Branes, monodromies and non-geometric backgrounds

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Motivation

- String theory symmetries suggest that there may exist backgrounds that cannot be described in terms of classical geometry.
- New exciting properties may arise, such as non-commutativity or non-associativity.
- It is still not well known how to fully describe properties of such backgrounds inside string theory.

What we do

- We look for solutions of the string background equations of sigma models at the limit $\alpha' \to 0$ ($\Rightarrow$ SUGRA equations of motion)
- In particular, we are interested in exotic brane solutions with non-geometric backgrounds around them using the semiflat approximation.
- We develop a method to construct such backgrounds in terms of torus fibrations with arbitrary monodromy.
- We discuss their general properties in terms of an exhaustive classification for all possible monodromies.
- We point out some possible interpretation of the new solutions.

Essential tool for our study: T-Duality

T-duality group in toroidal compactifications: $O(d,d,\mathbb{Z})$

2 dimensional case:

$$O(2,2,\mathbb{Z}) \sim SL(2,\mathbb{Z})_\rho \times SL(2,\mathbb{Z})_\tau \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$\text{Complex structure: } \tau = \frac{G_{12}}{G_{11}} + i\frac{\sqrt{\det G}}{G_{11}}$$

$$\text{Kähler structure: } \rho = B + i\sqrt{\det G}$$

$SL(2,\mathbb{Z})$ action:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$\rho \to \frac{a\rho+b}{c\rho+d}$$

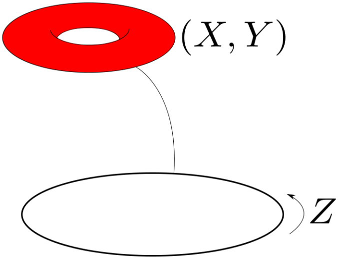
General case

T-duality can be generalised to other backgrounds if in one direction there is an isometry. $\Rightarrow$ **Buscher rules**

Introduction to non-geometric backgrounds.

T^3 with constant H flux and its duals

J. Shelton, W. Taylor, B. Wecht. [0508133]



T^3 with N units of H-flux

$$
\begin{aligned}
ds^2 &= dX^2 + dY^2 + dZ^2 \\
H &= N dX \wedge dY \wedge dZ
\end{aligned}
$$

N constant. Compactification:
$X \cong X + 1,\ Y \cong Y + 1,\ Z \cong Z + 1$

- This is not a solution of the equation of motion unless $N = 0$.
- Choose gauge: $B_{xy} = NZ$ and $B_{ij} = 0$ for $\{ij\} \neq x, y$.
- One has to impose the quantization condition $N \in \mathbb{Z}$.
- **Monodromy on the fiber:** $\boxed{\text{When } Z \to Z + 1 \Rightarrow \rho \to \rho + N}$

T-duality chain: $T^3 \xrightarrow{X}$ Nil-manifold $\xrightarrow{Y}$ T-fold

The Nil-manifold

$$\begin{aligned} ds^2 &= (dX - NZdY)^2 \\ &\quad + dY^2 + dZ^2 \\ B_{xy} &= 0 \end{aligned}$$

Monodromy on the fiber:

When $Z \to Z+1 \Rightarrow \tau \to \tau - N$

Geometric monodromy

The T-fold

$$\begin{aligned} ds^2 &= f(Z)(dX^2 + dY^2) + dZ^2 \\ B_{xy} &= f(Z)NZ \end{aligned}$$

$$f(Z) = \left[1 + (NZ)^2\right]^{-1}$$

Monodromy on the fiber:

$Z \to Z+1 \Rightarrow \rho^{-1} \to \rho^{-1} - N$

Non-geometric monodromy.
Volume and B-field get mixed

- Monodromies in τ are always geometric. Monodromies in ρ may be non-geometric

Genuine non-geometric backgrounds

Background with $\tau \to -1/\tau$, $\rho \to -1/\rho$

$$\begin{aligned} \tau(Z) &= \frac{\tau_0 \cos(fZ) + \sin(fZ)}{\cos(fZ) - \tau_0 \sin(fZ)} \\ \rho(Z) &= \frac{\rho_0 \cos(gZ) + \sin(gZ)}{\cos(gZ) - \rho_0 \sin(gZ)} \end{aligned}$$

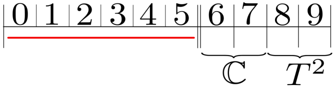
with $f, g \in \mathbb{Z} + \frac{1}{4}$ and $\rho_0, \tau_0 \in \mathbb{C}$.

- Not T-dual to any known geometric background.
- Fixed point of the transformation at $\tau_0 = \rho_0 = i$.
- Studied as an asymmetric $\mathbb{Z}_2$ orbifold CFT (*C.Condeescu, I. Florakis, D. Lüst. [1202.6366], [1307.0999]*).
- Constructed in DFT as a general twisted torus (*F. Haßler, D. Lüst. [1401.5068]*).
- Potential fixes τ_0 and ρ_0 to the fixed point.

Brane solutions

Our aims

- We will deal only with backgrounds that are solution of SUGRA equations of motion (Unlike above).
- Find a method to construct brane backgrounds in semiflat approximation with any given monodromy.



Semiflat approximation: Fibered torus with $U(1)^2$ isometries.

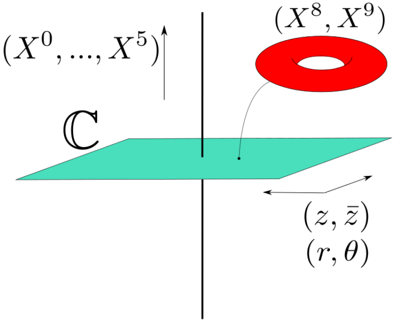
- Give an exhaustive classification of such backgrounds using the conjugacy classes of $SL(2,\mathbb{Z})$.
- Discuss the validity of such solutions.

General background solution in semiflat approximation

S. Hellerman, J. McGreevy, B. Williams. [0208174]; J. de Boer, M. Shigemori. [1209.6056]

Ansatz

$$
\begin{aligned}
ds^2_{10} &= f^2(-dX_0^2 + dX_{12345}^2) + g^2(z,\bar{z})dzd\bar{z} + G_{ab}(z,\bar{z})dX^a dX^b \\
H &= H_i dX_i \wedge dX_8 \wedge dX_9 \qquad i = 6,7 \quad a = 8,9
\end{aligned}
$$



Preservation of maximal compatible SUSY

$\Rightarrow$ SUGRA equations of motion

Solution

$$\begin{aligned} ds^2_{10} &= -dX_0^2 + dX^2_{12345} + g^2 dz d\bar{z} + G_{ab} dX^a dX^b \\ H &= H_i dX_i \wedge dX_8 \wedge dX_9 \qquad i = 6,7 \quad a = 8,9 \end{aligned}$$

With

$$\begin{aligned} g^2 &= \mathrm{e}^{2\varphi_1} \rho_2 \tau_2 \\ G_{ab} &= \frac{\rho_2}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix} \\ H_i &= \partial_i \rho_1 \end{aligned}$$

Where $\rho(z) = \rho_1 + i\rho_2$, $\tau(z) = \tau_1 + i\tau_2$ and $\varphi(z) = \varphi_1 + i\varphi_2$ are holomorphic functions of the complex plane

- We want to find backgrounds with any possible monodromy $M \in SL(2,\mathbb{Z})_\tau \times SL(2,\mathbb{Z})_\rho$: $\boxed{\theta \to \theta + 2\pi \Rightarrow (\tau,\rho) \to M[(\tau,\rho)]}$
- We classify all of them via $\tau(z)$ and $\rho(z)$.

Three possible cases

Non-trivial monodromy in ρ and $\tau = i$

One still have to fix the function $\varphi(z)$. Take $\rho(z)$ with monodromy

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

Studying the reduction from 10D to 8D one finds the extra condition:

$$e^{\varphi} \to (c\rho + d)e^{\varphi}, \text{ when } \theta \to \theta + 2\pi$$

Non-trivial monodromy in τ and $\rho = i$

One can study these cases using the symmetry $\tau \leftrightarrow \rho$.

Non-trivial monodromies in τ and ρ

Once one knows the solutions for the above cases, then $\varphi = \varphi^{(\rho)} + \varphi^{(\tau)}$

Constructing new backgrounds with $\tau = i$ and arbitrary monodromy in ρ

The construction procedure

1. Take a monodromy $e^m = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$. The matrix m always exist since $SL(2,\mathbb{Z})$ is connected.
2. Construct the matrix $e^{m\frac{\theta}{2\pi}} = \begin{pmatrix} \tilde{a}(\theta) & \tilde{b}(\theta) \\ \tilde{c}(\theta) & \tilde{d}(\theta) \end{pmatrix}$. It is an $SL(2,\mathbb{Z})$ matrix when $\theta = 2\pi n$, $n \in \mathbb{Z}$.
3. Construct the function $\rho(r,\theta) = \frac{\tilde{a}(\theta)\rho_0(r)+\tilde{b}(\theta)}{\tilde{c}(\theta)\rho_0(r)+\tilde{d}(\theta)}$.
4. Impose Cauchy-Riemann conditions on $\rho(r,\theta)$ to fix $\rho_0(r)$.
5. Construct the function $e^{\varphi(r,\theta)} = (\tilde{c}(\theta)\rho_0(r) + \tilde{d}(\theta))e^{\varphi_0(r)}$.
6. Construct the background.

Classification. The $SL(2,\mathbb{Z})$ conjugacy classes

- Conjugacy class of $a \in SL(2,\mathbb{Z})$:
  $[a] = \{M \,|\, M = gag^{-1},\ \ g \in SL(2,\mathbb{Z})\}$.
- A useful tool for studying conjugacy classes using the **Trace** of the matrix. Traces does not change under conjugation. However, two matrices with same trace can be in different conjugacy classes.

There are infinite conjugacy classes. They can be classified into:

Elliptic: $\lvert \mathrm{Tr}\ M \rvert < 2$	- Two conjugate complex eigenvalues - $M^n = \mathbb{I}$, for finite n (finite order) - M has fixed points
Parabolic: $\lvert \mathrm{Tr}\ M \rvert = 2$	- Eigenvalues: ± 1 - Infinite order
Hyperbolic: $\lvert \mathrm{Tr}\ M \rvert > 2$	- 2 real eigenvalues - Infinite order - When applying M several times ρ grows almost exponentially

Examples I: Parabolic monodromy in ρ. $\tau = i$

All elements in $\left[\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\right]$ can be written like:

$$M_{p,q} = \begin{pmatrix} 1+pq & p^2 \\ -q^2 & 1-pq \end{pmatrix} \quad p, q \in \mathbb{Z}$$

Function with the desired monodromy:

$$\rho(r,\theta) = -\frac{(pq\frac{\theta}{2\pi} + 1)\rho_0(r) + p^2\frac{\theta}{2\pi}}{q^2\frac{\theta}{2\pi}\rho_0(r) + pq\frac{\theta}{2\pi} - 1}$$

Imposing Cauchy Riemann conditions: (Define $H_q = \frac{q^2}{2\pi}$ and $H_p = \frac{p^2}{2\pi}$)

$q \neq 0$	$q = 0$
$\rho = -\sqrt{\frac{H_p}{H_q}} - \frac{1}{H_q\theta + i\left(1 + H_q \ln\left(\frac{\mu}{r}\right)\right)}$	$\rho = H_p\theta + i\left(1 + H_p \ln\left(\frac{\mu}{r}\right)\right)$

Background: (Define $h_{p,q}(r) = 1 + H_{p,q} \ln\left(\frac{\mu}{r}\right)$)

- $q \neq 0$

$$\begin{aligned} ds^2 &= dX_{012345}^2 + h_q(r) dX_{67}^2 + \frac{h_q(r)}{h_q^2(r) + (H_q\theta)^2} dX_{89}^2 \\ B &= \left(-\sqrt{\frac{H_p}{H_q}} - \frac{H_q\theta}{h_q^2(r) + (H_q\theta)^2} \right) dX_8 \wedge dX_9 \end{aligned}$$

- $q = 0$

$$\begin{aligned} ds^2 &= dX_{012345}^2 + h_p(r) dX_{67}^2 + h_p(r) dX_{89}^2 \\ B &= H_p\theta \;\, dX_8 \wedge dX_9 \end{aligned}$$

- $(p, q) = (1, 0) \leftrightarrow$ Smeared NS5
  $(p, q) = (0, 1) \leftrightarrow$ Q-brane (*J. de Boer, M. Shigemori. [1209.6056]; F. Haßler, D. Lüst [1303.1413]*)
- **Geometric case:** $(p, q) = (1, 0), (0, 1) \leftrightarrow$ Smeared KK-monopoles.
- Solutions only trustable up to some scale. (Volume becomes negative)

Examples II: Double elliptic case, $\tau \to -\frac{1}{\tau}, \rho \to -\frac{1}{\rho}$

Holomorphic function with the desired monodromy:

$$\begin{aligned} \rho(r,\theta) &= \frac{\rho_0(r)\cos(\theta/4)+\sin(\theta/4)}{\cos(\theta/4)-\rho_0(r)\sin(\theta/4)} \\ \tau(r,\theta) &= \frac{\tau_0(r)\cos(\theta/4)+\sin(\theta/4)}{\cos(\theta/4)-\tau_0(r)\sin(\theta/4)} \end{aligned}$$

$$\rho_0 = \tau_0 = -i\coth\left(\frac{1}{4}\ln(r)+C\right)$$

- The background is genuine non-geometrical. Not dual to any known geometric one.
- We were able to reproduce the genuine non-geometric background discussed above for the T^2 fibration over S^1.

Solution on the fiber

$$G_{ab} = \begin{pmatrix} 1 & \frac{\sin(\theta/2)}{\cos(\theta/2)-\cosh\left(C+\frac{\log(r)}{2}\right)} \\ \frac{\sin(\theta/2)}{\cos(\theta/2)-\cosh\left(C+\frac{\log(r)}{2}\right)} & \frac{2e^C\sqrt{r}\cos(\theta/2)+e^{2C}r+1}{-2e^C\sqrt{r}\cos(\theta/2)+e^{2C}r+1} \end{pmatrix}$$

$$B_{89} = \frac{\sin(\theta/2)}{\cos(\theta/2)-\cosh\left(C+\frac{\log(r)}{2}\right)}$$

Conclusions and open questions

- We investigated exotic branes with arbitrary monodromies.
- We gave an exhaustive classification of all of them.
- We derived explicit solutions for significant examples in each class.
- Picture with a brane source for the flux in contrast with turning on fluxes on a cycle.
- Monodromy becomes "charge" of the brane.
- The group structure of $SL(2,\mathbb{Z})$ suggests that this branes could be understood as bound states of NS5 and Q-brane.
- It is crucial to better understand the microscopic description of Q-branes in string theory.

THANK YOU

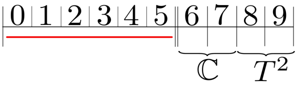
The smeared NS5 and its duals

The NS5 solution of supergravity equations in 10D

$$
\begin{aligned}
ds^2_{NS5} &= dX^2_{012345} + h(r_\perp) dX^2_{6789} \\
e^{2\phi} &= h(r_\perp) \\
H_{mnp} &= \epsilon_{mnpq} \partial_q h(r_\perp), \quad m, n, p, q \in \{6, 7, 8, 9\}
\end{aligned}
$$

$r_\perp$ transversal radial direction (6,7,8,9).

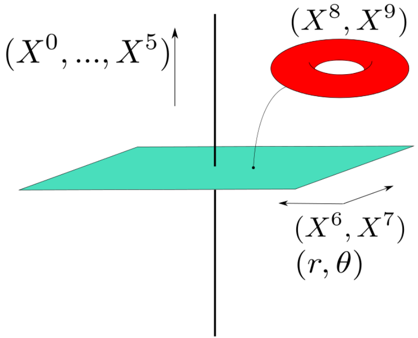
$$
h(r_\perp) = 1 + \frac{H}{r_\perp^2}, \quad H \text{ constant}
$$



This is a well defined object in string theory so it's a good starting point.

NS5 in semi-flat approximation

Compactify directions 8 and 9 and take the limit $r \gg R_8, R_9$:



Fibered torus with $U(1)^2$ isometries.

The smeared NS5

$$
\begin{aligned}
ds^2_{NS5} &= dX^2_{012345} \\
&\quad + h(r) dX^2_{67} + h(r) dX^2_{89} \\
e^{2\phi} &= h(r) \\
B &= H\theta \; dX_8 \wedge dX_9,
\end{aligned}
$$

With,

$$h(r) = 1 + H \ln \frac{\mu}{r}$$

Quantization condition: $2\pi H \in \mathbb{Z}$

Monodromy on the fiber:

When $\theta \to \theta + 2\pi \Rightarrow \rho \to \rho + N$

T-duality chain: Smeared NS5 $\xrightarrow{X^9}$ Smeared KK-monopole $\xrightarrow{X^8}$ Q brane

The KK monopole

$$
\begin{aligned}
ds^2_{KK} &= dX^2_{012345} + h(r)dX^2_{67} + \frac{1}{h(r)}(dX_9 + H\theta dX_8)^2 + h(r)dX^2_8 \\
e^{2\phi} &= 1 \\
H &= 0
\end{aligned}
$$

The Q brane

J. de Boer, M. Shigemori. [1209.6056]; F. Haßler, D. Lüst [1303.1413]

$$
\begin{aligned}
ds^2_{Q} &= dX^2_{012345} + h(r)dX^2_{67} + \frac{h(r)}{h^2(r) + (H\theta)^2}dX^2_{89} \\
e^{2\phi} &= \frac{h(r)}{h^2(r) + (H\theta)^2} \\
B &= -\frac{H\theta}{h^2(r) + (H\theta)^2}dX_8 \wedge dX_9,
\end{aligned}
$$