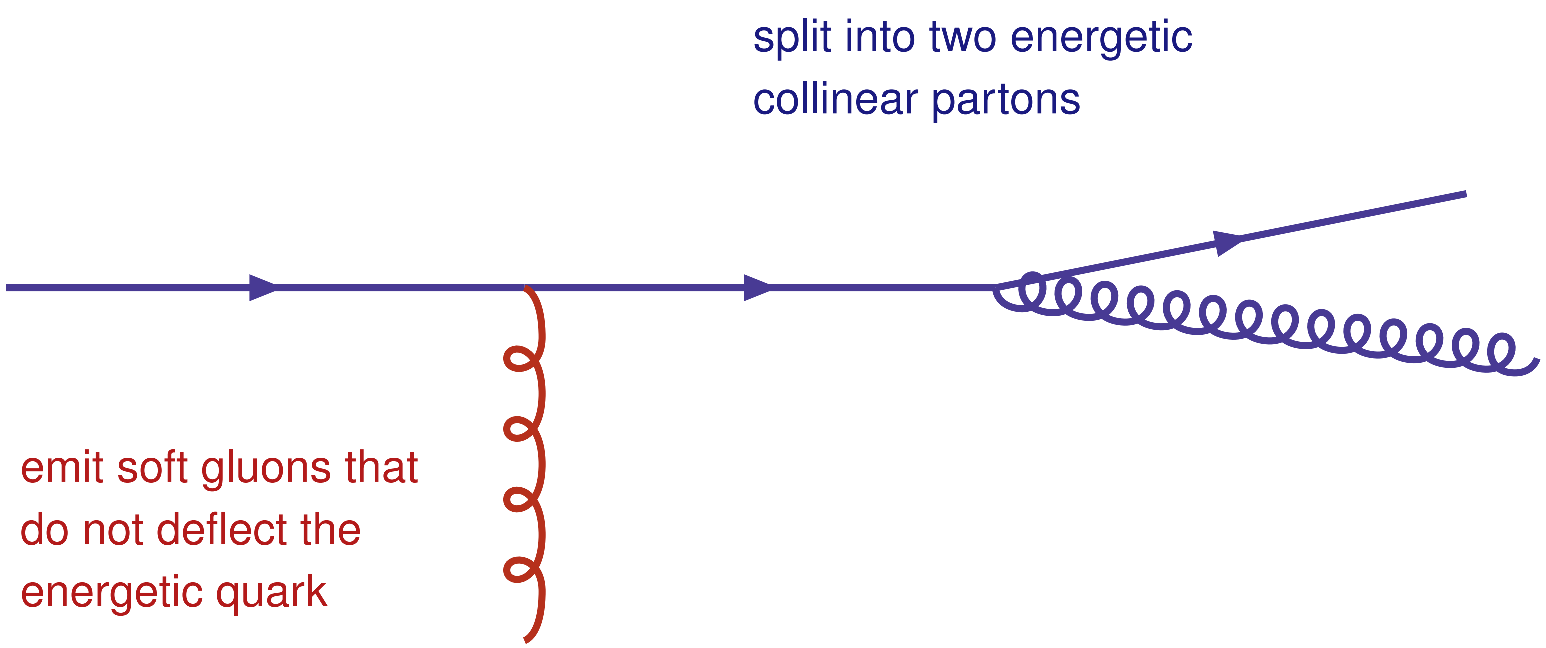

Automating Calculations in Soft Collinear Effective Theory

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Outline

1. ***Motivating an effective theory of QCD***
   (a) Collider physics and the separation of scales
   (b) Resummation examples: Z @ small p_T and thrust
2. ***A brief introduction to SCET***
   (c) Formalism, power counting, and momentum mode suppression
   (d) SCET Lagrangian
   (e) SCET dijet factorisation
   (f) Renormalisation group resummation (for thrust)
3. ***Universal soft functions at NLO***
   (g) Divergence structures, measurement functions, and Laplace space
   (h) naive subtraction
4. ***Soft automation to NNLO***
   (i) NLO vs. NNLO: the need for automation
   (j) Sector Decomposition, parameterising phase space, and *SecDec*
   (k) Analytic regulator at NNLO
   (l) Results

Why an EFT for QCD?

- A perturbative description of collider phenomena with widely separated momentum scales generically involves large logarithms of the scales' ratios—these must be *resummed*:

$$\alpha_s^n \ln^m \left(\frac{\mu_1}{\mu_2}\right)$$

- Traditional approaches in QCD based on coherent branching algorithm (CTTW) which sums probabilities of independent gluon emission diagrammatically

- Effective field theories allow for analytic resummation using renormalisation group techniques at the amplitude level…very efficient.

- Hierarchy of scales implemented at the level of the Lagrangian…

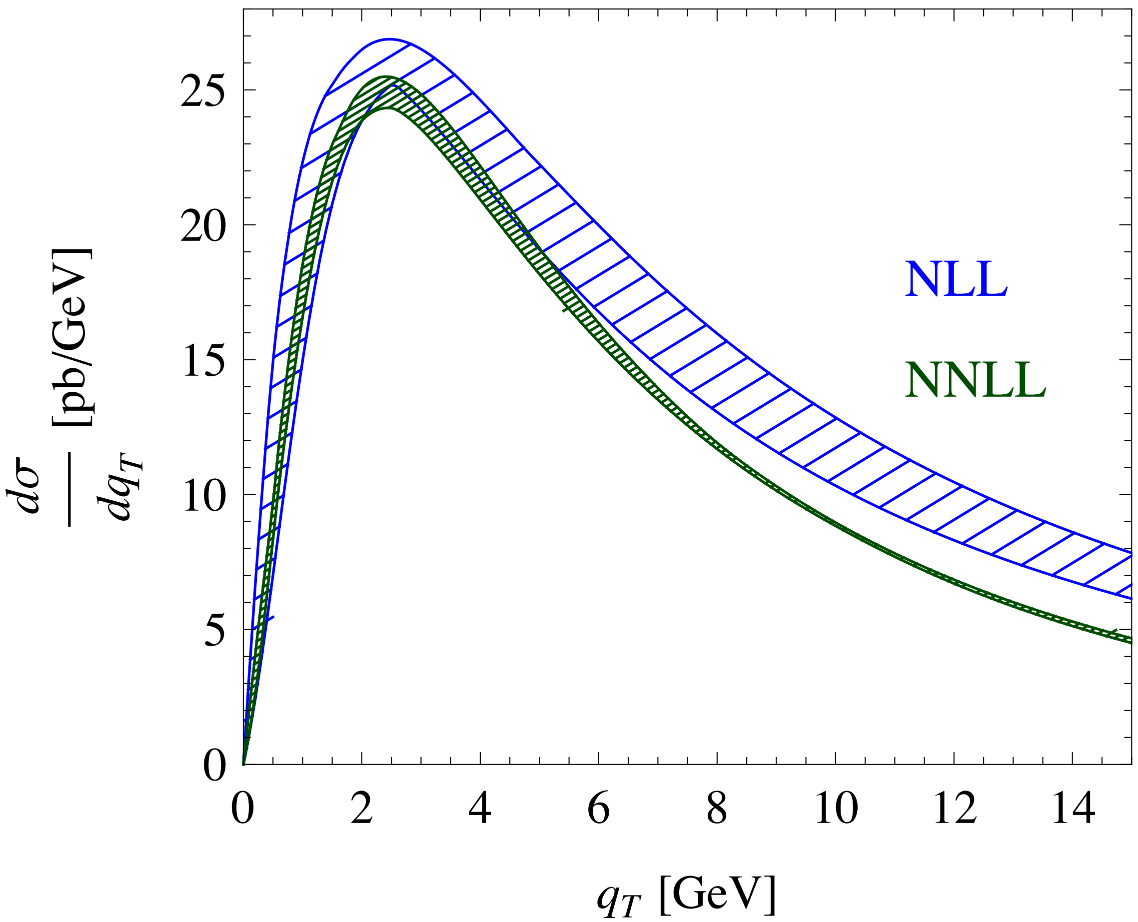
Resummation I: Z @ small p_T

1109.6027(Becher / Neubert / Wilhelm)

- For pp -> Z + X @ small p_T all radiation is confined to the beam or soft

$$\alpha_s^n \ln^{2n}\left(\frac{m_Z^2}{p_T^2}\right)$$

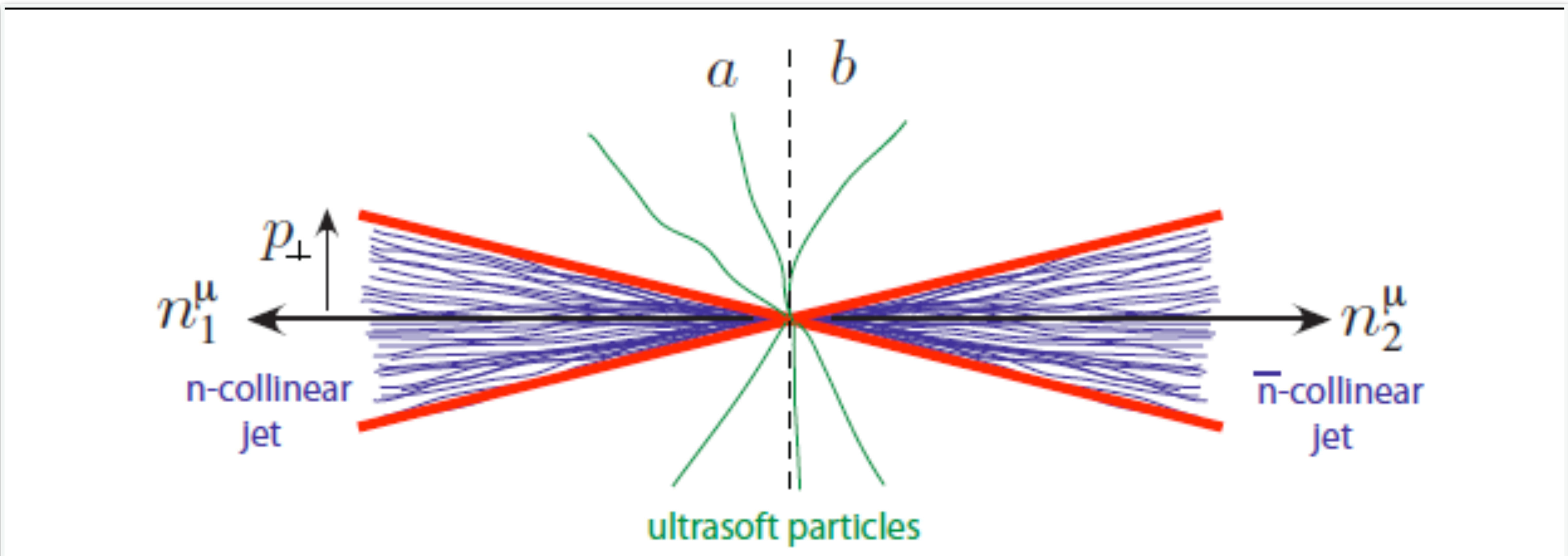
- Perturbative expansion plagued by large logarithms of m_Z / p_T — resummation required.
- Traditional QCD resummation using CSS (*N. Phys. B250*), incomplete *NNLL* *(hep-ph/0302104)*
- Becher, Neubert, and Wilhelm achieve *NNLL* resummation via SCET methods.



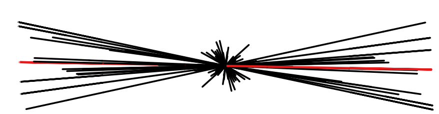
p_T distribution of Z-boson production @ LHC

Resummation II: thrust

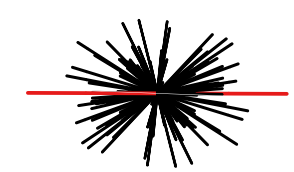
Thrust is an e^+e^- **event shape**—a geometric, dimensionless physical observable characterising the momentum distribution of particles



$$\textbf{thrust} \quad T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$



two-jet like: $T \simeq 1$



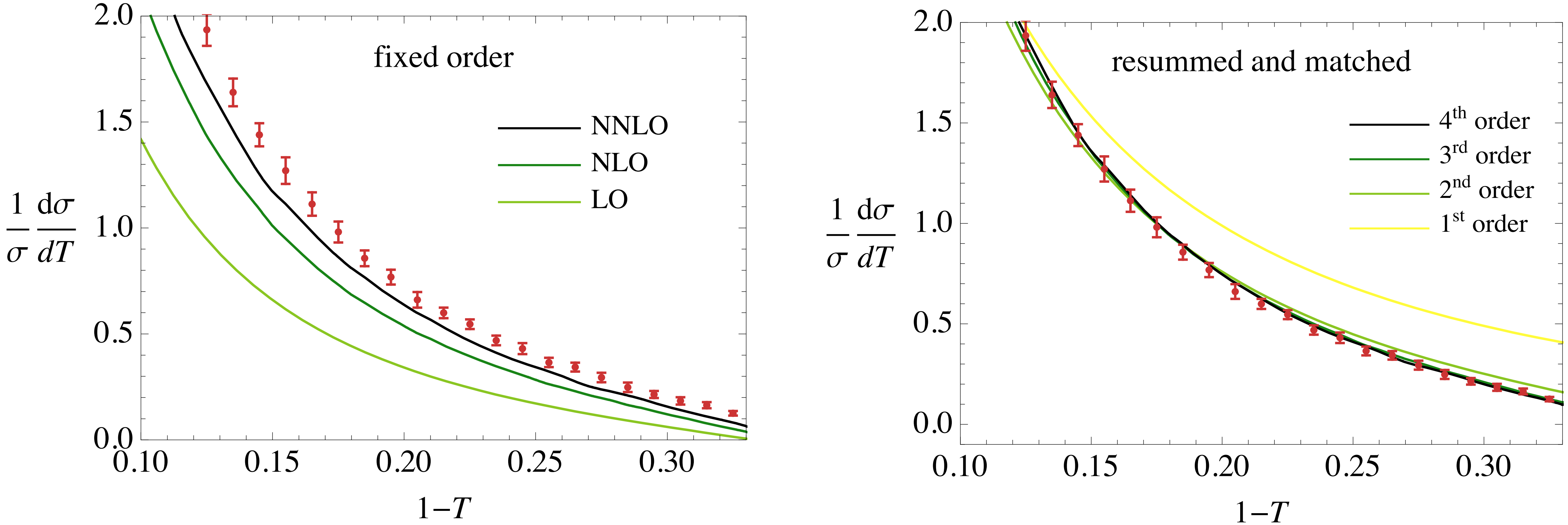
spherical: $T \simeq 1/2$

Resummation II: thrust

0803.0342v2(Becher / Schwartz)

- Traditional QCD resummation achieved by CTTW @ NLL *(Nucl. Phys. B407 [1993])*
- Recently extended to NNLL *(1105.4560; Monni, Gehrmann, Luisoni)*
- Becher, Schwartz achieve N^3LL resummation using SCET methods (kind of)

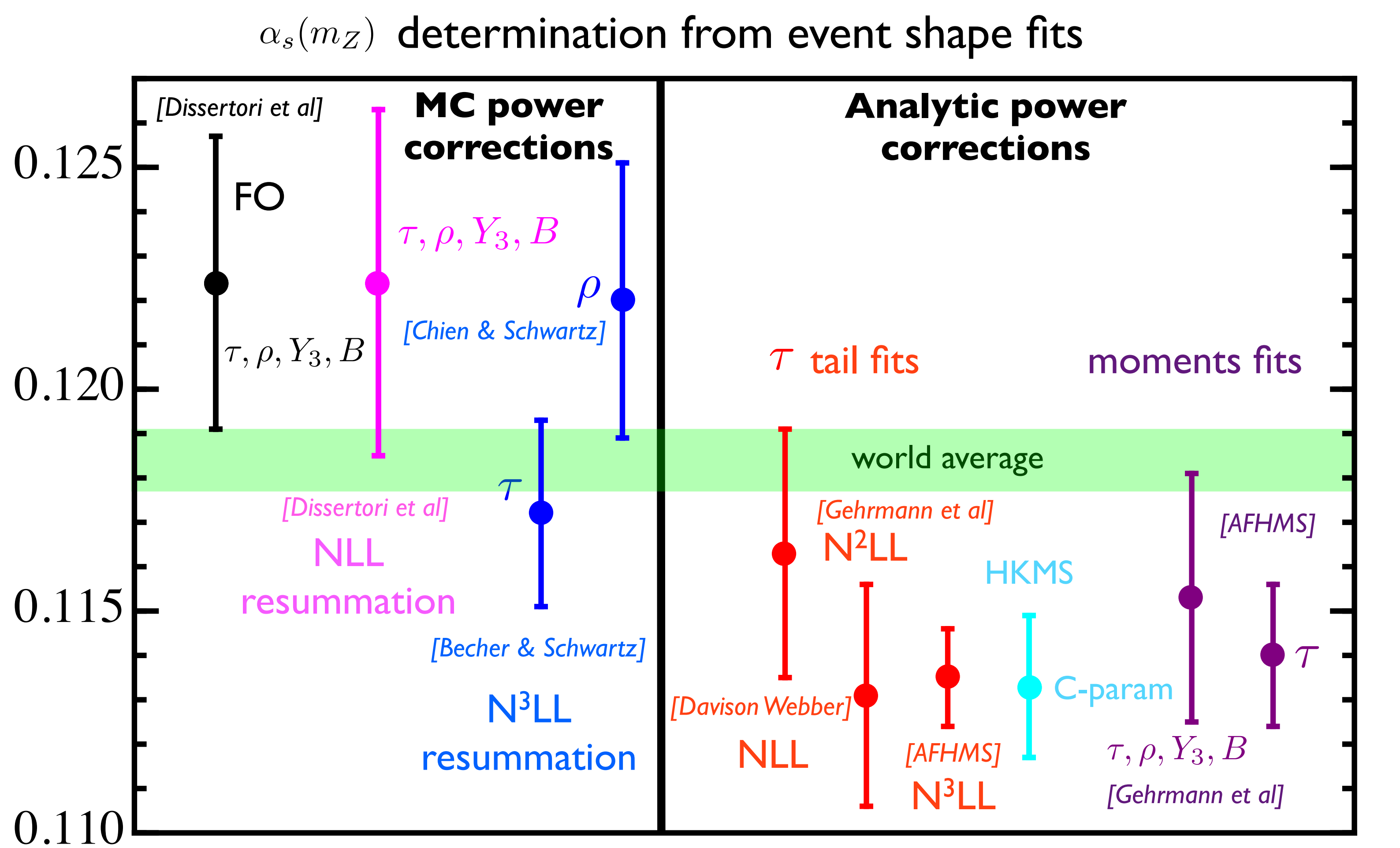
$$\tau = 1 - T \qquad\qquad \alpha_s^n \ln^{2n} \tau$$



Red = Aleph data

Resummation: α_s

- Resummed results used to perform precision extractions of strong coupling:

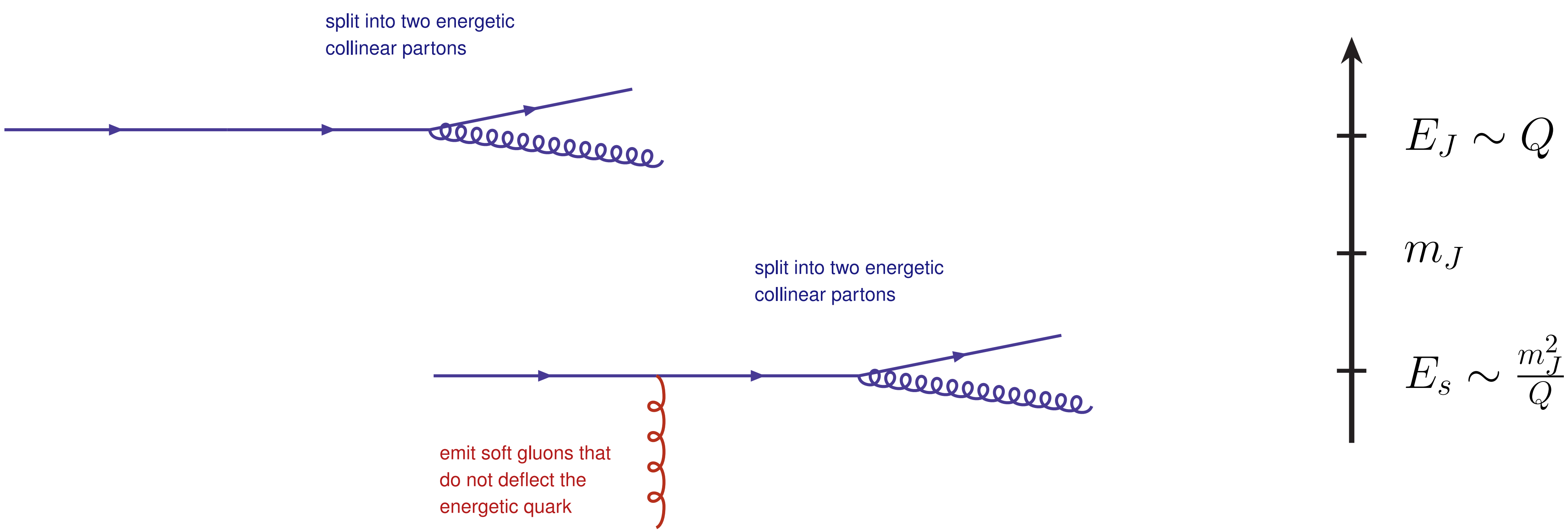


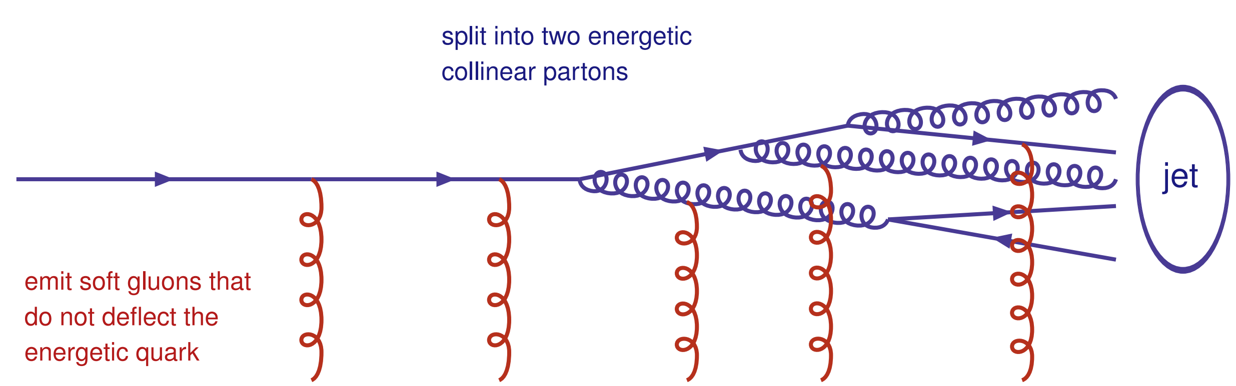
[compilation from A. Hoang, QCD workshop, Singapore, March 2013]

- -> Higher log resummations reduce uncertainties.
- -> Precision fits are lower than the world average.

Introducing SCET: intuition

- SCET is an effective theory whose degrees of freedom are soft and collinear partons





$\Rightarrow$ jet of collinear particles $m_J^2 \ll E_J^2$

soft large-angle radiation $E_s \ll E_J$

Introducing SCET: notation

- SCET is formulated in light-cone basis:

$$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$$
$$n \cdot n = 0 = \bar{n} \cdot \bar{n} \qquad n \cdot \bar{n} = 2$$

- Such that any vector or invariant can be parameterised as follows:

$$p^\mu = \frac{\bar{n}^\mu}{2} n \cdot p + \frac{n^\mu}{2} \bar{n} \cdot p + p^{\perp,\mu} \equiv (p^+, p^-, p^\perp)$$

$$p^2 = p^+ p^- + p_\perp^2$$

$$p \cdot q = \frac{1}{2} p^+ \cdot q^- + \frac{1}{2} p^- \cdot q^+ + p^\perp \cdot q^\perp$$

Introducing SCET: power-counting

- Separation of scales in EFTs characterised by power counting expansion parameter, in SCET this parameter changes depending on observable, e.g.:

(Z @ small p_T) $\lambda = \frac{p_T}{M_Z}$ $\lambda = \sqrt{\tau}$ (thrust)

- Momentum scaling is then determined for each relevant type of particle. Consider back-to-back light jets on the light cone, with background soft radiation:

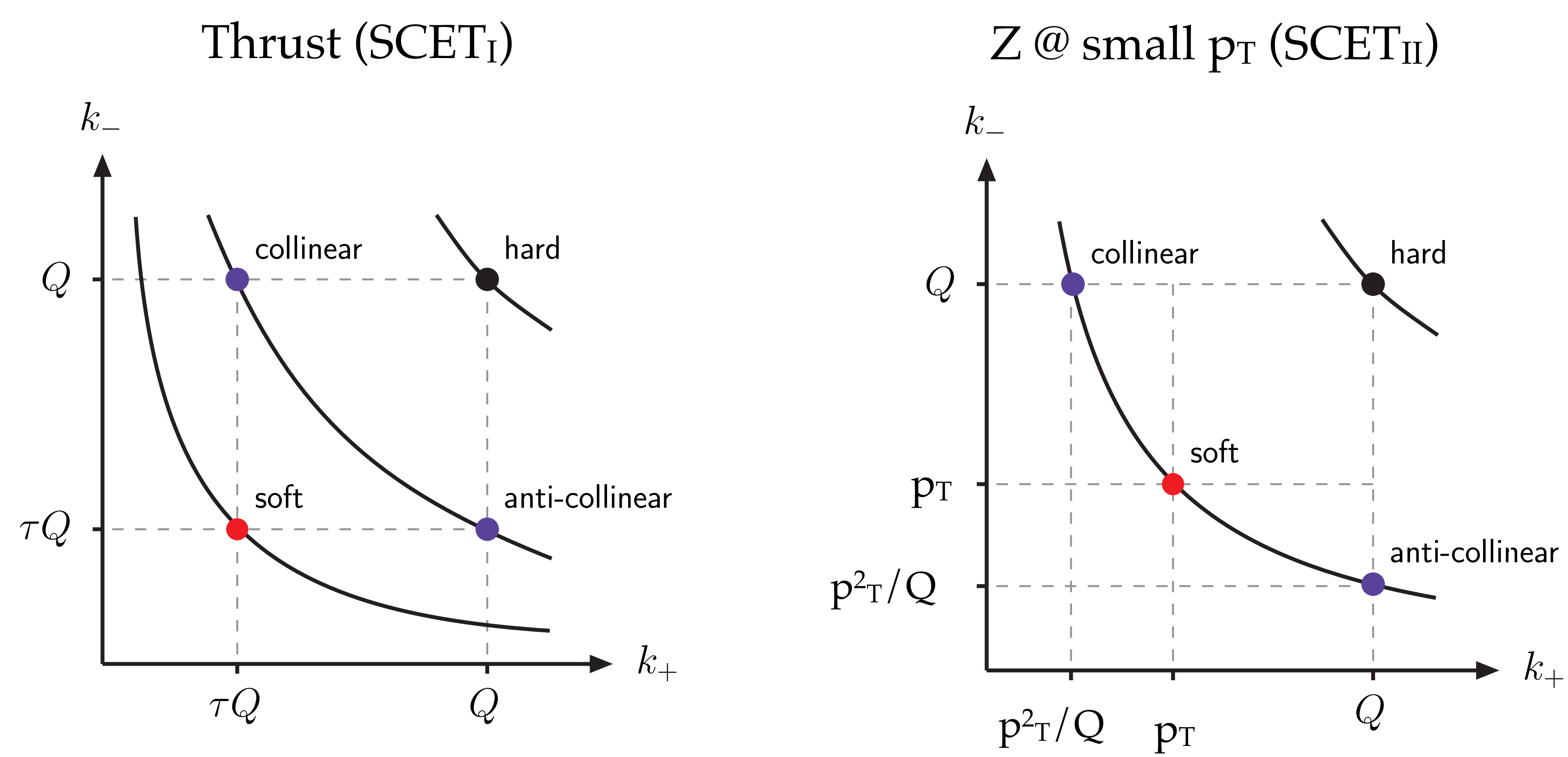
Collinear scaling along + $p^\mu \sim Q(1, \lambda^2, \lambda)_{+,-,\perp}$

Collinear scaling along - $q^\mu \sim Q(\lambda^2, 1, \lambda)$

Ultrasoft scaling $k^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

Introducing SCET: SCET$_{\mathrm{I}}$ vs. SCET$_{\mathrm{II}}$

- There are two types of SCET, depending on the relative scaling of soft and collinear modes:



- ▶ Thrust $p_s^2 \ll p_c^2$
- ▶ Z small p_T $p_s^2 \sim p_c^2$

- In SCET$_{\mathrm{II}}$ scaling alone does not suffice to differentiate—can only distinguish modes from their rapidity…

Introducing SCET: effective Lagrangian

- Begin with fundamental QCD fields and split into soft and collinear components:

$$A^\mu(x) \to A_c^\mu(x) + A_s^\mu(x) \qquad \Psi^\mu(x) \to \Psi_c^\mu(x) + \Psi_s^\mu(x)$$

- Further project collinear fermion into two components:

$$\zeta(x) = \frac{\not{n}\not{\bar{n}}}{4}\Psi_c(x), \quad \eta(x) = \frac{\not{\bar{n}}\not{n}}{4}\Psi_c(x)$$

- Now consider 2-pt correlators, and determine how field scales:

$$\langle 0|\{\zeta(x)\bar{\zeta}(0)\}|0\rangle \sim \lambda^2 \Rightarrow \zeta(x) \sim \lambda \qquad (\eta(x) \sim \lambda^2)$$

- Now, integrate out power suppressed modes. Note, this is not a traditional EFT! Let's look at the collinear portion of the SCET Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\Psi} i \not{D} \Psi \quad \Rightarrow \quad \mathcal{L}_{collinear} = \bar{\zeta}\left(in\cdot D + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c\right)\frac{\not{\bar{n}}}{2}\zeta$$

$$in\cdot D = in\cdot\partial + gn\cdot A_c + \underline{gn\cdot A_s}$$

only collinear-soft interaction at leading order in λ

Introducing SCET: dijet factorisation

- We can derive all order factorisation theorems in SCET. Two critical steps. *"Hard-Collinear factorisation"* (1) & *"Soft-decoupling"* (2):

$$(1) \quad \bar{\Psi}(0)\ \gamma^\mu\ \Psi(0) \to \int dsdt\ C_V(s,t)\ (\bar{\zeta}_{\bar{n}} W_{\bar{n}})(sn)\ \gamma_\perp^\mu\ (W_n^\dagger \zeta_n)(t\bar{n})$$

$$W_c = Pexp\left(ig\int_{-\infty}^{0} ds\bar{n}\cdot A_c(x+s\bar{n})\right)$$

- $C_V(s,t)$ is a Wilson coefficient to be determined in **matching QCD to SCET**
- Explicit **non-locality** along light-cone directions manifest -> **Wilson lines** necessary for gauge invariance. After a field redefinition, we obtain (some spatial dependence suppressed):

$$\zeta_n(x) = S_n(x_-)\zeta_n^0(x)$$

$$(2) \quad \bar{\Psi}(0)\ \gamma^\mu\ \Psi(0) \to \int dsdt\ C_V(s,t)\ \bar{\zeta}_{\bar{n}}^0\ W_{\bar{n}}^{0,\dagger} S_{\bar{n}}^\dagger\ \gamma_\perp^\mu\ W_n^0 S_n\ \zeta_n^0$$

- Now the Lagrangian contains no interactions between collinear and soft fields (at leading order), but the current still contains both…

Thrust w/ SCET: factorisation

$$\text{thrust } T = max_n \frac{\Sigma_i |p_i \cdot n|}{\Sigma_i |p_i|}$$

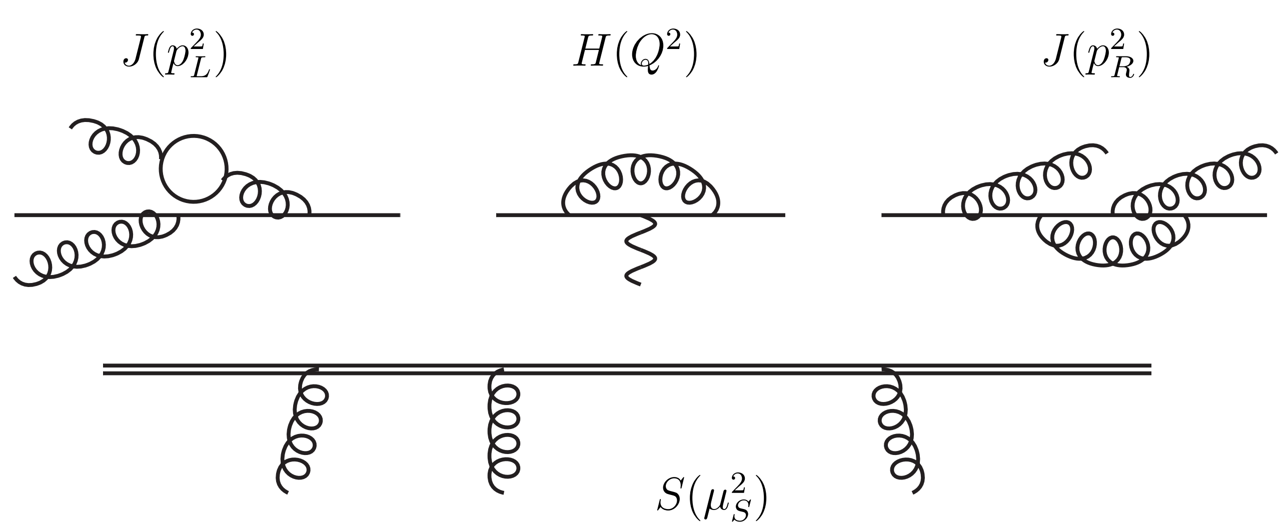
- We can thus **factorise** our matrix element for the dijet, two-fermion operator quite simply:

$$|C_V|^2 \sum_X |\langle 0|\mathcal{O}_{n\bar{n}}|X\rangle|^2$$

$$= |C_V|^2 \ \langle 0| \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right] \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right]^\dagger |0\rangle \ \langle 0| \left[W^0_n \zeta^0_n\right] \left[W^0_n \zeta^0_n\right]^\dagger |0\rangle \ \langle 0| \left[S^\dagger_{\bar{n}} S_n\right] \left[S^\dagger_{\bar{n}} S_n\right]^\dagger |0\rangle$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dp_L^2 \int dp_R^2 \ \ J(p_L^2,\mu) \ \ J(p_R^2,\mu) \ \ S(\tau Q - \frac{p_L^2 + p_R^2}{Q}, \mu)$$

(for dijet thrust)



"soft-decoupling"

Thrust w/ SCET: resummation

- H, J^i, and S contain logs of the form (respectively):

$$\ln \frac{\mu^2}{Q^2}, \quad \ln \frac{\mu^2}{\tau Q^2}, \quad \ln \frac{\mu^2}{\tau^2 Q^2}$$

- We evaluated H, J^i, and S at a common scale. Yet there are 'natural' scales at which the logarithms are no longer large:

$$\mu_h \sim Q \quad \mu_j \sim Q\sqrt{\tau} \quad \mu_s \sim Q\tau$$

- We thus wish to RG run our functions up to their natural scales. Take H as a simple example:

$$H(Q^2,\mu) = H(Q^2,\mu_h)\; U_h(\mu_h,\mu)$$

- Where the function U is a solution to the RG equation for the hard function:

$$\frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln\left(\frac{Q^2}{\mu^2}\right) + 4\gamma_H(\alpha_s)\right] H(Q^2,\mu)$$

- Which, at LL approximation, has the following form:

$$H(Q^2,\mu) = exp\left[\frac{4\pi\Gamma_0}{\beta_0^2}\frac{1}{\alpha_s(Q)}\left(1-\frac{1}{r}-\ln r\right)\right] = 1-\frac{\Gamma_0}{2}\frac{\alpha_s(Q)}{4\pi}\ln^2\left(\frac{Q^2}{\mu^2}\right)+\mathcal{O}(\alpha_s^2), \qquad r=\frac{\alpha_s(\mu)}{\alpha_s(Q)}$$

- Similar for jet and soft functions…

Resummation: Technicalities

Logarithmic Accuracy	Γ_{Cusp}	$\gamma_H,\ \gamma_J,\ \gamma_S$	$C_H,\ C_J,\ C_S$
LL	1-loop	tree	tree
NLL	2-loop	1-loop	tree
NNLL	3-loop	2-loop	1-loop
N3LL	4-loop	3-loop	2-loop

- **SCET Observables @ NNLL**: broadening, Z/W/H @ small p_T, jet-veto …
- **SCET Observables @ N^{3}LL**: thrust, C-parameter, Z/W/H @ large p_T, …
- Automated code for QCD resummation @NLL: *CAESAR* (Banfi, Salam, Zanderighi)
- Recently extended to NNLL for (Banfi, McAslan, Monni, Zanderighi)
- We want to automate soft functions in SCET…

Universal dijet soft functions

- We can write down a universal dijet soft function as the vacuum matrix element of a product of Wilson lines along the direction of energetic quarks.

$$S(\omega,\mu) = \sum_{X,reg.} \mathcal{M}(\omega,\{k_i\})|\langle X|S_n^\dagger(0)S_{\bar{n}}(0)|0\rangle|^2 \qquad S_n(x) = Pexp(ig_s\int_{-\infty}^{0} n\cdot A_s(x+sn)ds)$$

- The **matrix element** of soft wilson lines is *independent of the observable*. It contains the universal (implicit) UV/IR-divergences of the function.
- The **measurement function** (M) encodes all of the information of the particular observable at hand. It is *independent of the singularity structure*. Take thrust as an example:

$$\mathcal{M}_{thrust}(\omega,\{k_i\}) = \delta(\omega - \sum_{i\in L} k_i^+ - \sum_{i\in R} k_i^-)$$

- **Idea**: isolate singularities at each order and calculate the associated coefficient numerically:

$$\bar{S}(\tau) \sim 1 + \alpha_s\{\frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon^1} + c_0\} + \mathcal{O}(\alpha_s^2)$$

- The coefficients depend on the observable, we typically work in Laplace space.

Universal soft functions: NLO

- We work in **Laplace space**, so that our functions are not distribution valued. At 1-loop the virtual corrections are scaleless in DR and we can write the NLO soft function as:

$$\bar{S}^{(1)}(\tau,\mu) = \frac{\mu^{2\epsilon}}{(2\pi)^{d-1}} \int \delta(k^2)\ \theta(k^0)\ \mathcal{R}_\alpha(\nu;k_+,k_-)\ \left(\frac{16\pi\alpha_s C_F}{k_+k_-}\right)\ \bar{\mathcal{M}}(\tau,k)\ d^dk$$

- Where we use a symmetric version of the analytic $\mathrm{SCET_{II}}$ regulator (*Becher, Bell* / 1112.3907):

$$\mathcal{R}_\alpha(\nu,k_+,k_-) = \theta(k_- - k_+)(\nu/k_-)^\alpha + \theta(k_+ - k_-)(\nu/k_+)^\alpha$$

- We want to disentangle all of the UV and IR divergences. We thus split the integration region into two hemispheres and make the following physical substitutions:

$$k_- \to \frac{k_T}{\sqrt{y}} \qquad k_+ \to k_T\sqrt{y}$$

- We can now specify the measurement function *M*. We assume it can be written in terms of two dimensionless functions f & g:

$$\bar{\mathcal{M}}(\tau,k) = g(\tau k_T, y, \theta)\ exp(-\tau k_T f(y,\theta))$$

Universal soft functions: examples

$$\bar{\mathcal{M}}(\tau, k) = g(\tau k_T, y, \theta)\ exp(-\tau k_T f(y, \theta))$$

Obs.	$g(\tau k_T, y, \theta)$	$f(y, \theta)$
Thrust	1	$\sqrt{y}$
Angularities	1	$y^{(1-A)/2}$
C-Parameter	1	$\sqrt{y}/(1+y)$
Broadening	$\Gamma(1-\epsilon)\left(\frac{z\tau k_T}{4}\right)^{\epsilon}\mathcal{J}_{-\epsilon}\left(\frac{z\tau k_T}{2}\right)$	$1/2$
W/H @ large p_T	1	$\frac{1+y-2\sqrt{y}\cos\theta}{\sqrt{y}}$
Transverse Thrust	1	$\frac{1}{\lvert s\rvert}\{\sqrt{1+\frac{1}{4}\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)^2 s^2+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)cs\cos\theta - s^2\cos^2\theta}$ $-\lvert c\cos\theta+\frac{1}{2}\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)s\rvert\}$

Universal soft functions: NLO master formula

- We switch to a dimensionless variable (x) and extract the scaling of the observables in the collinear limit y $\Rightarrow$ 0:

$$\tau k_T f(y,\theta) \to x \qquad f(y,\theta) \to y^{\frac{n}{2}} \hat{f}(y,\theta)$$

- We are now in a position to write a master formula for the calculation of NLO dijet soft functions:

$$\bar{S}^{(1)}(\tau,\mu) \sim \int_{-1}^{1} \sin^{-1-2\epsilon}\theta \; d\cos\theta \int_0^\infty dx \int_0^1 dy \; x^{-1-2\epsilon-\alpha} \; y^{-1+n\epsilon+(n-1)\alpha/2} \; \hat{g}(x,y,\theta) \; [\hat{f}(y,\theta)]^{2\epsilon+\alpha} \; e^{-x}$$

- Note that n=0 corresponds to a SCET_{II} observable.

- We are in a position to apply a subtraction technique to extract the singularities. Consider a simple 1-D example:

$$\int_0^1 dx \; x^{-1-n\epsilon} f(x) = \int_0^1 dx \; x^{-1-n\epsilon} \{f(x) - f(0) + f(0)\}$$

divergent

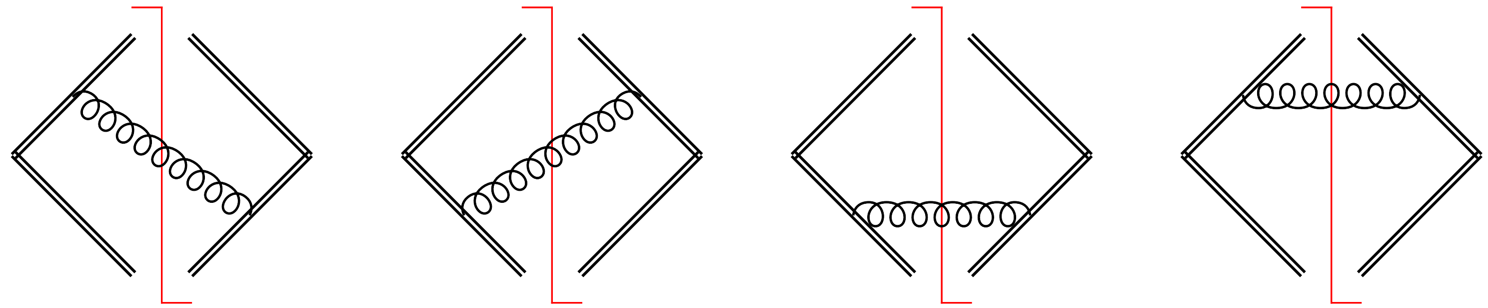
- finite / O(x)
- expand in ϵ
- integrate numerically

$\sim -\dfrac{1}{n\epsilon}$

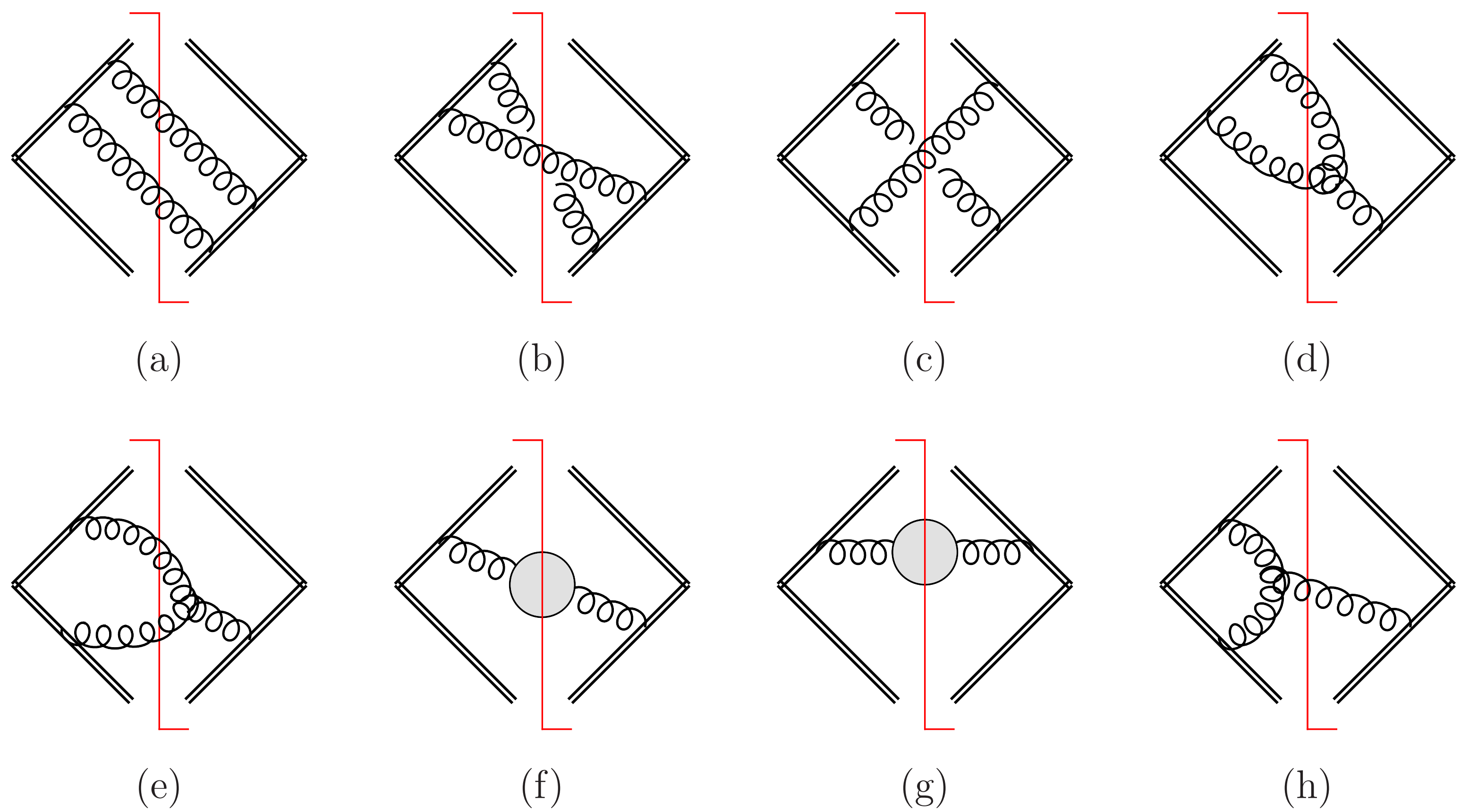
- singularity isolated

Automation: NLO vs. NNLO

NLO:



NNLO:



Automation: NLO vs. NNLO

- Consider the double real emission (and drop additional regulator):

$$\bar{S}^2_{RR}(\tau) = \frac{\mu^{4\epsilon}}{(2\pi)^{2d-2}} \int d^d k \; \delta(k^2)\; \theta(k^0) \int d^d l \; \delta(l^2)\; \theta(l^0)\; |\mathcal{A}(k,l)|^2 \; \bar{\mathcal{M}}(\tau, k, l)$$

- Decompose into light-cone coordinates and perform trivial integrations:

$$\bar{S}^{(2)}_{RR}(\tau) \sim \Omega_{d-3}\Omega_{d-4} \int_0^\infty dk_+ \int_0^\infty dk_- \int_0^\infty dl_+ \int_0^\infty dl_- \int_{-1}^1 d\cos\theta_k \;\; \sin^{d-5}\theta_k$$
$$\times \int_{-1}^1 d\cos\theta_l \;\; \sin^{d-5}\theta_l \int_{-1}^1 d\cos\theta_1 \;\; \sin^{d-6}\theta_1 \;\; (k_+k_-l_+l_-)^{-\epsilon} |\mathcal{A}(k,l)|^2 \; \bar{\mathcal{M}}(\tau, k, l)$$

- Consider, e.g., the $C_F T_F n_f$ color structure:

$$|\mathcal{A}(k,l)|^2 = 128\pi^2\alpha_s^2 C_F T_F n_f \frac{2k\cdot l(k_- + l_-)(k_+ + l_+) - (k_- l_+ - k_+ l_-)^2}{(k_- + l_-)^2(k_+ + l_+)^2(2k\cdot l)^2}$$

- It is clear the singularity structure is non-trivial, and that the singularities are overlapping…

Automation: sector decomposition

- Consider a simple integral over a unit hypercube with 'overlapping singularities' (singular as x,y simultaneously tend to 0):

$$I = \int_0^1 dx \int_0^1 dy (x+y)^{-2+\epsilon}$$

- We want to factorise such singularities. Split the hypercube with two sectors (x>y) and (y>x):

$$I = I_1 + I_2 = \int_0^1 dx \int_0^x dy (x+y)^{-2+\epsilon} + \int_0^1 dy \int_0^y dx (x+y)^{-2+\epsilon}$$

- Now substitute y = xt in first sector and x = yt in second:

$$I_1 = \int_0^1 dx \int_0^1 dt \; x^{-1+\epsilon}(1+t)^{-2+\epsilon}$$
$$I_2 = \int_0^1 dy \int_0^1 dt \; y^{-1+\epsilon}(1+t)^{-2+\epsilon}$$

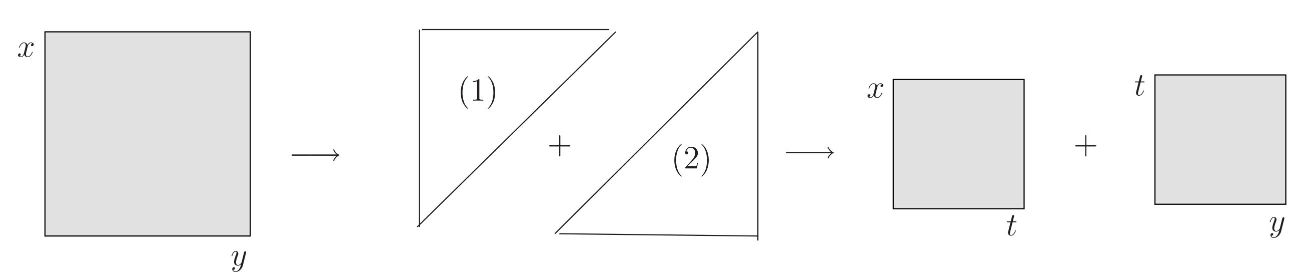


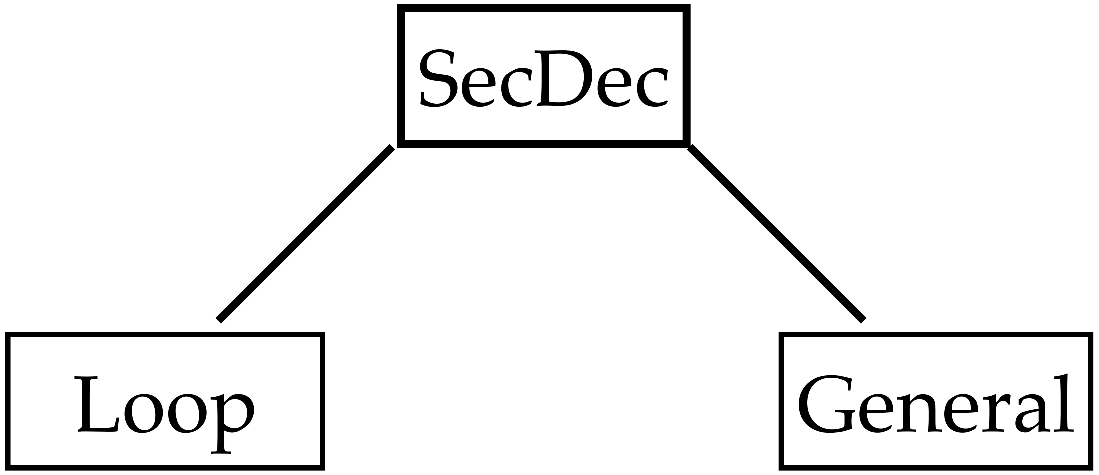
Image: J. Carter

Automation: *SecDec*

https://secdec.hepforge.org

Heinrich, Jones, Kerner, Borowka, Schlenk, Zirke

- A tool is already on the market that exploits the sector decomposition algorithm: ***SecDec***
- "A program to evaluate dimensionally regularised parameter integrals numerically"



- We use *SecDec*'s 'general' mode, as it allows the definition of *dummy functions*.
- SecDec provides: Simple interface to our NLO and NNLO master formulas (✔), numerical code output (✔), multiple numerical integrators for crosschecks (✔)
- Currently limited to $\mathrm{SCET_I}$ observables, though additional rapidity regulator in development.

Automation: NNLO parameterisation

- We thus need to find an appropriate phase space parameterisation that exposes the divergence structure and is amenable to sector decomposition (*SecDec*):

$$p_- = k_- + l_- \qquad a = \sqrt{\frac{k_- l_+}{k_+ l_-}} = e^{-(\eta_k - \eta_l)}$$

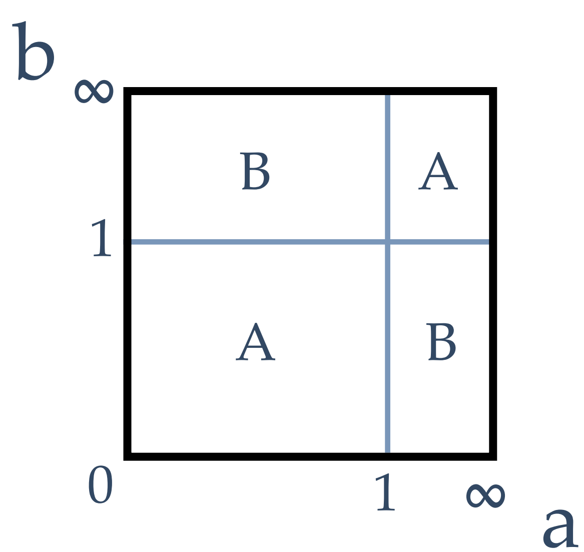
$$p_+ = k_+ + l_+ \qquad b = \sqrt{\frac{k_- k_+}{l_- l_+}} = \frac{k_T}{l_T}$$

- We further write the total momentum components in terms of p_T and y (as in NLO case):

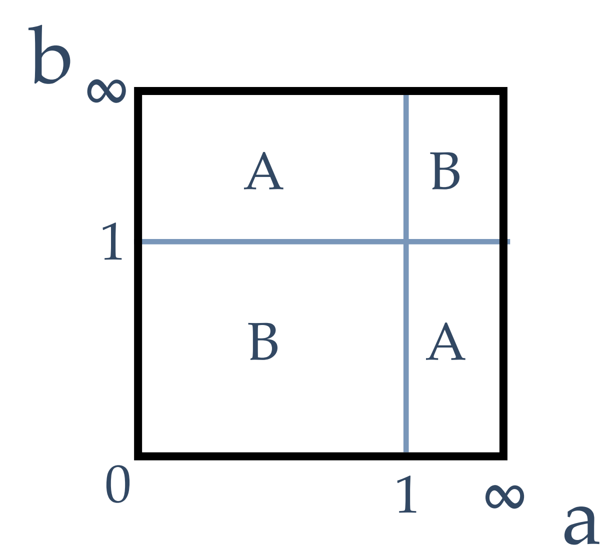
$$p_- \to \frac{p_T}{\sqrt{y}} \qquad p_+ \to p_T \sqrt{y}$$

- Finally, we map onto the unit hyper-cube:

$(y<1)$



$(y>1)$



Automation: NNLO master formula

- We again assume an exponentiated form for the (Laplace space) measurement function:

$$\tilde{\mathcal{M}}(\tau,k,l) = \exp\{-\tau\, p_T\, F(a,b,y,t_i(\theta_i))\}$$

- We again assume a factorised y-dependence in F(a,b,y):

$$F(a,b,y,t_i) = y^{\frac{N}{2}}\,\hat{F}(a,b,y,t_i)$$

- We arrive at an NNLO master formula (here shown for $C_F T_F n_f$ amplitude):

$$\begin{aligned}
\bar{\mathcal{S}}^{(2)}_{RR}(\tau) &= \left(\mu^2\tau^2 e^{\gamma}\right)^{2\epsilon}\,\frac{8\alpha_s^2 C_F T_F n_f}{\pi^2}\,\frac{\Gamma(-4\epsilon)}{\sqrt{\pi}\,\Gamma(1-\epsilon)\Gamma(\frac{1}{2}-\epsilon)} \\
&\times \int_0^1 da \int_0^1 db \int_0^1 dy \int_0^1 dt\, y^{-1+2N\epsilon}\, a^{2-2\epsilon}\, b^{-2\epsilon}\,(a+b)^{-2+2\epsilon}\,(1+ab)^{-2+2\epsilon}\,(4t\bar{t})^{-\frac{1}{2}-\epsilon} \\
&\times \int_0^1 dt_1 \int_0^1 dt_2\; 2^{-1-4\epsilon}\,\frac{\epsilon}{\pi}\,(t_1\bar{t}_1)^{-\frac{1}{2}-\epsilon}\,(t_2\bar{t}_2)^{-1-\epsilon} \\
&\times \frac{(1-a)^2(1-b)^2+4t(a+b)(1+ab)}{\left[(1-a)^2+4at\right]^2}\left\{\left[\hat{F}(a,b,y,t_i)\right]^{4\epsilon}+\left[\hat{F}(1/a,b,y,t_i)\right]^{4\epsilon}\right\}
\end{aligned}$$

$$\left[\bar{t}_{(i)} = \left(1-t_{(i)}\right)\right]$$

NNLO Measurement functions: examples

Observable	$\boldsymbol{F\left(y, a, b, t_i\left(\theta_i\right)\right)}$
Inclusive Drell-Yan	$\frac{1}{\sqrt{y}} + \sqrt{y}$
C-Parameter	$a\sqrt{y}\left(\frac{b}{a\left(a+b\right)+y\left(1+ab\right)} + \frac{1}{a+b+ay\left(1+ab\right)}\right)$
Thrust	$\sqrt{y} + \Theta\left(y - a\frac{1+ab}{a+b}\right)\left(\frac{a}{\sqrt{y}(a+b)} - \frac{\sqrt{y}}{1+ab}\right)$
Angularities	$\sqrt{y}^{1-A}\Bigg[b\left(\frac{a}{1+ab}\right)^{1-\frac{A}{2}}\left(\frac{1}{a+b}\right)^{\frac{A}{2}} + \left(\frac{1}{1+ab}\right)^{1-\frac{A}{2}}\left(\frac{a}{a+b}\right)^{\frac{A}{2}}$ $+\Theta\left(y - a\frac{(1+ab)}{a+b}\right)\left(y^{A-1}\left(\frac{a}{a+b}\right)^{1-\frac{A}{2}}\left(\frac{1}{1+ab}\right)^{\frac{A}{2}} - \left(\frac{1}{1+ab}\right)^{1-\frac{A}{2}}\left(\frac{a}{a+b}\right)^{\frac{A}{2}}\right)\Bigg]$
W/H at high p_T	$\frac{1}{\sqrt{y}} + \sqrt{y} - 2\sqrt{\frac{a}{(a+b)(1+ab)}}\left[1 - 2t_2 + b\left(1 - 2t + t_2 - 2tt_2 - 2\left(1-2t_1\right)\sqrt{t\bar{t}t_2\bar{t_2}}\right)\right]$
Recoil-free broadening	$\sqrt{\frac{a}{(a+b)(1+ab)}}(1+b)$

Analytic regulator for NNLO

- We extend the NLO definition to account for two gluons:

$$R_\alpha(\nu;k,l) = \left[\Theta(k_+ - k_-)\left(\frac{\nu}{k_+}\right)^{\frac{\alpha}{2}} + \Theta(k_- - k_+)\left(\frac{\nu}{k_-}\right)^{\frac{\alpha}{2}}\right] \times \left[\Theta(l_+ - l_-)\left(\frac{\nu}{l_+}\right)^{\frac{\alpha}{2}} + \Theta(l_- - l_+)\left(\frac{\nu}{l_-}\right)^{\frac{\alpha}{2}}\right]$$

- This definition still respects the $n \leftrightarrow \bar{n}$ and $k \leftrightarrow l$ exchange symmetries

- Using our NNLO parameterisation, the regulator function takes the form

$$R_\alpha^A(\nu;k,l) = \nu^\alpha\, p_T^{-\alpha} \left(\frac{1+ab}{b}\right)^{\frac{\alpha}{2}} \left[\Theta\left(y - \frac{a(a+b)}{1+ab}\right)\left(\frac{a+b}{b}\right)^{\frac{\alpha}{2}} + \Theta\left(\frac{a(a+b)}{1+ab} - y\right)\left(\frac{1+ab}{a}\right)^{\frac{\alpha}{2}} y^{\frac{\alpha}{2}}\right]$$

and similar for region B

Results: *Thrust*

$$\mathcal{M}_{thrust}(\omega, \{k_i\}) = \delta(\omega - \sum_{i\in L} k_i^+ - \sum_{i\in R} k_i^-)$$

- We use *SecDec* to calculate the double emission contribution. To obtain the renormalized soft function we have to add the counterterms, which are known analytically at the required order.
- We show the cancellation of the divergences for thrust, setting $\ln(\mu\bar{\tau}) \to 0$

$$\tilde{S}^{(2)}_{ren} = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\{C_A C_F\left(\frac{0}{\epsilon^4} - \frac{5.07333\times10^{-9}}{\epsilon^3} + \frac{1.07523\times10^{-6}}{\epsilon^2} + \frac{.0000102661}{\epsilon}\right) + C_F T_F n_f\left(-\frac{1.40667\times10^{-8}}{\epsilon^3} + \frac{6.83778\times10^{-8}}{\epsilon^2} - \frac{1.44697\times10^{-8}}{\epsilon}\right)\} + \tilde{S}^{(2)}_0$$

- We thus also have an indication of our numerical precision…
- For the finite portion, we find (setting again $\ln(\mu\bar{\tau}) \to 0$):

$$\tilde{S}^{(2)}_0 = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(48.7045 C_F^2 - 56.4992 C_A C_F + 43.3902 C_F T_F n_f\right)$$

- Versus the analytic expression calculated by *Kelley, Schabinger, Schwartz, Zhu* / 1105.3676 (see also *Monni, Gehrmann, Luisoni* / 1105.4560):

$$\tilde{S}^{(2)}_0 = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(48.7045 C_F^2 - 56.4990 C_A C_F + 43.3905 C_F T_F n_f\right)$$

Results: *C-parameter*

- *C-parameter* measurement function:

$$\mathcal{M}_C(\omega, \{k_i\}) = \delta(\omega - \sum_i \frac{k_+^i k_-^i}{k_+^i + k_-^i})$$

- For *C-parameter*, we obtain:

$$\tilde{S}_0^{(2)} = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(5.41162 C_F^2 - 57.9754 C_A C_F + 43.8179 C_F T_F n_f\right)$$

- Where *Hoang, Kolodrubetz, Mateu, Stewart* / 1411.6633 extracted (using EVENT2) the following:

$$\tilde{S}_0^{(2)} = \frac{\alpha_S^2(\mu)}{(4\pi)^2}\left((5.41162)\, C_F^2 - (58.16 \pm .26)\, C_F C_A + (43.74 \pm .06)\, C_F T_F n_f\right)$$

- We find similar numerical precision in the subtractions.

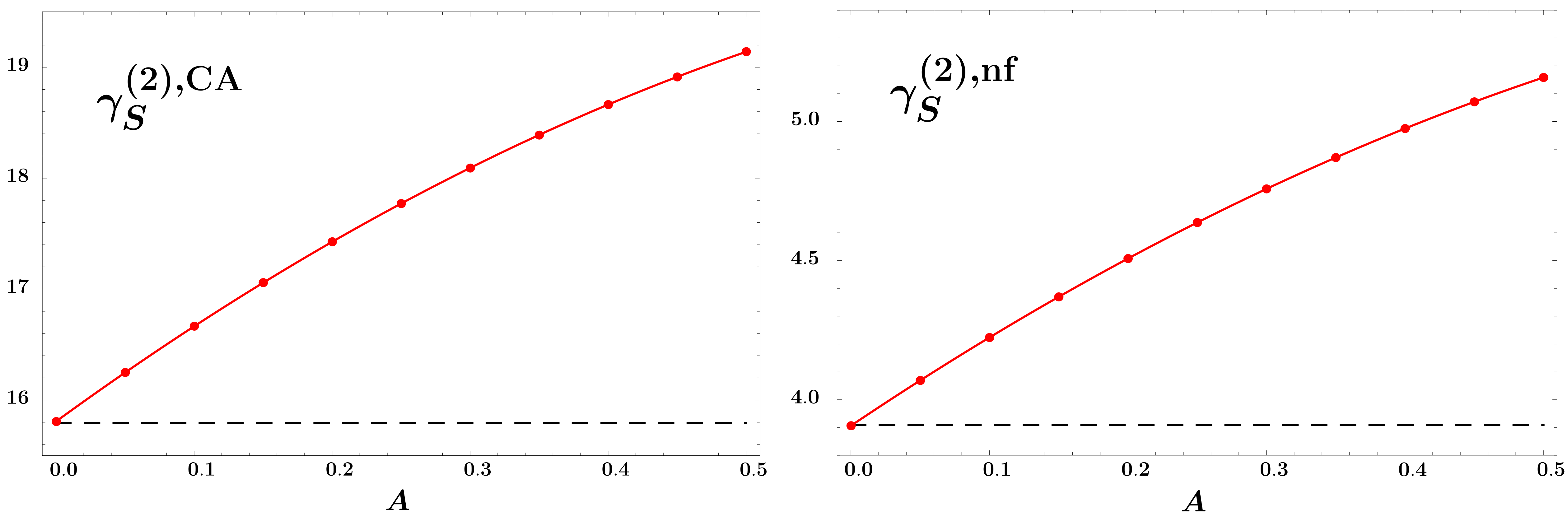
Results: *Angularities*

- *Angularities* measurement function:

$$\mathcal{M}_{Ang}(\omega,\{k_i\}) = \delta(\omega - \sum_{i\in L} k_{i,+}^{1-A/2}\, k_{i,-}^{A/2} - \sum_{i\in R} k_{i,+}^{A/2}\, k_{i,-}^{1-A/2})$$

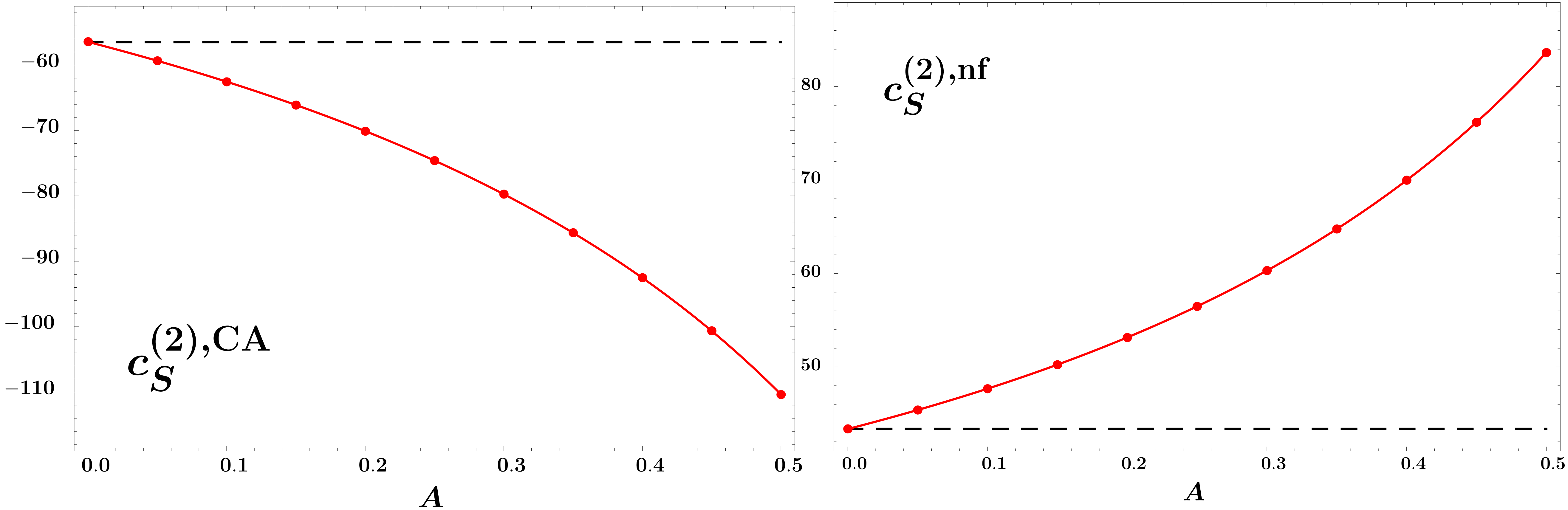
- The two-loop soft anomalous dimension is not known. We define in Laplace space:

$$\frac{d\tilde{S}(\tau)}{d\ln\mu} = -\frac{1}{(1-A)}\left[4\Gamma_{cusp}\ln(\mu\bar{\tau}) - 2\gamma_S\right]\tilde{S}(\tau)$$



Results: *Angularities*

$$\mathcal{M}_{Ang}(\omega,\{k_i\}) = \delta(\omega - \sum_{i\in L} k_{i,+}^{1-A/2}\ k_{i,-}^{A/2} - \sum_{i\in R} k_{i,+}^{A/2}\ k_{i,-}^{1-A/2})$$



Results: *Threshold Drell-Yan*

- *Drell-Yan production @ threshold:*

$$\mathcal{M}_{DY}(\omega, \{k_i\}) = \delta(\omega - \sum_i k_+^i + k_-^i)$$

- For *Drell-Yan*, we obtain:

$$\tilde{S}_0^{(2)} = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(5.41162 C_F^2 + 6.81281 C_A C_F - 10.6857 C_F T_F n_f\right)$$

- Whereas analytic expression calculated by *Belitsky* / 9808389 is:

$$\tilde{S}_0^{(2)} = \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left(5.41162 C_F^2 + 6.81287 C_A C_F - 10.6857 C_F T_F n_f\right)$$

- Again, similar precision found for pole cancellation.

Results: *W/H @ large* p_T

- With two beams and one recoiling jet the soft function depends on two initial state (1,2) and one final state (J) Wilson lines:

$$S(\omega) = \sum_X \delta(\omega - n_J \cdot p_X) \, |\langle X|S_1 S_2 S_J|0\rangle|^2$$

- However, due to rescaling invariance of light-cone vectors and colour conservation, the diagrams that contribute @ NNLO only involve attachments to the initial state Wilson Lines S_1 and S_2.

- Hence, up to NNLO, we encounter the same dijet matrix element as before.

- However, there is also now an angular dependence in the measurement function, giving six-dimensional integrals…

Results: W/H @ large p_T

- *W/H production @ large* p_T:

$$\mathcal{M}_{W/H}(\omega, \{k_i\}) = \delta(\omega - \sum_i (k_i^+ + k_i^- - 2k_i^T \cos\theta_i))$$

- We have similar color structures with the following definitions:

$$C_s = \begin{cases} C_F - C_A/2 & q\bar{q} \to g \\ C_A/2 & qg \to q \text{ and } gg \to g \end{cases}$$

- For *W/H production @ large* p_T, we obtain:

$$\tilde{S}_0^{(2)} = \frac{\alpha_s^2(\mu)}{(4\pi)^2} \left(48.7045\, C_s^2 + \left(\underbrace{107.12}_{bare} - \underbrace{111.40}_{ren} = -4.28 \right) C_A C_s - 25.2824\, C_s n_f T_F \right)$$

- Whereas *Becher, Bell, Marti* / 1201.5572 calculate:

$$\tilde{S}_0^{(2)} = \frac{\alpha_s^2(\mu)}{(4\pi)^2} \left(48.7045 C_s^2 - 2.6501 C_A C_s - 25.3073 C_s n_f T_F\right)$$

Conclusions and future work

- SCET provides an efficient, analytic approach to high-order resummations necessary for precision collider physics.
- We have presented an automated algorithm to compute dijet soft functions for a wide class of observables in SCET
- Our master formulas coupled with *SecDec* can quickly and easily produce predictions for a wide class of SCET_I soft functions at one and two-loops.
- This is an important ingredient for NNLL resummations in SCET…
- Next steps: Better understanding of the numerics, SCET_II observables, n-jet soft functions and a public code…

Thanks!