Poincaré invariance in pNRQCD

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MAX-PLANCK-GESELLSCHAFT

IMPRS YSW 2014 recap

Poincaré invariance in EFT (Wigner's induced representation)

- From higher to lower energy theory

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- Low energy EFT preserves the symmetry of its UV theory

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Poincaré invariance in EFT (Wigner's induced representation)

- From higher to lower energy theory
- Low energy EFT preserves the symmetry of its UV theory
- Symmetry yields constraints between operators
- Application: non-relativistic QCD (NRQCD)

From NRQCD to pNRQCD

Potential-NRQCD (pNRQCD): a nice theoretical tool for describing the dynamics of heavy quark-antiquark bound system (e.g., charmonium, bottomonium)

- Hierarchy of scales: $M \gg Mv \gg Mv^2$
  (M: heavy quark mass, Mv: inverse size of the bound system)

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- Operators come with Wilson coefficients

Wilson coefficients

Wilson coefficients c_n are undetermined scalar functions in front of the series of operators in EFT

$$\mathcal{L}_{EFT} = \sum_n \frac{c_n \mathcal{O}_n}{M^{d_n - 4}}, \quad \text{where} \quad [\mathcal{O}_n] = d_n$$

- Exact values of c_n determined via matching to its UV theory but formidable task
- Benefits from symmetry: reduce the task if not solved completely
- Poincaré invariance from the UV theory is the symmetry we use here

Outline

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- Field redefinition in pNRQCD
- Constraints in Wilson coefficients
- Open issues and outlook

pNRQCD - power counting

Potential non-relativistic QCD (pNRQCD) given by integrating out the relative momentum, $Mv \sim 1/\mathrm{r}$, from NRQCD

- Multipole expansion in $1/M$ and r

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- Coordinates: $\mathbf{R} \equiv (\mathbf{x}_1 + \mathbf{x}_2)/2$, $\mathbf{r} \equiv \mathbf{x}_1 - \mathbf{x}_2$, t

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- Multipole expansion in $1/M$ and r
- Coordinates: $\mathbf{R} \equiv (\mathbf{x}_1 + \mathbf{x}_2)/2$, $\mathbf{r} \equiv \mathbf{x}_1 - \mathbf{x}_2$, t
- Power counting:

$$\begin{aligned} \boldsymbol{\nabla}_r, \frac{1}{r} &\sim Mv \\ \partial_0, \boldsymbol{\nabla}_R, A_\mu &\sim Mv^2 \\ \mathbf{E}, \mathbf{B} &\sim M^2 v^4 \end{aligned}$$

pNRQCD - field contents

I. Quark-antiquark color singlet S and octet O^a configuration
II. 2×2 spin matrix S_{ij}, O^a_{ij}, for quark spin i and antiquark spin j
III. Coordinate dependence of the fields: $S = S(t, \mathbf{R}, \mathbf{r})$, $O^a = O^a(t, \mathbf{R}, \mathbf{r})$
IV. Multipole expanded gluon fields $A^a_\mu(t, \mathbf{R})$

Writing S and O^a as 3×3 matrices in color space

$$S \to \frac{1}{\sqrt{3}} S\, I_3, \quad O^a \to O = \sqrt{2} O^a T^a$$

so that $\mathrm{Tr}[S^\dagger S] = S^\dagger S$, $\mathrm{Tr}[O^\dagger O] = O^{a\dagger} O^a$, $\mathrm{Tr}[S^\dagger \mathbf{E} O] = S^\dagger \mathbf{E}^a O^a$.

pNRQCD Lagrangian

Schematic form of the Lagrangian up to quadratic order in singlet and octet:

$$\mathcal{L}_{pNRQCD} = \int d^3x \mathrm{Tr}\left\{ S^\dagger \mathcal{K}_{SS} S + O^\dagger \mathcal{K}_{OO} O + [S^\dagger \mathcal{K}_{SO} O + \mathrm{H.C.}] \right\} - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}(\mathbf{R}, t)$$

- Integration over relative distance r

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- Integration over relative distance r
- H.C. denotes hermitian conjugate
- $\mathcal{K}$ includes charge conjuate terms
- F^2: ultrasoft gluons
- We focus on the $S^\dagger \mathcal{K}_{SS} S$ for the simplicity in this talk

$S^\dagger \mathcal{K}_{SS} S$ [Brambilla, Gromes, Vairo, 2003]

$$
\begin{aligned}
S^\dagger \mathcal{K}_{SS} S = S^\dagger \Big(& i\partial_0 + \frac{1}{2M}\left\{ c_s^{(1,-2)}, \boldsymbol{\nabla}_r^2 \right\} + \frac{c_s^{(1,0)}}{4M}\boldsymbol{\nabla}_R^2 - V_S^{(0)} \\
& - \frac{V_S^{(1)}}{M} + \frac{V_{p^2Sa}}{8M^2}\boldsymbol{\nabla}_R^2 + \frac{1}{2M^2}\left\{ \boldsymbol{\nabla}_r^2, V_{p^2Sb} \right\} + \frac{V_{L^2Sa}}{4M^2r^2}(\mathbf{r}\times\boldsymbol{\nabla}_R)^2 \\
& + \frac{V_{L^2Sb}}{4M^2r^2}(\mathbf{r}\times\boldsymbol{\nabla}_r)^2 - \frac{V_{S_{12}S}}{M^2r^2}\Big(3(\mathbf{r}\cdot\boldsymbol{\sigma}^{(1)})(\mathbf{r}\cdot\boldsymbol{\sigma}^{(2)}) - \mathbf{r}^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\Big) \\
& - \frac{V_{S^2S}}{4M^2}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} + \frac{iV_{LSSa}}{4M^2}(\mathbf{r}\times\boldsymbol{\nabla}_R)\cdot(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}) \\
& + \frac{V_{LSSb}}{4M^2}(\mathbf{r}\times\boldsymbol{\nabla}_r)\cdot(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\Big) S
\end{aligned}
$$

- (a, b): a denotes order of $1/M$, b denotes order of r
- $\boldsymbol{\sigma}^{(1/2)}$: spin matrix for quark/antiquark

Spatial boost

We construct the most generalized form of boost transformation with following several criteria

- Right behaviors under C, P, T

$$\mathbf{k} \xrightarrow{P} -\mathbf{k}, \quad \mathbf{k} \xrightarrow{C} -\sigma_2 \mathbf{k}^* \sigma_2, \qquad \mathbf{k} \xrightarrow{T} \sigma_2 \mathbf{k} \sigma_2$$

- Proper order in truncation: include expressions up to $1/M$ and r
- Criteria apply both to singlet and octet

Generalized(!) boost transformation

$$S'(t,\mathbf{R},\mathbf{r}) = \Bigg(1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\left\{k_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\mathbf{r}, \boldsymbol{\nabla}_R\cdot\boldsymbol{\nabla}_r\right\}$$

$$+ \frac{i}{4M}\left\{k_{a''}^{(1,0)}\mathbf{r}\cdot\boldsymbol{\nabla}_R, \boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\right\} + \frac{i}{4M}\left\{k_{a'''}^{(1,0)}\mathbf{r}\cdot, \boldsymbol{\nabla}_r(\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R)\right\} + \frac{i}{4M}\left\{\frac{k_b^{(1,0)}}{r^2}(\boldsymbol{\eta}\cdot\mathbf{r}\right.$$

$$- \frac{k_c^{(1,0)}}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right) + \frac{k_{d'}^{(1,0)}}{8Mr^2}(\boldsymbol{\eta}\cdot\mathbf{r}\times\boldsymbol{\nabla}_R)\left(\mathbf{r}\cdot\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)$$

$$- \frac{k_{d''}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\mathbf{r}\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)(\mathbf{r}\cdot\boldsymbol{\nabla}_R) - \frac{1}{8M}\left\{k_a^{(1,-1)}, \boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right.\right.$$

$$+ \frac{1}{8M}\left\{\frac{k_{b'}^{(1,-1)}}{r^2}\left(\mathbf{r}\cdot\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)(\boldsymbol{\eta}\times\mathbf{r})\cdot, \boldsymbol{\nabla}_r\right\}$$

$$- \frac{1}{8M}\left\{\frac{k_{b''}^{(1,-1)}}{r^2}\left(\boldsymbol{\eta}\cdot\mathbf{r}\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)\mathbf{r}\cdot, \boldsymbol{\nabla}_r\right\}\Bigg) S(t',\mathbf{R}',\mathbf{r}')$$

Seemingly more problems from the boost...

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- Not so!
- Coefficients be removed by field redefinition
- Take field redefinition by unitary transformation!

Field redefinition by unitary transformation

Define new singlet via $S = \mathcal{U}_S \widetilde{S}$

- $\mathcal{U}_S$: unitary operator for singlet

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Field redefinition by unitary transformation

Define new singlet via $S = \mathcal{U}_S \widetilde{S}$

- $\mathcal{U}_S$: unitary operator for singlet
- Natural choice for unitary operator: exponential function with operators on the exponents
- Exponents are to be anti-hermitian
- What about the order of $1/M$ expansion on the exponents?

Boost transformation of the “new” field

Boost transformation of the new field $\widetilde{S}$ determines the order of $1/M$ expansions:

$$\begin{aligned}\widetilde{S}' =& \mathcal{U}_S'^{\dagger} S' = \mathcal{U}_S'^{\dagger}(1-\boldsymbol{\eta}\cdot\mathbf{k})\mathcal{U}_S\widetilde{S} \\ =& [1-\mathcal{U}_S^{\dagger}(i\boldsymbol{\eta}\cdot\mathbf{k})\mathcal{U}_S + (\delta\mathcal{U}_S^{\dagger})\mathcal{U}_S]\widetilde{S} \\ =& (1-i\boldsymbol{\eta}\cdot\mathbf{k} - [2iM\boldsymbol{\eta}\cdot\mathbf{R}, \ln\mathcal{U}_S] + \mathcal{O}(1/M^2))\widetilde{S}\end{aligned}$$

- $\mathcal{U}_S'^{\dagger} = \mathcal{U}_S + \delta\mathcal{U}_S$ assumed

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- $\mathcal{U}_S'^{\dagger} = \mathcal{U}_S + \delta\mathcal{U}_S$ assumed
- Exponents of the unitary operator to be in the order of $1/M^2$

Unitary operator for singlet

Motivated by [Brambilla, Gromes, Vairo, 2003]

$$
\begin{aligned}
\mathcal{U}_S = \exp\Bigg[& -\frac{1}{4M^2}\left\{q_{a'}^{(1,0)}\mathbf{r}\cdot\boldsymbol{\nabla}_R, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\} - \frac{1}{4M^2}\left\{q_{a''}^{(1,0)}\mathbf{r}\cdot\boldsymbol{\nabla}_R, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\} \\
& - \frac{1}{4M^2}\left\{q_{a'''}^{(1,0)}\mathbf{r}\cdot, \boldsymbol{\nabla}_r\right\}\boldsymbol{\nabla}_R^2 - \frac{1}{4M^2}\left\{\frac{q_b^{(1,0)}}{r^2}(\mathbf{r}\cdot\boldsymbol{\nabla}_R)^2\mathbf{r}\cdot, \boldsymbol{\nabla}_r\right\} \\
& + \frac{iq_{d''}^{(1,0)}}{8M^2r^2}\left(\mathbf{r}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)(\mathbf{r}\cdot\boldsymbol{\nabla}_R) \\
& + \frac{i}{8M^2}\left\{q_a^{(1,-1)}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right\} \\
& - \frac{i}{8M^2}\left\{\frac{q_{b'}^{(1,-1)}}{r^2}\left(\mathbf{r}\cdot\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)(\mathbf{r}\times\boldsymbol{\nabla}_R)\cdot, \boldsymbol{\nabla}_r\right\} \\
& + \frac{i}{8M^2}\left\{q_{b''}^{(1,-1)}\left(\mathbf{r}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)\mathbf{r}\cdot, \boldsymbol{\nabla}_r\right\}\Bigg]
\end{aligned}
$$

Shift in boost coefficients

Plugging this into $[2iM\boldsymbol{\eta}\cdot\mathbf{R}, \ln\mathcal{U}_S]$ from the boost transformation of $\widetilde{S}$, we observe following shifts in boost coefficients

$$
\begin{aligned}
&k_{a'}^{(1,0)} \to k_{a'}^{(1,0)} - 2q_{a'}^{(1,0)} - 2q_{a''}^{(1,0)} \\
&k_{a''}^{(1,0)} \to k_{a''}^{(1,0)} - 2q_{a''}^{(1,0)} - 2q_{a'}^{(1,0)} \\
&k_{a'''}^{(1,0)} \to k_{a'''}^{(1,0)} - 4q_{a'''}^{(1,0)} \\
&k_{b}^{(1,0)} \to k_{b}^{(1,0)} - 4q_{b}^{(1,0)} \\
&\ldots
\end{aligned}
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k_{b}^{(1,0)} &\to k_{b}^{(1,0)} - 4q_{b}^{(1,0)}\\
&\dots
\end{aligned}
$$

- q's are freely chosen
- Eliminate as many terms as possible to simplify the generalized boost

Intermezzo

After dropping tilde notation on singlet

$$
\begin{aligned}
S' = \Bigg(& 1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\left\{k_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\mathbf{r}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\} \\
& - \frac{k_c^{(1,0)}}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right) + \frac{k_{d'}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\mathbf{r}\times\boldsymbol{\nabla}_R\right)\left(\mathbf{r}\cdot\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right) \\
& - \frac{k_{d''}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\mathbf{r}\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)\left(\mathbf{r}\cdot\boldsymbol{\nabla}_R\right) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\Bigg) \\
& \times S(t,\mathbf{R}',\mathbf{r}') \equiv (1 - i\boldsymbol{\eta}\cdot\mathbf{k})S
\end{aligned}
$$

- 5 coefficients to be determined

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After dropping tilde notation on singlet

$$S' = \left(1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\left\{k_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\mathbf{r}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\}\right.$$
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$$\left. - \frac{k_{d''}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\mathbf{r}\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)\left(\mathbf{r}\cdot\boldsymbol{\nabla}_R\right) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)$$
$$\times S(t,\mathbf{R}',\mathbf{r}') \equiv (1 - i\boldsymbol{\eta}\cdot\mathbf{k})S$$

- 5 coefficients to be determined
- Any other constraints to impose?

1st constraint: commutation relation

- Commutation relation between boost generators **k**

$$[1 - i\boldsymbol{\xi}\cdot\mathbf{k}, 1 - i\boldsymbol{\eta}\cdot\mathbf{k}]S \stackrel{!}{=} i(\boldsymbol{\xi}\times\boldsymbol{\eta})\cdot\mathbf{j}S$$

fixes coefficients: $k_{a'}^{(1,0)} = k_c^{(1,0)} = 1$ and $k_{d'}^{(1,0)} = k_{d''}^{(1,0)} = 0$

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- Boost simplified further:

$$S' = \Bigg(1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\left\{\boldsymbol{\eta}\cdot\mathbf{r}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\} - \frac{1}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\Bigg) S$$

1st constraint: commutation relation

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- $k_D^{(1,0)}$ remains to be constrained

2nd constraint: Lorentz invariance

- After taking the boost transformation upon the Lagrangian

$$
\begin{aligned}
\delta\mathcal{L}_{2S} \quad = \quad & S^\dagger \Big(i\Big(1 - c_S^{(1,0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R - \frac{1}{2M}\Big(k_D^{(1,0)} - c_S^{(1,0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\partial_0 \\
& - \frac{i}{M}\Big(V_{p^2Sa} + V_{L^2Sa} + \frac{1}{2}V_S^{(0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R \\
& + \frac{i}{Mr^2}\Big(V_{L^2Sa} + \frac{r}{2}\partial_r V_S^{(0)}\Big)(\boldsymbol{\eta}\cdot\mathbf{r})(\mathbf{r}\cdot\boldsymbol{\nabla}_R) \\
& + \frac{1}{2M}\Big(V_{LSSa} + \frac{1}{2r}\partial_r V_S^{(0)}\Big)\boldsymbol{\eta}\cdot(\boldsymbol{\sigma}^{(1)} - \boldsymbol{\sigma}^{(2)})\times\mathbf{r}\Big) S
\end{aligned}
$$

is to vanish up to total derivatives.

2nd constraint: Lorentz invariance

- After taking the boost transformation upon the Lagrangian

$$
\begin{aligned}
\delta\mathcal{L}_{2S} \;=\;& S^\dagger\Big(i\Big(1-c_S^{(1,0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R-\frac{1}{2M}\Big(k_D^{(1,0)}-c_S^{(1,0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\partial_0\\
&-\frac{i}{M}\Big(V_{p^2Sa}+V_{L^2Sa}+\frac{1}{2}V_S^{(0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\\
&+\frac{i}{Mr^2}\Big(V_{L^2Sa}+\frac{r}{2}\partial_r V_S^{(0)}\Big)(\boldsymbol{\eta}\cdot\mathbf{r})(\mathbf{r}\cdot\boldsymbol{\nabla}_R)\\
&+\frac{1}{2M}\Big(V_{LSSa}+\frac{1}{2r}\partial_r V_S^{(0)}\Big)\boldsymbol{\eta}\cdot(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\times\mathbf{r}\Big)S
\end{aligned}
$$

  is to vanish up to total derivatives.
- Constraints on the Wilson coefficients are found:

$$
\begin{aligned}
k_D^{(1,0)} \;=\;& c_S^{(1,0)}=1,\quad V_{p^2Sa}+V_{L^2Sa}+\frac{1}{2}V_S^{(0)}=0,\\
V_{L^2Sa} \;=\;& -\frac{r}{2}\partial_r V_S^{(0)},\quad V_{LSSa}=-\frac{1}{2r}\partial_r V_S^{(0)}
\end{aligned}
$$

Coda

Finalized version of the boost after free parameters are chosen

$$S'(t,\mathbf{R},\mathbf{r}) = \Big(1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{i}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\{\boldsymbol{\eta}\cdot\mathbf{r}, \boldsymbol{\nabla}_R\cdot\boldsymbol{\nabla}_r\}$$
$$-\frac{1}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})$$
$$+\mathcal{O}(1/M^3)\Big)S(t',\mathbf{R}',\mathbf{r}')$$

- But why choose the free parmeters as was shown?

Coda

Finalized version of the boost after free parameters are chosen

$$S'(t,\mathbf{R},\mathbf{r}) = \Big(1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{i}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\{\boldsymbol{\eta}\cdot\mathbf{r},\boldsymbol{\nabla}_R\cdot\boldsymbol{\nabla}_r\}$$
$$-\frac{1}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})$$
$$+\mathcal{O}(1/M^3)\Big)S(t',\mathbf{R}',\mathbf{r}')$$

- But why choose the free parmeters as was shown?
- This matches to the one from *induced representation* from Wigner's little group formalism

Coda

Finalized version of the boost after free parameters are chosen

$$
S'(t,\mathbf{R},\mathbf{r}) = \Big(1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{i}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\{\boldsymbol{\eta}\cdot\mathbf{r}, \boldsymbol{\nabla}_R\cdot\boldsymbol{\nabla}_r\}
-\frac{1}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})
+\mathcal{O}(1/M^3)\Big)S(t',\mathbf{R}',\mathbf{r}')
$$

- But why choose the free parmeters as was shown?
- This matches to the one from *induced representation* from Wigner's little group formalism
- Bottomline: boost from induced representation is rather a particular choice in the non-interacting theory

Summary and Outlook

Summary

- Power counting and DOF in pNRQCD
- Generalized boost transformation in pNRQCD
- Field redefinition by unitary trnasformation
- Simplified boost form and constraints on the Wilson coefficients

Outlook

- Dark matter candidates (bottom-up approach in EFT)
- SCET (already known by reparametrization invariance)
- Sterile neutrinos (next goal)
- Gravitation? (last chapter of my thesis?)

References

N. Brambilla, D. Gromes, and A. Vairo (2003)
Poincaré invariance constrains on NRQCD and potential NRQCD
Phys. Lett. B 576, 314 - 327.

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Lorentz invariance in heavy particle effective theories
Phys. Rev. D 86, 094020 (2012).

Unitary operator: octet-octet

- Unitary operator for the octet - octet sector

$$\mathcal{U}_o = \exp\left[\frac{i}{2M}\left(-i\mathbf{D}_R\cdot(\tilde{\mathbf{k}}_{oo}^{(0,2)}+\tilde{\mathbf{k}}_{oo}^{(1,-1)}+\tilde{\mathbf{k}}_{oo}^{(1,0)}\right)+h.c.\right]$$

- Boost on the octet after the field redefinition:

$$\widetilde{O}' = \Bigg(1 - i\boldsymbol{\eta}\cdot\mathbf{k} - \frac{i}{8}\tilde{k}_a^{(0,2)}(\mathbf{r}\cdot g\mathbf{E})(\mathbf{r}\cdot\boldsymbol{\eta}) - \frac{i}{8}\tilde{k}_b^{(0,2)}\mathbf{r}^2(\boldsymbol{\eta}\cdot g\mathbf{E})$$
$$-\frac{i}{2M}\{\tilde{k}_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\mathbf{r},\boldsymbol{\nabla}_r\cdot\mathbf{D}_R\} - \frac{i}{2M}\{\tilde{k}_{a''}^{(1,0)}\mathbf{r}\cdot\mathbf{D}_R,\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\}$$
$$+\cdots+\mathcal{O}\left(\frac{1}{M^2}\right)\Bigg)\widetilde{O}$$

- Similar shift in the parameters

Constraints: octet-octet sector (1/2)

Octet - octet sector includes:

$$
\begin{aligned}
\mathcal{L}_{2O} \ni \; & O^\dagger \Big(\frac{i}{8M} V^{(1,0)}_{OOa}(r) \{ \boldsymbol{\nabla}_r \cdot, \mathbf{r} \times g\mathbf{B} \} \\
& + \frac{1}{16M^2} \{ (\boldsymbol{\nabla}_r \cdot \mathbf{D}_R), V^{(2,0)}_{OOc'}(r)(\mathbf{r} \cdot g\mathbf{E}) \} \\
& + \frac{1}{16M^2} \{ \boldsymbol{\nabla}^i_r \mathbf{D}^j_R, V^{(2,0)}_{OOc''}(r) \mathbf{r}^j g\mathbf{E}^i \} \\
& + \frac{1}{16M^2} \{ \boldsymbol{\nabla}^i_r \mathbf{D}^j_R, V^{(2,0)}_{OOc'''}(r) \mathbf{r}^i g\mathbf{E}^j \} \\
& + \frac{1}{16M^2} \{ \boldsymbol{\nabla}^i_r \mathbf{D}^j_R, \frac{V^{(2,0)}_{OOd}(r)}{r^2} \mathbf{r}^i \mathbf{r}^j (\mathbf{r} \cdot \mathbf{E}) \} \Big) O + C.C.
\end{aligned}
$$

Constraints: octet-octet sector (2/2)

Constraints under the little group:

$$\begin{aligned} V^{(1,0)}_{OOa} + V^{(2,0)}_{OOc'} &= 0 \\ V^{(1,0)}_{OOa} - V^{(2,0)}_{OOc''} &= 2 \\ rV^{(2,0)}_{OOc'''} &= 0 \\ V^{(2,0)}_{OOd} &= 0 \end{aligned} \quad (1)$$