Holographic entanglement entropy

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YSW 2015 - Ringberg

take home message

entanglement entropy is a measure for quantum entanglement ● it has a holographic dual which is easier to compute ● it is an important quantity in theoretical physics but also in condensed matter physics

reminder: statistical entropy

$$S_{\mathrm{stat}} = -k_{\mathrm{B}} \sum_i p_i \log(p_i)$$

. it can be interpreted as a measure for the lack of knowledge about the system

. vanishes at zero temperature

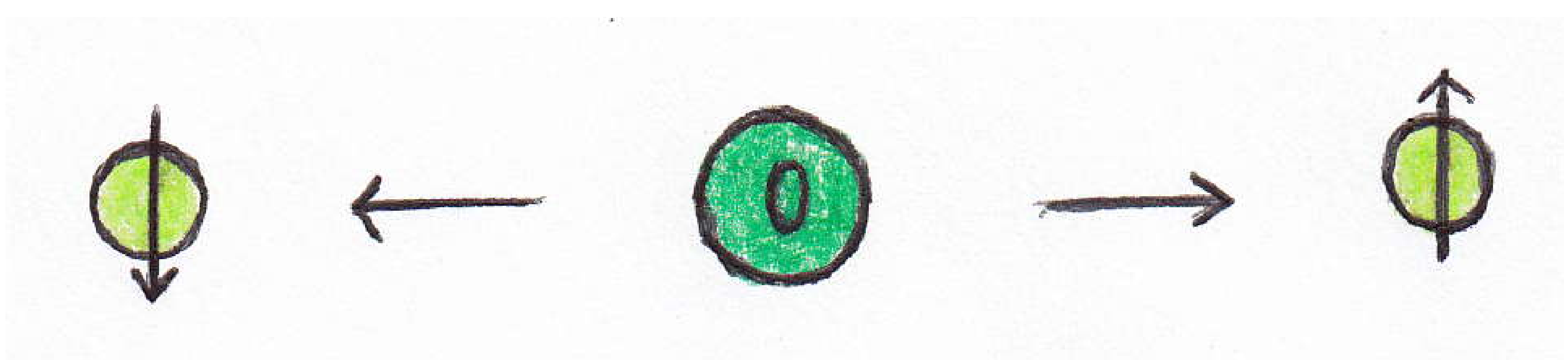
. related to the degrees of freedom

about quantum entanglement

. the quantum state of each constituent cannot be described independently of the others

. the system then has to be described as a whole

. quantum entanglement is not local



entanglement entropy in words

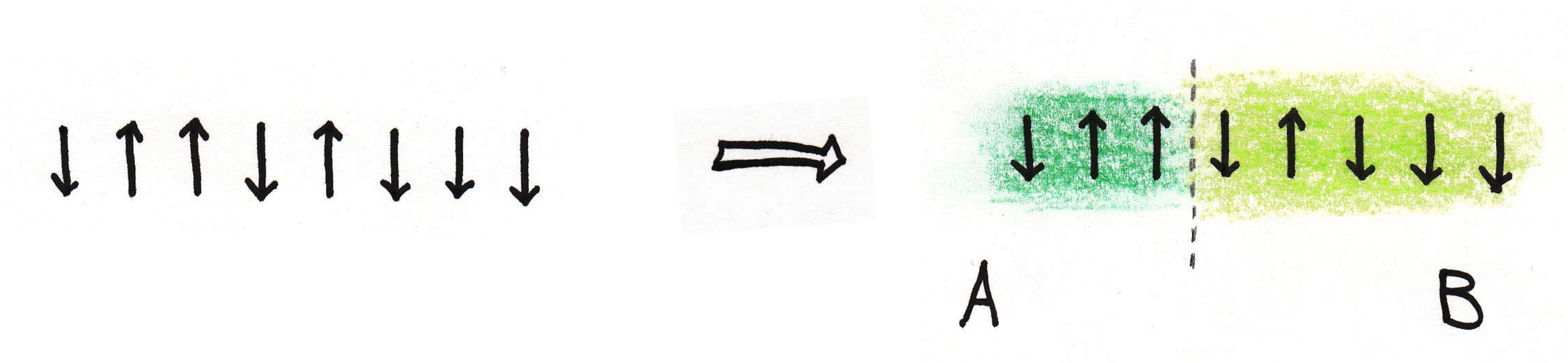
. generalisation of statistical entropy taking quantum effects into account

. measure for the information that you cannot access when you only consider a part of the total quantum system

. non-vanishing at zero temperature

. related to the degrees of freedom of the system

entanglement entropy in formulae



. the whole system has a density matrix ρ

. reduced density matrix: ‘smearing out the knowledge about B’

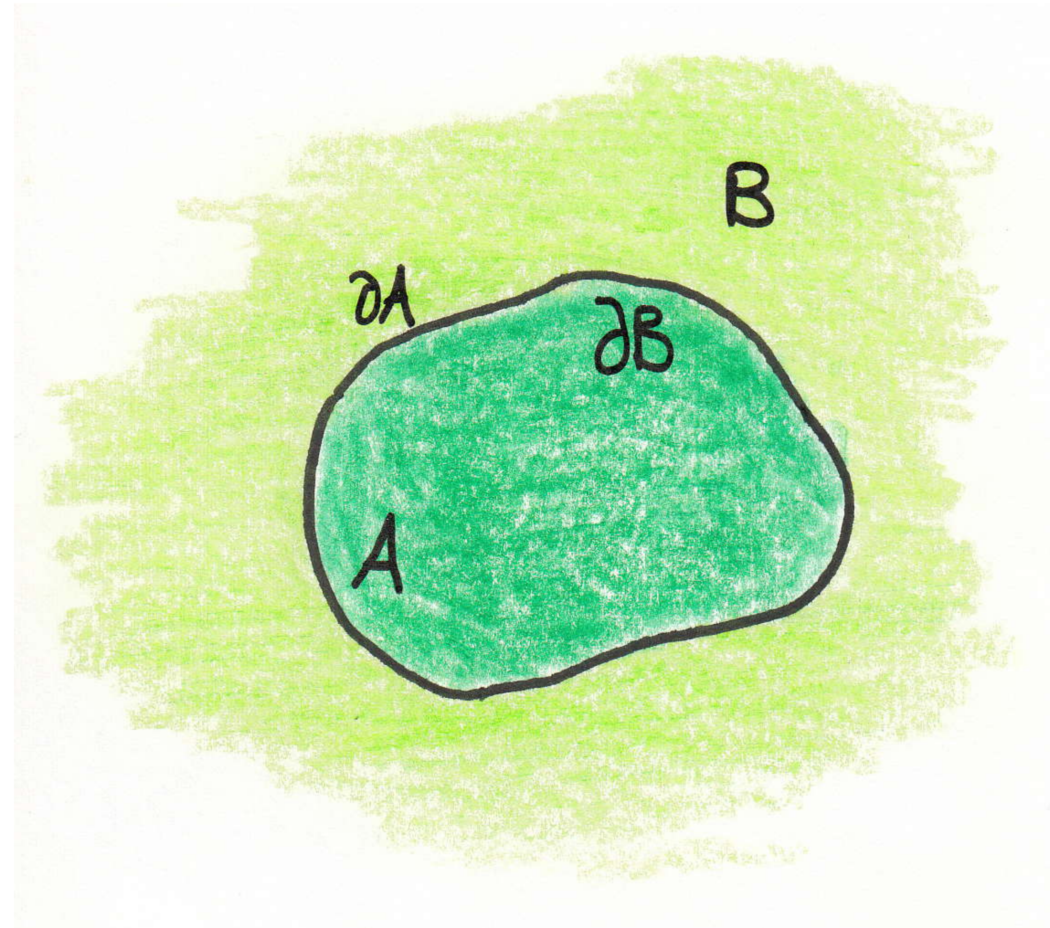
$$\rho_A = \mathrm{Tr}_B(\rho)$$

. EE is a functional of the reduced density matrix

$$S(A) = -\mathrm{Tr}(\rho_A \log(\rho_A))$$

what about continuum QFTs?

. at zero temperature: $S(A) = S(B)$



. exact results for 2D CFTs

. EE of continuum QFTs is divergent

. introducing a UV cutoff: area-law

$$S(A) \sim \frac{\mathrm{Area}(\partial \mathrm{A})}{\epsilon^{d-1}}$$

in short

. EE is a generalisation of statistical entropy taking
  quantum effects into account

. EE has certain properties

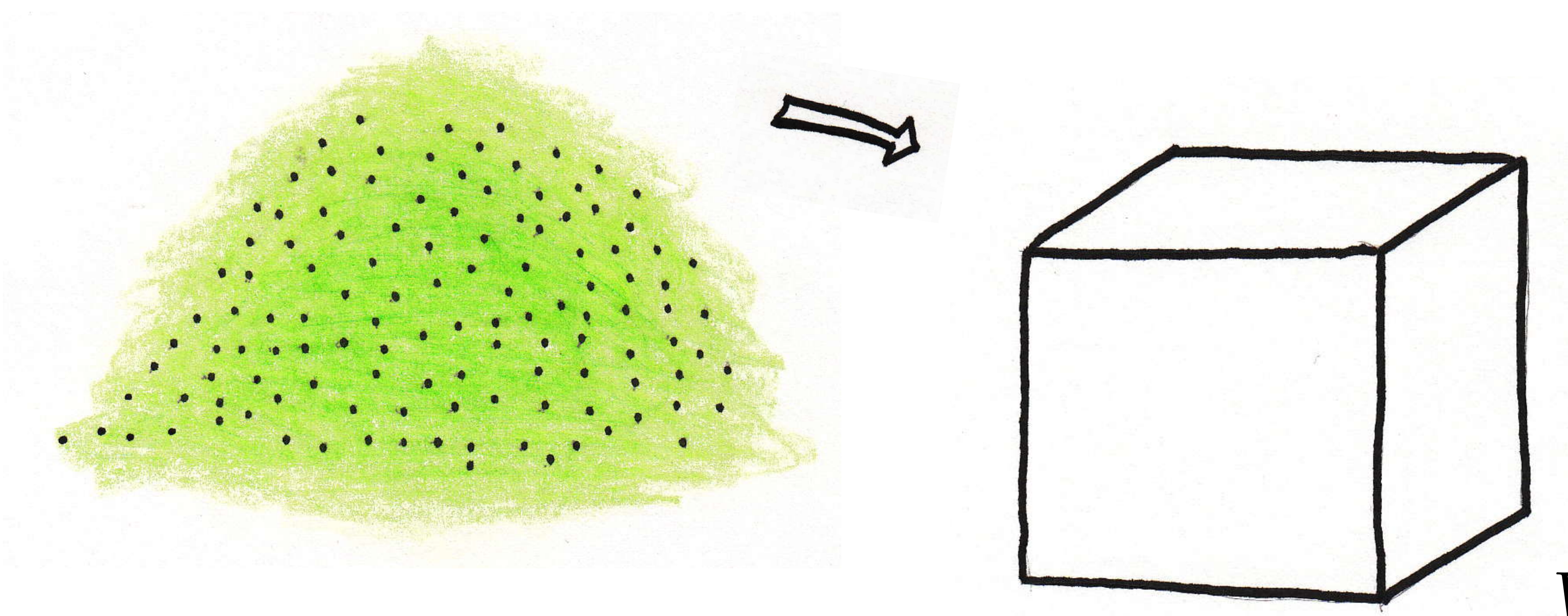
. EE is an important quantity in theoretical physics

. EE has applications in condensed matter physics

. EE in general is hard to compute

the holographic principle 't Hooft, Susskind '93

. let's start without gravity…



$V_0 \sim L^3$

the holographic principle 't Hooft, Susskind '93

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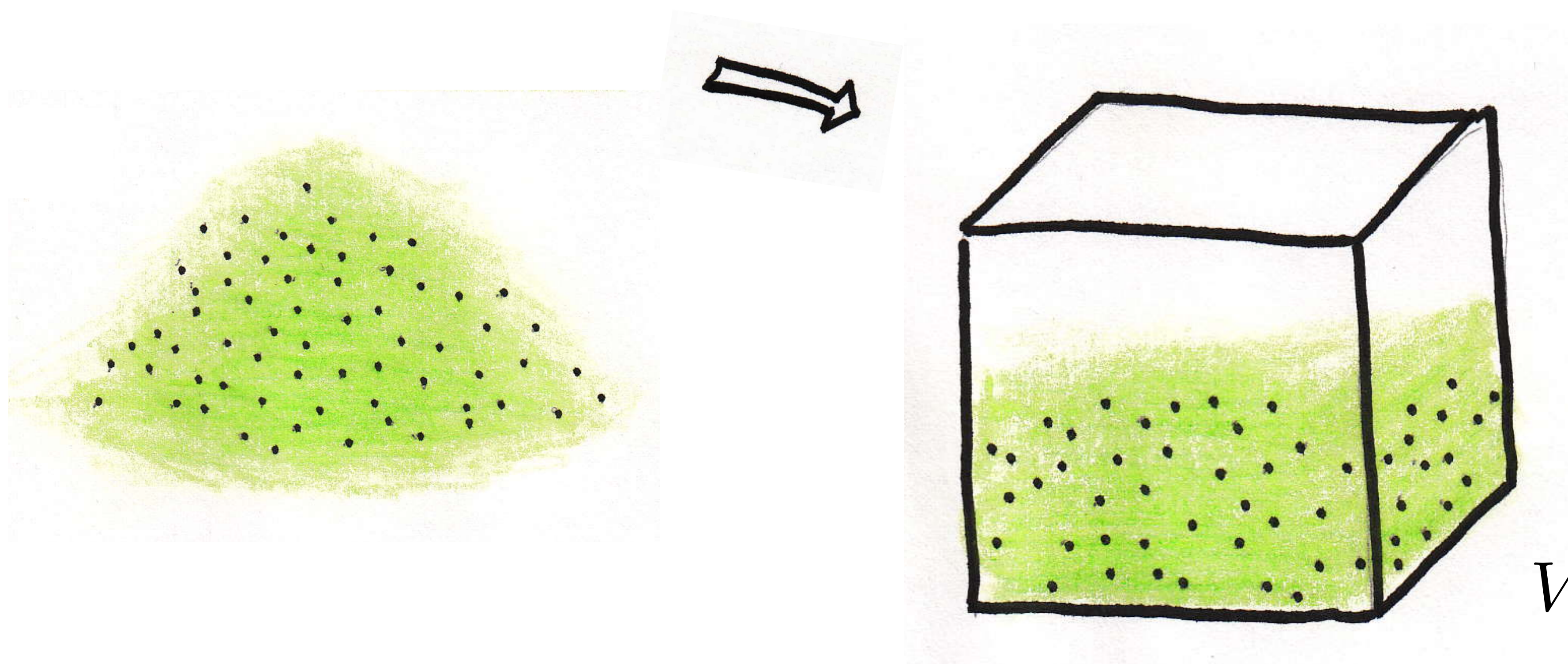

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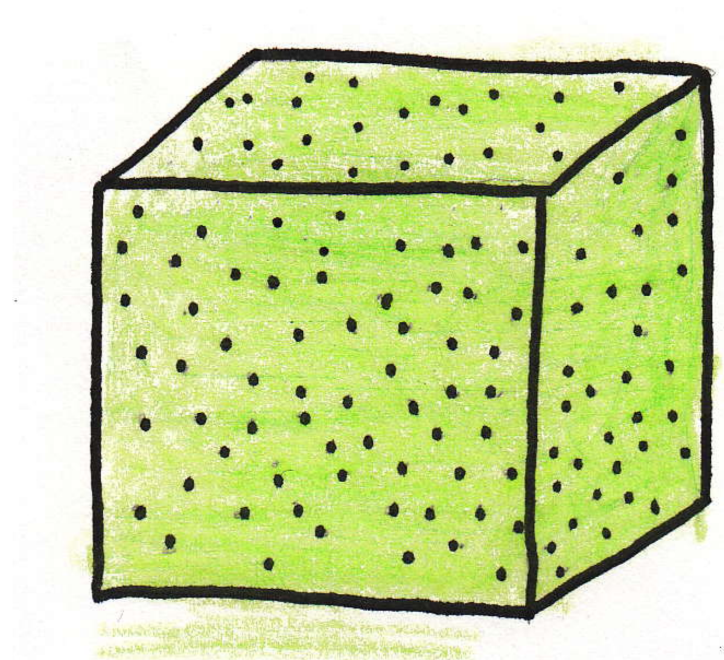
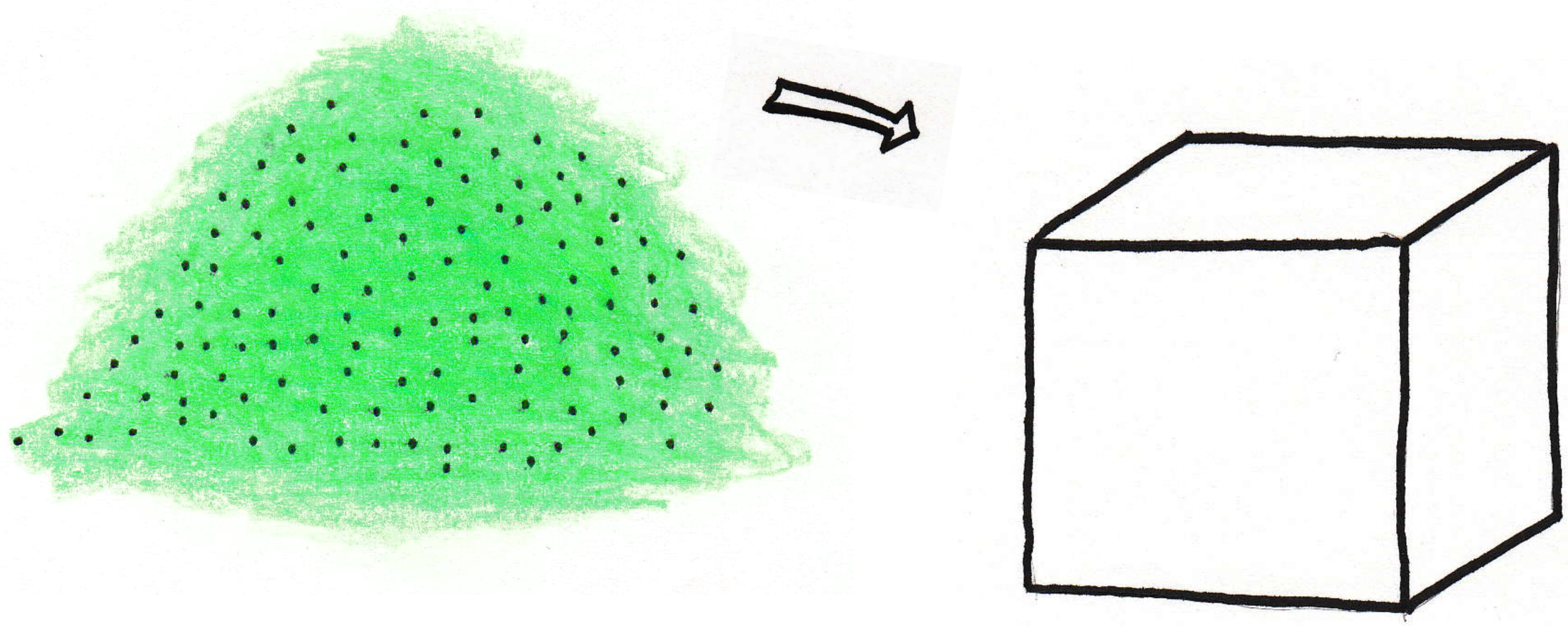


$V_0 \sim L^3$

$S_{\text{stat}} \sim V_0$

the holographic principle 't Hooft, Susskind '93

. what changes if we consider a gravitating system?



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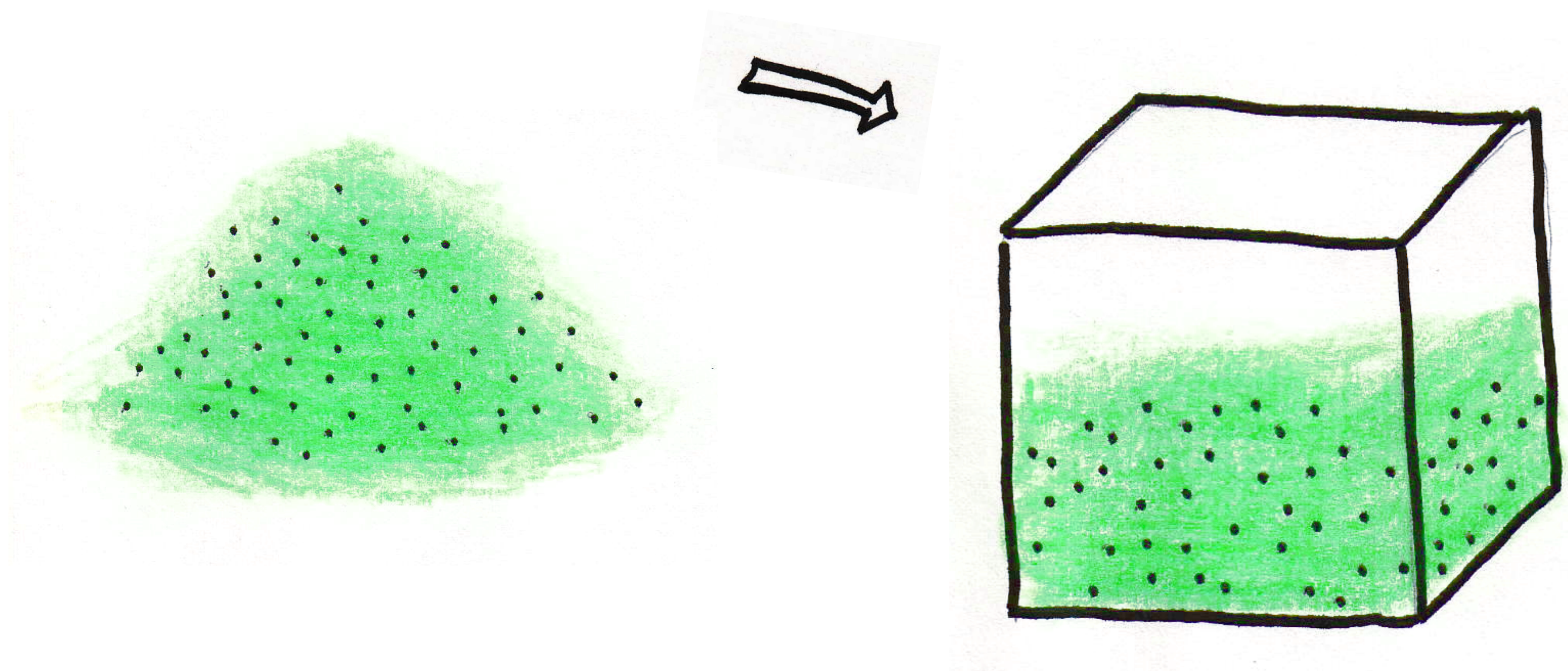

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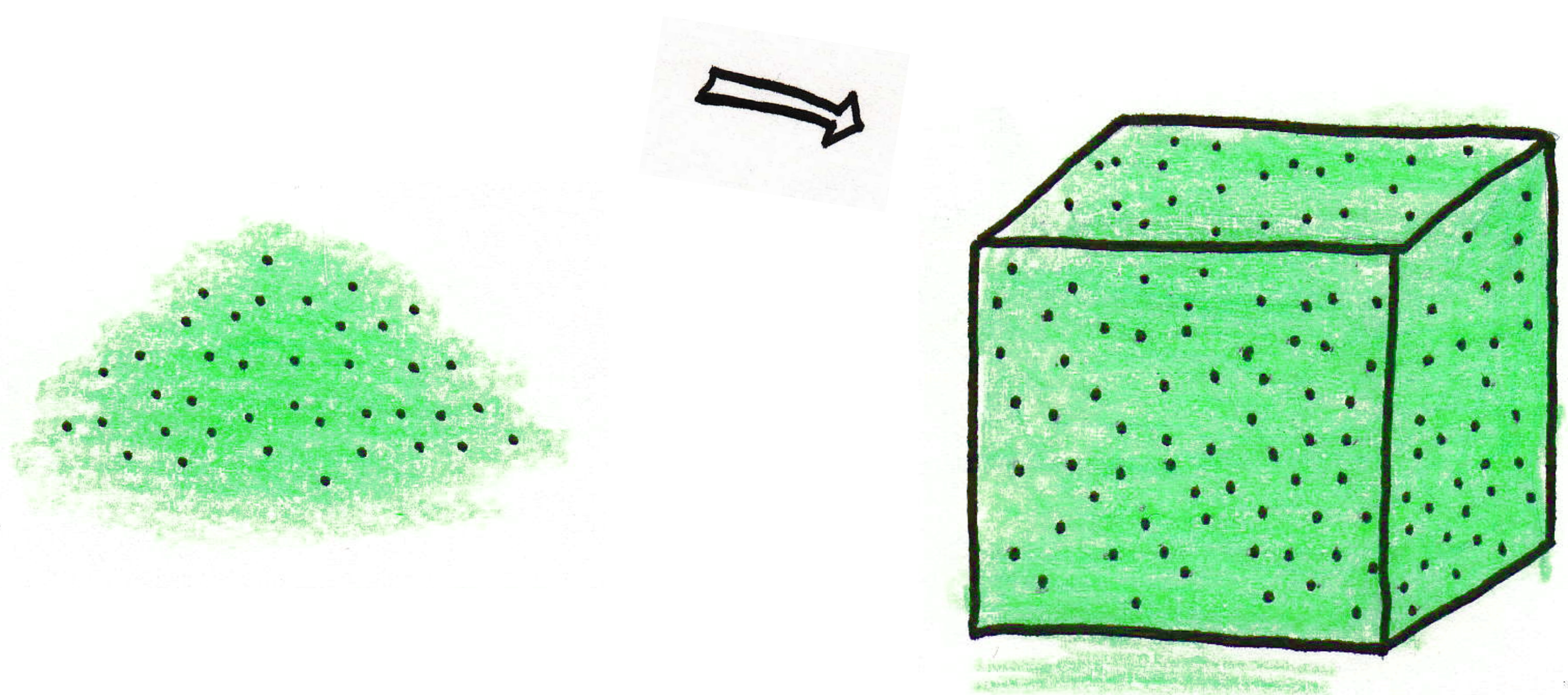

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the holographic principle 't Hooft, Susskind '93

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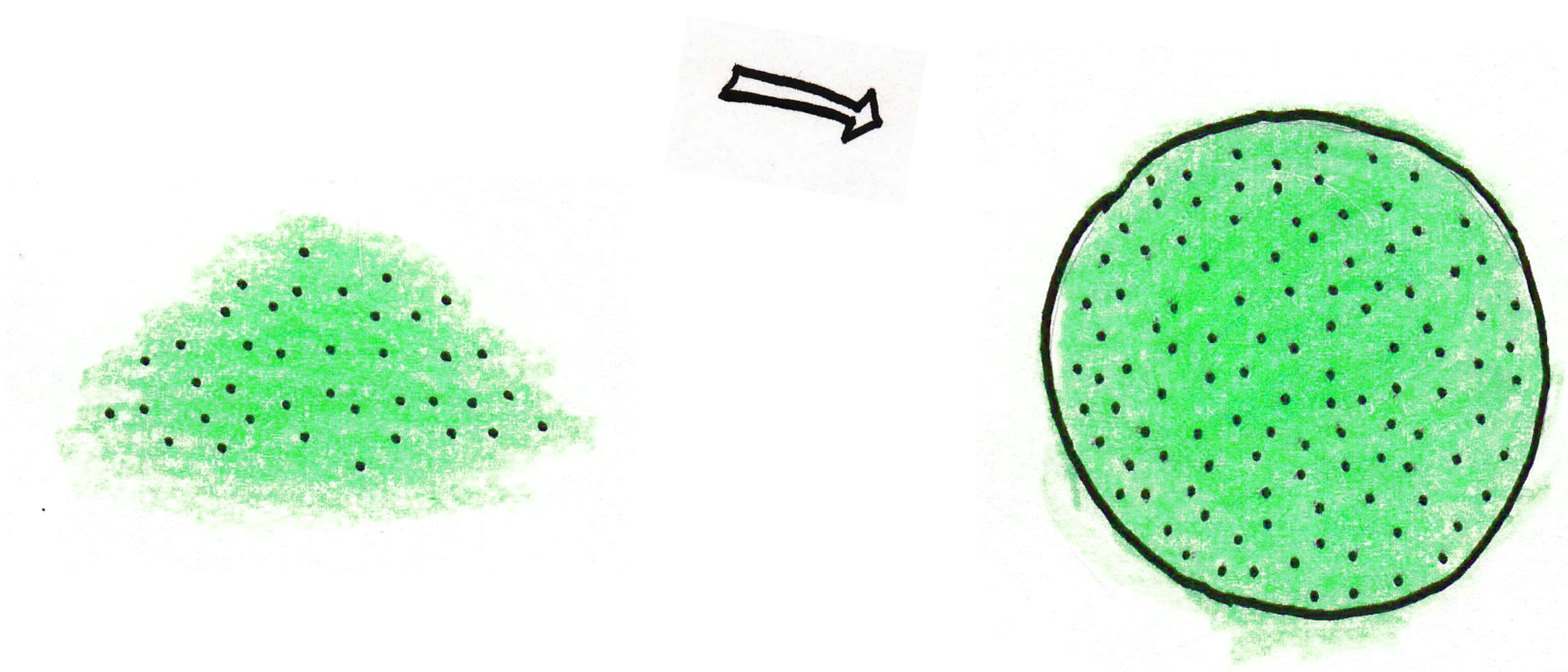


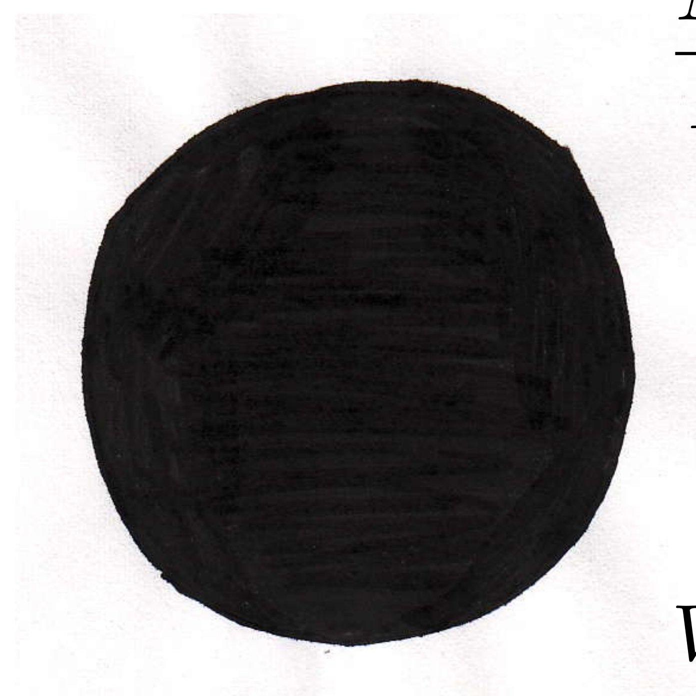
$V_0 \sim R^3$

the holographic principle

Bekenstein '73
't Hooft, Susskind '93

. the description of a volume of space can in some way be encoded on the boundary of that volume

$$\frac{M}{R} \geq \mathrm{const}$$



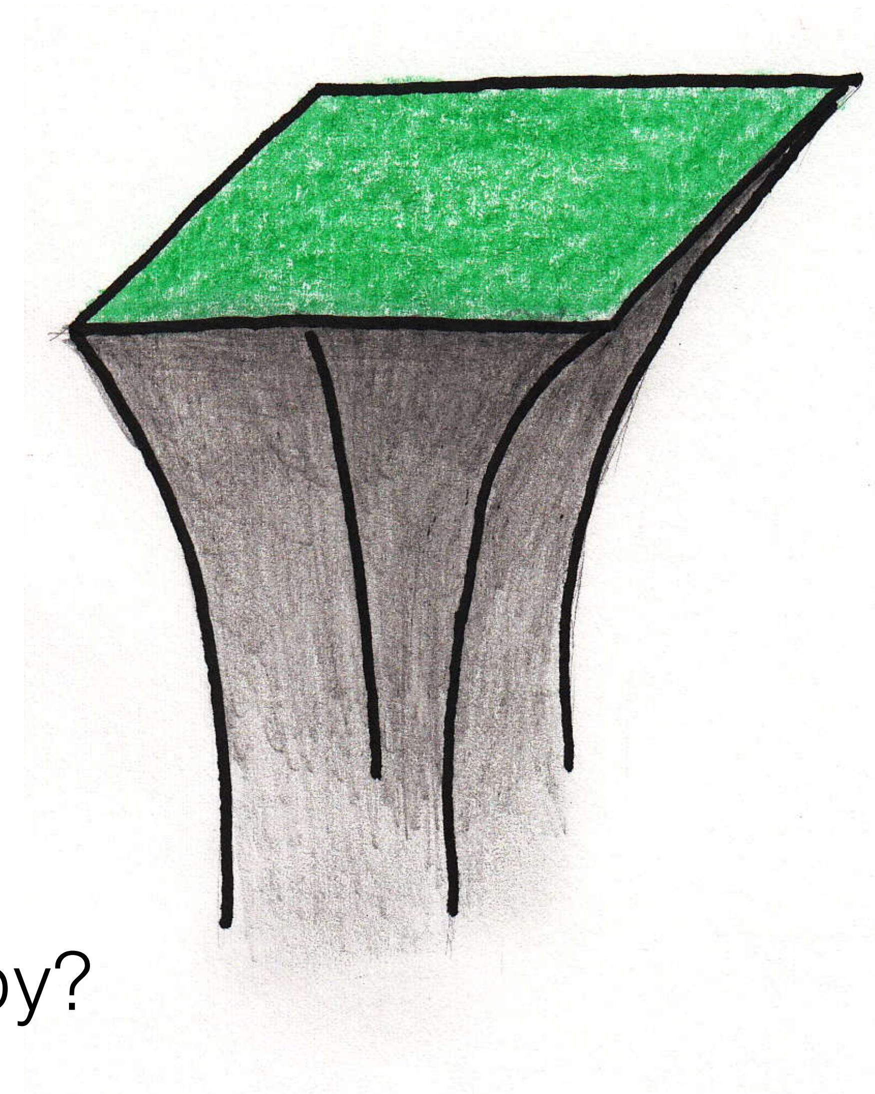
$$V_0 \sim R^3$$

$$S_{\mathrm{BH}} \sim \mathrm{Surface}(\mathrm{V_0}) = \mathrm{Area}(\mathrm{BH})$$

how can AdS/CFT help? Maldacena '97

. statistical entropy of the CFT is given by the classical BH entropy

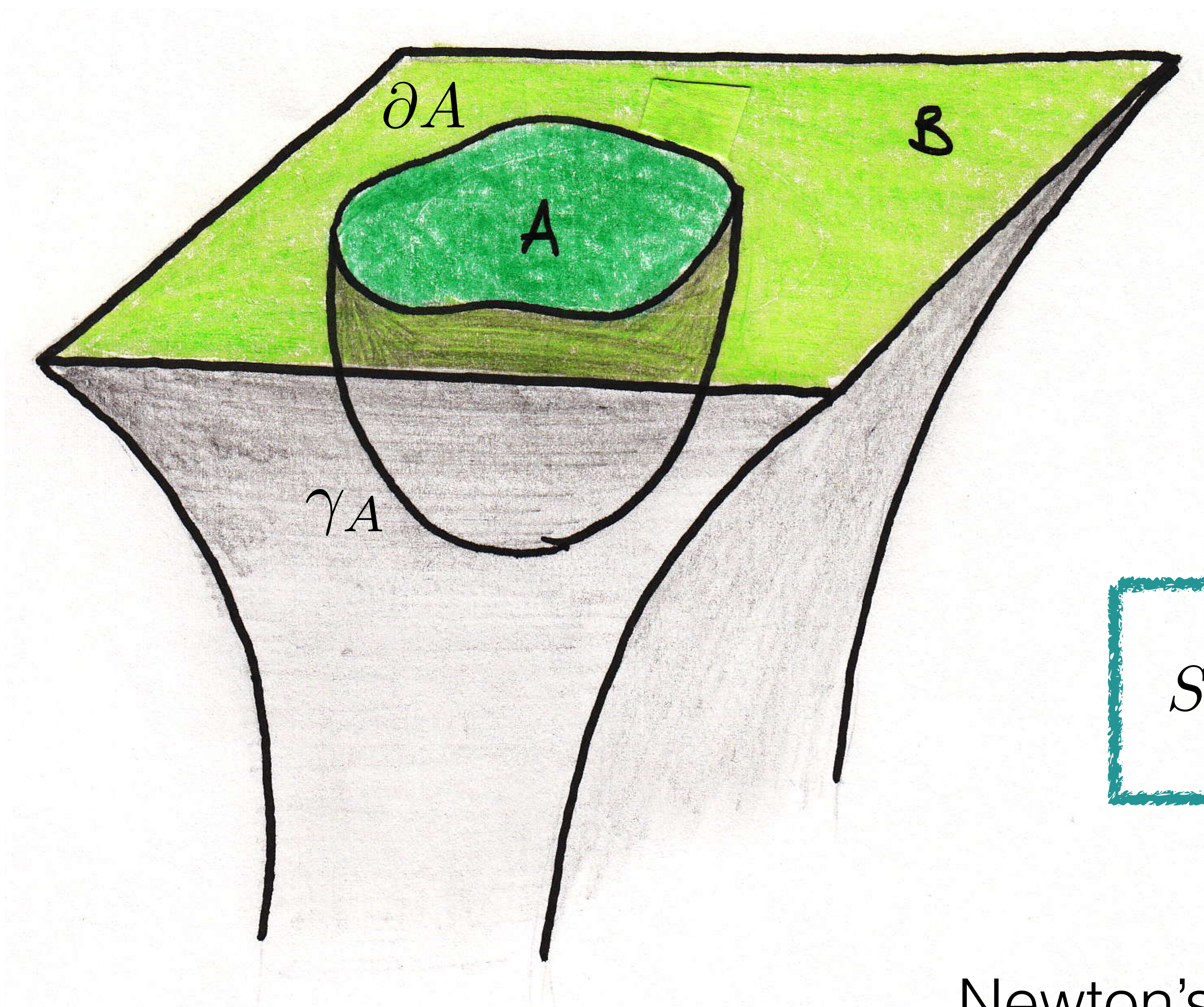
$$S_{\rm BH} = \frac{\rm Area(BH)}{4G_N}$$



. is there some surface on the gravity side corresponding to the entanglement entropy?

holographic entanglement entropy

Ryu, Takayanagi '06



minimal surface γ_A

$$S(A) = \frac{\text{Area}(\gamma_A)}{4G_N}$$

Newton's constant

testing the conjecture I

. known 2D CFT results are exactly reproduced

. in higher dimensions: indications and checks supporting the conjecture

non-negative by definition $S \geq 0$

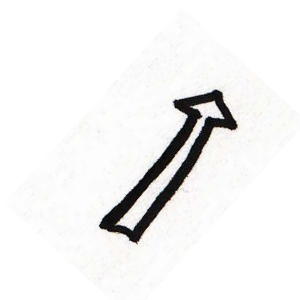
area-law divergence $S_A \sim \mathrm{Area}(\partial A)\epsilon^{-(d-1)}$

strong subadditivity $S(A \cup B) + S(B \cup C) \geq S(A \cup B \cup C) + S(B)$

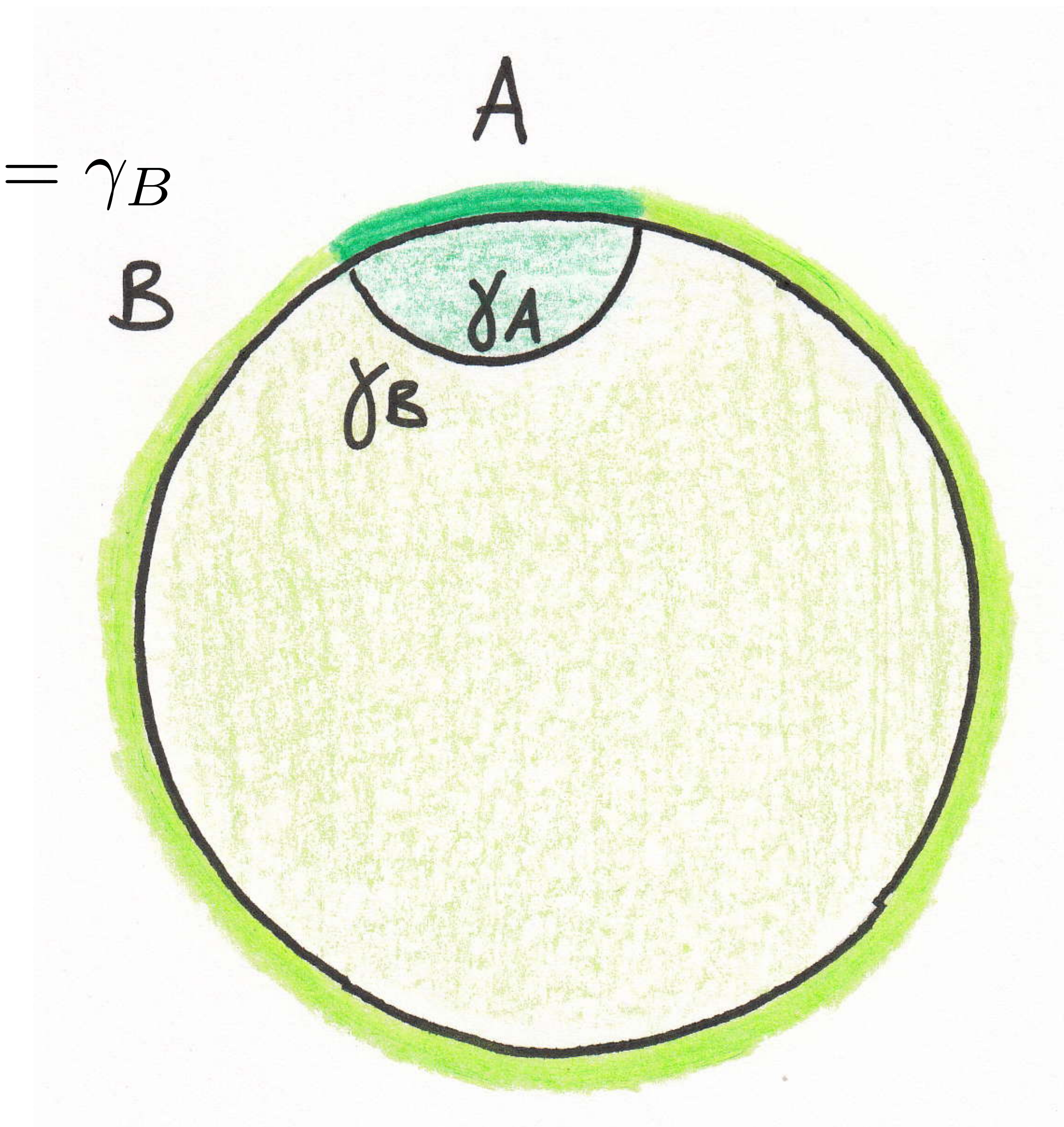
testing the conjecture II

. at zero temperature we have $\gamma_A = \gamma_B$

$$S(A) = S(B)$$



$$S(A) = \frac{\text{Area}(\gamma_A)}{4G_N}$$



. for Einstein gravity the formula is proven Lewkowycz, Maldacena '13

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