Conformal defects in Supergravity

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based on work with R. Janik and J. Jankowski

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Main Motivation: AdS/CMT

- AdS/CMT
- "purely AdS" solutions – translational invariance spoils computations of transport properties (like DC conductivity)
- a solution proposed by [G. T. Horowitz, J. E. Santos, D. Tong] – translational invariance broken by introduction of spatially modulated scalar field ("Holographic lattice")
    - extensively investigated (ex. [M. Blake, D. Tong, D. Vegh] , [A. Donos, J.P. Gauntlett])
    - the lattice is mimicked by a spatially spread ("wide") source $\sim \cos(kx)$

Our modifications:

1. idea: replace "wide" source with local, point- or line-like source $\sim \delta(x)$
2. use solutions with local sources to study point-like defect
3. try to obtain lattice constructed from such defects – source $\sim \sum_n \delta(x - nx_l)$ – holographic realisation of the Kronig-Penney model of condensed matter physics

Framework: AdS$_4$/CFT$_3$

- action

$$S = \frac{1}{16\pi G_n}\int \sqrt{|g|}\,(R - 1/2\nabla_a\phi\nabla^a\phi - V(\phi))$$

- potential $V(\phi) = -6 - \phi^2 \Leftrightarrow$ cosmological constant & mass $m^2 = -2$
- $\phi(x,y,z) \Leftrightarrow$ operator $\mathcal{O}(x,y)$ of dimension $\Delta = 2$, deforming CFT
- near-boundary asymptotic of scalar
  $\phi(x,y,z) = \boldsymbol{\phi_1(x,y)}z + \phi_2(x,y)z^2 + \ldots$ (in Poincaré coordinates)
- the operator deforms CFT by a shift of Lagrangian:

$$\mathcal{L} = \mathcal{L}_{CFT_3} + \boldsymbol{\phi_1(x,y)}\mathcal{O}(x,y)$$

  Its expectation value reads $\langle\mathcal{O}\rangle = \phi_2(x,y)$
- focus on single defect concentrated along some line (eg. x=0)

Goal:

Implement the boundary condition of type $\boldsymbol{\phi_1(x,y)} = \eta\delta(x)$ (Dirac delta on line $x = 0$) in Einstein equations generated by given Action

Previous works with discontinuous BCs in (super)gravity

- $\phi_1 = \theta(x)$ and $m_\phi = 0$ analytical Janus solutions [D. Bak, M. Gutperle, S. Hirano]
- $\phi_1 = \theta(x)$ and $m_\phi = 0$ at $T > 0$ numerical and analytical Janus black holes in $d = 2 + 1$ [D. Bak, M. Gutperle, R. A. Janik]
- $\phi_1 = \delta(x)$ and $m_\phi^2 = -2$ with SUSY, analytical and scale invariant [E. D'Hoker *et al.*]

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  $\rightarrow$ hard to generalise to black hole case ($T > 0$)

  $\rightarrow$ not very useful in AdS/CMT

"Emergent" supergravity

- linearised analysis gives $\phi_{lin} = \frac{\eta z^2}{\pi(x^2+z^2)}$ $\rightarrow$ suggests conformal symmetry along defect line ($SO(2,2)$)
- new coordinates

$$r^2 = x^2 + z^2, \tan\alpha = x/z$$

  $(\phi_{lin}(\alpha) = \frac{\eta}{\pi}\cos^2(\alpha))$

- full solution with this symmetry cannot be found! *dynamical* generation of source $\boldsymbol{\phi_1} \sim \delta(x) + 1/|x| + ...!$
- a way out – modification of the scalar potential $V(\phi)$

"Emergent" supergravity

Supersymmetric potential

1. $SO(2,2)$ symmetry fixes *uniquely* $V(\phi)$

$$V(\phi) = -6\cosh(\phi/\sqrt{3})$$

2. the same potential arises from reduction & truncation of D=11 SUGRA on $\mathrm{AdS}_4 \times S^7$![M. Cvetic et al.]
3. with such potential $\boldsymbol{\phi_1(x,y)} = \eta\delta(x)$ & $SO(2,2)$ symmetry can be both fulfilled

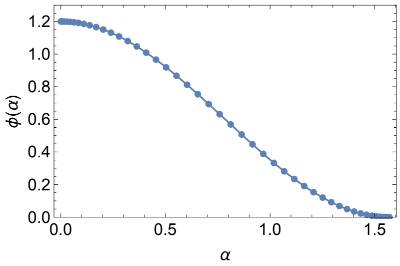
$T = 0$ (no horizon)

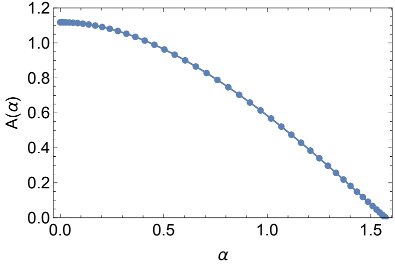
- we take the supersymmetric potential $V(\phi) = -6\cosh(\phi/\sqrt{3})$ in action
- metric ansatz:

$$ds^2 = \frac{1}{A(\alpha)^2}\left(\frac{d\alpha^2}{p^2} + \frac{dr^2 - dt^2 + dy^2}{r^2}\right)$$

- solving both using numerics (pseudospectral collocation method on Chebyschev grid) and perturbative expansion in parameter $\eta = \phi(0)$

$T = 0$ (no horizon)





Metric and scalar field for $\phi(0) = 1.2$
Points $\rightarrow$ numerical solution with $N = 47$ spectral grid
Lines $\rightarrow$ fourth order perturbative solution

$T > 0$ case

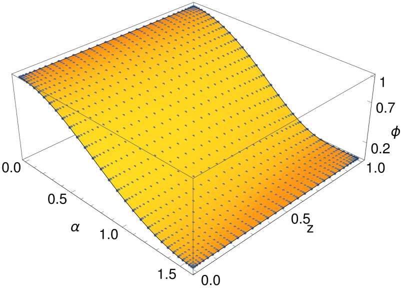
- problem no longer 1-dimensional – we only replace x coordinate with $\alpha = \tan(x/z)$
- we use the most general metric ansatz:

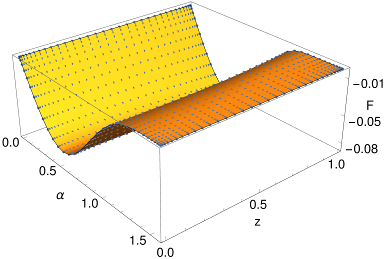
$$
\begin{aligned}
ds^2 &= \frac{1}{z^2}\Bigg[-(1-z)G(z)H_1(\alpha,z)dt^2 + \frac{H_2(\alpha,z)dz^2}{(1-z)G(z)} \\
&+ S_1(\alpha,z)(d\alpha + F(\alpha,z)dz)^2 + S_2(\alpha,z)dy^2\Bigg]
\end{aligned}
$$

with $G(z) = 1 + z + z^2$. DeTurk method stands for gauge-fixing [M. Headrick, et al.]

- numerical method was based on spectral collocation method on Chebyschev grid [P. Grandclement and J. Novak]

$T > 0$ case



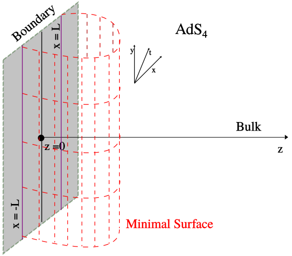


Scalar field (right) and metric component $F(\alpha, z)$ (left) for $\phi(0) = 1.0$.

An observable: Entanglement Entropy

Holographic entanglement entropy

[S. Ryu, T. Takayanagi] – EE of some region is proportional to the area of minimal a surface whose boundary is boundary of that region. For strip of width $2L$ around the defect the generic form of EE should be: $S = \frac{1}{\epsilon} - \frac{B}{L}$



A strip for which we calculated entanglement entropy, with a sketch of minimal surface used in calculation.

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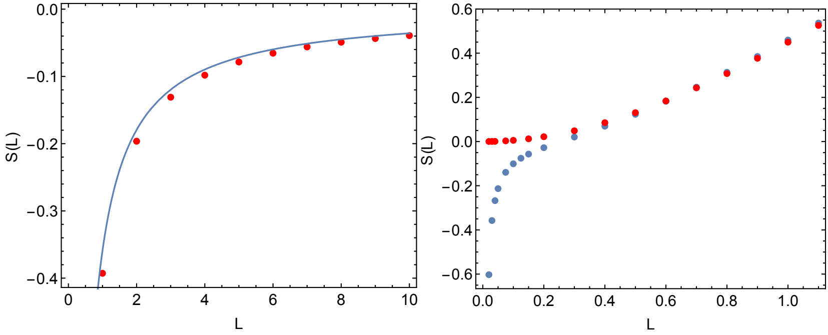

Left: EE of pure AdS (line) and defect geometry $\phi(0) = 2$. Right: EE difference between pure AdS and: standard AdS-black hole (red points), defected black hole (blue dots).

Conclusions

- We examined a novel setup in numerical GR, and developed methods to handle it
- It turned out that conformal defect exists only in Supergravity (scalar potential is fixed to be $V(\phi) = -6\cosh(\phi/\sqrt{3})$)
- In the theory with defect, entanglement entropy of a strip is lower than in theory without it

Further directions

- Construction of holographic lattice from such local defects
- Introduction of nonzero chemical potential (gauge field in bulk)
- Computation of various quantities – i.e. optical conductivity or heat transport

Thank you for your attention

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