From parton model to bound states

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Outline

1. Quantum bound states
   1. The description
   2. A simple application: the ρ-meson
2. Decay rates of unflavoured mesons
   1. Starting point: para-exotic atoms
   2. Effective potential: an intuitive approach
   3. The mesonic wave-function at ground state
3. Results and discussion
   1. Comparison with experimental data
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The description of quantum bound states

Let $\hat{\mathcal{J}}$ be some current operator with all quantum numbers and symmetries of the bound state,

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$$\Longrightarrow \quad \boxed{|\mathcal{B}\rangle = \sum\!\!\!\!\!\!\!\int_{\mathcal{L}} \Psi_{\mathcal{B}}(\mathcal{L})\hat{\mathcal{J}}|\Omega\rangle}$$

A simple application: the ρ-meson

We demand the non-vanishing matrix element

$$f_\rho \epsilon_\mu^{s*}(p) = \langle p; s | \hat{j}_\mu(0) | \Omega \rangle$$

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Therefore, the expansion in momentum space yields

$$|\rho\rangle = -\frac{1}{f_\rho}\int d^4x \int \frac{d^3p}{(2\pi)^3}\; \phi(\mathbf{p})\; e^{-ip\cdot x} \sum_s \epsilon^s_\mu(\mathbf{p})\hat{j}^\mu(x)|\Omega\rangle$$

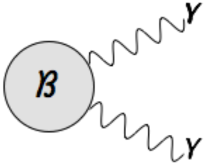
with dispersion relation

$$p^2 = M_\rho^2$$

Starting point: para-exotic atoms

We compute at lowest order of perturbation theory the pure QED channel

$$|\langle 2\gamma|\mathcal{B}_l;0\rangle|^2$$



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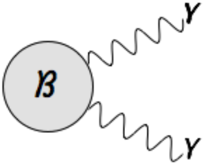

which leads to

$$\Gamma_{\mathcal{B}_l\to 2\gamma} = \frac{16\pi\alpha^2}{M^2_{\mathcal{B}_l}}|\psi(0)|^2$$

with

$$M_{\mathcal{B}_l} \gg |\mathbf{p}|$$

Leptonic wave-function and results

Wave-function ansatz?

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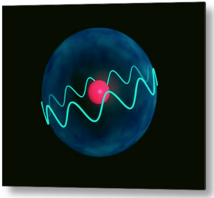
2-body system motivation:

$\Longrightarrow$ ground state of Hydrogen atom



Therefore

$$\Gamma_{\mathcal{B}_l \to 2\gamma} = \frac{\alpha^5}{2} m_l$$

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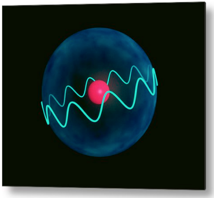

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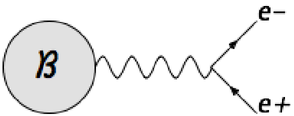
Comparison with experimental data:

1^1S_0:	e^-e^+	$\mu^-\mu^+$	$\tau^-\tau^+$
$\tau[10^{-10}\text{s}]$	1.24	6.04×10^{-3}	3.57×10^{-4}
$\tau^{\text{exp.}}[10^{-10}\text{s}]$	1.24	5.95×10^{-3}	NA

Application on unflavoured vector mesons

Leading process in α:

$$|\langle e^- e^+ | \mathcal{B}_q ; 1\rangle|^2$$



Recall i.e.

$$i\mathcal{M}_{q_L \bar{q}_R \to e^-_L e^+_R} = 2i \frac{e^2 Q_q}{|\langle 12\rangle|^2} \langle 23\rangle [41]$$

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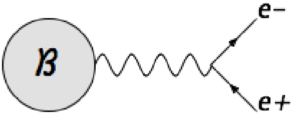

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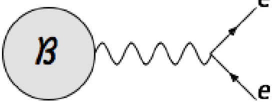

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Claim

The mesonic wave-function should be universal (same form for any unflavoured meson at the ground state): It depends only on the mass of bound state $M_{\mathcal{B}_q}$.

Effective mesonic potential

In Born-approximation, 1-gluon exchange for $q\bar{q}$ singlet scattering gives

$$\mathcal{V}_{q\bar{q}}(r) = -C_2 \frac{\alpha_s}{r} \qquad \text{(attractive)}$$

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Cornell-potential: asymptotic freedom and 1-loop running of α_s

$$\mathcal{V}_{\text{meson}}(r) = \frac{8\pi}{3b_0} \frac{1}{r \ln\left(\Lambda_{\text{QCD}} r\right)} + F_0 r \simeq \frac{A}{2}(r - r_0)^2 + B$$

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insufficient predictive power!

Effective potential: an intuitive approach

Claim

- Perturbative effects are dominant around the origin.
- Non-perturbative effects do not contribute at full scale

$\longrightarrow$ confining effects are parametrized as a deviation from $Z = 1$.

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In fact picking

$$\mathcal{V}_{\mathcal{B}_q}(r) = -C_2 \frac{Z\alpha_s(M_{\mathcal{B}_q})}{r} \quad \text{with} \quad \delta Z \equiv Z - 1 \simeq 0.75 \quad \text{(anti-screening)}$$

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Interpretation

Mesons with $M_{\mathcal{B}_q} \gg \Lambda_{\text{QCD}}$ behave effectively like 2-body systems of heavy non-relativistic "QCD-clouds".

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Hydrogen atom

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confining parametrization

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$$\phi_{\mathcal{B}_q}(\mathbf{p}) = N(M_{\mathcal{B}_q}) \frac{\lambda(M_{\mathcal{B}_q})}{\left[|\mathbf{p}|^2 + \lambda^2(M_{\mathcal{B}_q})\right]^2}$$

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i.e. for Υ_{1S}:

$$\langle \hat{r} \rangle_{1S} \simeq 0.266\text{fm}$$

$\longrightarrow$ very close to the experimental value of 0.2fm.

Comparison with experimental data

Predictive formula for decay rates in pure QED channel is analytically computed,

$$\Gamma_{\mathcal{B}_q \to X}(M_{\mathcal{B}_q})$$

where $X = e^- e^+, 2\gamma$ for vector and scalar mesons respectively.

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Impressive matching with experimental values:

LML-mesons:	Υ	J/ψ	ρ	ϕ	η_c	η'	η_b
Γ[keV]	1.42	5.12	6.07	1.32	6.85	3.28	0.55
$\Gamma^{\text{exp.}}$[keV]	1.34	5.55	7.04	1.26	5.06	4.34	0.45
$M_{\mathcal{B}}$[GeV]	9.46	3.097	0.775	1.019	2.98	0.958	9.40

$\xrightarrow{\text{deviations}}$ Z should also run, i.e. $Z(\alpha_s) = \sum_{n=0}^{\infty} C_n \left(\frac{\alpha_s}{\pi}\right)^n$

Outlook

1. QCD related topics:
   (i) Flavoured mesons (weak channels)
   (ii) Excited states
   (iii) (2)Glue-ball application
2. Gravity and black holes as parton model

Further Reading

- S. Hofmann, *Lecture notes: QCD and Standard Model SoSe 2010*.
- S. Weinberg, *The Quantum Theory of Fields, Volume 1 & 2*. CUP; 2005.
- C. Itzykson and J.B. Zuber, *Quantum Field Theory*. Dover Pubn Inc; 2006.
- W. Greiner, S. Schramm and E. Stein, *QCD*. Springer; 2007.
- Ta-Pei Cheng, *Gauge Theory Of Elementary Particle Physics*. OUP; 1995.
- K. Hashimoto, *D-Brane: Superstrings and New Perspective of Our World*. Springer; 2012.

Starting point: para-exotic atoms

Consider a bound state in QED of a charged lepton $l : (k_1; s_1)$ and $l^c : (k_2; s_2)$.

For a para-exotic atom state:

$$|\mathcal{B}_l; 0\rangle = \sqrt{2M_{\mathcal{B}_l}} \int \frac{d^3k}{(2\pi)^{3/2}} \phi(\mathbf{k}) \sqrt{2E_l \; 2E_{l^c}}^{\,-1} |k_1, k_2; s_1, s_2\rangle$$

For convenience we used

$$\langle \mathcal{B}_l; 0 | \mathcal{B}_l; 0\rangle = 2M_{\mathcal{B}_l} V$$

QCD sum rules: perturbation theory revisited

Our knowledge for bound states with $M_{\mathcal{B}_q} \gg \Lambda_{\text{QCD}}$:

Perturbative effects are the most dominant ones,

$$m_\rho^2(M^2) = M^2 \frac{1.1\left(1 - (1 + \frac{s_0}{M^2})e^{-s_0/M^2}\right) - 0.12\left(\frac{0.6\text{GeV}^2}{M^2}\right)^2}{1.1\left(1 - e^{-s_0/M^2}\right) + 0.12\left(\frac{0.6\text{GeV}^2}{M^2}\right)^2}$$

Sending condensates to zero

$$m_\rho^2(M^2) \overset{\text{large } M^2}{\longrightarrow} \frac{s_0}{2}$$

Impressive fitting of mesonic wave-function

We define the reduced wave-function as

$$|\psi_q^R(0)|^2 \equiv \frac{|\psi_q(0)|^2}{M_{\mathcal{B}_q}^3}$$

Simple power-ansatz:

$$|\psi_q^R(0)|^2 = \tilde{\kappa}\alpha_s^\lambda(s)$$

Impressive fitting for LML-unflavoured vector mesons:

$\boxed{\lambda \simeq 3.04}$ (same as in $\mathcal{B}_l$ case) and $\boxed{\tilde{\kappa} \simeq (15.8)^{-1}}$

which are independent of $\mathcal{B}_q$.

The wave-function and a first test

Final form of mesonic wave-function:

$$\phi_{\mathcal{B}_q}(\mathbf{p}) = N(M_{\mathcal{B}_q}) \frac{\lambda(M_{\mathcal{B}_q})}{\left[|\mathbf{p}|^2 + \lambda^2(M_{\mathcal{B}_q})\right]^2}$$

where

$$\lambda(M_{\mathcal{B}_q}) = C_2 \frac{Z\pi M_{\mathcal{B}_q}}{2b_0 \ln\left(\frac{M_{\mathcal{B}_q}}{\Lambda_{\text{QCD}}}\right)} \quad \text{and} \quad N(M_{\mathcal{B}_q}) = \frac{\lambda^{3/2}(M_{\mathcal{B}_q})}{Z\sqrt{\pi}}$$

i.e. for Υ_{1S}:

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Predictive formula and results

Decay formula for pure QED channel:

$$\Gamma_{\mathcal{B}_q \to X}(M_{\mathcal{B}_q}) = \frac{128\pi^3 f_q^{S/V} \alpha^2 Z^3 M_{\mathcal{B}_q}}{9 b_0^3(M_{\mathcal{B}_q}) \ln^3\left(\frac{M_{\mathcal{B}_q}}{\Lambda_{\text{QCD}}}\right)}$$

with effective color charge $Z = 1 + 0.75$.
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$$f_q^S = \begin{cases} \frac{16}{81}, & \text{for } \eta_c, \\ \frac{2}{27}, & \text{for } \eta', \\ \frac{1}{81}, & \text{for } \eta_b \end{cases}, \quad X = 2\gamma \quad \text{or} \quad 3f_q^V = \begin{cases} \frac{1}{9}, & \text{for } \Upsilon, \\ \frac{4}{9}, & \text{for } J/\psi, \\ \frac{1}{9}, & \text{for } \phi, \\ \frac{5}{18}, & \text{for } \rho^0 \end{cases}, \quad X = e^- e^+$$

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