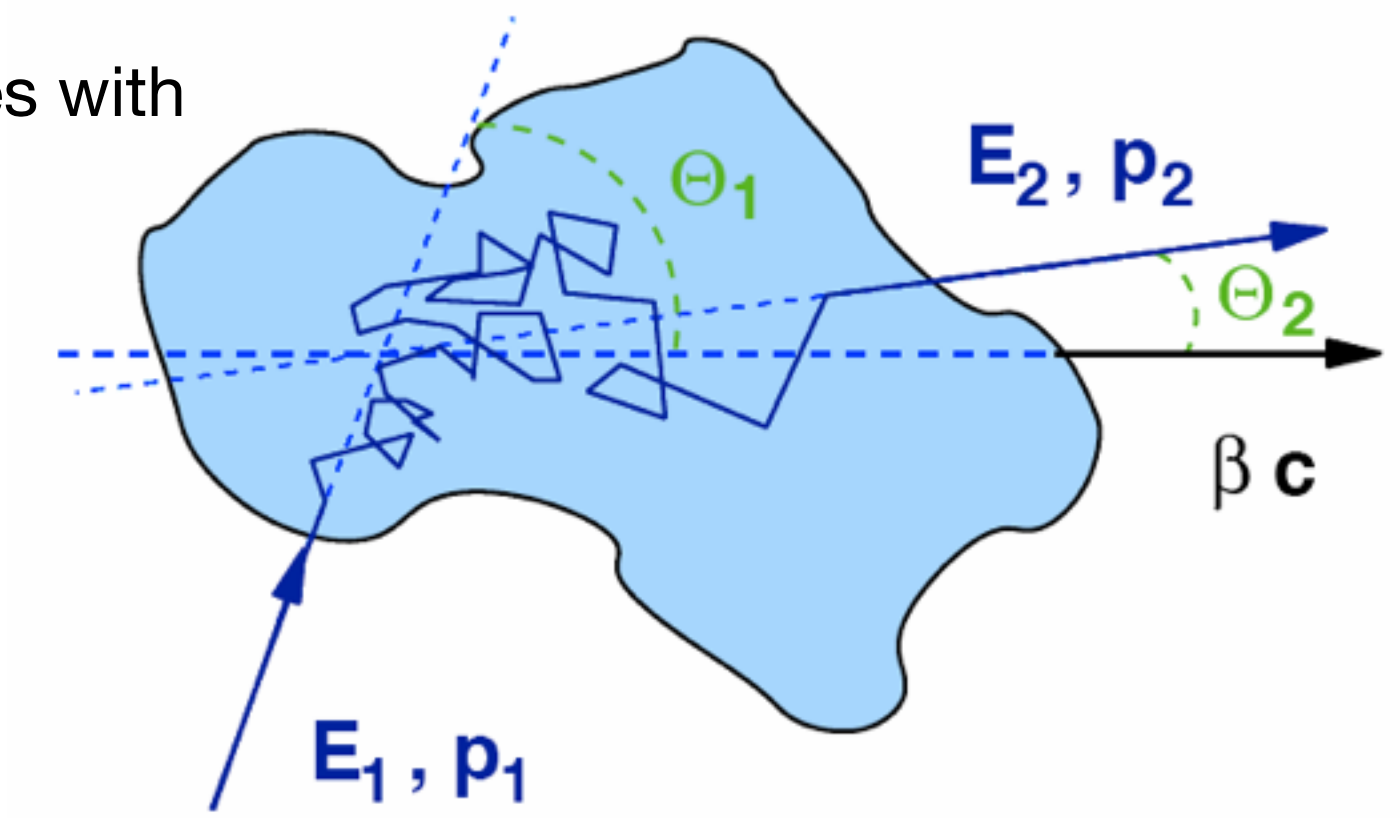
Second Order Fermi Acceleration

- Proposed by Enrico Fermi 1949
- The principle: Collisions of particles with interstellar gas clouds
  - Particle speed ~ c
  - Cloud speed βc
  - entrance and exit angle relative to cloud direction Θ_1, Θ_2



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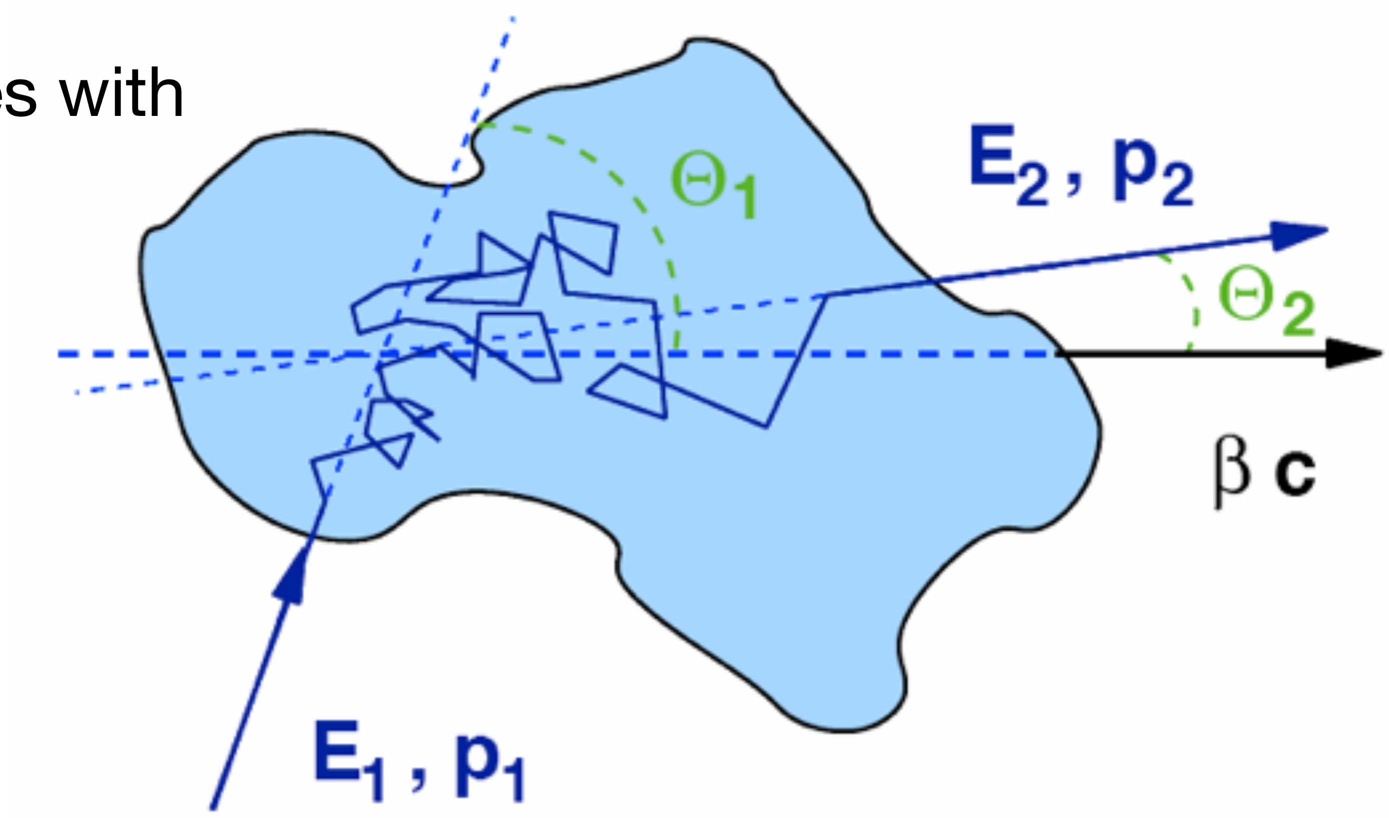

- Boost into cloud rest frame:

$$E_1' = \gamma E_1 - \beta\gamma p_{||} \;=\; \gamma E_1 - \beta\gamma cos\Theta_1\, p \approx\; \gamma E_1(1 - \beta cos\Theta_1)$$

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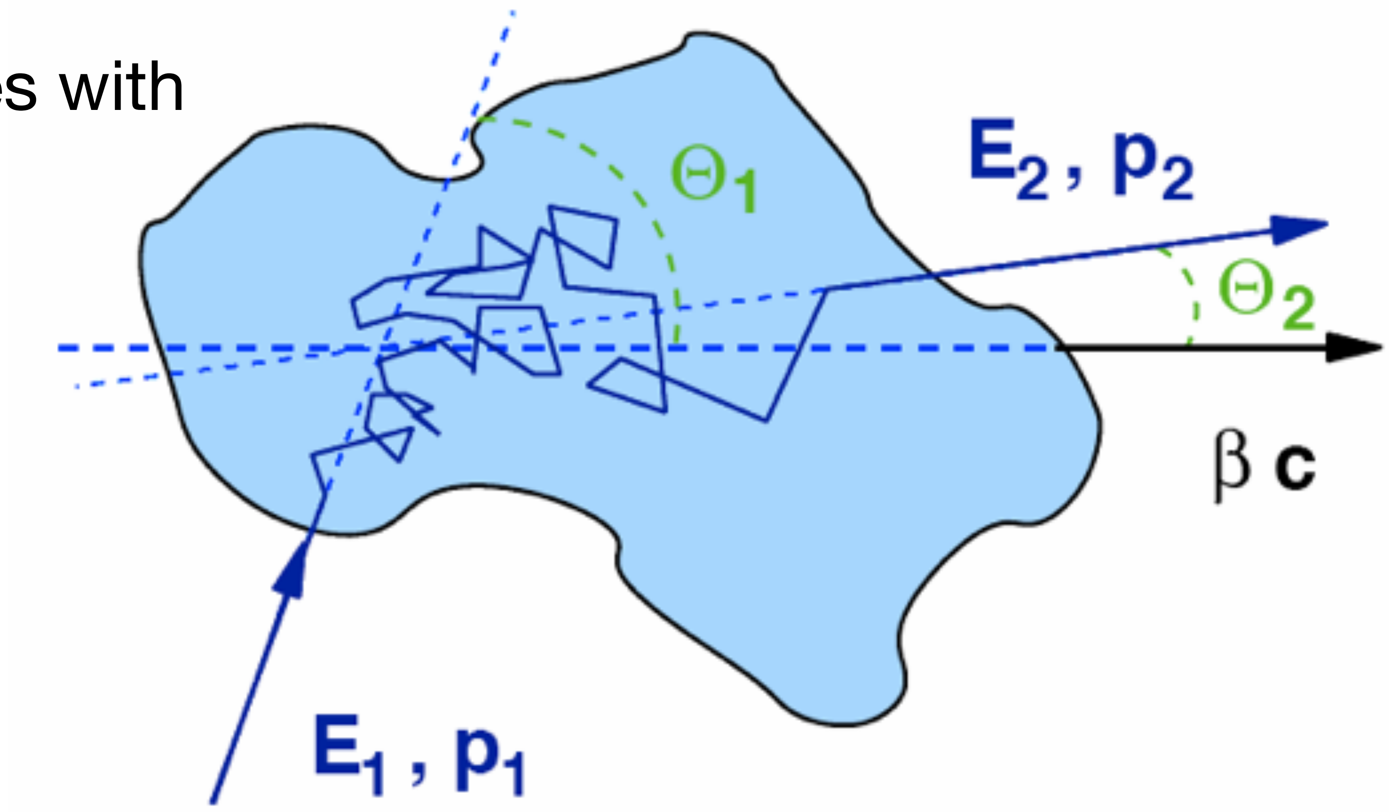

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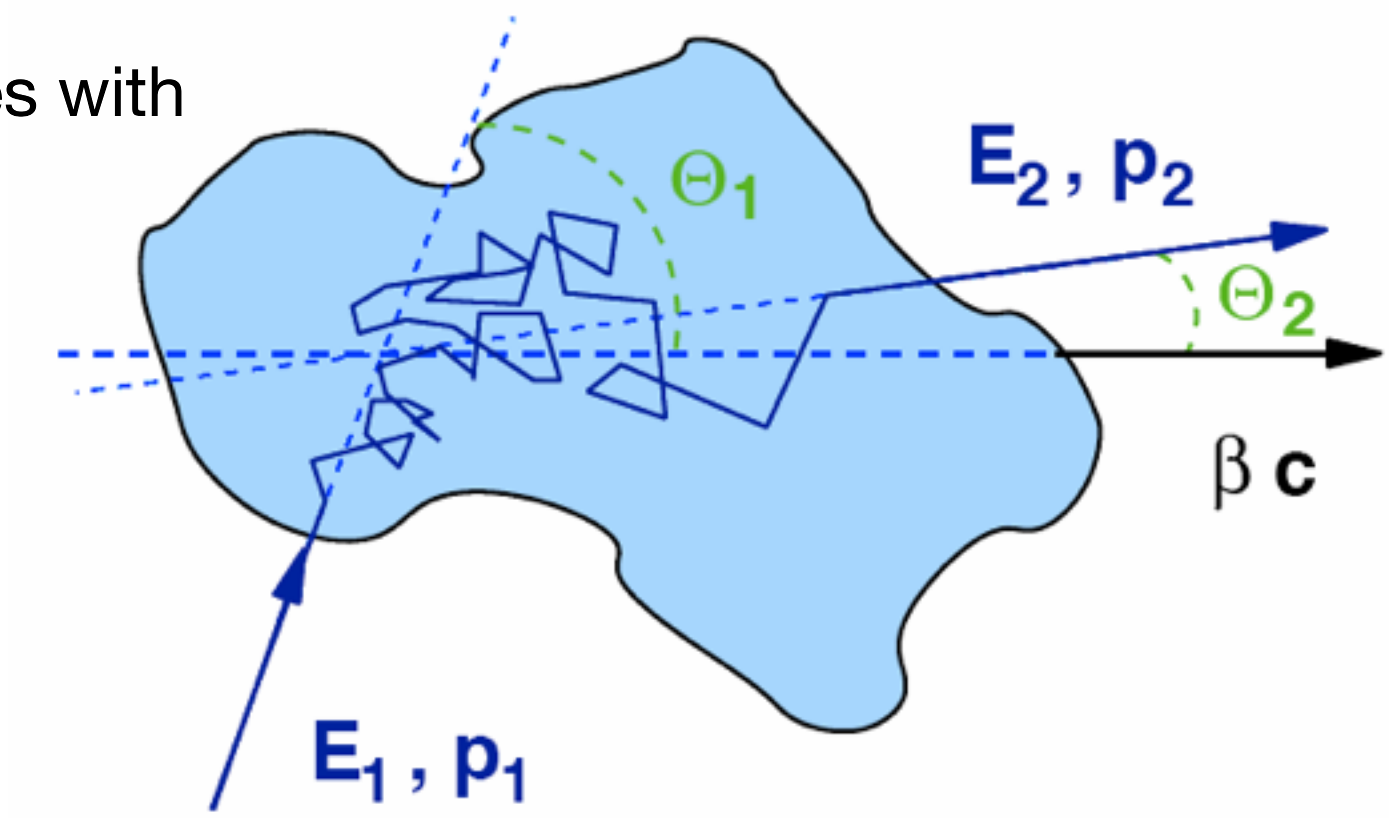

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- Energy difference $$\frac{\Delta E}{E} = \frac{E_2 - E_1}{E_1} = \frac{E_2}{E_1} - 1$$

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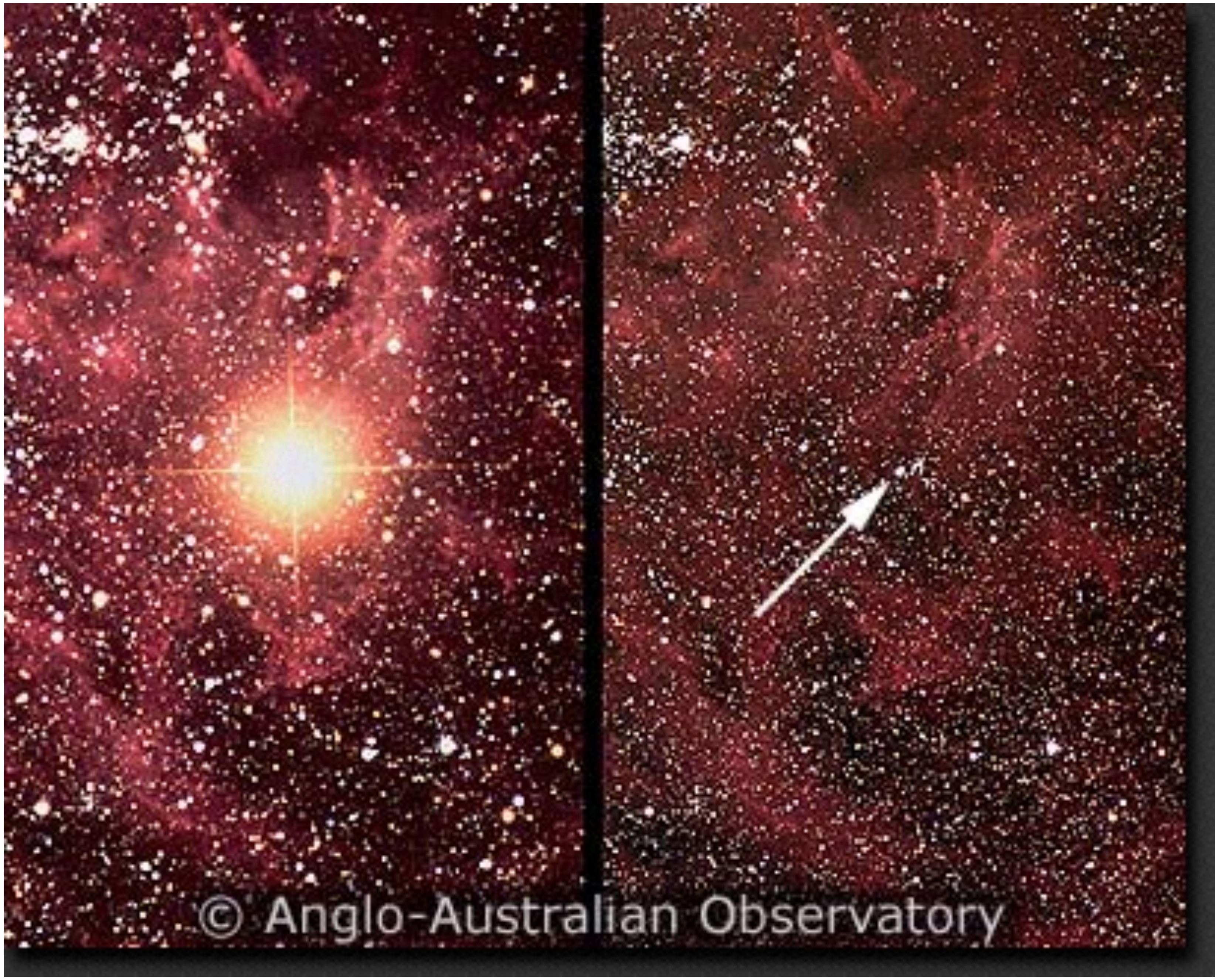
Characteristic time: $$\frac{3\tau}{4\beta^2} \approx 6\times 10^{10} a$$

Supernovae

- Classification in to main types
  - SN I: no hydrogen lines in spectrum
    - SN Ia collapse of an accreting white dwarf in a binary star system to a neutron star
  - SN II: hydrogen lines visible
    - Gravitational collapse of a massive star at the end of its life
    - Star burns up to the formation of iron in the core, then no radiation pressure to counter gravitation
      - Atoms are converted to neutrons via electron capture
      - Star collapses with a speed of ~ 0.1 c
      - Matter is reflected at the stable neutron star in the core
      - A shock wave runs outwards
      - An enormous number of neutrinos is produced ($\sim 10^{58}$), despite their small interaction cross section they further drive the shock wave

Supernova SN1987a

- Supernova explosion 1987 in the great Magellanic Cloud (small partner galaxy of the Milky Way)



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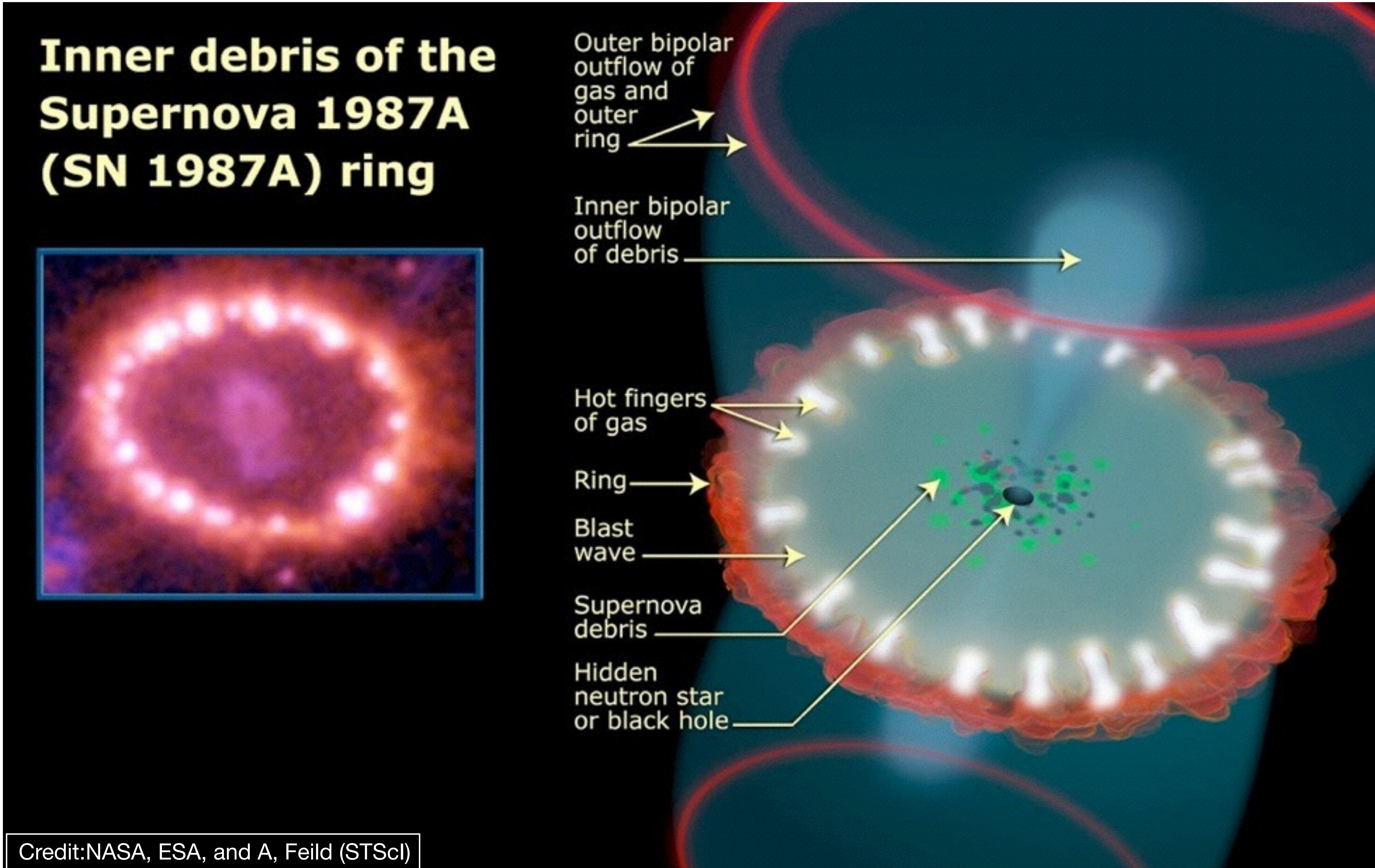
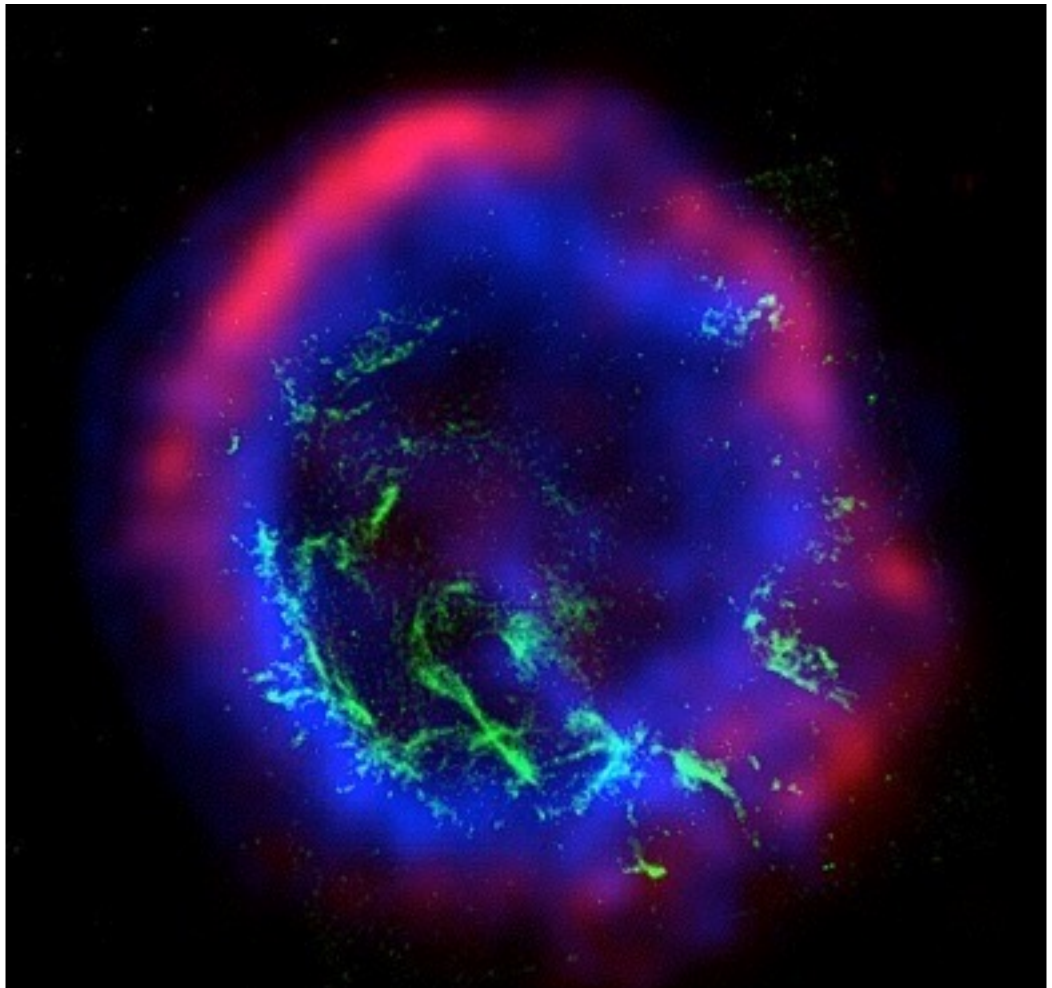


Credit:NASA, ESA, and A, Feild (STScI)

First Order Fermi Acceleration

- Extension of Fermi acceleration concept to supernova shocks
- Formation of the shock wave:
  - SN ejects large amounts of material (several solar masses) with high velocity into the interstellar medium
  - $v_{material} >> v_{sound(ISM)}$, $v_{material}$ ~ 10^7 m/s , $v_{sound(ISM)}$ ~ 10^4 m/s
  - ▸ Since the matter is much faster than the speed of sound a shock front develops

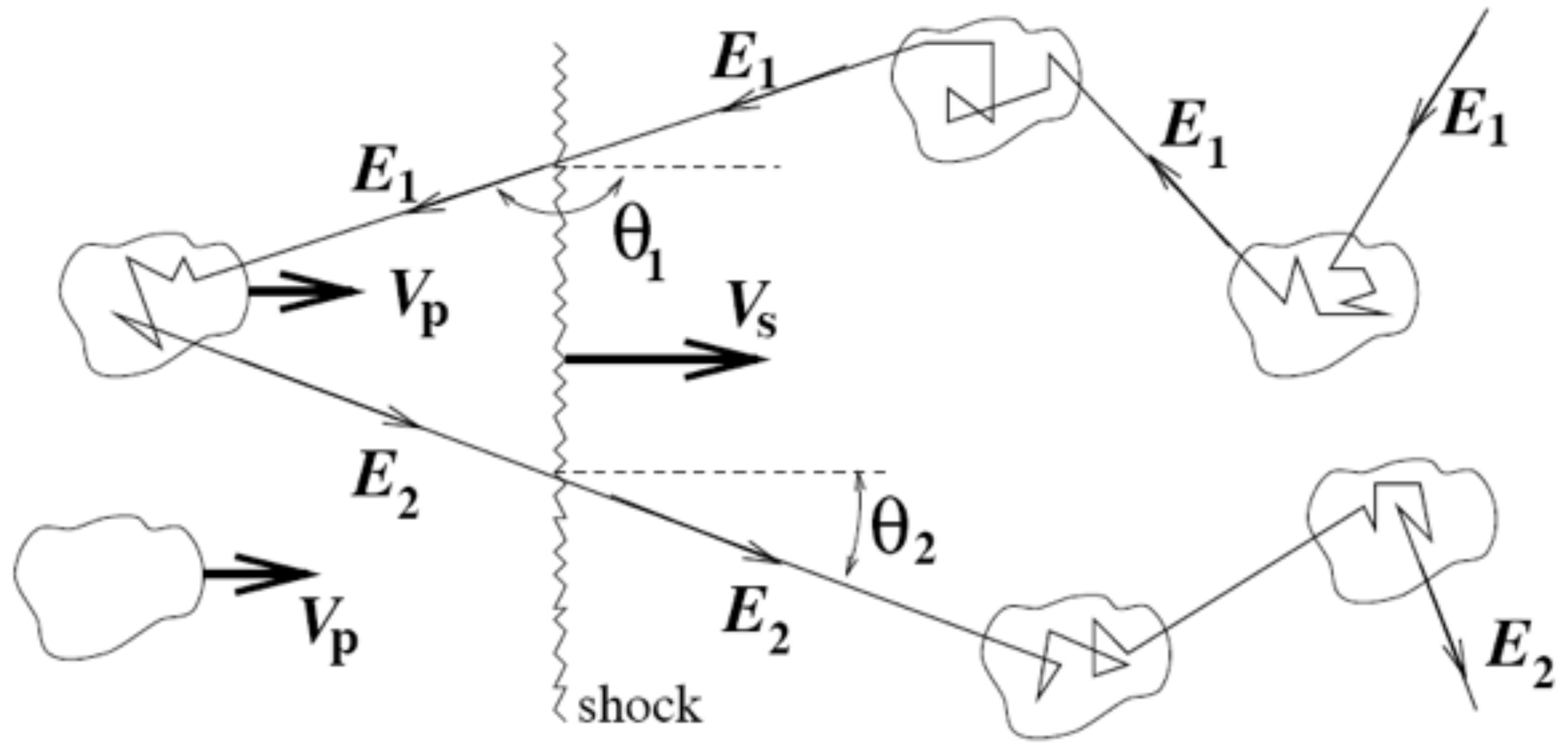

Credit: X-ray: NASA/CXC/SAO, Optical: NASA/HST, Radio: CSIRO/ATNF/ATCA

  - The shock wave propagates in the ionized plasma of the ISM (single atoms, specific heat 5/3)
  - Hydrodynamics can show that the speed of the shock wave is:
    $v_{shock}/v_{material}$ ~ 4/3

First Order Fermi Acceleration

- Particle acceleration by repeated crossing of shock fronts

- As for second order Fermi Acceleration the particles are scattered by magnetic field inhomogeneities / turbulences
  - Key “feature”: behind the shock, these turbulences move with the speed of the ejected matter



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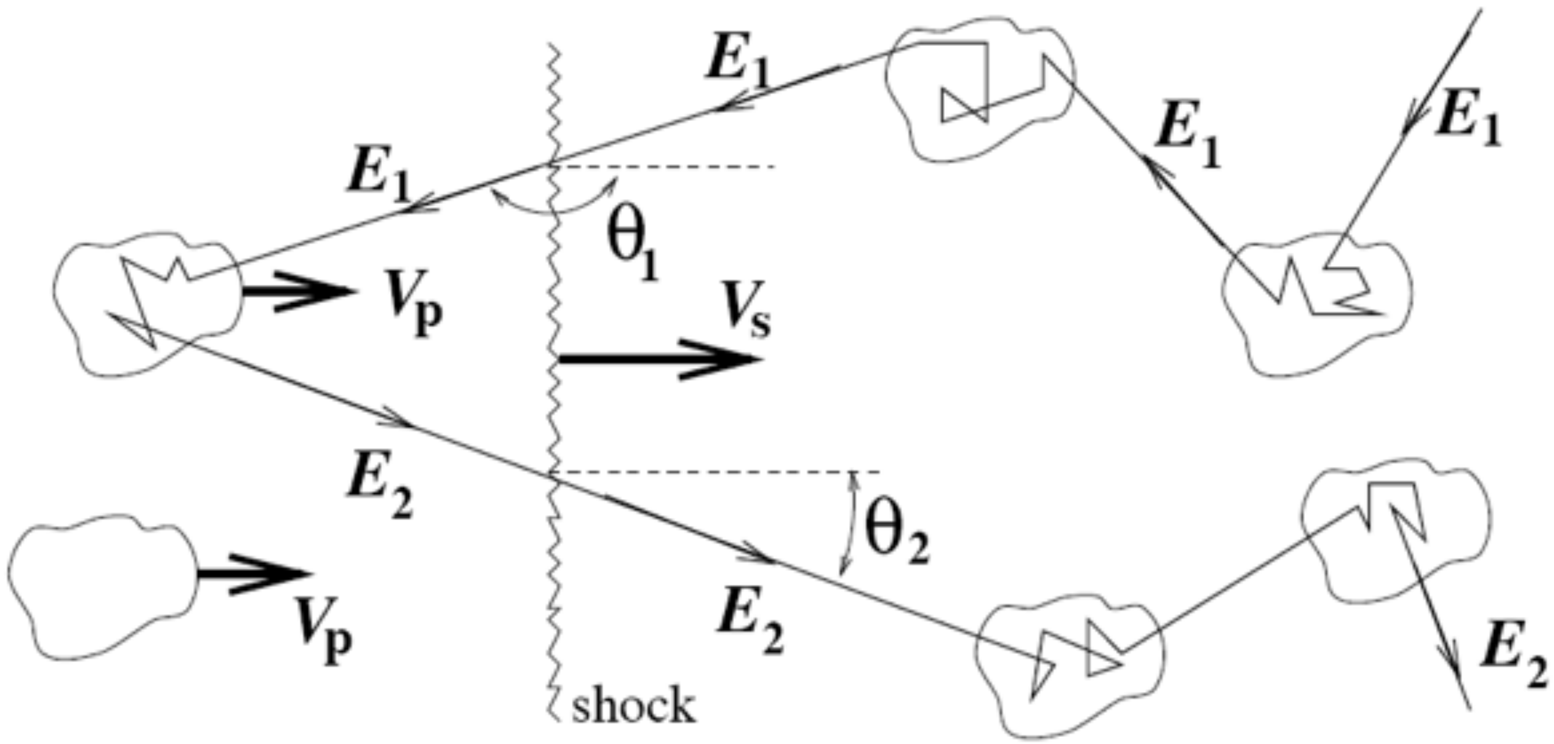

- Considerations concerning incidence angles of particles:
  - Shock front introduces directional motion - projection of flux onto front

Effective area $A' = -A\,cos\Theta_1$

Flux crossing shock $\frac{dN}{dcos\Theta_1} \propto -cos\Theta_1$

First Order Fermi Acceleration

- Mean value of angles (key point here: Crossing of shock only for $\cos\Theta_1 < 0$, other particles do not contribute!):

$$\langle cos\Theta_1 \rangle = \frac{\int_{-1}^{0} \frac{dN}{dcos\Theta_1} \, cos\Theta_1 \, dcos\Theta_1}{\int_{-1}^{0} \frac{dN}{dcos\Theta_1} \, dcos\Theta_1}$$

$$= \frac{\int_{-1}^{0} -cos^2\Theta_1 \, dcos\Theta_1}{\int_{-1}^{0} -cos\Theta_1 \, dcos\Theta_1} = -\frac{2}{3}$$

analog: $\langle cos\Theta_2 \rangle = \frac{2}{3}$

First Order Fermi Acceleration

- Mean energy change (analogous to second order FA):

$$\frac{\Delta E}{E} = \frac{(1-\beta cos\Theta_1)(1+\beta cos\Theta_2')}{1-\beta^2} - 1$$

β: Speed of matter behind shock wave

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- for β << 1

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- Substantially more efficient than second order acceleration due to two effects:
  - large velocity differences of material before and after shock front
  - directed motion of shock instead of random drifting
- $\beta \sim 3 \times 10^{-2}$, acceleration linear in β (first order in β)

Energy Spectrum

- Energy gain per cycle

$$\frac{\langle \Delta E \rangle}{E} = \zeta \approx \frac{4}{3} \beta_{Plasma} \approx \beta_{Schock}$$

$$\Rightarrow \quad E = E_0 \, (1 + \zeta)^k$$ after k cycles

- Reminder:
  $v_{shock}/v_{material}$ ~ 4/3
  for strong shocks

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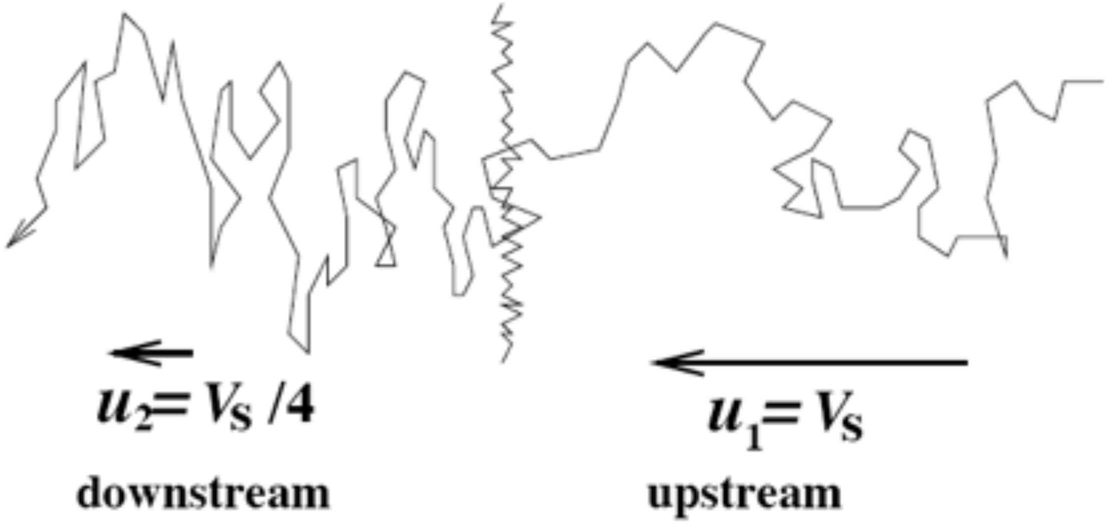
  - particles can be lost downstream of the shock

behind the shock the plasma is $v_s/4$ slower than the shock wave itself, particles "diffuse" in the plasma

=> A particle can get lost if it falls to far behind the shock



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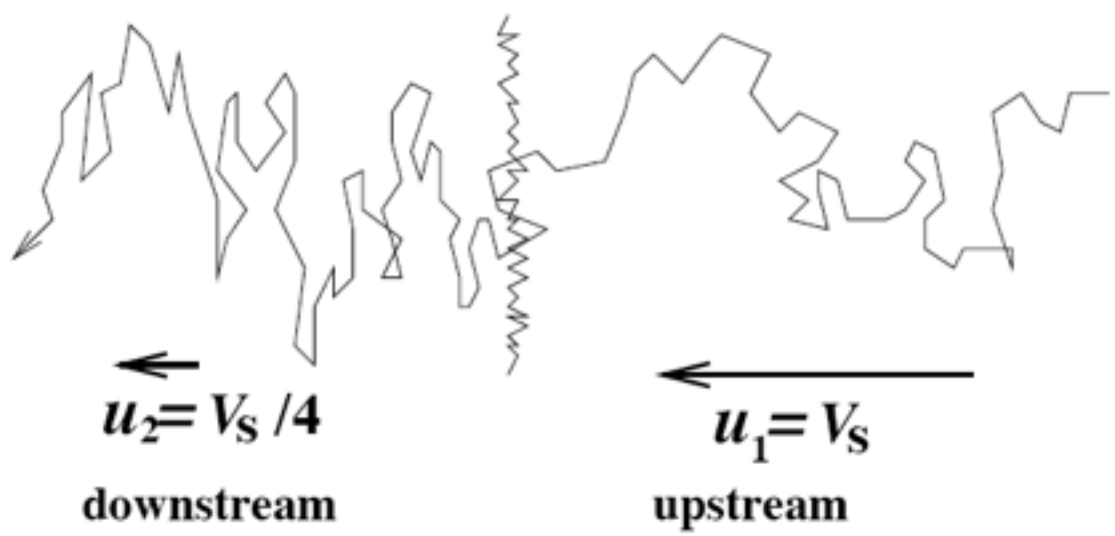

In general: the material behind the shock is 1/4 slower than the front itself, the loss rate depends on that difference:

$$R_{loss} = n_{CR} v_s / 4 \qquad n_{CR} \text{ is the particle density}$$

Energy Spectrum

- Particle flux from "upstream" through the shock:

  Particle movement relative to shock (particle velocity v_t)

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$$R_{cross} = n_{CR} \frac{1}{4\pi} \int_{-v_s/v_t}^{1} (v_s + v_t cos\theta) 2\pi \, dcos\theta \;\approx\; v_t n_{CR}/4$$

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Frank Simon (fsimon@mpp.mpg.de)

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NB: $v_t \sim c$

The probability to cross the shock front at least k times is:

$$P_{cross>k} \;=\; (1 - P_{escape})^k \;=\; \left(1 - \frac{v_s}{v_t}\right)^k \;\approx\; (1 - \beta_{Schock})^k$$

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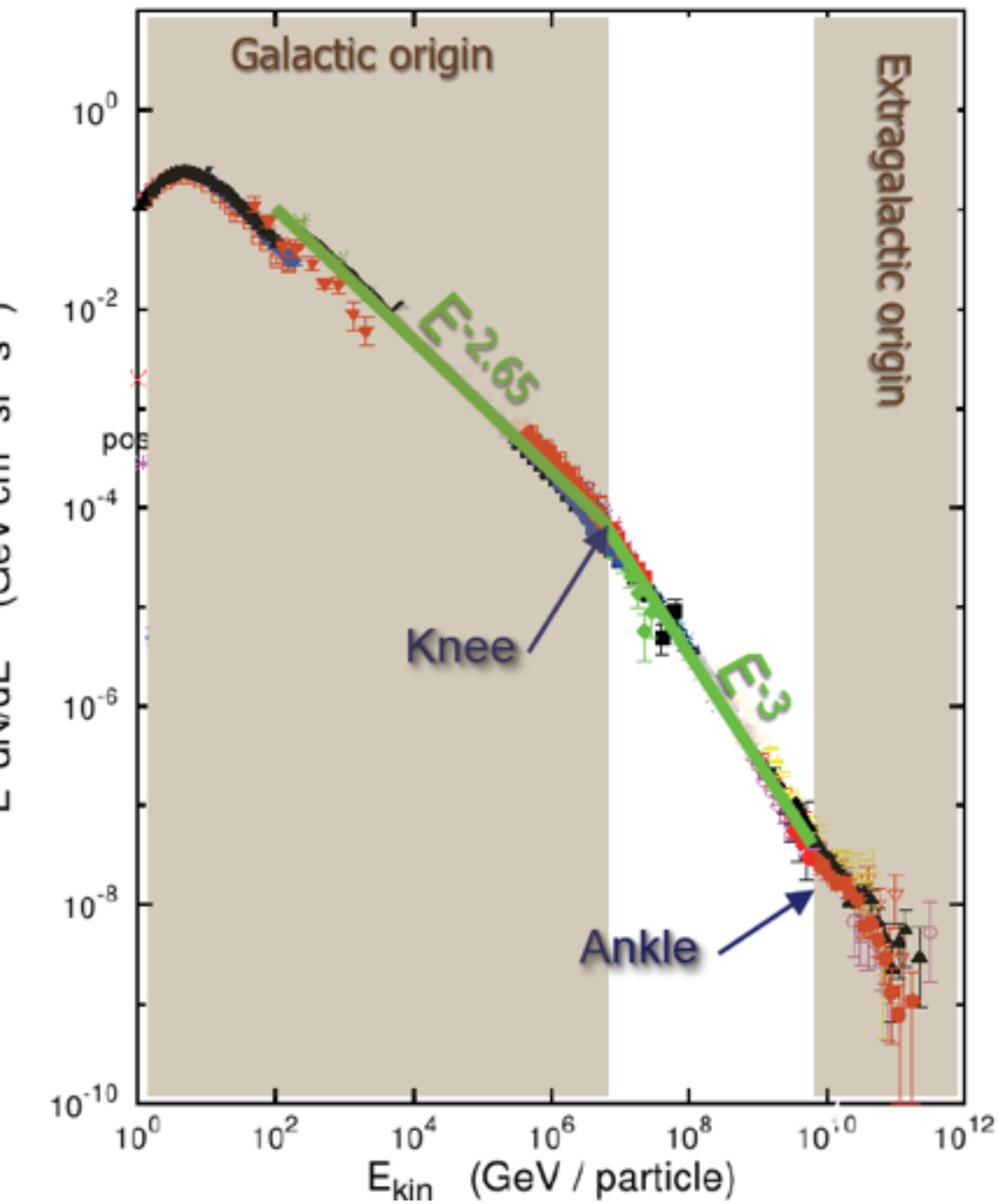
- Differential particle spectrum

$$\frac{dN}{dE} \;\propto\; E^{-2}$$

Maximum Energy

- First order Fermi Acceleration can reach energies up to ~ 10^{14} eV for shock waves originating from supernova explosions (incomplete derivation in Backup)



based on a shock lifetime of ~ 1000 years a shock speed of 0.03 c, and a B field in the nT range

➯ Extends up to the knee of the cosmic ray spectrum

Supernova shock acceleration is well established as a source for cosmic rays

But: What is the origin of the very highest energies above 10^{18} GeV?

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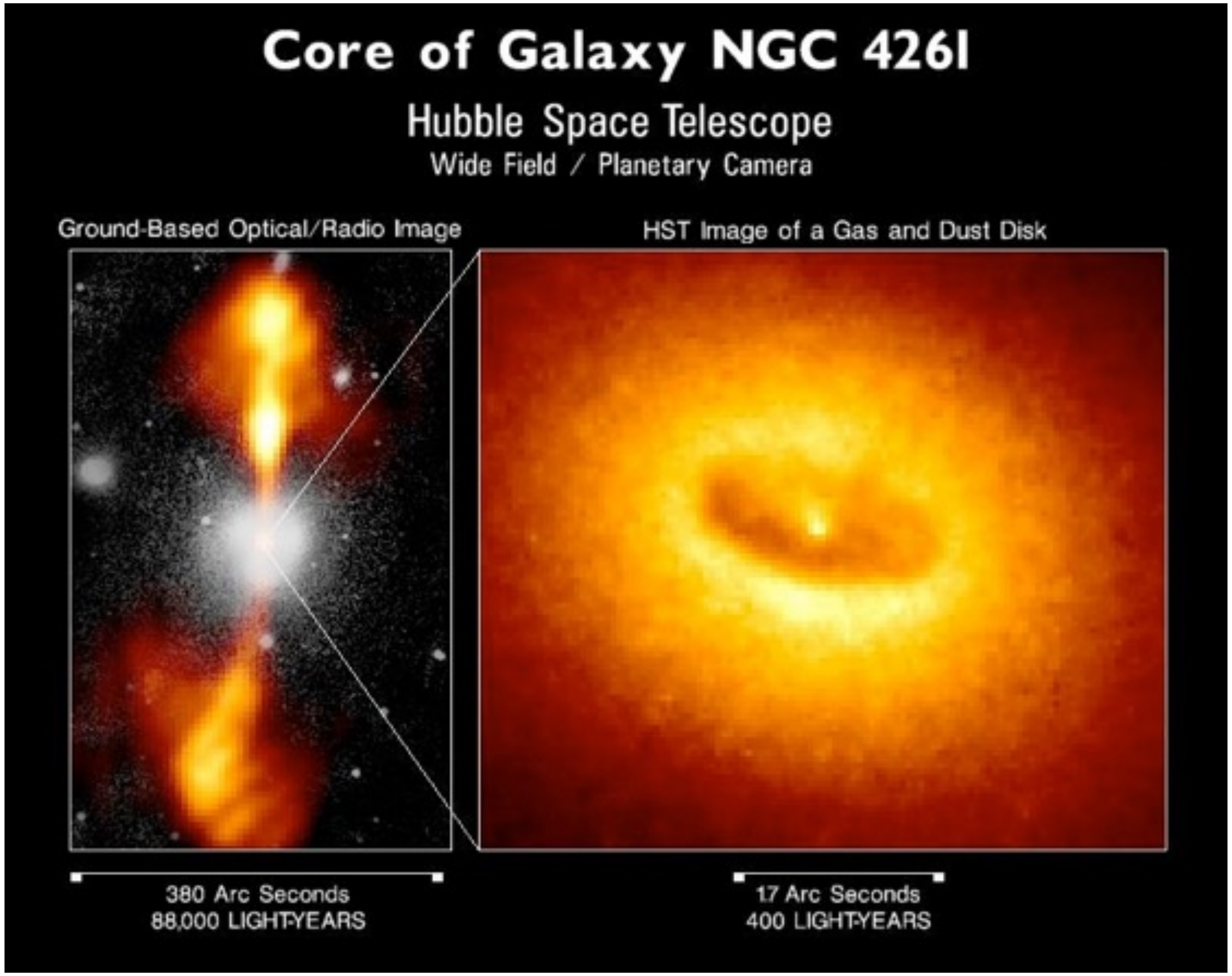
Highest Energies?

- How are energies > 1 PeV reached?

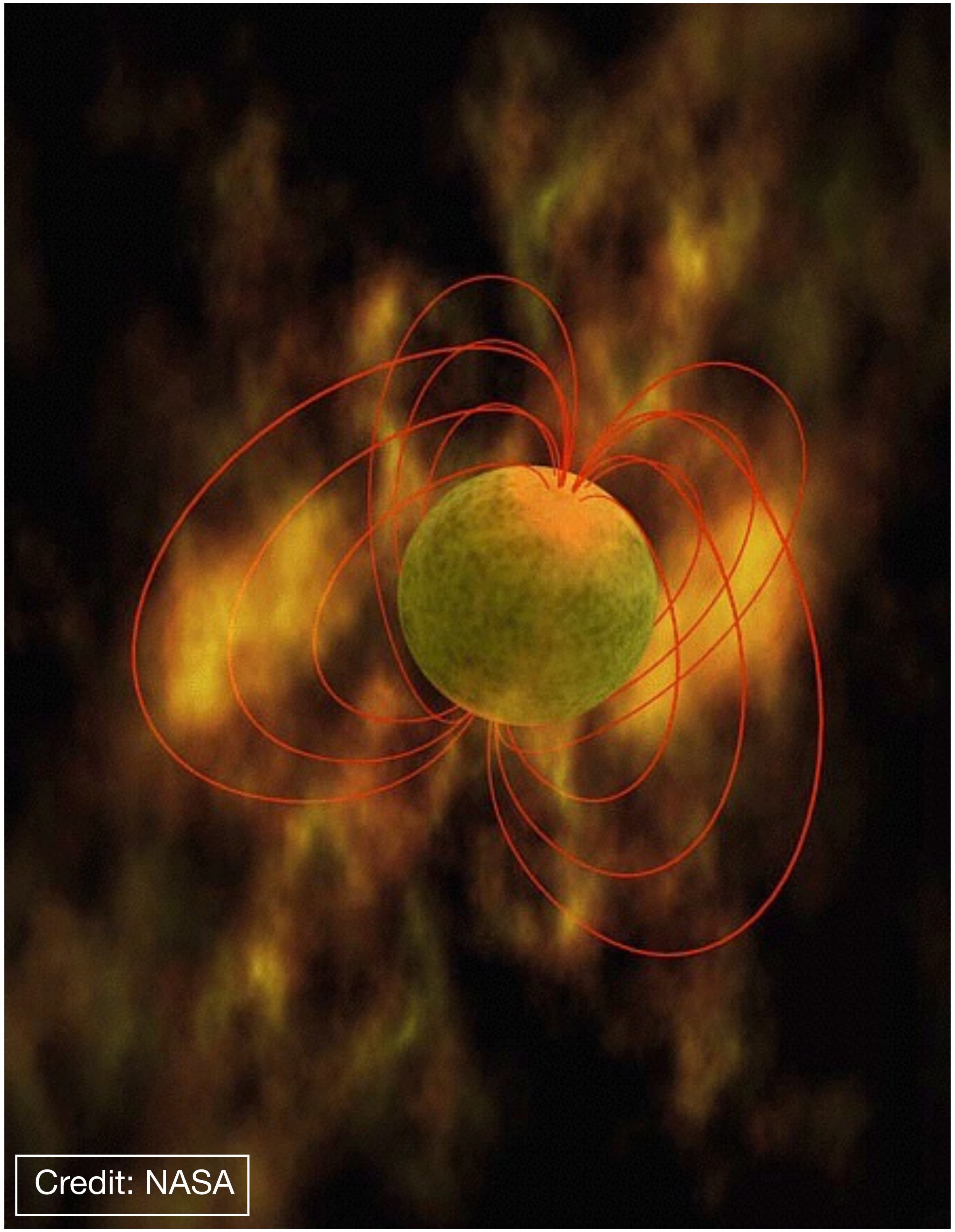


- More energetic events
  - Active galactic nuclei
  - Pulsars (neutron stars)
  - GRB's

➯ Extreme magnetic fields

➯ Shock acceleration in highly relativistic jets: additional γ - Factor

One Example: Neutron Stars



- Neutron stars: compact remnants of supernova explosions
  - radius ~ 10 km
  - extreme rotation: up to ~ 40 000 RPM
  - magentic fields up to ~ 10^8 T
  - mass ~ 1.4 M_{Sun}

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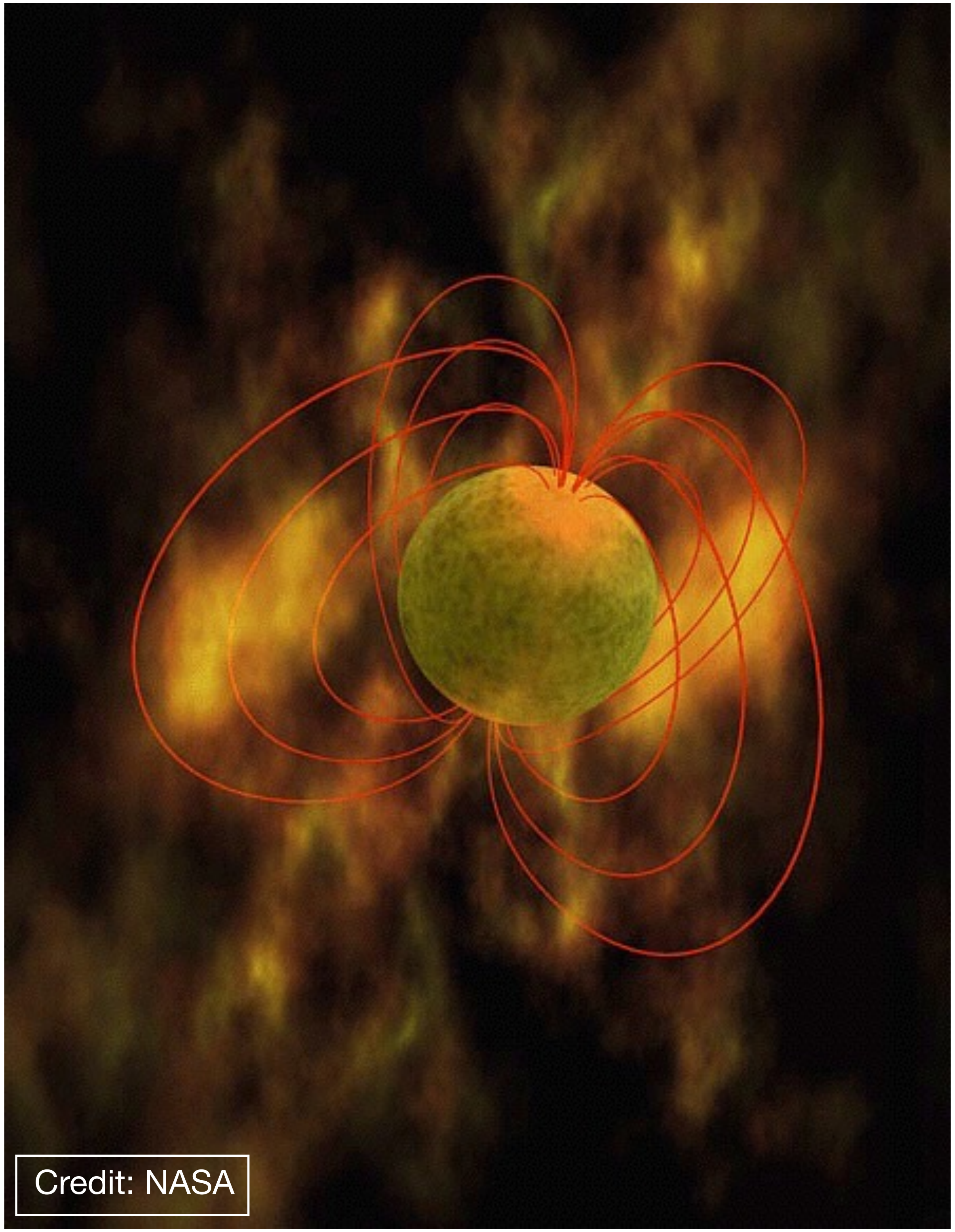

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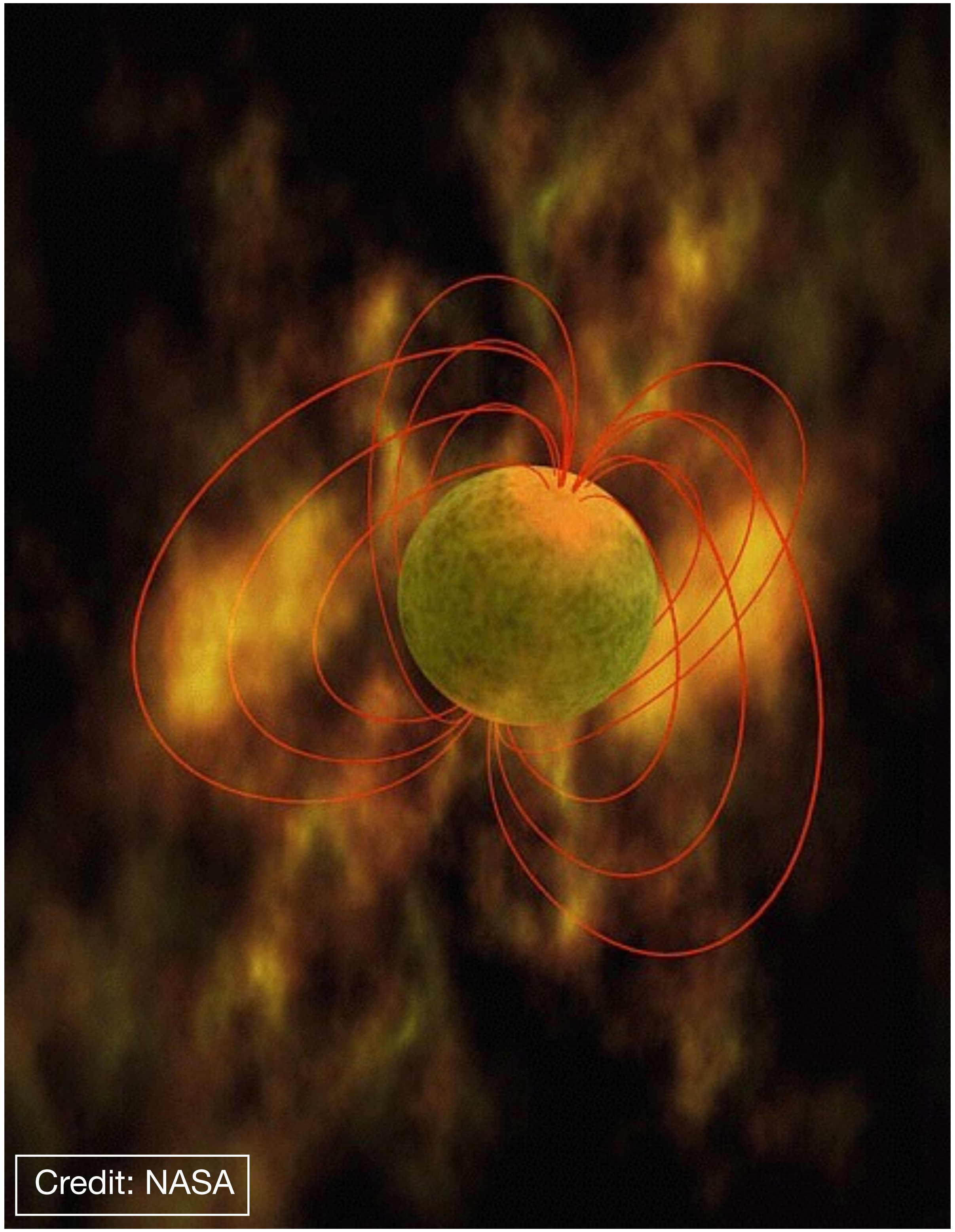

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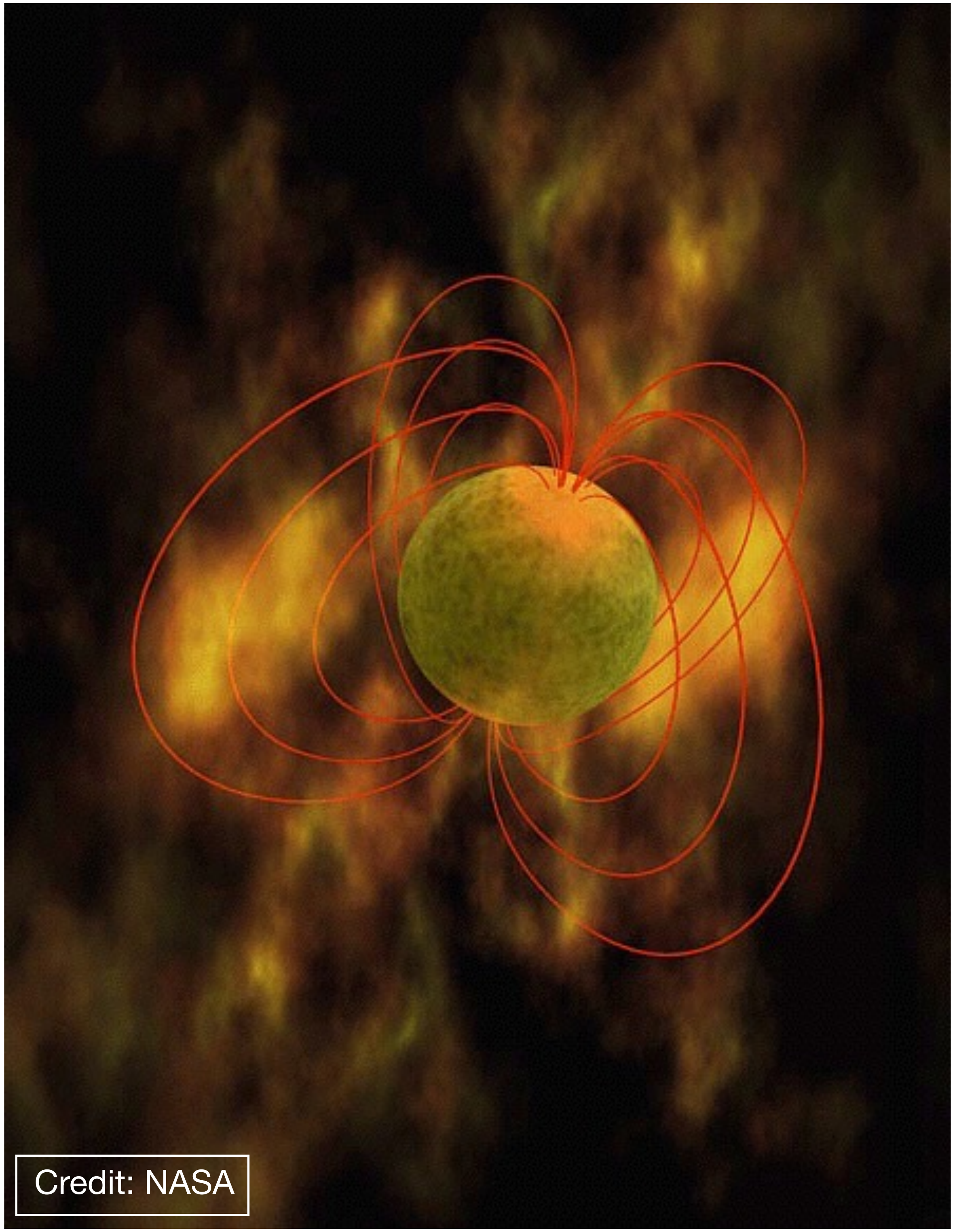

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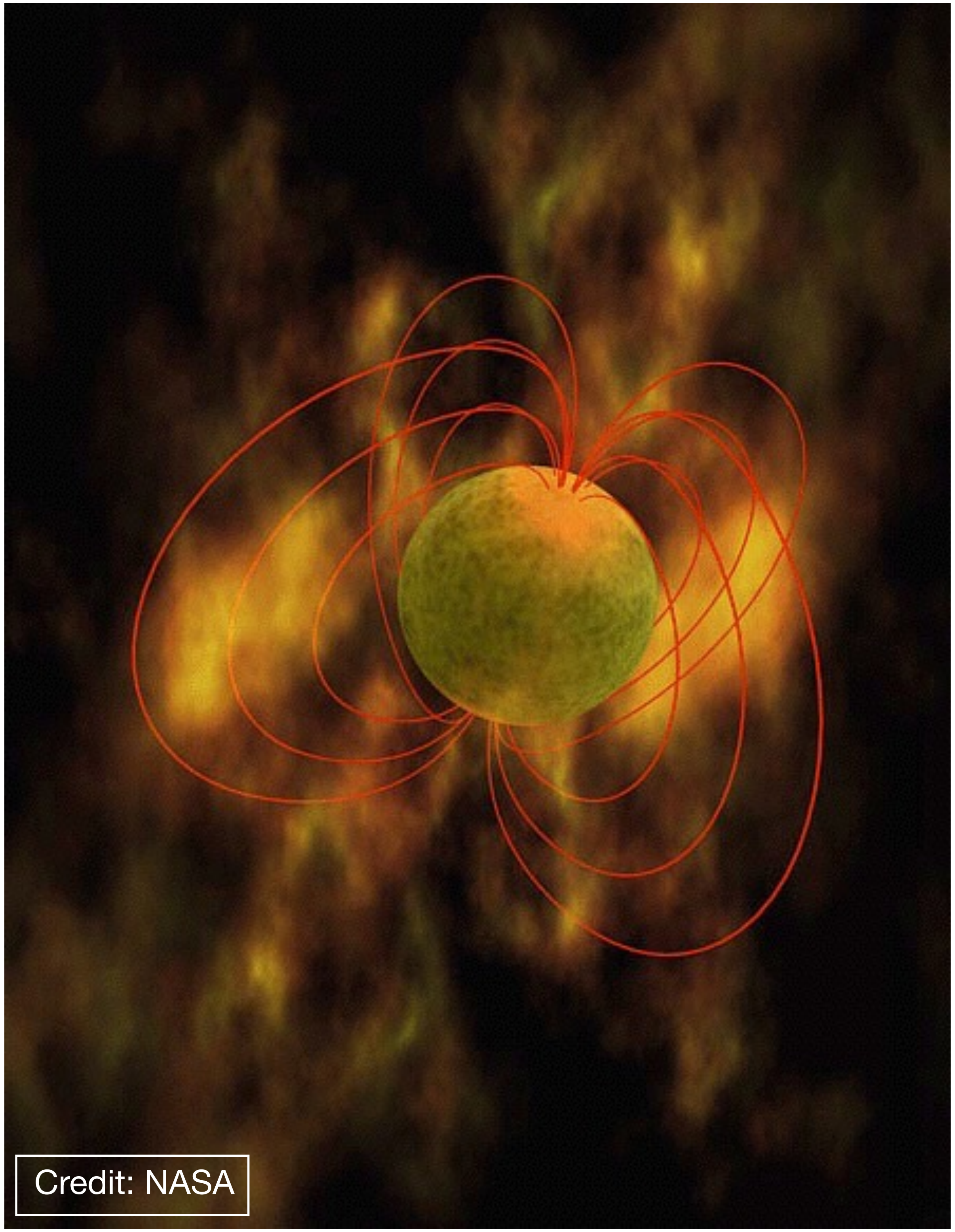

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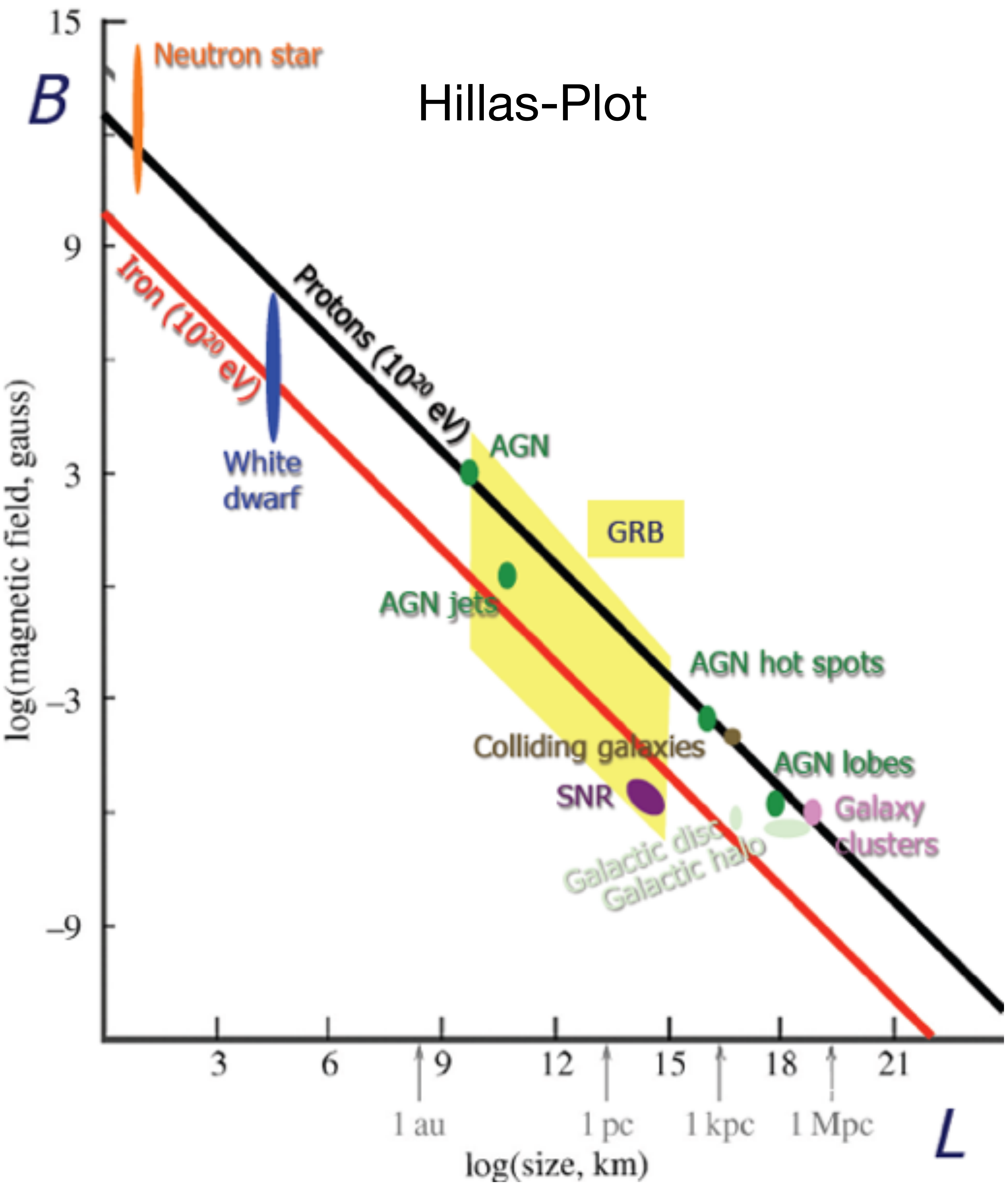
For Z = 1 (protons):

E_{max} ~ 5 J = 3 x 10^{20} eV

Candidates for the Highest Energies



Particles are accelerated as long as they stay in the high field region: $r_L < L$

$$E_{max} \approx 10^{20} eV \, Z B_{\mu G} L_{100 kpc}$$

For Iron nuclei 26 x higher energies are possible relative to protons!

S. Coutu, TIPP11

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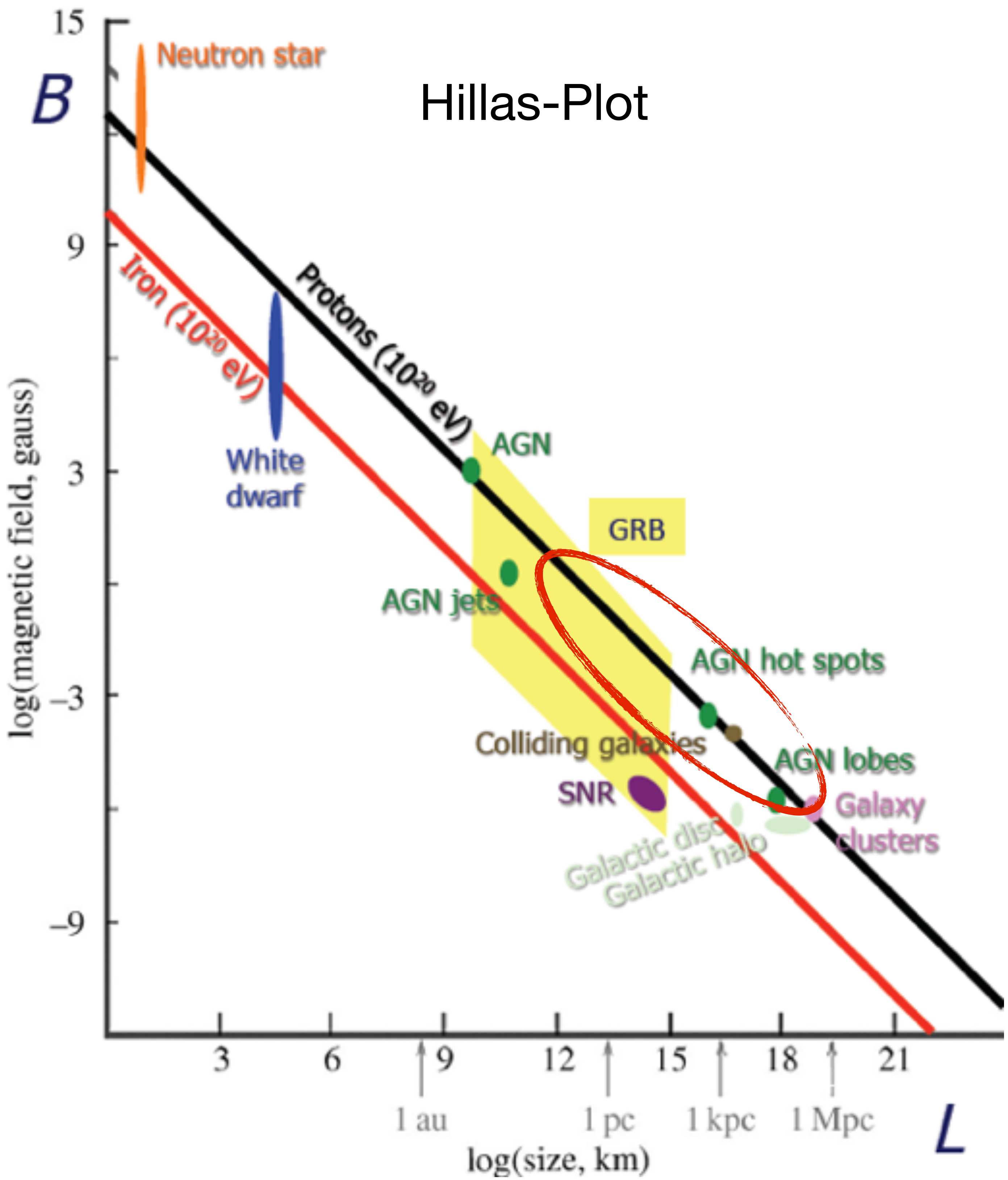

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Beyond this simple consideration:
Radiation losses in the source have to be taken into account:
synchotron radiation, photo reactions

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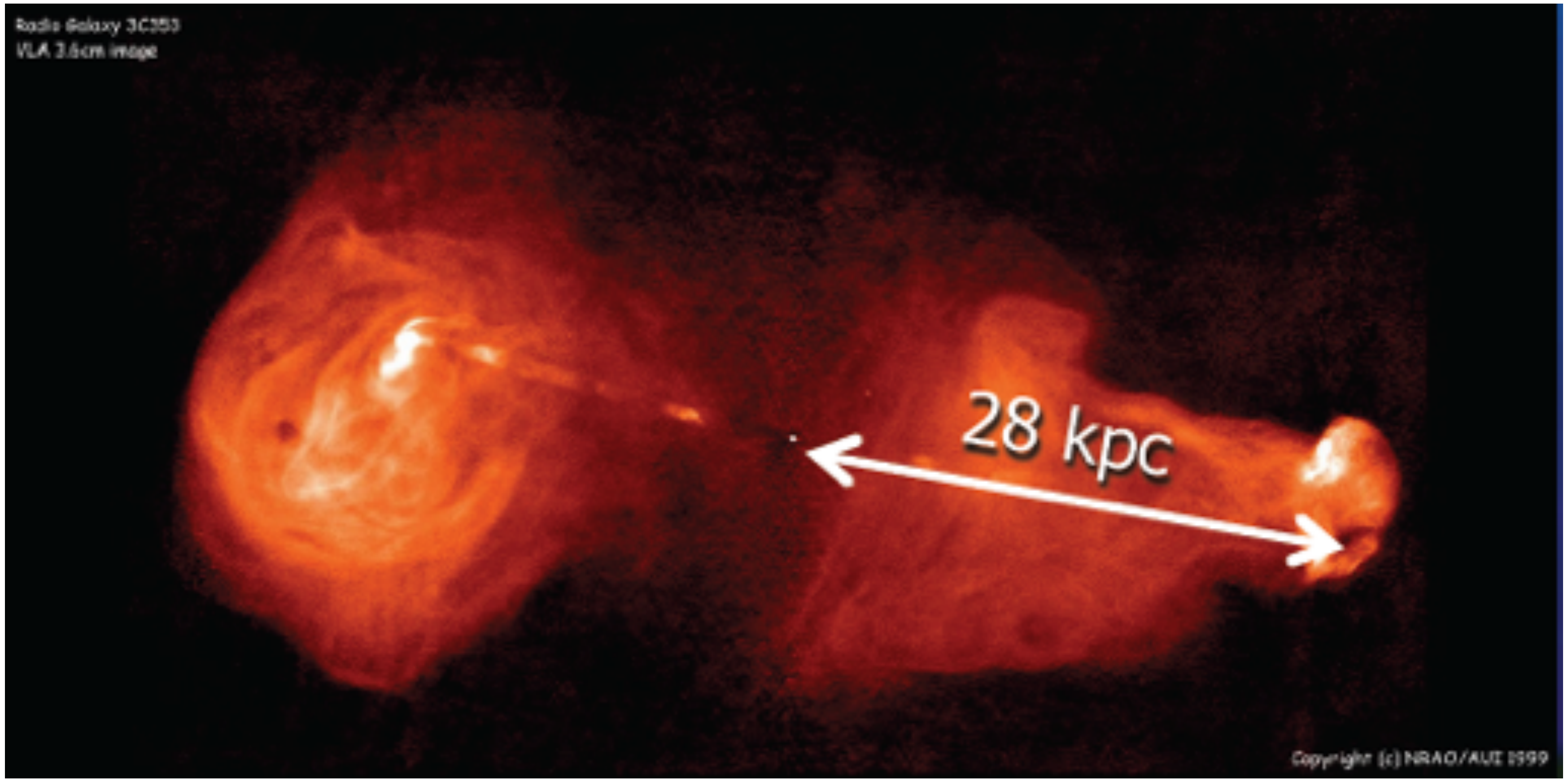
One Example

- 3C353 - Active galaxy 130 Mpc away



… more about the highest energies next week!

Propagation of Cosmic Rays

- The source spectrum of shock acceleration follows an E^{-2} distribution, but we observe $E^{-2.7}$, why?
- ‣ Energy-dependent loss of particles when travelling through the galaxy

- Important contributions:
  - diffusion
  - convection
  - acceleration

  transport in turbulent galactic magnetic fields

  - decay of unstable particles and nuclei
  - collisions
  - cascade production, spallation of heavy nuclei

  loss processes

Leaky Box Model

- Very simple model assuming cosmic rays propagate freely in the galaxy with a constant escape / loss probability

$$N(E,t) = N_0(E) e^{-t/\tau_{escape}}$$

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- Adaptation to reality - escape / loss probability is energy dependent:
  - particles with higher energy can more easily leave the “magnetic confinement” in the galaxy and are more likely to participate in inelastic reactions
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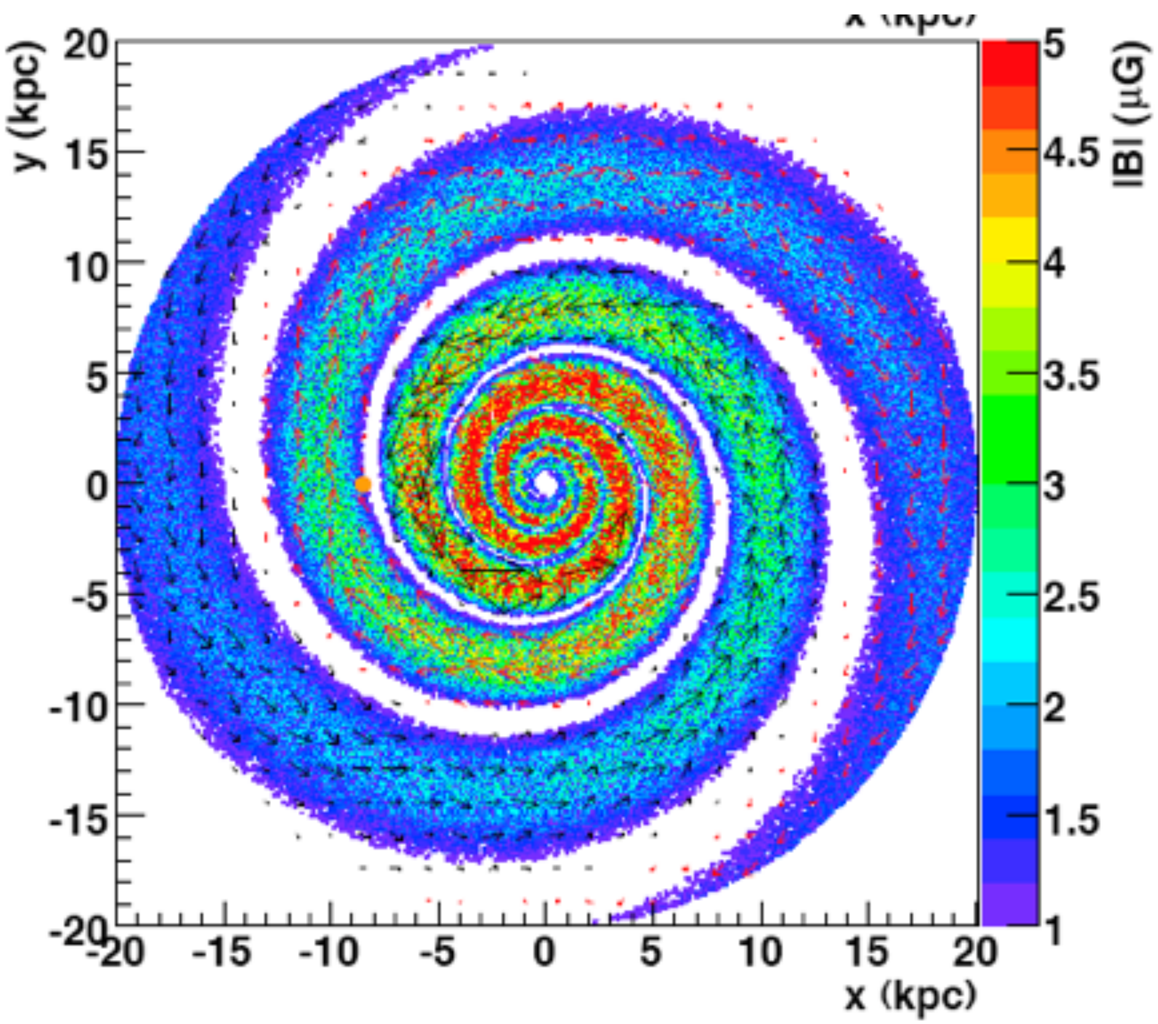
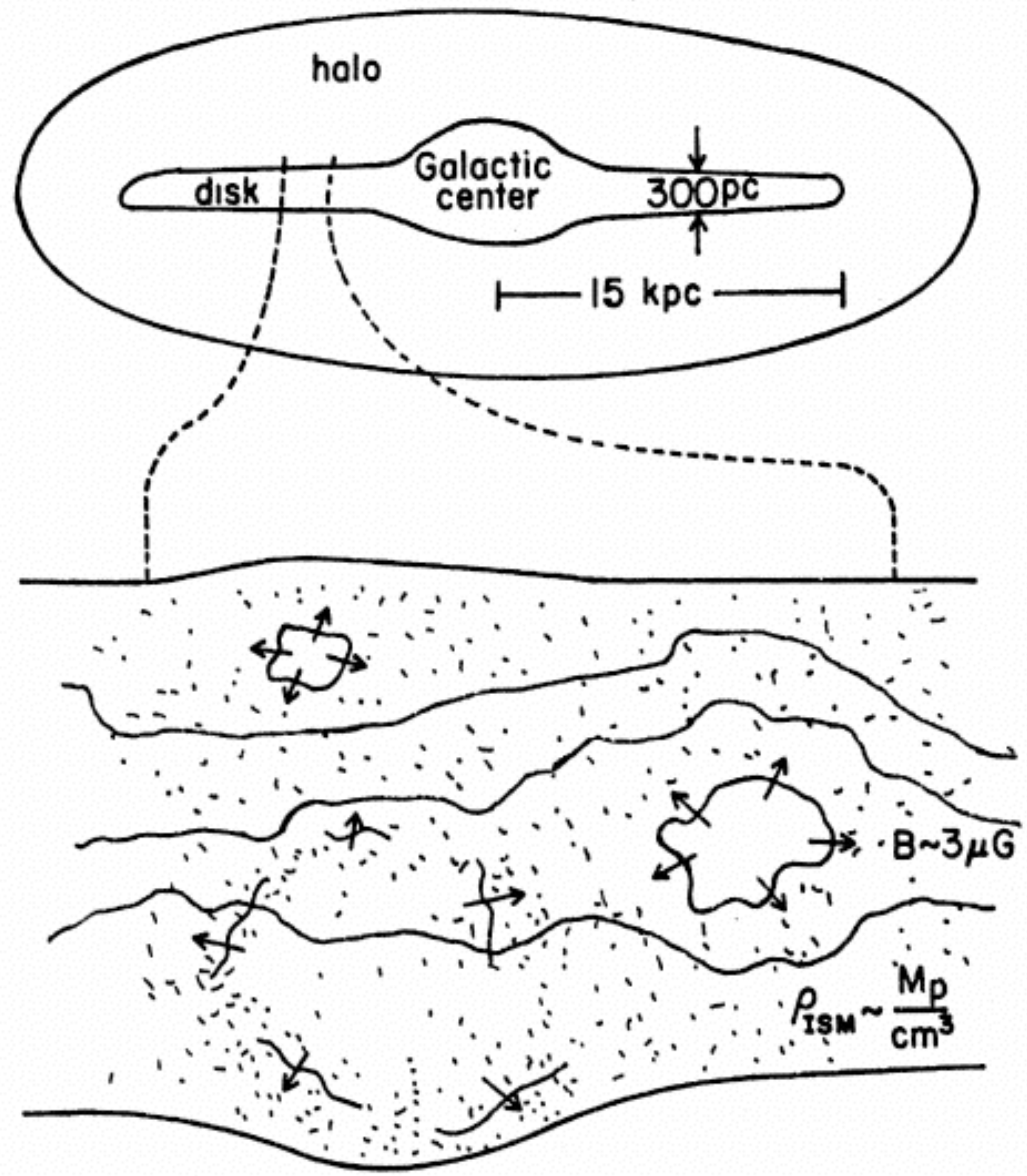
  - ‣ The observed spectrum gets steeper than the source spectrum: $E^{-2.7}$
- Loss probability due to inelastic reactions depends on amount of traversed matter
  - Density of the ISM in the galaxy: ~ 1 proton/cm^3 ~ 1.7 x 10^{-24} g/cm^3
  - ‣ per year one particle traverses ~ 1.5 x10^{-6} g/cm^2
  - ‣ loss after traversing ~ 5 - 10 g/cm^2 (derived from observed composition)
  - ‣ Particles stay in the galaxy for about 5 x 10^6 years

Magnetic Fields in the Galaxy

- Magnetic field in the galaxy along spiral arms, with additional turbulent contributions overlaid
- typical strength ~0.1 nT

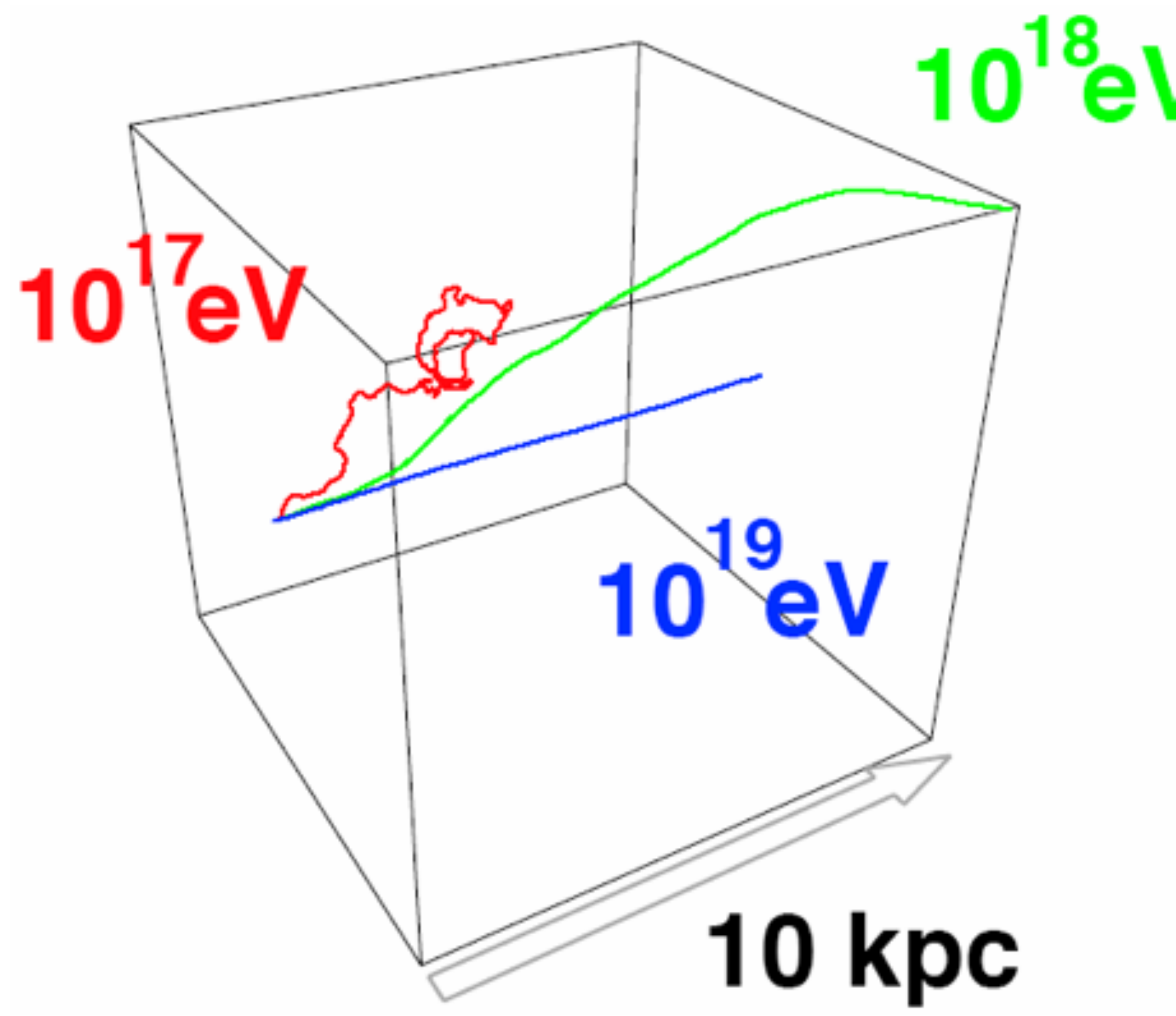




Propagation of Particles in Magnetic Fields

- Charged particles are deflected by cosmic magnetic fields

- To demonstrate: toy simulation with magnetic fields of ~0.1 nT and a coherence length of ~100 pc
  - Particles start from the left center with different energies



- Only the very highest energies ($E \sim 10^{19}$ eV) can show the way to their sources - all other particles get substantially deflected and arrive from random directions

Summary

- Cosmic rays are known since 100 years
  - Discovered by Victor Hess on balloon flights
- Acceleration mechanism via scattering on randomly moving cosmic clouds proposed by Fermi 60 years ago (second order Fermi Acceleration)
  - A proof of principle, but insufficient to reach high energies
- Acceleration in shock fronts created by supernovae (first order Fermi Acceleration) can explain energies at least up to ~10^{14} eV
  - Shock front acceleration is most likely also responsible for even higher energies in objects such as pulsars, blazars, AGNs...

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Next Lecture: 08.06., “Cosmic Rays I”, F. Simon

Topics - Overview

13.04.	Einführung / Introduction
20.04.	**Achtung - keine Vorlesung! No Lecture!**
27.04.	Erdgebundene Beschleuniger / Accelerators
04.05.	Detektoren in der Nicht-Beschleuniger-Physik / Detectors
11.05.	Das Standardmodell / The Standard Model
18.05.	QCD und Jet Physik an Lepton Beschleunigern
25.05.	Pfingsten - Keine Vorlesung! No Lecture
01.06.	Kosmische Beschleuniger / Cosmic Accelerators
08.06.	Kosmische Strahlung I / Cosmic Rays I
15.06.	Kosmische Strahlung II / Cosmic Rays II
22.06.	Präzisionsexperimente (g-2) / Precision Experiments
29.06.	Neutrinos I
06.07.	Neutrinos II
13.07.	Dunkle Materie & Dunkle Energie / Dark Matter & Dark Energy

Backup

Erreichbare Energie

- Die Rate des Energiezuwaches ist gegeben durch die Dauer eines Zyklus und durch den Zuwachs pro Zyklus:

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Hinter dem Schock:
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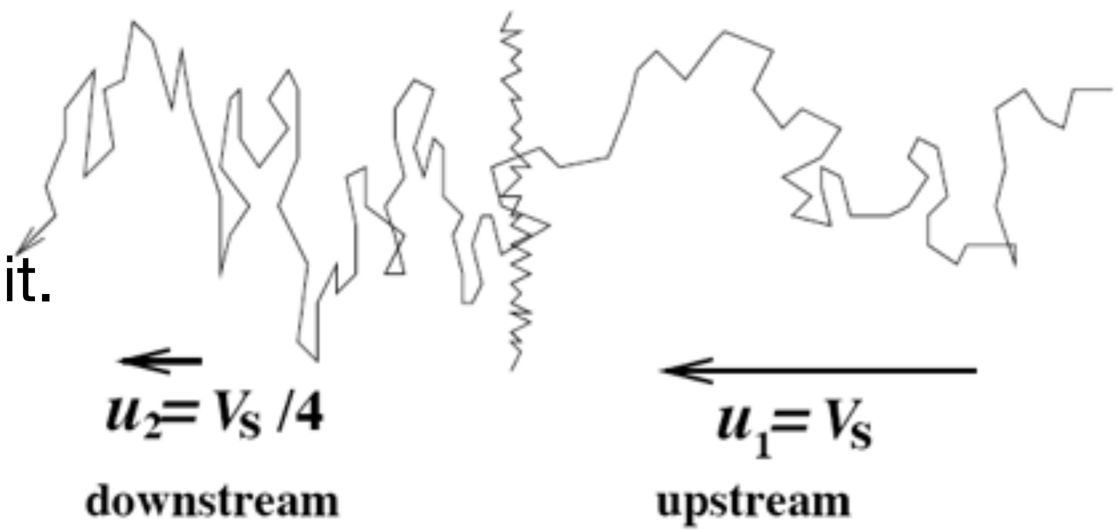
Verweildauer hinter dem Schock: t



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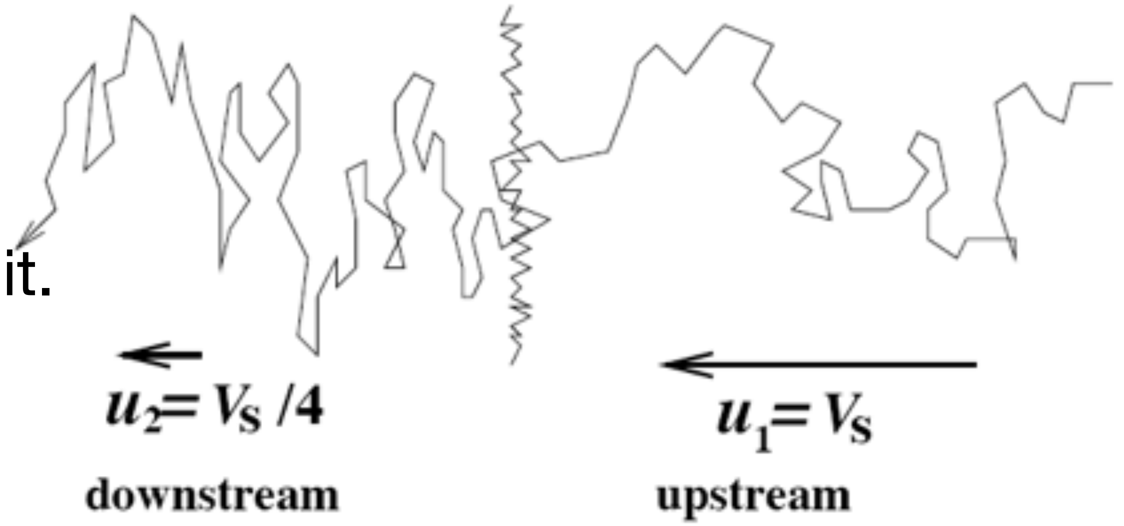

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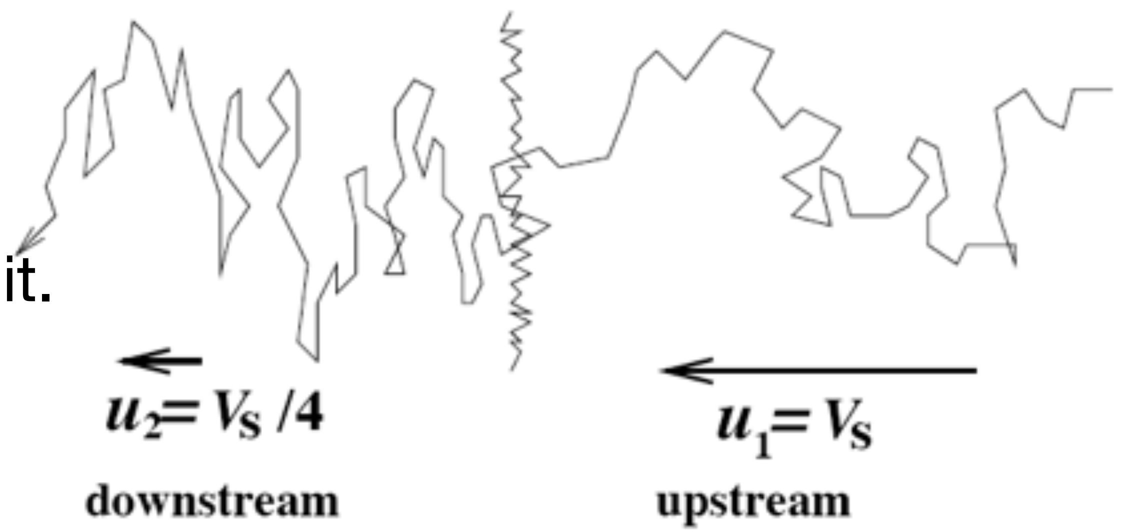

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Hohe Wahrscheinlichkeit, den Schock für immer zu verlassen: $\sqrt{k_2 t} \ll u_2 t$

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- "Grenze", die entscheidet, ob ein Teilchen "verloren" ist: $\sim \; k_2/u_2$

- Verweildauer hinter dem Schock aus Teilchendichte und Übergangsrate:

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  - k_1/u_1 markiert hier die Grenze zwischen Teilchen, die schon von hinter dem Schock gekommen und und solchen, die noch nie durch den Schock gegangen sind

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- Damit ergibt sich die Zyklus-Dauer:

$$t_{cycle} \; = \; t_1 + t_2 \; \approx \; \frac{4}{c}\left(\frac{k_1}{u_1} + \frac{k_2}{u_2}\right) = \frac{4}{\beta_{Schock}c^2}(k_1 + 4k_2)$$

Erreichbare Energie

- Die Diffusionskonstante hängt vom Magnetfeld ab (Diffusionslänge muss mindestens so groß sein wie der Lamor-Radius, damit Streuung an Magnetfeldänderungen funktioniert) - Bohm-Diffusions-Koeffizient:

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- Es folgt für die Zyklus-Dauer für $k_1 = k_2$

$$t_{cycle} = \frac{20}{3} \frac{E}{c^2 \beta_{Schock} ZeB} \propto E$$

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$$k = \frac{1}{3} r_L \, c \; , \; r_L = \frac{p}{ZeB} \sim \frac{E}{cZeB}$$

- Es folgt für die Zyklus-Dauer für $k_1 = k_2$

$$t_{cycle} = \frac{20}{3} \frac{E}{c^2 \beta_{Schock} ZeB} \propto E$$

- Erreichbare Energie:

$$E_{max} = \int_0^{t_{acc}} \frac{dE}{dt} dt = \int_0^{t_{acc}} \frac{E \beta_{Schock}}{\frac{20E}{3c^2 \beta_{Schock} ZeB}} = \frac{3}{20} \beta_{Schock}^2 c^2 ZeB t_{acc}$$

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- Für typische Werte (β_{Schock} ~ 0.03, B ~ 0.3 nT, t_{acc} ~ 1000 Jahre) E_{max} ~ 10^{14} eV (für Protonen)

‣ bis zum Knie der Verteilung