

Gravitational scattering in a corpuscular picture

Sebastian Zell

As part of a master thesis
at the Ludwig-Maximilians-Universität München
supervised by Prof. Dr. Georgi Dvali

IMPRS workshop
Max-Planck-Institute for Physics

30th June 2015

- Idea: Classical metric = collective effect of gravitons¹

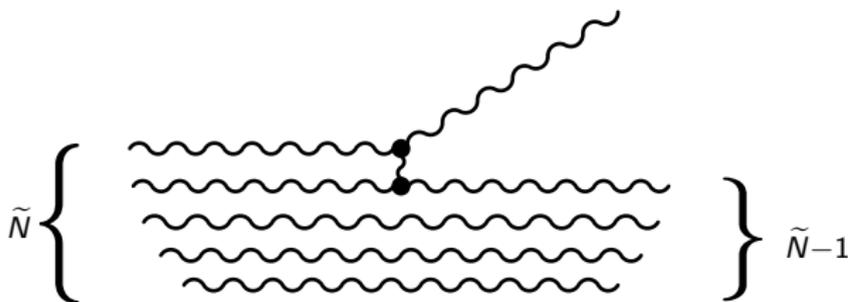
¹ G. Dvali and C. Gomez, *Black Hole's Quantum N-Portrait*, arXiv:1112.3359.

- Idea: Classical metric = collective effect of gravitons¹
 - Black hole as bound state of gravitons
 - Occupation number: $\tilde{N} = M^2/M_p^2$

¹ G. Dvali and C. Gomez, *Black Hole's Quantum N-Portrait*, arXiv:1112.3359.

Corpuscular approach

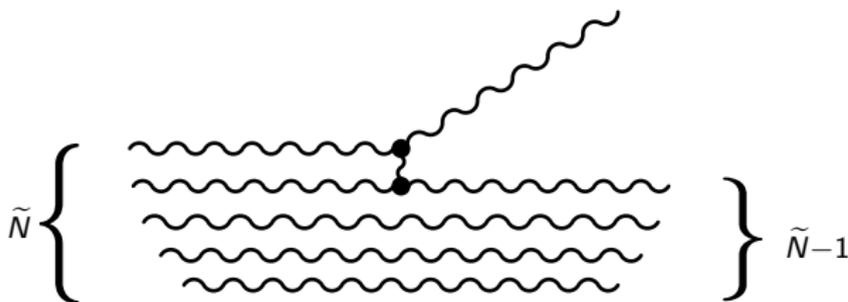
- Idea: Classical metric = collective effect of gravitons¹
 - Black hole as bound state of gravitons
 - Occupation number: $\tilde{N} = M^2/M_p^2$
- Example: Hawking radiation



¹ G. Dvali and C. Gomez, *Black Hole's Quantum N-Portrait*, arXiv:1112.3359.

Corpuscular approach

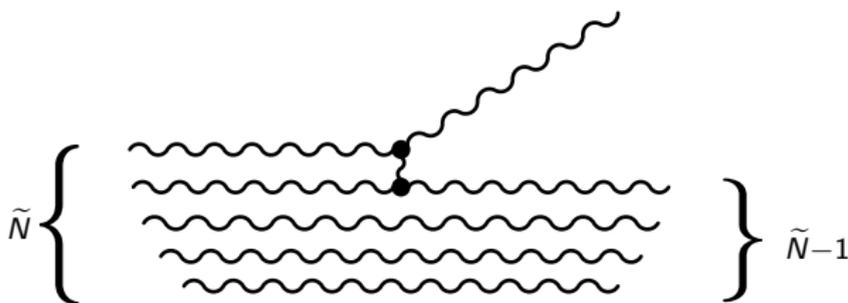
- Idea: Classical metric = collective effect of gravitons¹
 - Black hole as bound state of gravitons
 - Occupation number: $\tilde{N} = M^2/M_p^2$
- Example: Hawking radiation
 - Back reaction: $\tilde{N}' = \tilde{N} - 1 = \tilde{N} \left(1 - \frac{1}{\tilde{N}}\right)$



¹ G. Dvali and C. Gomez, *Black Hole's Quantum N-Portrait*, arXiv:1112.3359.

Corpuscular approach

- Idea: Classical metric = collective effect of gravitons¹
 - Black hole as bound state of gravitons
 - Occupation number: $\tilde{N} = M^2/M_p^2$
- Example: Hawking radiation
 - Back reaction: $\tilde{N}' = \tilde{N} - 1 = \tilde{N} \left(1 - \frac{1}{\tilde{N}}\right)$
 - $1/\tilde{N}$ -correction could resolve information paradox



¹ G. Dvali and C. Gomez, *Black Hole's Quantum N-Portrait*, arXiv:1112.3359.

- 1 Corpuscular weak gravity
- 2 Application to Scattering
- 3 Outlook

Classical corpuscles

- Scalar weak gravity:

$$g_{00} = 1 + 2\frac{\Phi}{M_p}$$

Classical corpuscles

- Scalar weak gravity:

$$g_{00} = 1 + 2\frac{\Phi}{M_p}$$

- Classical self-energy:

$$|E[\Phi]| = \frac{1}{2} \int d^3\vec{x} \left| \vec{\nabla}\Phi(\vec{x}) \right|^2$$

Classical corpuscles

- Scalar weak gravity:

$$g_{00} = 1 + 2\frac{\Phi}{M_p}$$

- Classical self-energy:

$$\begin{aligned} |E[\Phi]| &= \frac{1}{2} \int d^3\vec{x} \left| \vec{\nabla}\Phi(\vec{x}) \right|^2 \\ &\propto \int d^3\vec{k} \vec{k}^2 \left| \mathcal{F}(\Phi)(\vec{k}) \right|^2 \end{aligned}$$

Classical corpuscles

- Scalar weak gravity:

$$g_{00} = 1 + 2 \frac{\Phi}{M_p}$$

- Classical self-energy:

$$\begin{aligned} |E[\Phi]| &= \frac{1}{2} \int d^3 \vec{x} \left| \vec{\nabla} \Phi(\vec{x}) \right|^2 \\ &\propto \int d^3 \vec{k} \underbrace{\vec{k}^2}_{\epsilon(\vec{k})} \left| \mathcal{F}(\Phi)(\vec{k}) \right|^2 \end{aligned}$$

Classical corpuscles

- Scalar weak gravity:

$$g_{00} = 1 + 2 \frac{\Phi}{M_p}$$

- Classical self-energy:

$$\begin{aligned} |E[\Phi]| &= \frac{1}{2} \int d^3 \vec{x} \left| \vec{\nabla} \Phi(\vec{x}) \right|^2 \\ &\propto \int d^3 \vec{k} \underbrace{\vec{k}^2}_{\epsilon(\vec{k})} \left| \mathcal{F}(\Phi)(\vec{k}) \right|^2 \end{aligned}$$

- Idea: energy carried by gravitons (with free dispersion):

$$n(\vec{k}) = \frac{\epsilon(\vec{k})}{|\vec{k}|}$$

Classical corpuscles

- Scalar weak gravity:

$$g_{00} = 1 + 2 \frac{\Phi}{M_p}$$

- Classical self-energy:

$$\begin{aligned} |E[\Phi]| &= \frac{1}{2} \int d^3 \vec{x} \left| \vec{\nabla} \Phi(\vec{x}) \right|^2 \\ &\propto \int d^3 \vec{k} \underbrace{\vec{k}^2}_{\epsilon(\vec{k})} \left| \mathcal{F}(\Phi)(\vec{k}) \right|^2 \end{aligned}$$

- Idea: energy carried by gravitons (with free dispersion):

$$n(\vec{k}) = \frac{\epsilon(\vec{k})}{|\vec{k}|} \propto |\vec{k}| \left| \mathcal{F}(\Phi)(\vec{k}) \right|^2$$

Quantization

- Conditions:
 - ① Distribution over different modes: $n(\vec{k})$

Quantization

- Conditions:
 - ① Distribution over different modes: $n(\vec{k})$
 - ② Distribution in fixed mode: coherent state

Quantization

- Conditions:

- ① Distribution over different modes: $n(\vec{k})$
- ② Distribution in fixed mode: coherent state

⇒ Unique quantum state $|n(\vec{k})\rangle$

Quantization

- Conditions:

- ① Distribution over different modes: $n(\vec{k})$
- ② Distribution in fixed mode: coherent state

⇒ Unique quantum state $|n(\vec{k})\rangle$

- Expectation value:

$$\langle n(\vec{k}) | \hat{\Phi} | n(\vec{k}) \rangle$$

Quantization

- Conditions:

- ① Distribution over different modes: $n(\vec{k})$
- ② Distribution in fixed mode: coherent state

⇒ Unique quantum state $|n(\vec{k})\rangle$

- Expectation value:

$$\begin{aligned} & \langle n(\vec{k}) | \hat{\Phi} | n(\vec{k}) \rangle \\ &= \langle n(\vec{k}) | \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n(\vec{k}) \rangle \end{aligned}$$

Quantization

- Conditions:

- ① Distribution over different modes: $n(\vec{k})$
- ② Distribution in fixed mode: coherent state

⇒ Unique quantum state $|n(\vec{k})\rangle$

- Expectation value:

$$\begin{aligned}
 & \langle n(\vec{k}) | \hat{\Phi} | n(\vec{k}) \rangle \\
 &= \langle n(\vec{k}) | \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \left(\sqrt{n(\vec{k})} e^{-ikx} + \sqrt{n(\vec{k})} e^{ikx} \right)
 \end{aligned}$$

Quantization

- Conditions:

- ① Distribution over different modes: $n(\vec{k})$
- ② Distribution in fixed mode: coherent state

⇒ Unique quantum state $|n(\vec{k})\rangle$

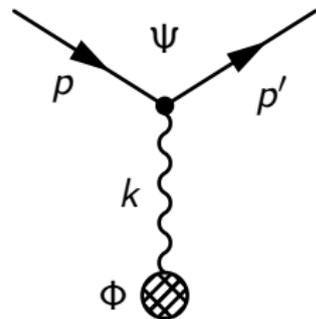
- Expectation value:

$$\begin{aligned}
 & \langle n(\vec{k}) | \hat{\Phi} | n(\vec{k}) \rangle \\
 &= \langle n(\vec{k}) | \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \left(\sqrt{n(\vec{k})} e^{-ikx} + \sqrt{n(\vec{k})} e^{ikx} \right) \\
 &= \Phi
 \end{aligned}$$

Cross section

Semi-classical:

$$\frac{d\sigma_{sc}}{d\Omega} \propto |\mathcal{F}(\Phi)|^2 (\vec{p}' - \vec{p})$$



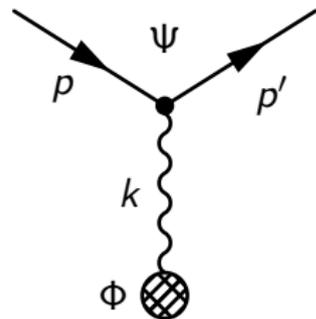
Cross section

Semi-classical:

$$\frac{d\sigma_{sc}}{d\Omega} \propto |\mathcal{F}(\Phi)|^2 (\vec{p}' - \vec{p})$$

Fully quantum:

$$\frac{d\sigma}{d\Omega} \propto n_i(\vec{p}' - \vec{p})$$



Cross section

Semi-classical:

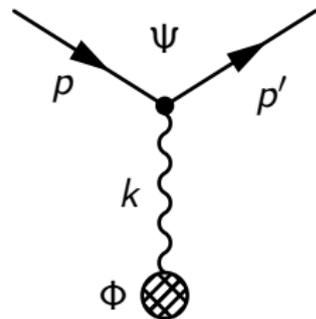
$$\frac{d\sigma_{sc}}{d\Omega} \propto |\mathcal{F}(\Phi)|^2 (\vec{p}' - \vec{p})$$

Fully quantum:

$$\frac{d\sigma}{d\Omega} \propto n_i (\vec{p}' - \vec{p})$$

- Reinterpretation: Momentum transfer = graviton momentum

$$\vec{p}' - \vec{p} = \vec{k}$$



Cross section

Semi-classical:

$$\frac{d\sigma_{sc}}{d\Omega} \propto |\mathcal{F}(\Phi)|^2 (\vec{p}' - \vec{p})$$

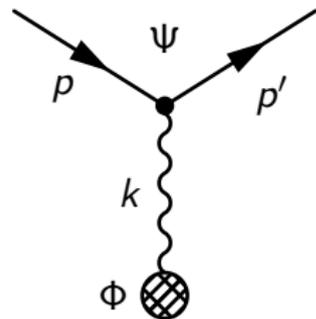
Fully quantum:

$$\frac{d\sigma}{d\Omega} \propto n_i(\vec{p}' - \vec{p})$$

- Reinterpretation: Momentum transfer = graviton momentum

$$\vec{p}' - \vec{p} = \vec{k}$$

- Back reaction: $n_i(\vec{k}) \neq n_f(\vec{k})$



Cross section

Semi-classical:

$$\frac{d\sigma_{sc}}{d\Omega} \propto |\mathcal{F}(\Phi)|^2 (\vec{p}' - \vec{p})$$

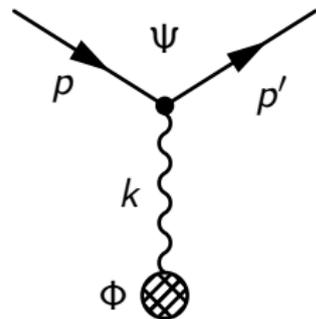
Fully quantum:

$$\frac{d\sigma_q}{d\Omega} \propto n_i(\vec{p}' - \vec{p}) \exp \left\{ - \int d^3\vec{k} \left(\sqrt{n_i(\vec{k})} - \sqrt{n_f(\vec{k})} \right)^2 \right\}$$

- Reinterpretation: Momentum transfer = graviton momentum

$$\vec{p}' - \vec{p} = \vec{k}$$

- Back reaction: $n_i(\vec{k}) \neq n_f(\vec{k})$
 \Rightarrow Quantum correction



Concrete setup

- Potential created by a shell of mass M :

$$\Phi(\vec{x}) \propto \frac{\sqrt{\tilde{N}}}{\max\{R, |\vec{x}|\}}$$

Concrete setup

- Potential created by a shell of mass M :

$$\Phi(\vec{x}) \propto \frac{\sqrt{\tilde{N}}}{\max\{R, |\vec{x}|\}}$$
$$n(\vec{k}) \propto \tilde{N} \frac{\sin^2(|\vec{k}|R)}{|\vec{k}|^5 R^2}$$

Concrete setup

- Potential created by a shell of mass M :

$$\Phi(\vec{x}) \propto \frac{\sqrt{\tilde{N}}}{\max\{R, |\vec{x}|\}}$$

$$n(\vec{k}) \propto \tilde{N} \frac{\sin^2(|\vec{k}|R)}{|\vec{k}|^5 R^2}$$

$$\Rightarrow \tilde{N} \neq N = \infty$$

Concrete setup

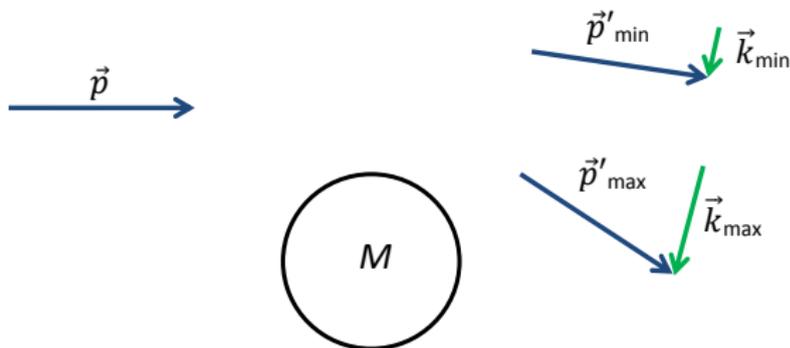
- Potential created by a shell of mass M :

$$\Phi(\vec{x}) \propto \frac{\sqrt{\tilde{N}}}{\max\{R, |\vec{x}|\}}$$

$$n(\vec{k}) \propto \tilde{N} \frac{\sin^2(|\vec{k}|R)}{|\vec{k}|^5 R^2}$$

$$\Rightarrow \tilde{N} \neq N = \infty$$

- Graviton momentum not fixed:



Most probable final state

- Constraint: Momentum conservation

$$K[n_i] = K[n_f] + \bar{k}$$

Most probable final state

- Constraint: Momentum conservation

$$K[n_i] = K[n_f] + \bar{k}$$

- Maximization of

$$\sigma_q = \sigma_{sc} \cdot \exp \left\{ - \int_{k_{min} \leq |\vec{k}| \leq k_{max}} d^3 \vec{k} \left(\sqrt{n_i(\vec{k})} - \sqrt{n_f(\vec{k})} \right)^2 \right\}$$

Most probable final state

- Constraint: Momentum conservation

$$K[n_i] = K[n_f] + \bar{k}$$

- Maximization of

$$\sigma_q = \sigma_{sc} \cdot \exp \left\{ - \int_{k_{min} \leq |\vec{k}| \leq k_{max}} d^3 \vec{k} \left(\sqrt{n_i(\vec{k})} - \sqrt{n_f(\vec{k})} \right)^2 \right\}$$

⇒ Result:

$$\sigma_q \propto \sigma_{sc} \left(1 - \frac{1}{\tilde{N}} \cdot \frac{\bar{k}}{k_{max} - k_{min}} \right)$$

Most probable final state

- Constraint: Momentum conservation

$$K[n_i] = K[n_f] + \bar{k}$$

- Maximization of

$$\sigma_q = \sigma_{sc} \cdot \exp \left\{ - \int_{k_{min} \leq |\vec{k}| \leq k_{max}} d^3 \vec{k} \left(\sqrt{n_i(\vec{k})} - \sqrt{n_f(\vec{k})} \right)^2 \right\}$$

⇒ Result:

$$\sigma_q \propto \sigma_{sc} \left(1 - \underbrace{\frac{1}{\tilde{N}}}_{\text{background}} \cdot \underbrace{\frac{\bar{k}}{k_{max} - k_{min}}}_{\text{photon}} \right)$$

Outlook

Summary

- Corpuscular picture of weak gravitational background
- Natural interpretation of scattering
- Explicit computation of $1/\tilde{N}$ -correction

Outlook

Summary

- Corpuscular picture of weak gravitational background
- Natural interpretation of scattering
- Explicit computation of $1/\tilde{N}$ -correction

Future research

- Black holes
- De Sitter
- Inflationary scenarios

Quantization of the Newtonian potential

- Interaction picture: Free fields

Quantization of the Newtonian potential

- Interaction picture: Free fields
- Analogy of weak gravity and electrodynamics:
Newtonian potential \cong electric potential

Quantization of the Newtonian potential

- Interaction picture: Free fields
- Analogy of weak gravity and electrodynamics:
Newtonian potential \cong electric potential

$$\hat{\Phi} = \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^{\dagger} e^{ikx} \right)$$

with $\int_{\vec{k}} \propto \int \frac{d^3 \vec{k}}{\sqrt{|\vec{k}|}}$

Coherent States

$$\hat{\Phi} = \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^{\dagger} e^{ikx} \right)$$

Coherent States

$$\hat{\Phi} = \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^{\dagger} e^{ikx} \right)$$

- Number eigenstate not classical

$$\langle m_{\vec{k}} | \hat{a}_{\vec{k}} | m_{\vec{k}} \rangle = 0 \quad \Rightarrow \quad \Phi = 0$$

Coherent States

$$\hat{\Phi} = \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^{\dagger} e^{ikx} \right)$$

- Number eigenstate not classical

$$\langle m_{\vec{k}} | \hat{a}_{\vec{k}} | m_{\vec{k}} \rangle = 0 \quad \Rightarrow \quad \Phi = 0$$

- Arbitrary state

$$|\psi\rangle = \sum_{m=0}^{\infty} c_m |m_{\vec{k}}\rangle$$

Coherent States

$$\hat{\Phi} = \int_{\vec{k}} \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^{\dagger} e^{ikx} \right)$$

- Number eigenstate not classical

$$\langle m_{\vec{k}} | \hat{a}_{\vec{k}} | m_{\vec{k}} \rangle = 0 \quad \Rightarrow \quad \Phi = 0$$

- Arbitrary state

$$|\psi\rangle = \sum_{m=0}^{\infty} c_m |m_{\vec{k}}\rangle$$

- Maximize expectation value

$$|\langle \psi | \hat{a}_{\vec{k}} | \psi \rangle| = \left| \sum_{m=0}^{\infty} c_m^* \sqrt{m+1} c_{m+1} \right|$$

Coherent States

- Cauchy-Schwarz

$$|\langle \psi | \hat{a}_{\vec{k}} | \psi \rangle| \leq \left(\sum_{m=0}^{\infty} |c_m|^2 \right)^{\frac{1}{2}} \left(\sum_{m=0}^{\infty} (m+1) |c_{m+1}|^2 \right)^{\frac{1}{2}}$$

Coherent States

- Cauchy-Schwarz

$$|\langle \psi | \hat{a}_{\vec{k}} | \psi \rangle| \leq \left(\sum_{m=0}^{\infty} |c_m|^2 \right)^{\frac{1}{2}} \left(\sum_{m=0}^{\infty} (m+1) |c_{m+1}|^2 \right)^{\frac{1}{2}}$$

- Linear dependence for equality

$$c_m \propto \sqrt{m+1} c_{m+1} \quad \Rightarrow \quad c_m = c \frac{\alpha^m}{\sqrt{m!}}$$

Coherent States

- Cauchy-Schwarz

$$|\langle \psi | \hat{a}_{\vec{k}} | \psi \rangle| \leq \left(\sum_{m=0}^{\infty} |c_m|^2 \right)^{\frac{1}{2}} \left(\sum_{m=0}^{\infty} (m+1) |c_{m+1}|^2 \right)^{\frac{1}{2}}$$

- Linear dependence for equality

$$c_m \propto \sqrt{m+1} c_{m+1} \quad \Rightarrow \quad c_m = c \frac{\alpha^m}{\sqrt{m!}}$$

- Normalization and expectation value of particle number:

$$|N_{\vec{k}}; c\rangle := |\Psi\rangle = e^{-\frac{m}{2}} \sum_{m=0}^{\infty} \frac{N^{\frac{m}{2}}}{\sqrt{m!}} |m_{\vec{k}}\rangle$$

Construction of $n(\vec{k})$

- Coherent states in fixed mode

$$|N_{\vec{k}}; c\rangle := e^{-\frac{N}{2}} \sum_{m=0}^{\infty} \frac{N^{\frac{m}{2}}}{\sqrt{m!}} |m_{\vec{k}}\rangle$$

Construction of $n(\vec{k})$

- Coherent states in fixed mode

$$|N_{\vec{k}}; c\rangle := e^{-\frac{N}{2}} \sum_{m=0}^{\infty} \frac{N^{\frac{m}{2}}}{\sqrt{m!}} |m_{\vec{k}}\rangle$$

- Discretize number density (with resolution Δk)

$$n(\vec{k})(\Delta k)^3$$

Construction of $n(\vec{k})$

- Coherent states in fixed mode

$$|N_{\vec{k}}; c\rangle := e^{-\frac{N}{2}} \sum_{m=0}^{\infty} \frac{N^{\frac{m}{2}}}{\sqrt{m!}} |m_{\vec{k}}\rangle$$

- Discretize number density (with resolution Δk)

$$| \left(n(\vec{k})(\Delta k)^3 \right)_{\vec{k}}; c \rangle$$

Construction of $n(\vec{k})$

- Coherent states in fixed mode

$$|N_{\vec{k}}; c\rangle := e^{-\frac{N}{2}} \sum_{m=0}^{\infty} \frac{N^{\frac{m}{2}}}{\sqrt{m!}} |m_{\vec{k}}\rangle$$

- Discretize number density (with resolution Δk)

$$\bigotimes_{\substack{\vec{k} = \vec{z} \Delta k \\ \vec{z} \in \mathbb{Z}^3}} | (n(\vec{k})(\Delta k)^3)_{\vec{k}}; c \rangle$$

Construction of $n(\vec{k})$

- Coherent states in fixed mode

$$|N_{\vec{k}}; c\rangle := e^{-\frac{N}{2}} \sum_{m=0}^{\infty} \frac{N^{\frac{m}{2}}}{\sqrt{m!}} |m_{\vec{k}}\rangle$$

- Discretize number density (with resolution Δk)

$$|n(\vec{k})\rangle := \lim_{\Delta k \rightarrow 0} \bigotimes_{\substack{\vec{k} = \vec{z} \Delta k \\ \vec{z} \in \mathbb{Z}^3}} | \left(n(\vec{k})(\Delta k)^3 \right)_{\vec{k}}; c \rangle$$

Construction of $n(\vec{k})$

- Coherent states in fixed mode

$$|N_{\vec{k}}; c\rangle := e^{-\frac{N}{2}} \sum_{m=0}^{\infty} \frac{N^{\frac{m}{2}}}{\sqrt{m!}} |m_{\vec{k}}\rangle$$

- Discretize number density (with resolution Δk)

$$|n(\vec{k})\rangle := \lim_{\Delta k \rightarrow 0} \bigotimes_{\substack{\vec{k} = \vec{z} \Delta k \\ \vec{z} \in \mathbb{Z}^3}} | \left(n(\vec{k})(\Delta k)^3 \right)_{\vec{k}}; c \rangle$$

- ⇒ Distribution over different modes determined by $n(\vec{k})$
- ⇒ Distribution in one modes coherent

Quantum time evolution

- As in one mode:

$$\hat{a}_{\vec{k}} |n(\vec{k}')\rangle = \sqrt{n(\vec{k})} |n(\vec{k}')\rangle$$

Quantum time evolution

- As in one mode:

$$\hat{a}_{\vec{k}} |n(\vec{k}')\rangle = \sqrt{n(\vec{k})} |n(\vec{k}')\rangle$$

- Consistent with classical limit at $t = 0$:

$$\langle n(\vec{k}) | \hat{\Phi}(0, \vec{x}) | n(\vec{k}) \rangle = \int_{\vec{k}} \sqrt{n(\vec{k})} \left(e^{-i\vec{k}\vec{x}} + e^{i\vec{k}\vec{x}} \right) = \Phi(0, \vec{x})$$

Quantum time evolution

- As in one mode:

$$\hat{a}_{\vec{k}} |n(\vec{k}')\rangle = \sqrt{n(\vec{k})} |n(\vec{k}')\rangle$$

- Consistent with classical limit at $t = 0$:

$$\langle n(\vec{k}) | \hat{\Phi}(0, \vec{x}) | n(\vec{k}) \rangle = \int_{\vec{k}} \sqrt{n(\vec{k})} \left(e^{-i\vec{k}\vec{x}} + e^{i\vec{k}\vec{x}} \right) = \Phi(0, \vec{x})$$

- Unknown time evolution

$$\Phi(t, \vec{x}) = \int_{\vec{k}} \left(e^{-i\vec{k}\vec{x}} e^{i|\vec{k}|t} \langle n(\vec{k}, t) | \hat{a}_{\vec{k}}^\dagger | n(\vec{k}, t) \rangle + \text{c.c.} \right)$$

Quantum time evolution

- As in one mode:

$$\hat{a}_{\vec{k}} |n(\vec{k}')\rangle = \sqrt{n(\vec{k})} |n(\vec{k}')\rangle$$

- Consistent with classical limit at $t = 0$:

$$\langle n(\vec{k}) | \hat{\Phi}(0, \vec{x}) | n(\vec{k}) \rangle = \int_{\vec{k}} \sqrt{n(\vec{k})} \left(e^{-i\vec{k}\vec{x}} + e^{i\vec{k}\vec{x}} \right) = \Phi(0, \vec{x})$$

- Unknown time evolution

$$\Phi(t, \vec{x}) = \int_{\vec{k}} \left(e^{-i\vec{k}\vec{x}} e^{i|\vec{k}|t} \langle n(\vec{k}, t) | \hat{a}_{\vec{k}}^\dagger | n(\vec{k}, t) \rangle + \text{c.c.} \right)$$

Quantum time evolution

- As in one mode:

$$\hat{a}_{\vec{k}} |n(\vec{k}')\rangle = \sqrt{n(\vec{k})} |n(\vec{k}')\rangle$$

- Consistent with classical limit at $t = 0$:

$$\langle n(\vec{k}) | \hat{\Phi}(0, \vec{x}) | n(\vec{k}) \rangle = \int_{\vec{k}} \sqrt{n(\vec{k})} \left(e^{-i\vec{k}\vec{x}} + e^{i\vec{k}\vec{x}} \right) = \Phi(0, \vec{x})$$

- Unknown time evolution

$$\Phi(t, \vec{x}) = \int_{\vec{k}} \left(e^{-i\vec{k}\vec{x}} \underbrace{e^{i|\vec{k}|t} \langle n(\vec{k}, t) | \hat{a}_{\vec{k}}^\dagger | n(\vec{k}, t) \rangle}_{\stackrel{!}{=} \sqrt{n(\vec{k})}} + \text{c.c.} \right)$$

Quantum time evolution

- As in one mode:

$$\hat{a}_{\vec{k}} |n(\vec{k}')\rangle = \sqrt{n(\vec{k})} |n(\vec{k}')\rangle$$

- Consistent with classical limit at $t = 0$:

$$\langle n(\vec{k}) | \hat{\Phi}(0, \vec{x}) | n(\vec{k}) \rangle = \int_{\vec{k}} \sqrt{n(\vec{k})} \left(e^{-i\vec{k}\vec{x}} + e^{i\vec{k}\vec{x}} \right) = \Phi(0, \vec{x})$$

- Unknown time evolution

$$\Phi(t, \vec{x}) = \int_{\vec{k}} \left(e^{-i\vec{k}\vec{x}} \underbrace{e^{i|\vec{k}|t} \langle n(\vec{k}, t) | \hat{a}_{\vec{k}}^\dagger | n(\vec{k}, t) \rangle}_{\stackrel{!}{=} \sqrt{n(\vec{k})}} + \text{c.c.} \right)$$

⇒ Non-gravitational interaction needed to stabilize configuration

Amplitude

- Scattering of massless scalar Ψ

Amplitude

- Scattering of massless scalar Ψ
- First-order S-matrix

$$\langle f | S_1 - 1 | i \rangle \propto \int d^4x e^{ip'x} e^{-ipx} \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle$$

Amplitude

- Scattering of massless scalar Ψ
- First-order S-matrix

$$\langle f | S_1 - 1 | i \rangle \propto \int d^4x e^{ip'x} e^{-ipx} \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle$$

Amplitude

- Scattering of massless scalar Ψ
- First-order S-matrix

$$\langle f | S_1 - 1 | i \rangle \propto \int d^4x e^{ip'x} e^{-ipx} \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle$$

- ⇒ Case $|n_f(\vec{k})\rangle = |n_i(\vec{k})\rangle$ (no back reaction)
⇒ Semi-classical limit.

Assumption of mild time dependence

- Matrix element of the background field

$$\begin{aligned} & \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle \\ &= \int_{\vec{k}} \langle n_f(\vec{k}) | \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n_i(\vec{k}) \rangle \end{aligned}$$

Assumption of mild time dependence

- Matrix element of the background field

$$\begin{aligned} & \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle \\ &= \int_{\vec{k}} \langle n_f(\vec{k}) | \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n_i(\vec{k}) \rangle \\ &= \int_{\vec{k}} \left(\sqrt{n_i(\vec{k})} e^{-i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} + \sqrt{n_f(\vec{k})} e^{i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} \right) \\ & \quad \cdot \langle n_f(\vec{k}) | n_i(\vec{k}) \rangle \end{aligned}$$

Assumption of mild time dependence

- Matrix element of the background field

$$\begin{aligned}
 & \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \langle n_f(\vec{k}) | \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n_i(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \left(\sqrt{n_i(\vec{k})} e^{-i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} + \sqrt{n_f(\vec{k})} e^{i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} \right) \\
 & \quad \cdot \underbrace{\langle n_f(\vec{k}) | n_i(\vec{k}) \rangle}_{\text{}} \\
 & \exp \left\{ -\frac{1}{2} \int d^3 \vec{k}' \left(\sqrt{n_i(\vec{k}')} - \sqrt{n_f(\vec{k}')} \right)^2 \right\}
 \end{aligned}$$

Assumption of mild time dependence

- Matrix element of the background field

$$\begin{aligned}
 & \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \langle n_f(\vec{k}) | \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n_i(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \left(\sqrt{n_i(\vec{k})} e^{-i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} + \sqrt{n_f(\vec{k})} e^{i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} \right) \\
 & \quad \cdot \underbrace{\langle n_f(\vec{k}) | n_i(\vec{k}) \rangle}_{\text{}} \\
 & \exp \left\{ -\frac{1}{2} \int d^3 \vec{k}' \left(\sqrt{n_i(\vec{k}')} - \sqrt{n_f(\vec{k}')} \right)^2 \right\}
 \end{aligned}$$

- Assumption: $n_i(\vec{k}) \neq n_f(\vec{k})$, but $n_i(\vec{k}) \approx n_f(\vec{k})$

Assumption of mild time dependence

- Matrix element of the background field

$$\begin{aligned}
 & \langle n_f(\vec{k}) | \hat{\Phi} | n_i(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \langle n_f(\vec{k}) | \left(\hat{a}_{\vec{k}} e^{-ikx} + \hat{a}_{\vec{k}}^\dagger e^{ikx} \right) | n_i(\vec{k}) \rangle \\
 &= \int_{\vec{k}} \left(\sqrt{n_i(\vec{k})} e^{-i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} + \sqrt{n_f(\vec{k})} e^{i(\omega_{\text{eff}}(|\vec{k}|)t - \vec{k} \cdot \vec{x})} \right) \\
 & \quad \cdot \underbrace{\langle n_f(\vec{k}) | n_i(\vec{k}) \rangle}_{\text{}} \\
 & \exp \left\{ -\frac{1}{2} \int d^3 \vec{k}' \left(\sqrt{n_i(\vec{k}')} - \sqrt{n_f(\vec{k}')} \right)^2 \right\}
 \end{aligned}$$

- Assumption: $n_i(\vec{k}) \neq n_f(\vec{k})$, but $n_i(\vec{k}) \approx n_f(\vec{k})$
 $\Rightarrow 0 \neq \omega_{\text{eff}}(|\vec{k}|) \ll |\vec{k}|$

Non-interference of absorption and emission

- Splitting of amplitude

$$\begin{aligned} \langle f | S_1 - 1 | i \rangle &\propto \exp \left\{ -\frac{1}{2} \int d^3 \vec{k}' \left(\sqrt{n_i(\vec{k}')} - \sqrt{n_f(\vec{k}')} \right)^2 \right\} \\ &\quad \left(\delta(p'_0 - p_0 + \omega_{\text{eff}}(|\vec{p} - \vec{p}'|)) \sqrt{n_f(\vec{p}' - \vec{p})} \right. \\ &\quad \left. + \delta(p'_0 - p_0 - \omega_{\text{eff}}(|\vec{p} - \vec{p}'|)) \sqrt{n_i(\vec{p} - \vec{p}')} \right) \end{aligned}$$

Non-interference of absorption and emission

- Splitting of amplitude

$$\begin{aligned}
 \langle f | S_1 - 1 | i \rangle &\propto \exp \left\{ -\frac{1}{2} \int d^3 \vec{k}' \left(\sqrt{n_i(\vec{k}')} - \sqrt{n_f(\vec{k}')} \right)^2 \right\} \\
 &\quad \left(\underbrace{\delta(p'_0 - p_0 + \omega_{\text{eff}}(|\vec{p} - \vec{p}'|)) \sqrt{n_f(\vec{p}' - \vec{p})}}_{\text{Emission of a graviton}} \right. \\
 &\quad \left. + \underbrace{\delta(p'_0 - p_0 - \omega_{\text{eff}}(|\vec{p} - \vec{p}'|)) \sqrt{n_i(\vec{p} - \vec{p}')}}_{\text{Absorption of a graviton}} \right)
 \end{aligned}$$

Non-interference of absorption and emission

- Splitting of amplitude

$$\begin{aligned}
 \langle f | S_1 - 1 | i \rangle &\propto \exp \left\{ -\frac{1}{2} \int d^3 \vec{k}' \left(\sqrt{n_i(\vec{k}')} - \sqrt{n_f(\vec{k}')} \right)^2 \right\} \\
 &\quad \left(\underbrace{\delta(p'_0 - p_0 + \omega_{\text{eff}}(|\vec{p} - \vec{p}'|)) \sqrt{n_f(\vec{p}' - \vec{p})}}_{\text{Emission of a graviton}} \right. \\
 &\quad \left. + \underbrace{\delta(p'_0 - p_0 - \omega_{\text{eff}}(|\vec{p} - \vec{p}'|)) \sqrt{n_i(\vec{p} - \vec{p}')}}_{\text{Absorption of a graviton}} \right)
 \end{aligned}$$

- We only consider absorption

Determination of the average graviton momentum

- Physical observable: finite cross section

$$\sigma_+ = \int_{\theta_{min}}^{\theta_{max}} d\Omega \frac{d\sigma_+}{d\Omega}$$

Determination of the average graviton momentum

- Physical observable: finite cross section

$$\sigma_+ = \int_{\theta_{min}}^{\theta_{max}} d\Omega \frac{d\sigma_+}{d\Omega} = \int_{k_{min}}^{k_{max}} dk P(k)$$

with $k :=$ momentum transfer

Determination of the average graviton momentum

- Physical observable: finite cross section

$$\sigma_+ = \int_{\theta_{min}}^{\theta_{max}} d\Omega \frac{d\sigma_+}{d\Omega} = \int_{k_{min}}^{k_{max}} dk P(k)$$

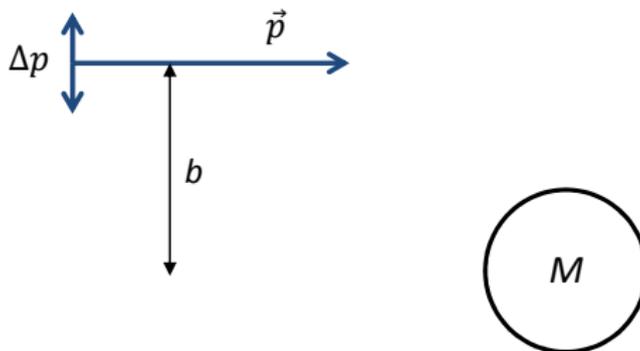
with $k :=$ momentum transfer

- Average graviton momentum

$$\bar{k} = N \int_{k_{min}}^{k_{max}} dk k P(k)$$

Maximal effect

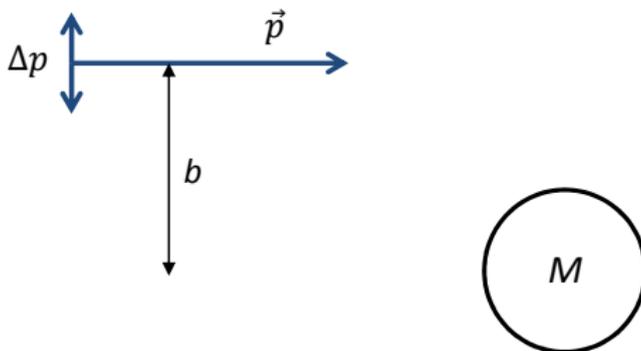
- First order: Deterministic process



Maximal effect

- First order: Deterministic process
- Heisenberg's uncertainty principle $\Delta p \Delta b \gtrsim 1$:

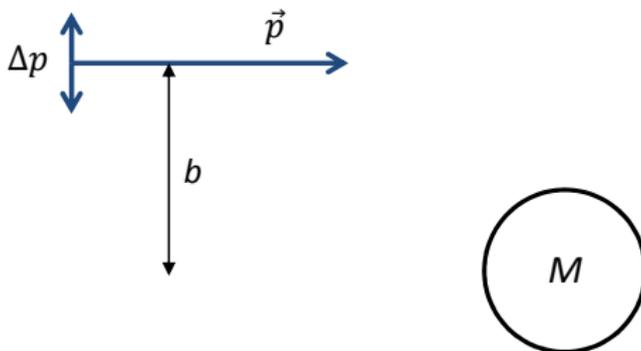
$$|\text{dev}| \lesssim \frac{1}{\tilde{N}} \frac{4|\vec{p}|r_g}{1 + 4\sqrt{|\vec{p}|r_g}}$$



Maximal effect

- First order: Deterministic process
- Heisenberg's uncertainty principle $\Delta p \Delta b \gtrsim 1$:

$$|\text{dev}| \lesssim \frac{1}{\tilde{N}} \underbrace{\frac{4|\vec{p}|r_g}{1 + 4\sqrt{|\vec{p}|r_g}}}_{\lesssim 1(\text{weak coupling})}$$



Source-free gravity

Semi-classical

Fully-quantum

Source $\rho \Rightarrow g_{\mu\nu} \propto \Phi$

$|\text{in}\rangle = \hat{a}_{\vec{k}}^\dagger |0\rangle$

$\sigma \propto |\mathcal{F}(\Phi)|^2$

Source-free gravity

Semi-classical

Fully-quantum

Source $\rho \Rightarrow g_{\mu\nu} \propto \Phi$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$|\text{in}\rangle = \hat{a}_{\vec{k}}^\dagger |0\rangle$$

$$|\text{in}\rangle = \hat{a}_{\vec{k}}^\dagger |n(\vec{k})\rangle$$

$$\sigma \propto |\mathcal{F}(\Phi)|^2$$

$$\sigma \propto n$$

Source-free gravity

Semi-classical

Fully-quantum

$$\text{Source } \rho \Rightarrow g_{\mu\nu} \propto \Phi$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$|\text{in}\rangle = \hat{a}_{\vec{k}}^\dagger |0\rangle$$

$$|\text{in}\rangle = \hat{a}_{\vec{k}}^\dagger |n(\vec{k})\rangle$$

$$\sigma \propto |\mathcal{F}(\Phi)|^2$$

$$\sigma \propto n$$

-
- The collective effects of the gravitons $|n(\vec{k})\rangle$ mimics the classical source.

Source-free gravity

Semi-classical

Fully-quantum

Source $\rho \Rightarrow g_{\mu\nu} \propto \Phi$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$|\text{in}\rangle = \hat{a}_{\vec{k}}^\dagger |0\rangle$$

$$|\text{in}\rangle = \hat{a}_{\vec{k}}^\dagger |n(\vec{k})\rangle$$

$$\sigma \propto |\mathcal{F}(\Phi)|^2$$

$$\sigma \propto n$$

-
- The collective effects of the gravitons $|n(\vec{k})\rangle$ mimics the classical source.
 - The classical source ρ is not quantized, but does not exist on quantum level.

Normalization of number eigenstates

- Identify

$$[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}}^\dagger] = \delta^{(3)}(\vec{0}) = \frac{1}{(2\pi)^3} \int d^3\vec{x} e^{-i0\vec{x}} = \frac{V}{(2\pi)^3}$$

Normalization of number eigenstates

- Identify

$$[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}}^\dagger] = \delta^{(3)}(\vec{0}) = \frac{1}{(2\pi)^3} \int d^3\vec{x} e^{-i0\vec{x}} = \frac{V}{(2\pi)^3}$$

- Calculate

$$\langle 0 | \hat{a}_{\vec{k}}^N \hat{a}_{\vec{k}}^{\dagger N} | 0 \rangle = \frac{V^N N!}{(2\pi)^{3N}}$$

Normalization of number eigenstates

- Identify

$$[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}}^\dagger] = \delta^{(3)}(\vec{0}) = \frac{1}{(2\pi)^3} \int d^3\vec{x} e^{-i0\vec{x}} = \frac{V}{(2\pi)^3}$$

- Calculate

$$\langle 0 | \hat{a}_{\vec{k}}^N \hat{a}_{\vec{k}}^{\dagger N} | 0 \rangle = \frac{V^N N!}{(2\pi)^{3N}}$$

- Normalized number eigenstate

$$|N_{\vec{k}}\rangle = \sqrt{\frac{(2\pi)^{3N}}{V^N N!}} \hat{a}_{\vec{k}}^{\dagger N} |0\rangle$$