Numerical Evaluation of Two-Loop Master Integrals for Higgs pair production through Gluon fusion

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Outline

Why do we study HH production?

HH production mechanisms

Virtual gg$\rightarrow$ HH Amplitude at NLO

Some Results

Why do we study HH production?

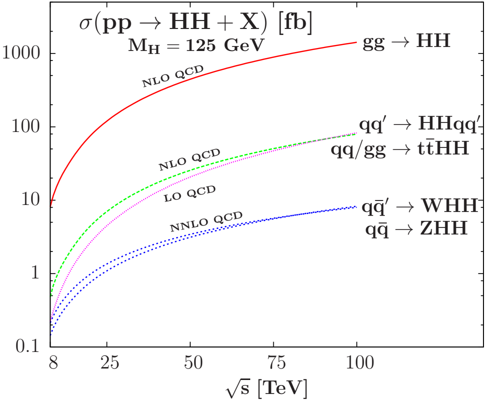
- SM electroweak gauge group: $SU(2)_L \otimes U(1)_Y$
- Higgs mechanism breaks this group to $U(1)_{em}$
- Consequence: 3 massive gauge bosons : $W^{\pm}$ and Z
  $+$ 1 massless gauge boson: γ
- Additionally: 1 scalar Higgs boson

$$\mathcal{L}_H = \frac{1}{2}(D^\mu H)(D_\mu H) - \frac{m_H^2}{2}H^2 - \underbrace{\frac{m_H^2}{2v}}_{\lambda_{3H}}H^3 - \frac{m_H^2}{8v^2}H^4$$

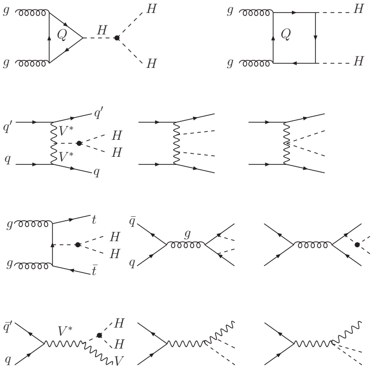
  its mass m_H is a free parameter in the SM
- measurement of triple Higgs selfcoupling λ_{3H} would validate Higgs mechanism

HH production mechanisms

[Baglio, Djouadi et al 2013, 1212.5581]



LO diagrams



QCD factorization

- Origin: strong coupling α_s is
  small for large momentum transfer $\leftrightarrow$ small distances and
  large for small momentum transfer $\leftrightarrow$ large distances
- Consequence: Factorize the hadronic cross section at scale μ

$$\sigma_{pp\to X} = \sum_{i,j\in\{q,\bar{q},g\}} \underbrace{f_i(\mu^2) \otimes f_j(\mu^2)}_{\text{protons' PDF's}} \otimes \hat{\sigma}_{ij\to X}(Q^2, \alpha_s(\mu^2), \mu^2)$$

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- $\hat{\sigma}$ can be calculated perturbatively in powers of α_s:

$$\hat{\sigma} = \int d\Pi_n \; |\mathcal{M}(p_A, p_B \to p_1 ... p_n)|^2$$

$$|\mathcal{M}|^2 = \alpha_s^2 \underbrace{\left|\mathcal{M}^{LO}\right|^2}_{\to\hat{\sigma}^{LO}} + \alpha_s^3 \underbrace{\left|\mathcal{M}^{NLO}\right|^2}_{\to\hat{\sigma}^{NLO}} + \mathcal{O}(\alpha_s^4)$$

- use Feynman rules to get all diagrams for $\mathcal{M}$

HH production mechanisms

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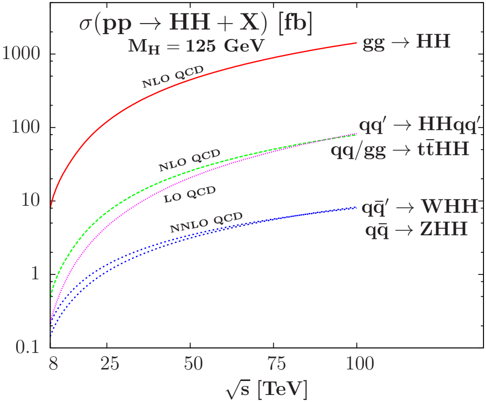

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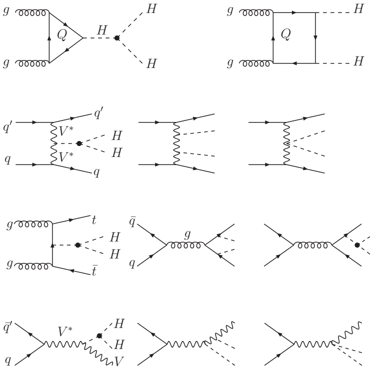

Feynman diagrams

$$\text{Fdiagram}_{D=4} = \prod_{l=1}^{L} \int \frac{d^4 k_l}{i\pi^2} \frac{\text{tensorstructure}}{\prod_{n=1}^{P} (q_n^2(\{k,p\}) - m_n^2 + i\delta)^{\nu_n}}$$

can diverge in the following cases

- $k_l \to \infty$: *ultraviolet (UV)* singularity,
- *infrared (IR)* singularity:
  - $k_l \to 0$: *soft* singularity
  - *collinear* singularity

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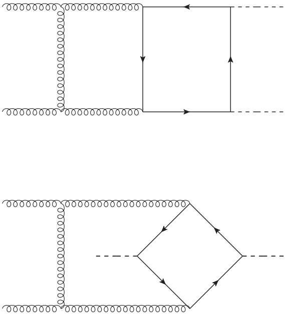
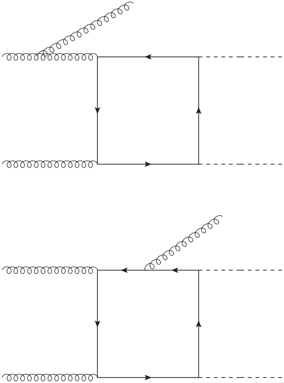
Solution: Dimensional Regularization

- shift dimension of integration: $4 \to D = 4 - 2\,\varepsilon$
- renders both UV and IR divergences finite → poles in $1/\varepsilon$
- Integrals can be safely manipulated, poles must finally cancel

gg→HH: Some Virtual and Real NLO diagrams



analytically not feasible



automatated 1 Loop tools exist, e.g. GoSam [Cullen, van Deurzen, Greiner, Heinrich, Luisoni et al]

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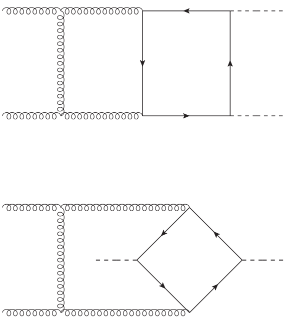

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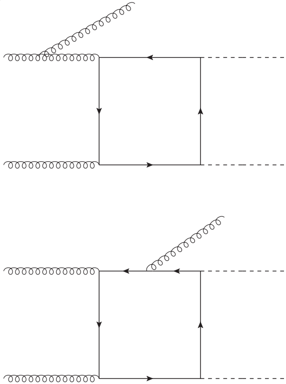

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$\sigma_V^{NLO} + \sigma_R^{NLO}$ is IR finite

Virtual Amplitude $\mathcal{M}_V(gg \to HH)$

- number of diagrams ≈ 120
- two loop four point functions
- four mass scales: m_t, m_H, 2 kinematic invariants

Virtual Amplitude $\mathcal{M}_V(gg \to HH)$

- number of diagrams $\approx$ 120
- two loop four point functions
- four mass scales: m_t, m_H, 2 kinematic invariants

Procedure

1. Perform the traces over Fermion loops in the numerator
2. group diagrams with similar denominators
3. use Computer algebra programs to reduce them to a small set of *master integrals (MI)*

Numerical Integration with SecDec

$$\mathrm{MI} = \int d^D k_1 d^D k_2 \frac{1}{\prod_{n=1}^{7} (q_n^2(\{k,p\}) - m_n^2 + i\delta)}$$

- analytical integration not feasible due to four mass scales
- use SecDec: a program to evaluate dimensionally regulated parameter integrals numerically [Borowka, Heinrich, Jones, Kerner, Schlenk, Zirke]
- the dimensionally regularized 2-loop master integrals are expressible as a Laurent series in ε:

$$\mathrm{MI} = \sum_{i=-n}^{2L=4} \frac{C_{-i}}{\varepsilon^i} + \mathcal{O}(\varepsilon^{n+1}) = \ldots + \frac{C_{-2}}{\varepsilon^2} + \frac{C_{-1}}{\varepsilon} + C_0 + C_1\varepsilon + \ldots$$

- C_i: integrals; depend on $(m_t, m_H, 2$ kinematic invariants$)$

Kinematics

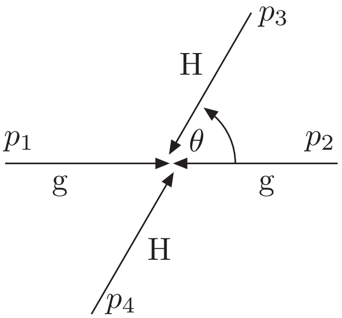
$$p_1^\mu = \frac{\sqrt{\hat{s}}}{2}(1, 0, 0, 1)$$

$$p_2^\mu = \frac{\sqrt{\hat{s}}}{2}(1, 0, 0, -1)$$

$$p_3^\mu = -\frac{\sqrt{\hat{s}}}{2}(1, 0, \beta \sin(\theta), \beta \cos(\theta))$$

$$p_4^\mu = \frac{\sqrt{\hat{s}}}{2}(-1, 0, \ \beta \sin(\theta), \ \beta \cos(\theta))$$

$$\beta = \sqrt{1 - \frac{4m_H^2}{\hat{s}}}$$



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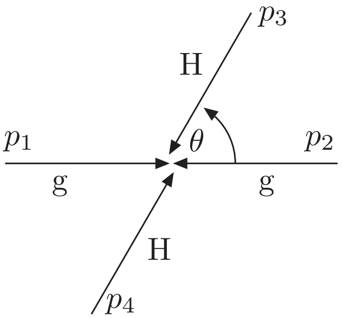
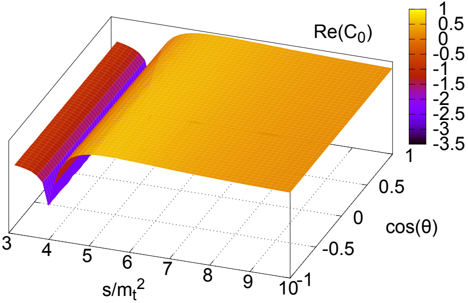
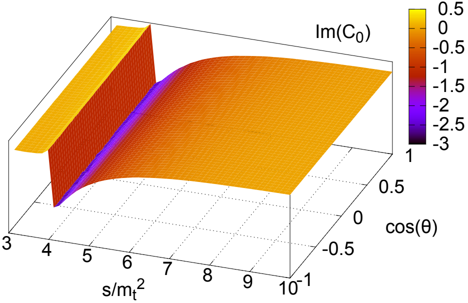
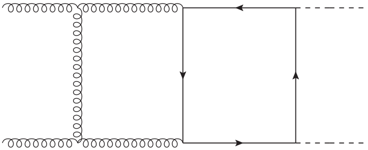


Parametrize the process through: partonic center-of-mass energy $\hat{s}$ and angle $\cos(\theta)$ between gluon and Higgs

Plot of a MI over $\hat{s}$ and $\cos(\theta)$

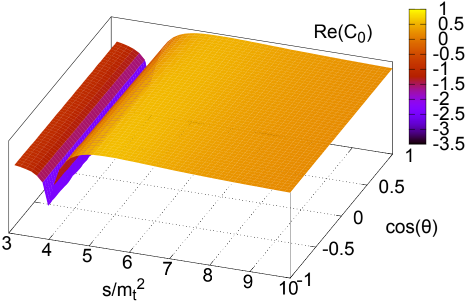


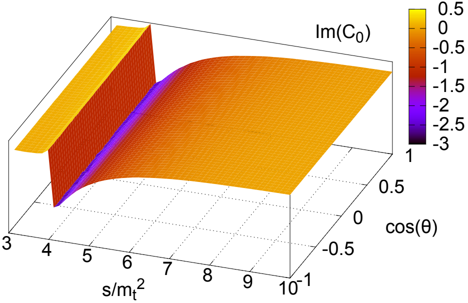


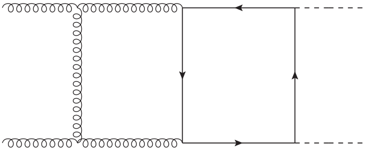


$$\mathrm{MI} = \frac{C_{-2}}{\varepsilon^2} + \frac{C_{-1}}{\varepsilon} + C_0 + \mathcal{O}(\varepsilon)$$

Plot of a MI over ŝ and cos(θ)







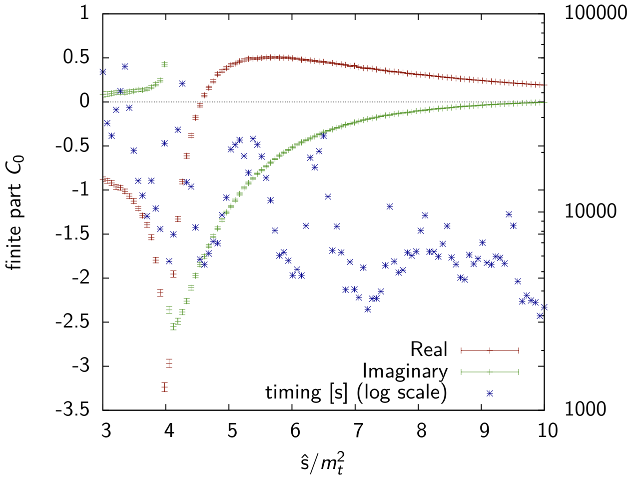
$\text{MI} = \frac{C_{-2}}{\varepsilon^2} + \frac{C_{-1}}{\varepsilon} + C_0 + \mathcal{O}(\varepsilon)$

threshold: $\hat{s} = (2m_t)^2$

produce 2 real top quarks

discontinuity in imaginary part

Scan of same MI over ŝ only



$\cos(\theta)$	$1/\sqrt{2}$
m_H^2/m_t^2	0.75
# points	$50 \cdot 10^6$
rel. acc.	10^{-2}
abs. acc.	10^{-4}

Conclusions

- aim of the thesis: look at some master integrals that appear in the virtual gg→HH amplitude at NLO
- 2 loop, 4 point master integrals, involving 4 mass scales
- calculate them numerically, see where thresholds might appear such that the integrator struggles
- main problems occur at top threshold $\hat{s} = (2m_t)^2$:
  long timings for both real and imaginary part

- Thank you for your attention!

- Backup slides

SecDec: Algebraic Part

1. Introduce Feynman parameters

$$\frac{1}{P_1 P_2} = \int_0^\infty \frac{dx_1 dx_2 \delta(1 - x_1 - x_2)}{x_1 P_1 + x_2 P_2}$$

2. integrate out the loop momenta k_l analytically

$$\mathrm{FI} = \Gamma(N_\nu - LD/2) \int_0^\infty \prod_{i=1}^{P} dx_i x_i^{\nu_i - 1} \delta\left(1 - \sum_{j=1}^{P} x_j\right) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}}{\mathcal{F}^{N_\nu - LD/2}} \, .$$

3. extract the poles in ε through the sector decomposition algorithm:

$$\mathrm{FI} = \Gamma(N_\nu - LD/2) \sum_{i=-n}^{2L} \underbrace{\frac{C_{-i}}{\varepsilon^i} + \mathcal{O}(\varepsilon^{n+1})}_{\ldots + \frac{C_{-2}}{\varepsilon^2} + \frac{C_{-1}}{\varepsilon} + C_0 + C_1 \varepsilon + \ldots}$$

Kinematical Thresholds

- integrate the coefficients C_i numerically
- Besides IR and UV divergences, there can be another problem: $\mathcal{F}$ can vanish inside the x_i integration region $\rightarrow$ kinematical thresholds: they are integrable, by distorting the integration contour