Mirror symmetry of Calabi-Yau four-folds with non-trivial cohomology of odd degrees

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Master's Thesis supervised by Thomas Grimm

Max-Planck-Institut für Physik

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Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Introduction

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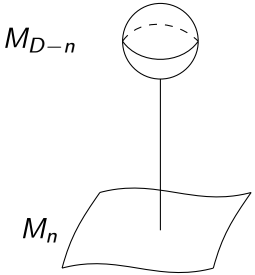
Of special interest to phenomenology is F-Theory, since it allows insights into strongly-coupled behavior of string-theory. This (in some sense) twelve-dimensional string theory needs to be compactified on a so called (elliptically fibered) Calabi-Yau four-fold. (CY_4)

Outline

1. Dimensional Reduction
2. Moduli
3. Mirror Symmetry of the Torus
4. Mirror Symmetry on CY_4

Dimensional Reduction



D-dim. gravity theory coupled to matter

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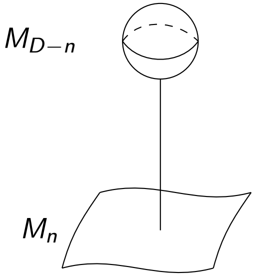

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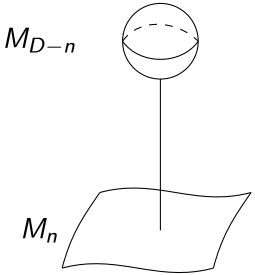


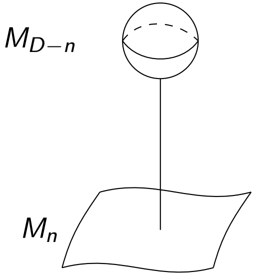
D-dim. gravity theory coupled to matter

$\Rightarrow$ n-dim. effective theory

At every point of the curved spacetime M_n there is a very small deformable internal manifold M_{D-n} whose eigenmodes around a stable ground state correspond to fields on M_n.

$$\text{mass} \sim \frac{1}{\text{size}\ (M_{D-n})}$$

Dimensional Reduction



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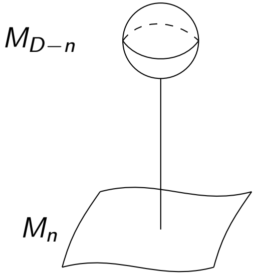

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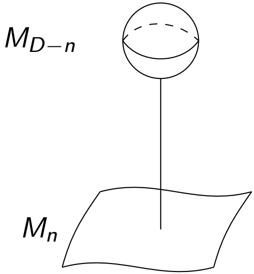

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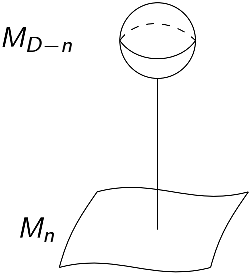

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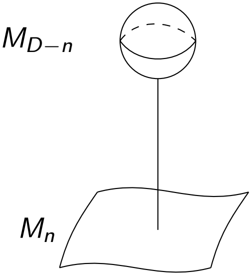

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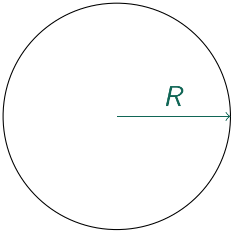
$\Rightarrow$ Study moduli!

Moduli - Circle



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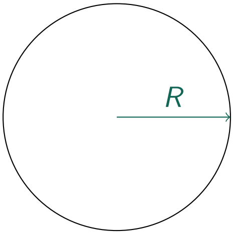

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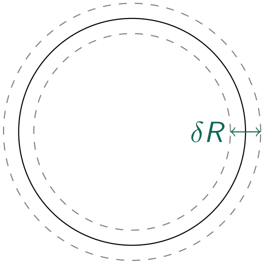
Therefore, only the deformations δR are possible.



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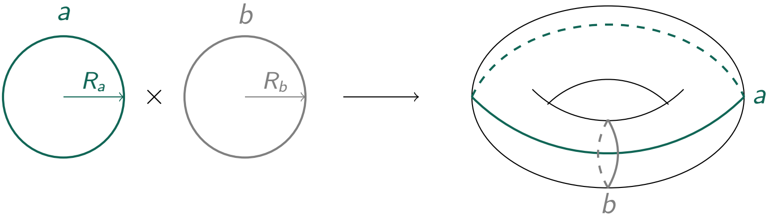

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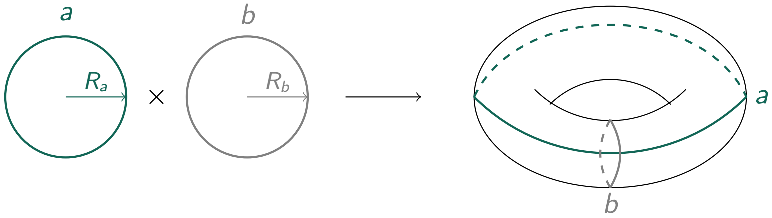

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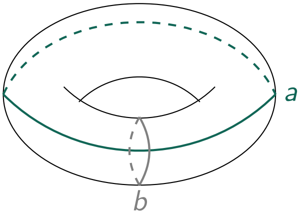
$\Rightarrow$ Two independent eigenmodes to excite!

Moduli - Torus



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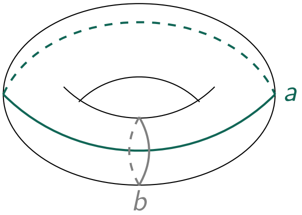

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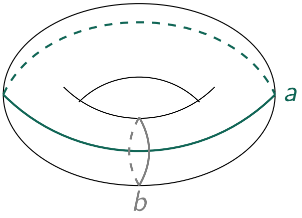

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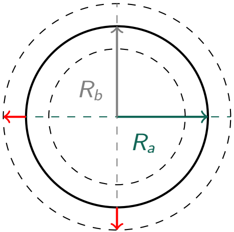
$$\text{Volume:} \quad \mathcal{V} = R_a \cdot R_b$$

and

$$\text{Ratio:} \quad \mathcal{R} = \frac{R_a}{R_b}$$

Moduli - Torus

We have two basic eigenmodes:



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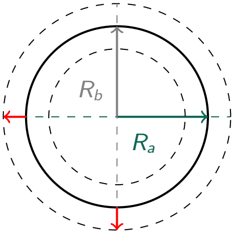

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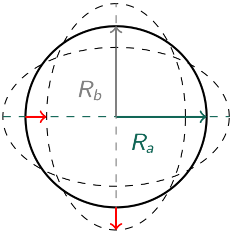

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(opposite phases)

Mirror Symmetry

Consider now the torus $\hat{T}^2$ defined by

$$\hat{R}_b = \frac{1}{R_b}, \quad \hat{R}_a = R_a. \quad \Rightarrow \hat{\mathcal{V}} = \hat{R}_a \hat{R}_b = \frac{R_a}{R_b} = \mathcal{R}!$$

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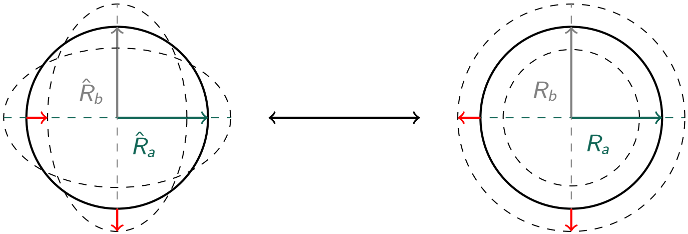
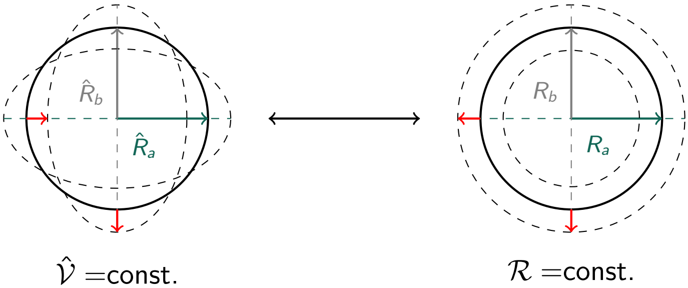
Therefore, we have the correspondence of eigenmodes:



$\hat{\mathcal{V}}$ =const. $\mathcal{R}$ =const.

Mirror Symmetry



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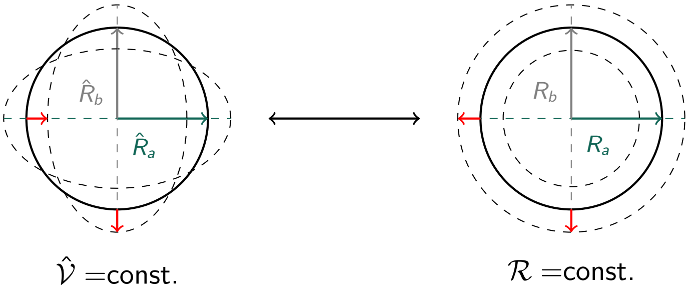

Due to this correspondence, on both geometries (stable ground states) we have the same content of massless eigenmodes.

$\Rightarrow$ We have the same **physics** on both configurations!

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This symmetry is called **mirror symmetry** and $\hat{T}^2$ is the **mirror** of T^2.

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$$\delta R_b \gg 1, \quad \Rightarrow \quad \delta \hat{R}_b \sim \frac{1}{\delta R_b} \ll 1,$$

i.e. **non-perturbative** effects map to **perturbative** effects and are therefore acessible for calculations.

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With the same argument, **quantum** effects can be mapped to **classical** effects and are treatable this way.

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However, in higher dimensions mirror symmetry is highly **non-trivial**, but in the acessible cases it gave us a number of insights into the non-perturbative behavior of string theory.

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Unfortunately, the 8-dimensional case, CY_4, is hardly studied, beside the analogues of lower dimensional properties.

Mirror Symmetry on CY_4

In our present work, we discuss the so called $h^{2,1}$-moduli (N^I), which where not acessible before, due to the lack of a good base choice for these moduli.

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In particular, we found that the Kählerpotential receives corrections for $N^l \neq 0$:

$$K = -\log \mathcal{V} - \log(\int \Omega \wedge \overline{\Omega}) - ReN_l(\int J \wedge \Psi^l \wedge \beta^m) ReN_m$$

where

$$\Psi^l = \frac{1}{2} Re(f)^{lm}(\alpha_m - i\overline{f}_{mk}\beta^k) \in H^{2,1}(Y_4) \quad \text{[Grimm]}$$

with α_m, β^k a basis of $H^3(Y_4, \mathbb{R})$ and

$$f_{lm} = f_{lm}(z), \quad \frac{df_{lm}}{d\overline{z}_{\overline{K}}} = 0.$$

Mirror symmetry on CY_4

Using mirror symmetry we found that the new correction at large volume/complex structure behaves like

$$ReN_l(\int J \wedge \Psi^l \wedge \beta^m)ReN_m \sim \frac{v}{Re(z)}.$$

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This implies that the A-model and the B-model of topological string theory on the **same** CY_4 get **coupled** and can therefore no longer be treated independently.

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On CY_4 **new** moduli arise, that have no lower-dimensional analogue. Mirror symmetry allows us to make first statements about their properties, that are currently not available in the literature. (publication planned)

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- many more!

Thank you for your attention!