Top Quark Physics: on its mass mostly …

André H. Hoang

University of Vienna

Young Physicists Workshop @ Ringberg, July 6 - 10, 2015

Outline

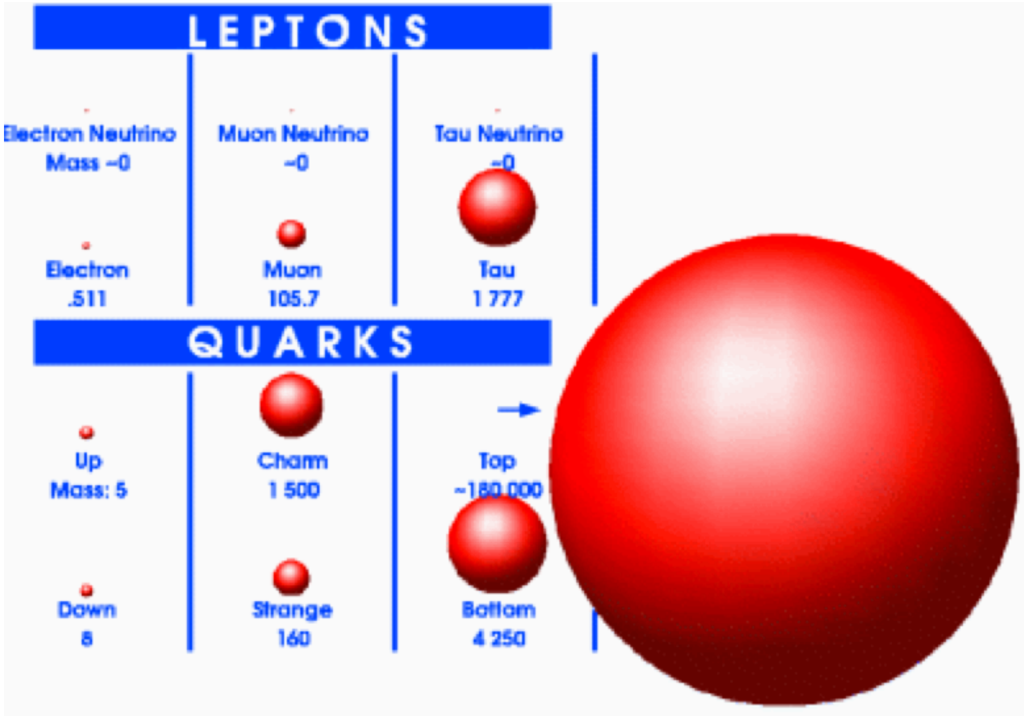
Today:

- Introduction
- Importance of precise top quark mass measurements
- Indirect ways to determine the top quark mass
- Direkt ways to determine the top quark
- Top threshold at a future lepton collider

Tomorrow:

- Top mass reconstruction using Monte-Carlo generators
- Fixing the top mass parameter in Monte-Carlos m_t^{MC}
- Status & preliminary results

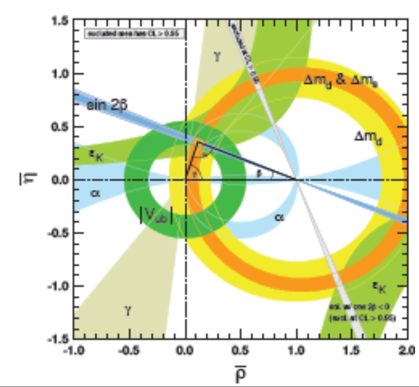
Why the top quark is not just heavy



- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background it new physics searches
- One of crucial motivations for SUSY

Orthogonal to good approximation

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$



Motivation

Example: additional quarks:

Mass Mixing and Heavy Quark Couplings to Higgs

Chiral Doublet

$$-\mathcal{L}_Q = Y_U^{ij}\bar{Q}_L\tilde{\Phi}U_R + Y_D^{ij}\bar{Q}_L\Phi D_R + h.c.$$

- **SU(2) singlet**

SM Yukawa | FCNC Yukawa | Explicit Dirac mass

- Up-type $\quad -\mathcal{L}_T = Y_t\,\overline{q_{0L}}\,\tilde{\Phi}\,t_{0R} + Y_T\,\overline{q_{0L}}\,\tilde{\Phi}\,T_{0R} + M_T\,\overline{T_{0L}}T_{0R} + \text{H.c.}$
- Down-type $\quad -\mathcal{L}_B = Y_b\,\overline{q_{0L}}\,\Phi\,b_{0R} + Y_B\,\overline{q_{0L}}\,\Phi\,B_{0R} + M_B\,\overline{B_{0L}}B_{0R} + \text{H.c.}$

del Aguila, Perez-Victoria, Santiago (2000)

- **SU(2) doublet**

$$-\mathcal{L}_Q = Y_t\,\overline{q_{0L}}\,\tilde{\Phi}\,t_{0R} + Y_T\,\overline{Q_{0L}}\,\tilde{\Phi}\,t_{0R} + Y_B\,\overline{Q_{0L}}\,\Phi\,b_{0R} + M\,\overline{Q_{0L}}Q_{0R} + \text{H.c.}$$

$$-\mathcal{L}_{Q'} = Y_t\,\overline{q_{0L}}\,\tilde{\Phi}\,t_{0R} + Y_T\,\overline{Q'_{0L}}\,\Phi\,t_{0R} + M\,\overline{Q'_{0L}}Q'_{0R} + \text{H.c.}$$

$$Q_{0L} = \begin{pmatrix} T_{0L} \\ B_{0L} \end{pmatrix},\ Q_{0R} = \begin{pmatrix} T_{0R} \\ B_{0R} \end{pmatrix} \quad Q'_{0L} = \begin{pmatrix} Y \\ T_{0L} \end{pmatrix},\ Q'_{0R} = \begin{pmatrix} Y \\ T_{0R} \end{pmatrix}$$

Angular-Saavedra (2009)

- **SU(2) triplet**

Exotic Q=5/3 fermion

$$-\mathcal{L}_\Sigma = Y_t\,\overline{q_{0L}}\,\tilde{\Phi}\,t_{0R} + Y_T\,\overline{q_{0L}}\,\tau^a\,\tilde{\Phi}\,\Sigma_{0R} + M\,\overline{\Sigma_{0L}}\Sigma_{0R} + \text{H.c.}$$

$$-\mathcal{L}_{\Sigma'} = Y_t\,\overline{q_{0L}}\,\tilde{\Phi}\,t_{0R} + Y_T\,\overline{q_{0L}}\,\tau^a\,\Phi\,\Sigma'_{0R} + M\,\overline{\Sigma'_{0L}}\Sigma'_{0R} + \text{H.c.}$$

$$\Sigma_{0L} = \begin{pmatrix} X_{0L} \\ T_{0L} \\ B_{0L} \end{pmatrix},\ \Sigma_{0R} = \begin{pmatrix} X_{0R} \\ T_{0R} \\ B_{0R} \end{pmatrix} \quad \Sigma'_{0L} = \begin{pmatrix} T_{0L} \\ B_{0L} \\ X_{0L} \end{pmatrix},\ \Sigma'_{0R} = \begin{pmatrix} T_{0R} \\ B_{0R} \\ X_{0R} \end{pmatrix}$$

Exotic Q=-4/3 fermion

Cacciapaglia, Deandrea, Harada, Okada (2010)

/ Quarks, 20-21 Dec 2011 Koji Tsumura (ntu) 4

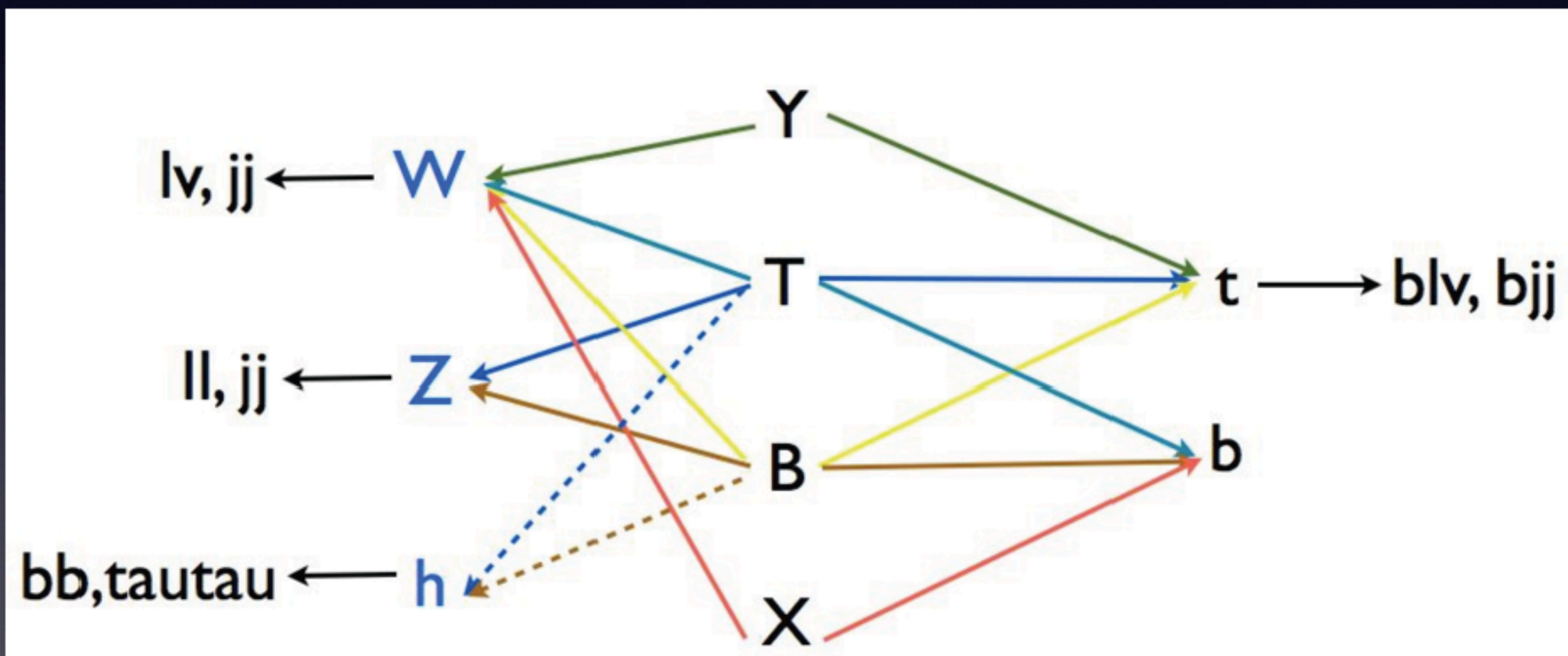
4

Motivation

Example: additional quarks:

$$T \to W^+b/Zt/Ht \qquad Y \to W^+t$$

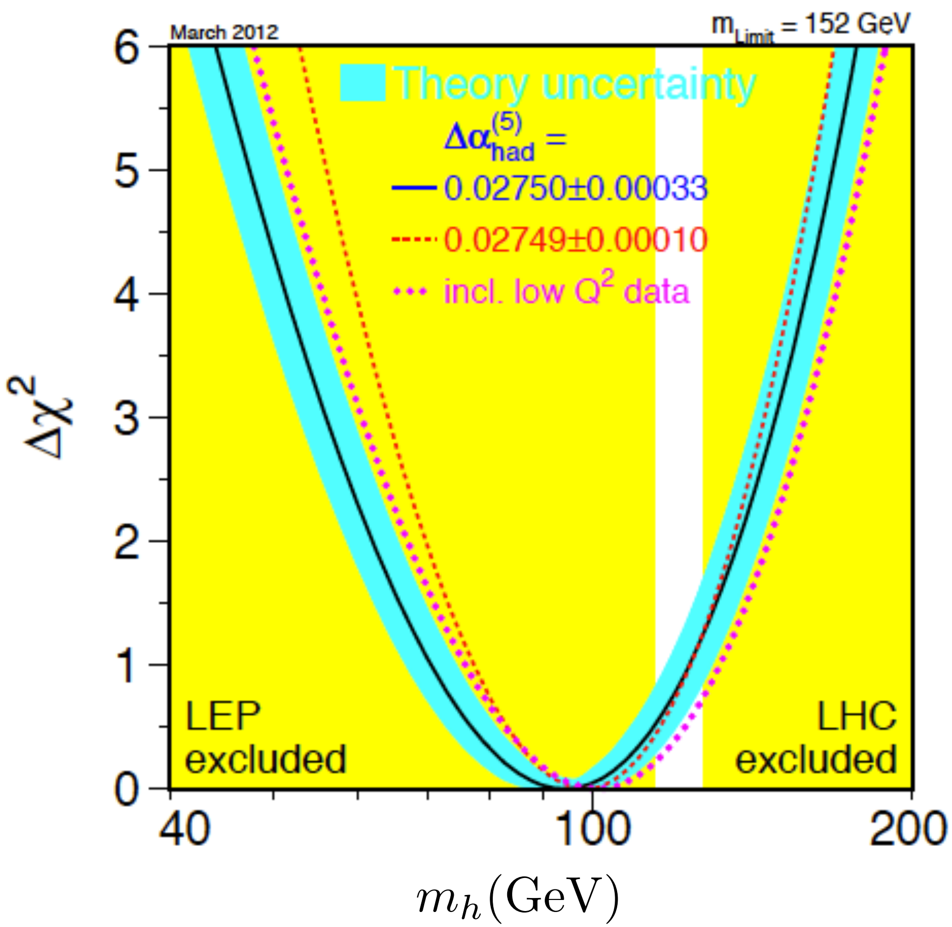
$$B \to W^+t/Zb/Hb \qquad X \to W^-b$$



Very Rich Collider Signatures

Motivation

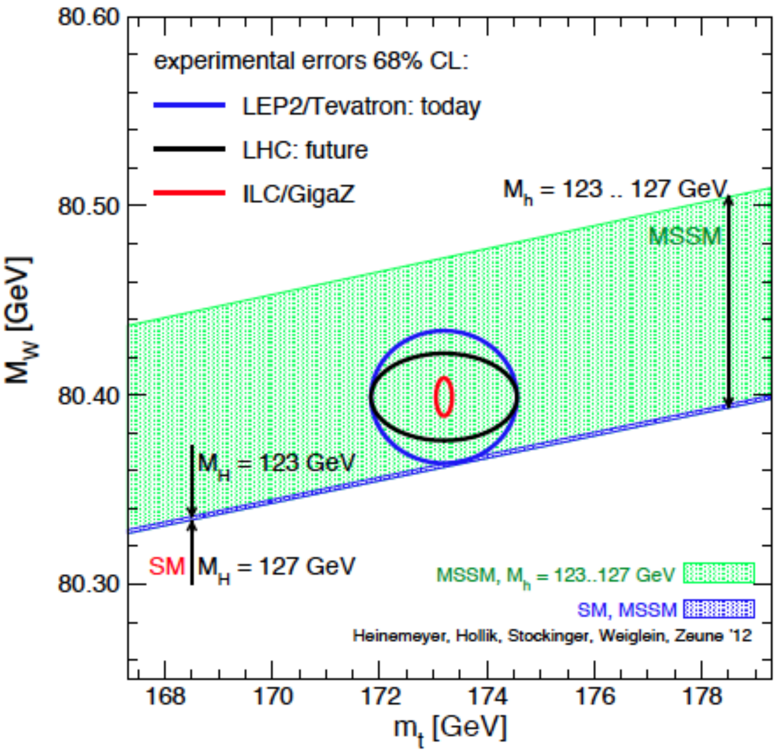
Indirekt search for new physics:



- Top crucial ingredient for global fits within the Standard Model
- Largest impact on indirect evidence for the Higgs

Motivation

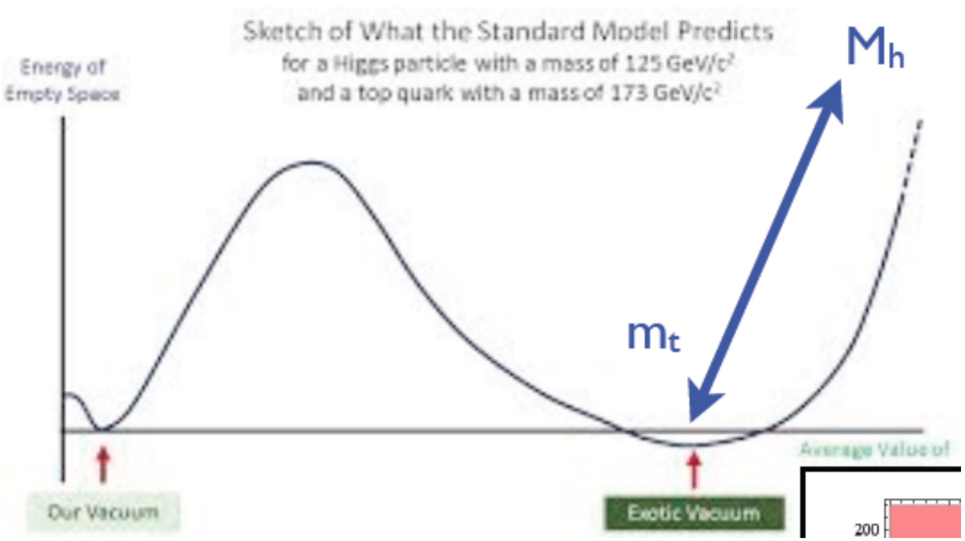
Indirekt search for new physics:



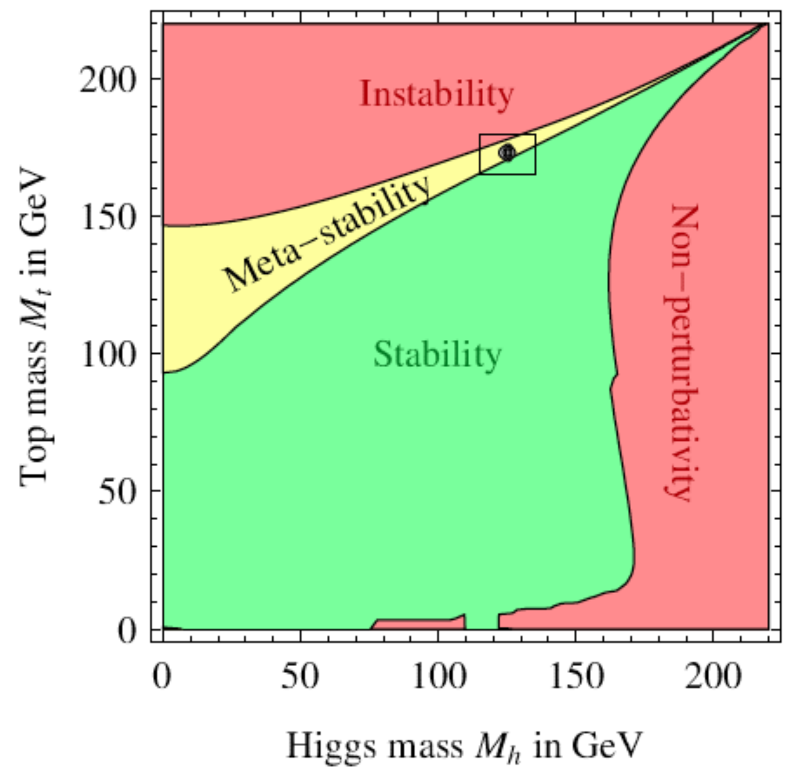
- Top crucial ingredient for global fits within the Standard Model
- Relations among electroweak precision observables put stringent constraints on BSM models

Motivation

Role in the fate of our universe (?):

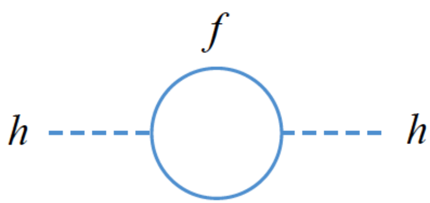


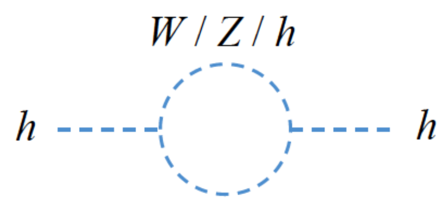
- Strong dependence of SM Higgs potential on top quark mass
- Existence of a lower energy vacuum state (if calculations apply)
- Dependence on higher-dimension Higgs interactions

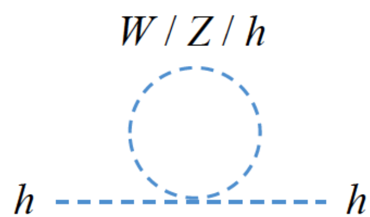


Motivation

Hierarchy problem:





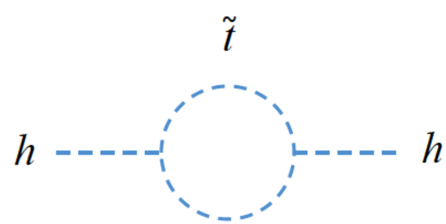


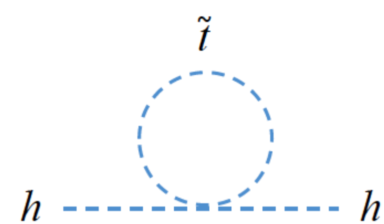
Largest contribution to Higgs mass divergence – from top

$$\Delta m_h^2 \sim -\frac{\alpha}{\pi}\left[\sum_f m_f^2 - \frac{3}{4}\left(m_W^2 + m_Z^2 + m_h^2\right)\right]\frac{\Lambda^2}{m_W^2}$$

- Top mostly involved in the resolution of the hierarchy problem
- Overall, BSM particles associated to the top tend to be the easiest to be discovered because they tend to be relatively light.
- Very strong sensitivity of the lightest MSSM Higgs boson mass on the top quark mass (m_t^4-dependence).

Stops cancel top divergences



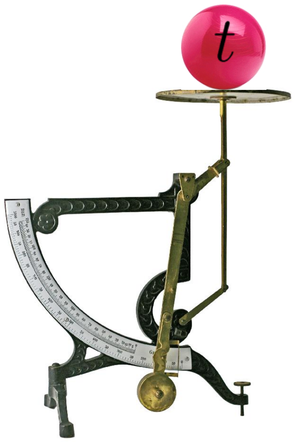


Motivation

quantity	CDF/DØ		ATLAS/CMS	
$\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}}$	11% with 1 fb^{-1}	[554]	5%–10% luminosity systematics dominated	[286, 288]
$\Delta\sigma_{\text{single-top}}/\sigma_{\text{single-top}}$	26% with 1 fb^{-1}	[554]	10% (< 2% stat. error with 10 fb^{-1}	[288])
$B(t \to Wb)$	3.3% with 1 fb^{-1}	[554]		
V_{tb} from $\sigma_{\text{single-top}}$	14% with 1 fb^{-1}	[554]	6.5%	[528]
V_{tb} from $B(t \to Wb)$	> 0.22 with 1 fb^{-1}	[554]	0.2% (stat. only)	[286]
single-top polarisation	–		1.6% with 10 fb^{-1}	[288]
$\Delta m_{\text{top}}/m_{\text{top}}$	$\leq 2\,\text{GeV/c}^2$	Sect. 7	$\approx 1\,\text{GeV/c}^2$	[286, 288]
spin correlation θ	40% (2 fb^{-1})	[538]	7% ($\ell\ell \oplus \ell+$jets) for 10 fb^{-1}	[538]
spin correlation ϕ	–		4% ($\ell\ell \oplus \ell+$jets) for 10 fb^{-1}	[538]
W-helicity $\mathcal{F}_0$	6.5% with 1 fb^{-1}	[554]	2%–5% with 10 fb^{-1}	[527, 537, 538]
W-helicity $\mathcal{F}_+$	2.6% with 1 fb^{-1}	[554]	1% with 10 fb^{-1}	[538]
electric charge q_t	distinguish $\frac{2}{3}$ and $\frac{4}{3}$ cases with 1 fb^{-1}	Sect. 7.2	distinguish $\frac{2}{3}$ and $\frac{4}{3}$ cases with 10 fb^{-1}	[536]
Yukawa coupling y_t	–		4.8σ, 16% (12%) with 30(100) fb^{-1}	[548, 549]
FCNC $B(t \to gq)$	$< 1.9 \times 10^{-2}$ with 2 fb^{-1}	[288, 555]	$< 1 \quad \times 10^{-5}$ - $< 1.4 \times 10^{-3}$ (10 fb^{-1})	[288, 556]
FCNC $B(t \to Zq)$	$< 1.5 \times 10^{-2}$ with 1 fb^{-1}	[554]	$< 6.5 \times 10^{-4}$ - 1.3×10^{-3} with 10 fb^{-1}	[286, 288, 556]
FCNC $B(t \to \gamma q)$	$< 3.0 \times 10^{-3}$ with 1 fb^{-1}	[554]	$< 8.6 \times 10^{-5}$ - 1.9×10^{-4} with 10 fb^{-1}	[286, 288, 556]
FCNC $B(t \to WbZ)$	–		$< 10^{-7}$ with 100 fb^{-1}	[553]
$\Delta\sigma^{M_{Z'}=1\,\text{TeV/c}^2}$ $B(Z' \to t\bar{t})$	100 fb with 1 fb^{-1}	[554]	700 fb with 30 fb^{-1}	[286, 288]
anom. coupling	$F_{2L}{}^{>+0.55}_{<-0.18}$	[553]	$F_{2L}{}^{>+0.097}_{<-0.052}$	[553]
	$F_{2R}{}^{>+0.25}_{<-0.24}$	[553]	$F_{2R}{}^{>+0.13}_{<-0.12}$	[553]
$\Delta F^Z_{1V,A}$	–	[542]	15%–85% (300 fb^{-1})	[542]
$\Delta F^\gamma_{1V,A}$	${}^{<+1.03\ldots+2.60}_{>-1.17\ldots-1.88}$ (8 fb^{-1})	[542]	15%–50% (30 fb^{-1}), 4%–7% (300 fb^{-1})	[542]
$\Delta F^\gamma_{2V,A}$	–	[542]	35% (30 fb^{-1}), 20% (300 fb^{-1})	[542]
$\Delta F^Z_{2V,A}$	–	[542]	55% (300 fb^{-1})	[542]

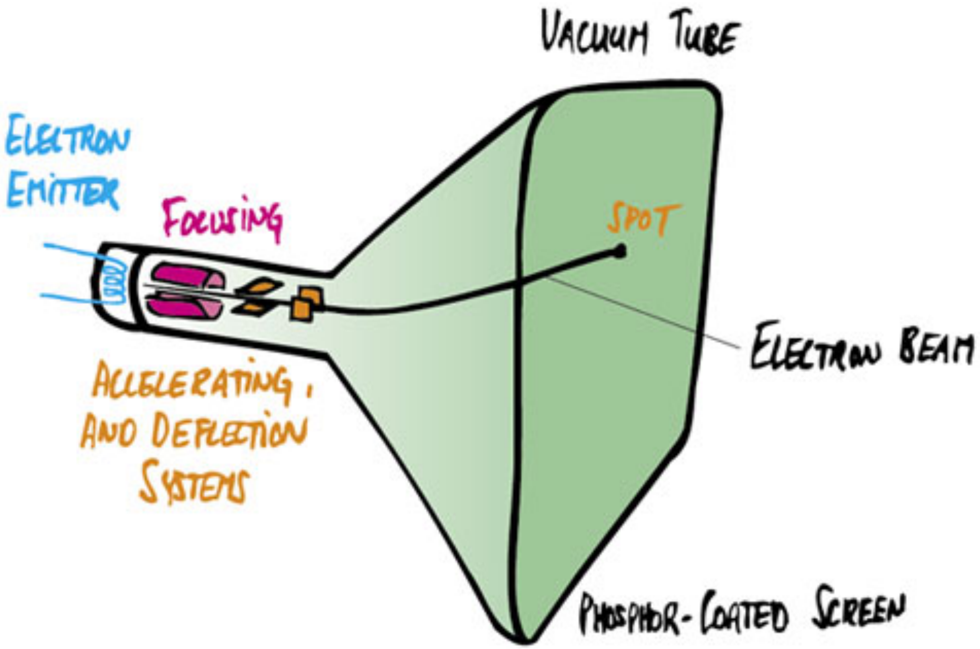
A. Quadt, Top quark physics at hadron collilders, Springer Verlag, 2007

The Top Quark Mass



The Top Quark Mass

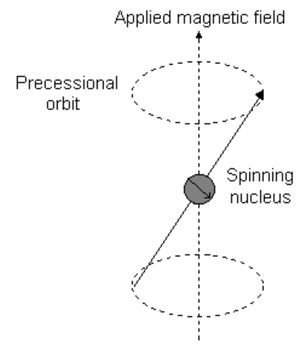
Mass of the elektrons:

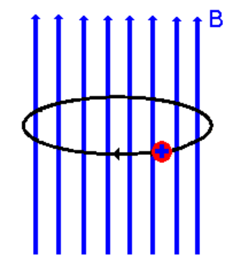


The Top Quark Mass

Mass of the elektrons:

$$m_\mathrm{e} = \frac{\omega_\mathrm{c}}{\omega_\mathrm{L}} \frac{g\,|e|}{2q} m_\mathrm{ion}$$





Larmor- and Cyclotron frequency of elektrons bound into ions

Quantum electro dynamics (QED)

$$g(nS) = \underbrace{2 - \frac{2\,(Z\alpha)^2}{3\,n^2} + \frac{(Z\alpha)^4}{n^3}\left(\frac{1}{2n} - \frac{2}{3}\right) + \mathcal{O}(Z\alpha)^6}_{\text{Breit (1928), Dirac theory}}$$

$$+ \underbrace{\frac{\alpha}{\pi}\left\{2 \times \frac{1}{2}\left(1 + \frac{(Z\alpha)^2}{6n^2}\right) + \frac{(Z\alpha)^4}{n^3}\left\{a_{41}\ln[(Z\alpha)^{-2}] + a_{40}\right\} + \mathcal{O}(Z\alpha)^5\right\}}_{\text{one-loop correction}}$$

$$+ \underbrace{\left(\frac{\alpha}{\pi}\right)^2\left\{-0.656958\left(1 + \frac{(Z\alpha)^2}{6n^2}\right) + \frac{(Z\alpha)^4}{n^3}\left\{b_{41}\ln[(Z\alpha)^{-2}] + b_{40}\right\} + \mathcal{O}(Z\alpha)^5\right\}}_{\text{two-loop correction}}$$

$$+ \mathcal{O}(\alpha^3)$$

$$\alpha = \frac{1}{137.035999679(94)} \ll 1$$

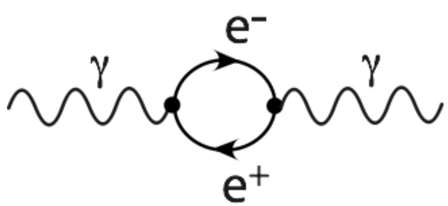
$$\boxed{m_e = 0.51099892(4)\ \mathrm{MeV}}$$

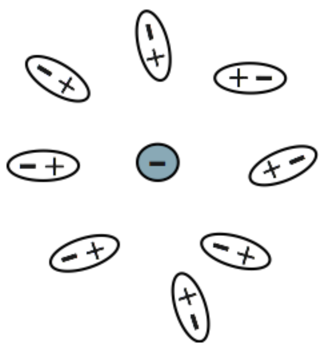
Ruhemasse des Elektrons

Beier, Häffner etal. 2002

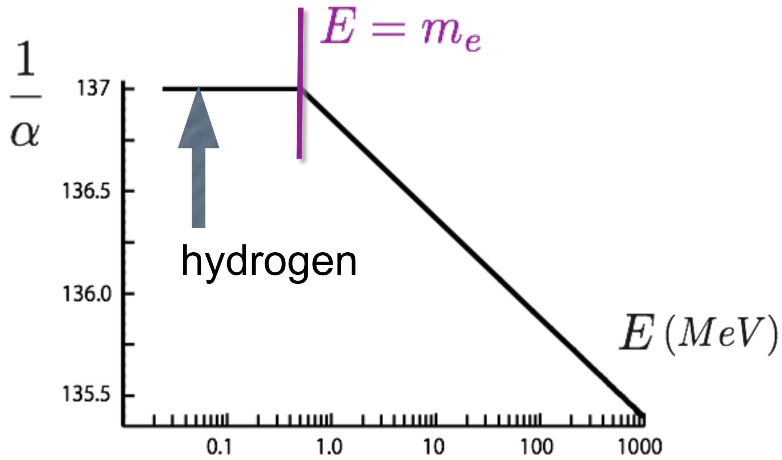
The Top Quark Mass

Vacuum polarisation (due to elektron):





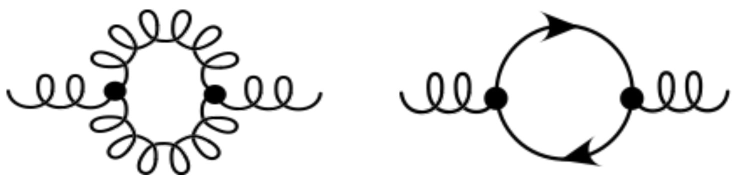
“Screening”

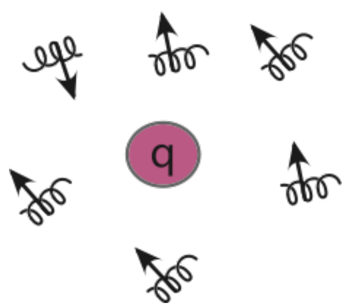


QED-Quantenkorrekturen sind störungstheoretisch sehr gut berechenbar

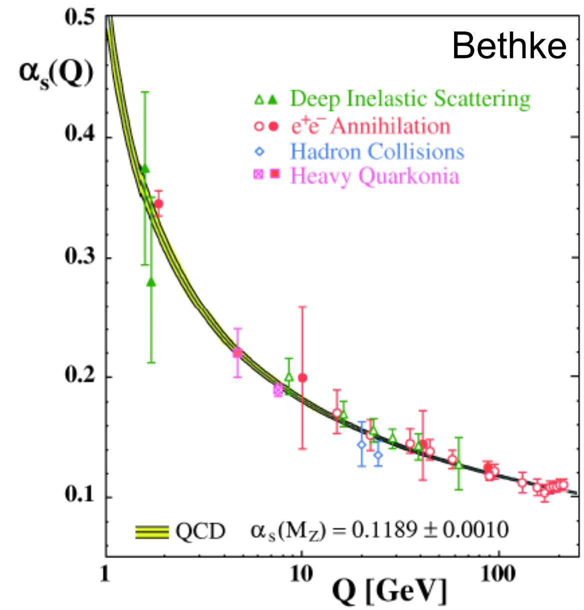
The Top Quark Mass

Vacuum polarisation (QCD):





“anti screening”



High Energies: QCD corrections are perturbatively calculable
“Asymtotic Freedom”

Low energies: QCD effects are non-perturbative **“Confinement”**

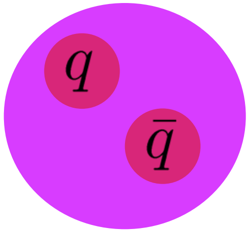
$(\Lambda_{\rm QCD} \approx 0.3 \text{ GeV})$

The Top Quark Mass

Confinement:

Mesonen

$\pi, K, \rho, B, \ldots$

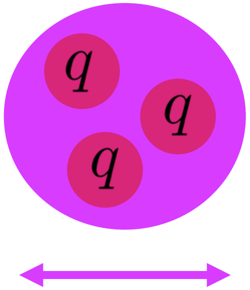


Free quarks do not occur in conditions of our daily life.

Stable quarks on arise as bound constituents of the quarks.

Baryonen

$p, n, \Sigma, \Delta, \ldots$



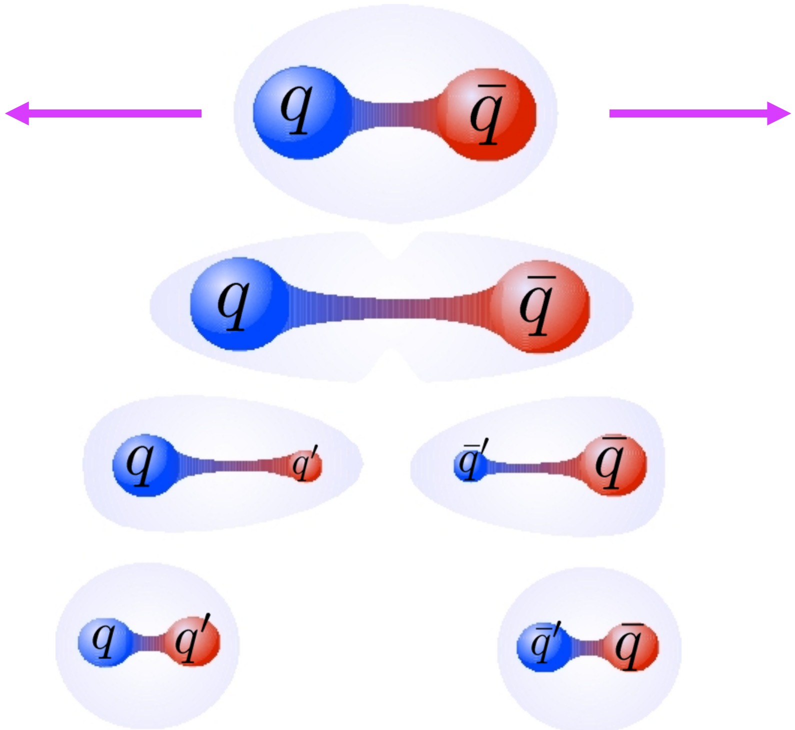
$$
\begin{aligned}
r &= \Lambda_{\mathrm{QCD}}^{-1} \\
&\simeq 1 \text{ fm}
\end{aligned}
$$

Hadronisation time:

$$\tau_{\mathrm{had}} = 7 \times 10^{-24} \text{ s}$$

The Top Quark Mass

Confinement: "String breaking"



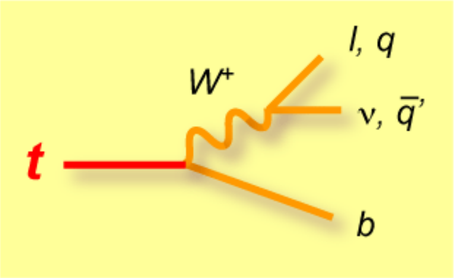
Hadronisation time:

$$\tau_{\mathrm{had}} = 7 \times 10^{-24}\ \mathrm{s}$$

The Top Quark Mass

Weak decay of the top quark:

$$V_{tb} \approx 1$$



$$\Gamma(t \to bW) \approx 1.5 \text{ GeV}$$

Top quark decays before it can form stable hardrons.

Top quarks behave in some (!) respects like uncolored particles.

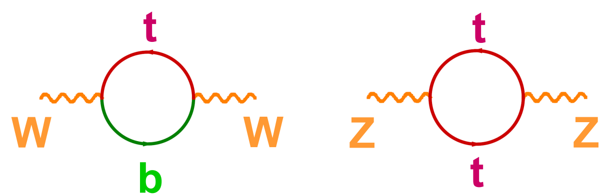
Average life time:

$$\tau_{\text{had}} = 10^{-24} \text{ s}$$

Top quark mass measurements with precision smaller than the hadronization scale need to be carried out and interpreted with great care.

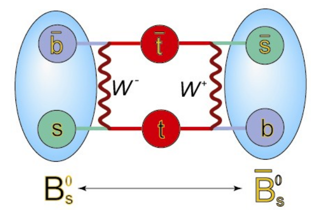
Indirect ways to determine the top quark mass

Electroweak quantum corrections:



$$m_Z^2 - m_W^2 \sim \alpha\, m_t^2$$

Flavor violating processes:

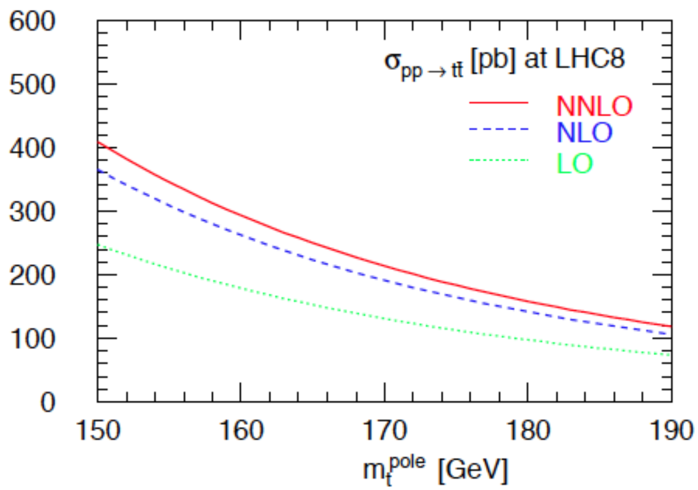


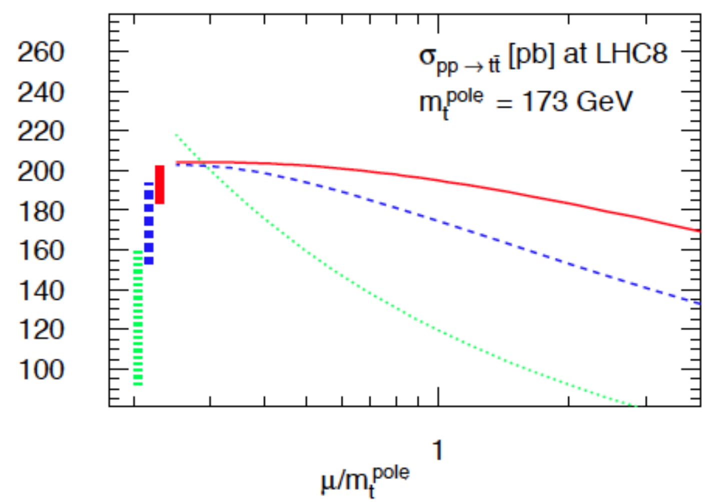
$$\sim \alpha\, m_t^2 / m_W^2$$

$$1993/1994\!: \; m_t = 130 - 200 \text{ GeV}$$

Total top-antitop cross section @ LHC:

Czakon, Fiedler, Mitov '13

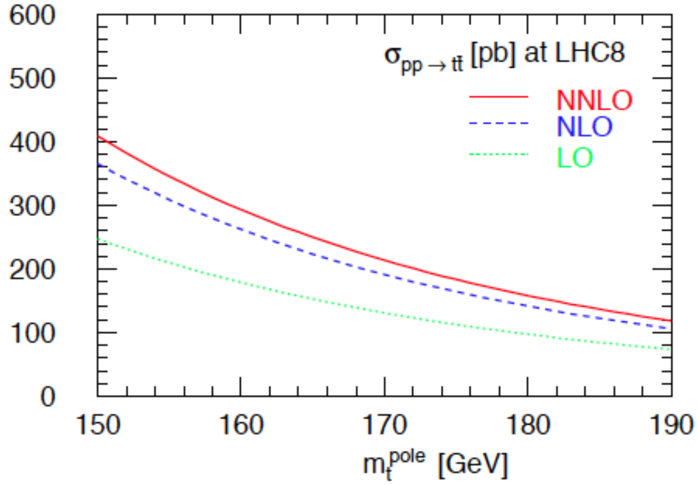


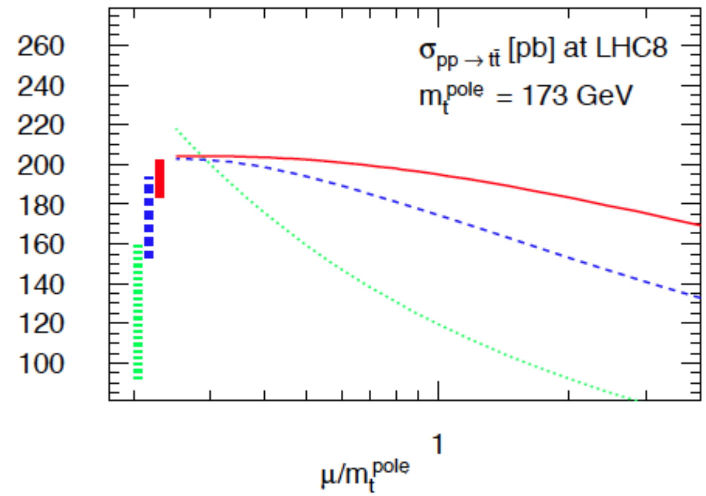


Indirect ways to determine the top quark mass

Total top-antitop cross section @ LHC:

Czakon, Fiedler, Mitov '13





- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$; scale stability of $\mathcal{O}(\pm 5\%)$
- Beyond NNLO
  - theory improvements with soft gluon resummation [many people]
  - K-factor (NNLO $\to$ resummed) small; scale stability further improved

Indirect ways to determine the top quark mass

- Intrinsic limitation of sensitivity in total cross section

$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

- QCD factorization for cross section

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

  - joint dependence on non-perturbative parameters:
    parton distribution functions f_i, strong coupling α_s, masses m_X

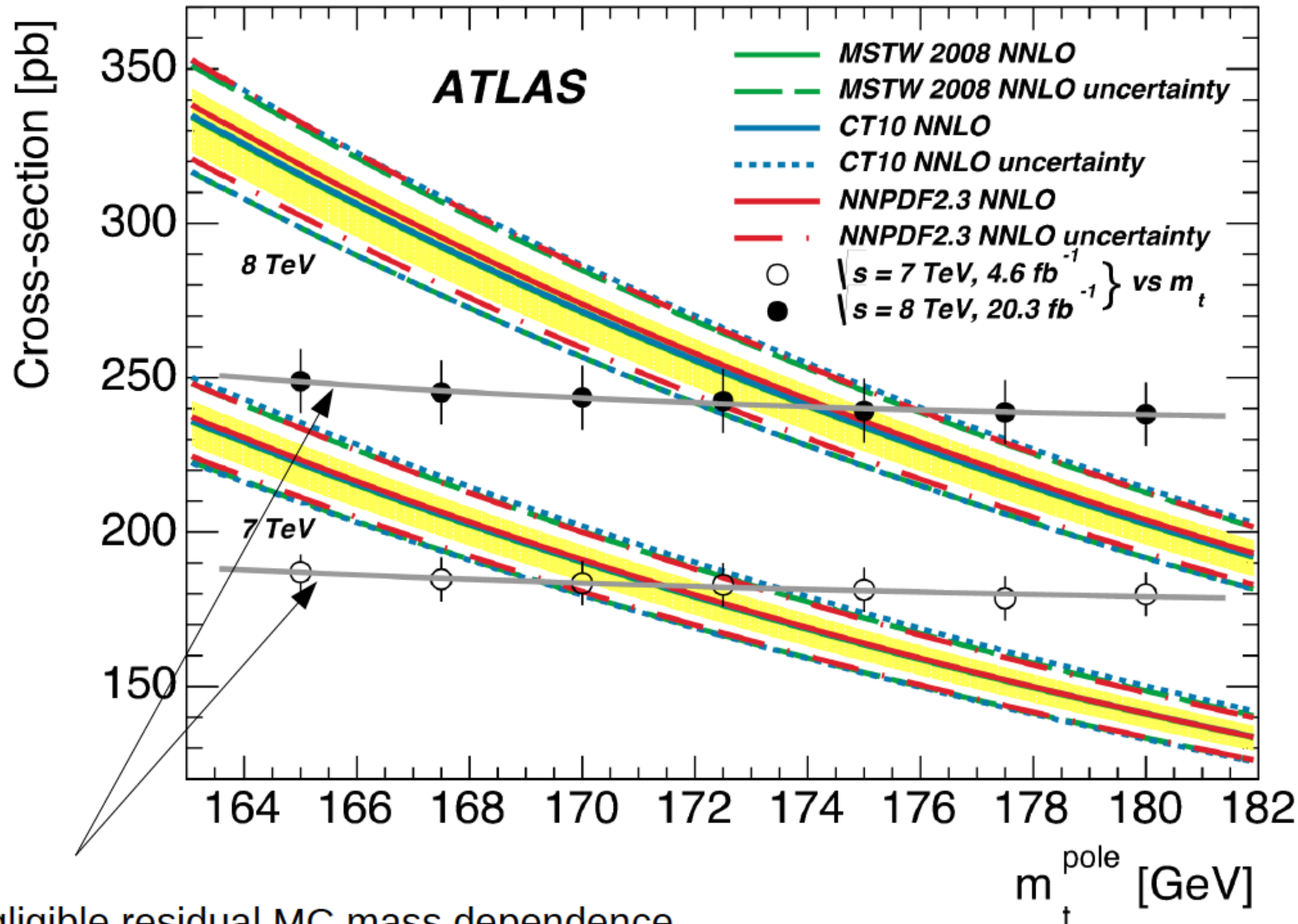
Correlations are essential

- Cross section at LHC has correlation of m_t, $\alpha_S(M_Z)$ and gluon PDF
  $\sigma_{t\bar{t}} \sim \alpha_s^2\, m_t^2\, g(x) \otimes g(x)$
  - effective parton $\langle x\rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$
  - fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
    Czakon, Mangano, Mitov, Rojo '13
  - fit with PDF re-weighting and fixed values of m_t insufficient
    Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12

Indirect ways to determine the top quark mass

Total top-antitop cross section @ LHC:

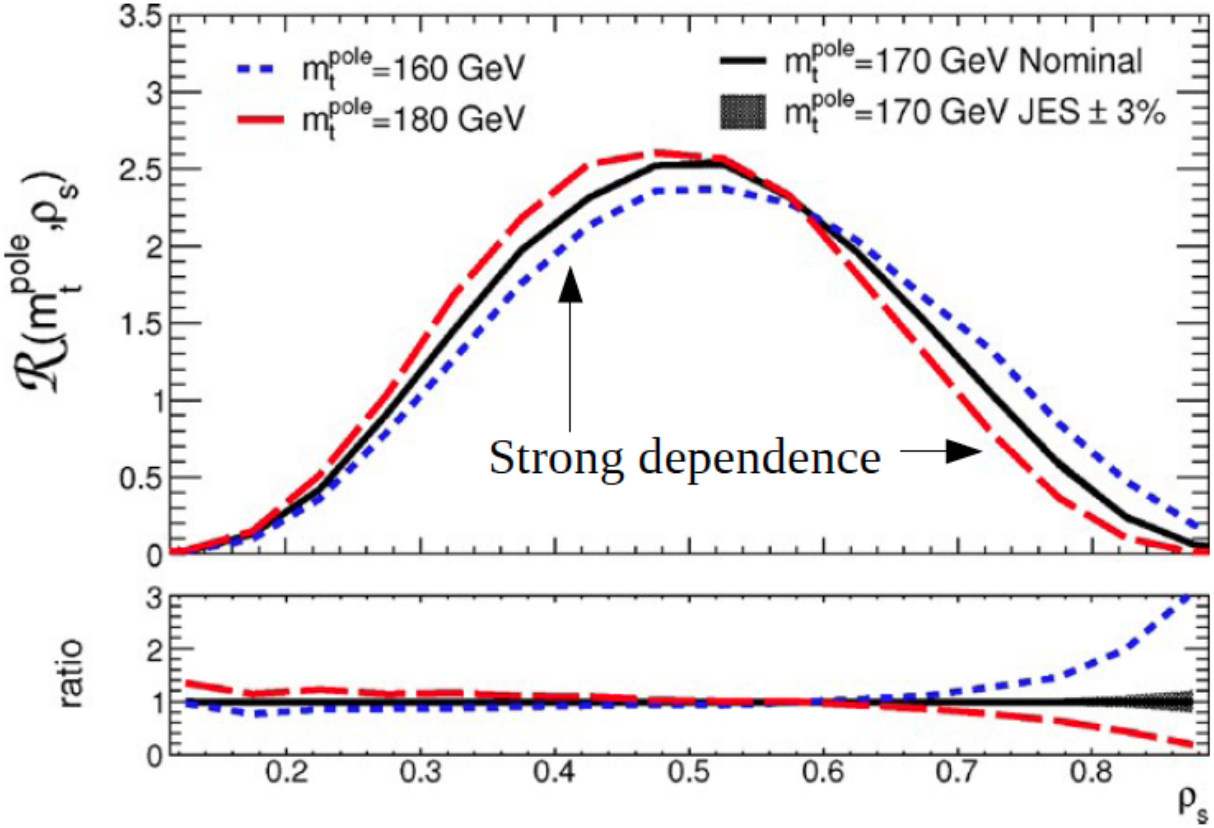
$\Delta m_t^{pole} = 2.5$ GeV



Nearly flat, negligible residual MC mass dependence

Indirect ways to determine the top quark mass

Top-antitop+Jets invariant mass @ LHC:

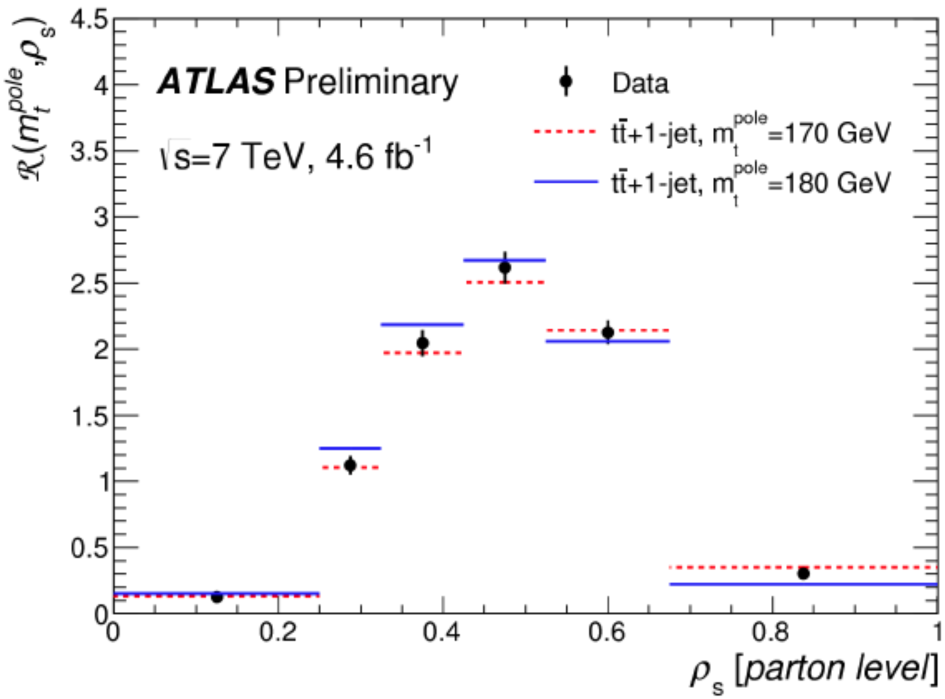


- NLO calculations (pole mass scheme currently)
- Distribution has a intrinsic peak in the distribution
- Mass determination less sensitive to PDF (< 1 GeV)

Indirect ways to determine the top quark mass

Top-antitop+Jets invariant mass @ LHC:

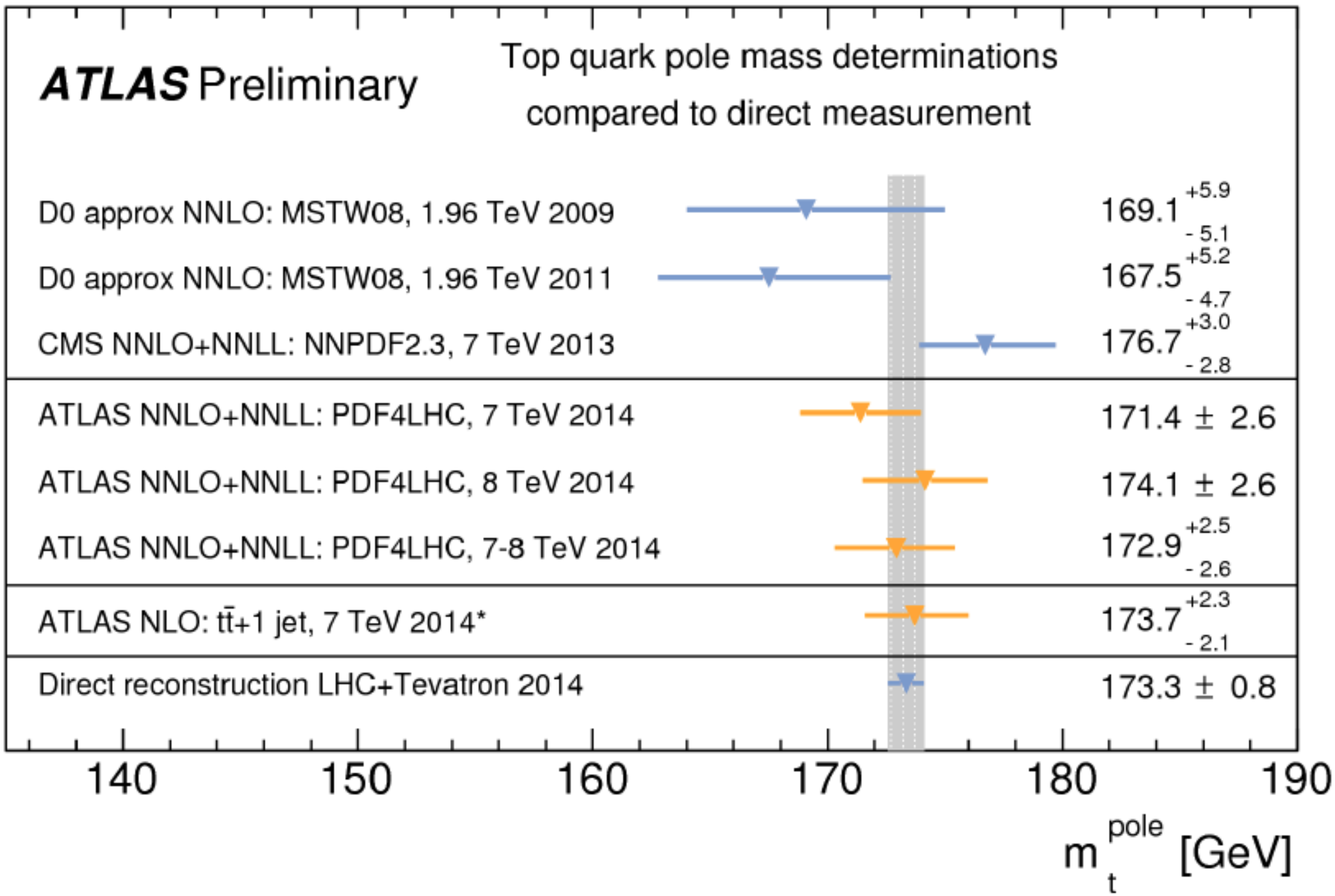
ATLAS-CONF-2014-053



Fit with tt+jet NLO+PS theory

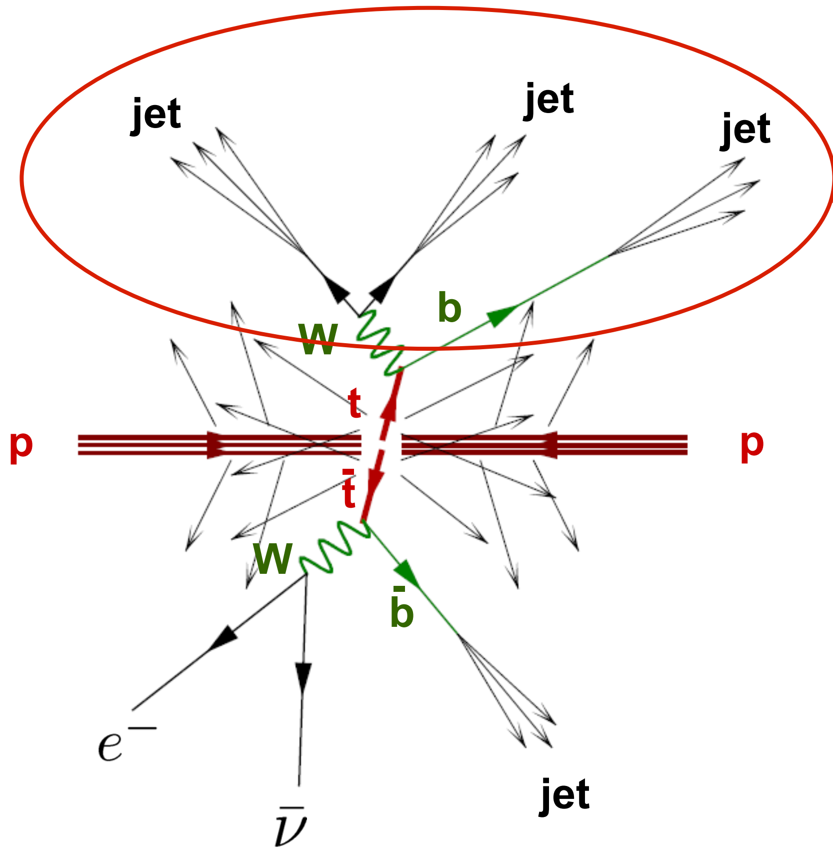
Is this the pole mass? Yes!
Scheme fixed in NLO calculation
(difference NLO vs. NLO+PS ~ 300 MeV)

Indirect ways to determine the top quark mass



Top quark mass reconstruction

LHC:



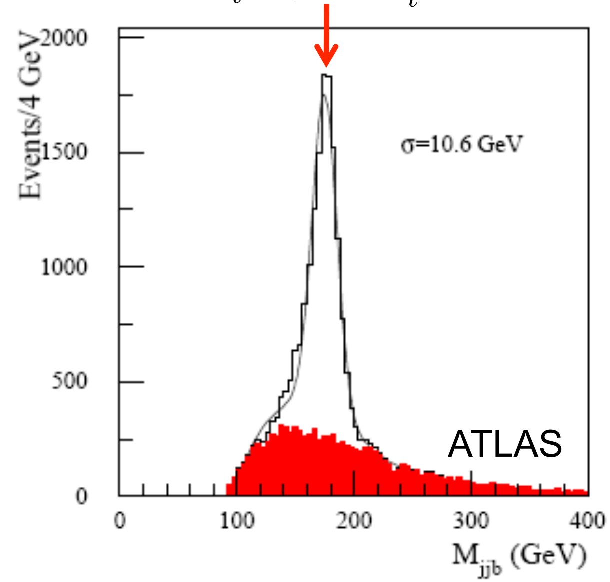
Principle of mass measurements:

Identification of the top decay products

$$\text{“}\ m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ \text{”}$$

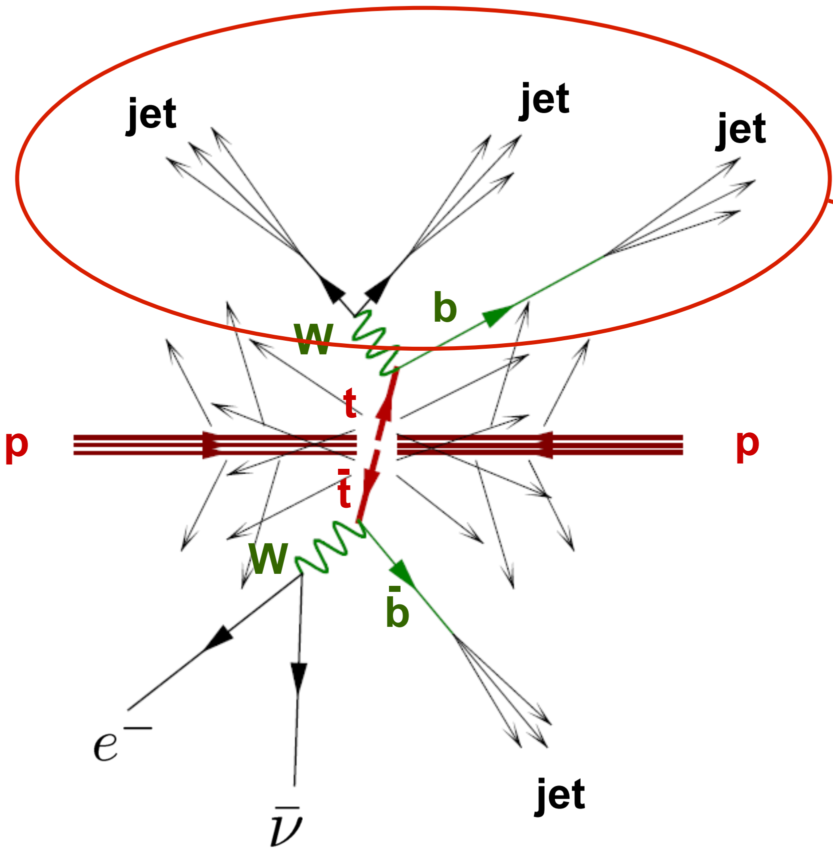
Invariant mass distribution

$m_t \ + \ \Delta m_t^{\text{non-pert.}}$



Top quark mass reconstruction

LHC:



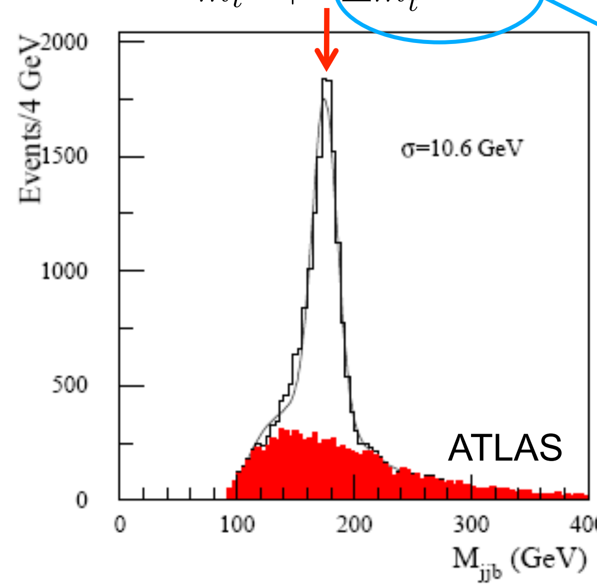
Principle of mass measurements:

Identification of the top decay products

$$“\; m_{\rm top}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2 \;”$$

Invariant mass distribution

$$m_t \; + \; \Delta m_t^{\rm non-pert}$$

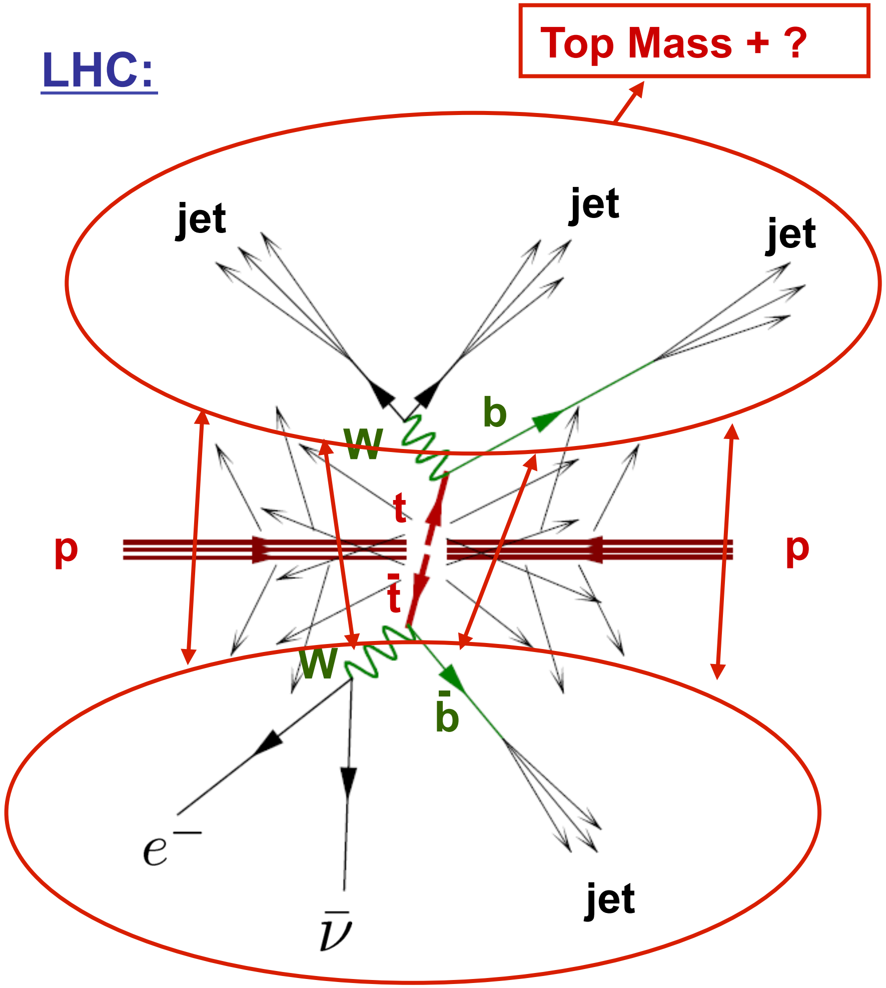


depends on:

* mass definition
* jet definition
* Monte Carlo

Top quark mass reconstruction

LHC:



Principle of mass measurements:

Identification of the top decay products

$$“\ m_{\rm top}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ ”$$

Problem is non-trivial !

- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects !!**

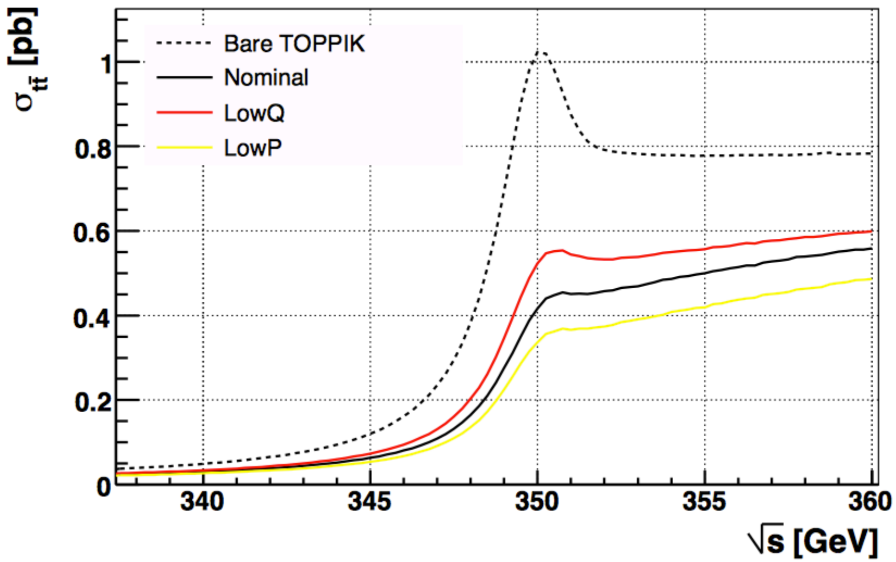
The idea of a - by itself - well defined object having a well defined mass is incorrect !!

Details and uncertainties of the parton shower and the hadronization models in den MC's influence the measured top quark mass.

Top Reconstruction + Total Cross Section

Top pair total cross section at a lepton collider:

$$\sigma(e+e- \to t\bar{t}+X) \text{ at } E_{cm} \approx 2m_t$$



Principle: **m_t from $\sigma_{tt}(m_t)$**

Advantages:

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood (renormalons, summations)

- Top decay protects from non-pert effects

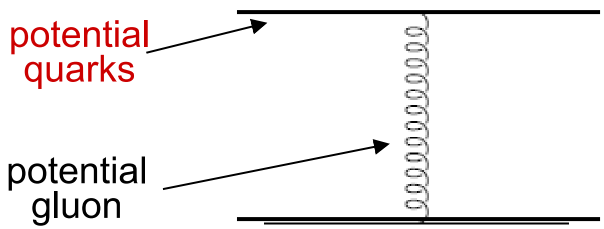
- Remnant of a topionium resonance (“postronium of QCD”)
- Crucial to control e+e- luminosity spectrum
- Binding energy about twice the top quark width: $$E_{\text{bind}} \approx \frac{\alpha_s^2 m_t}{2} \approx 2\Gamma_t$$
- Can be calculated in pQCD (nonrelativistic expansion)
- True final state: WWbb (includes single-top + nonresonant background)

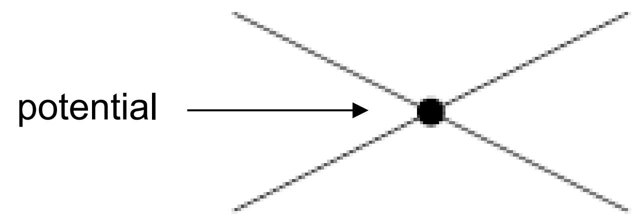
Top Threshold Theory

Nonrelativistic QCD (EFT):

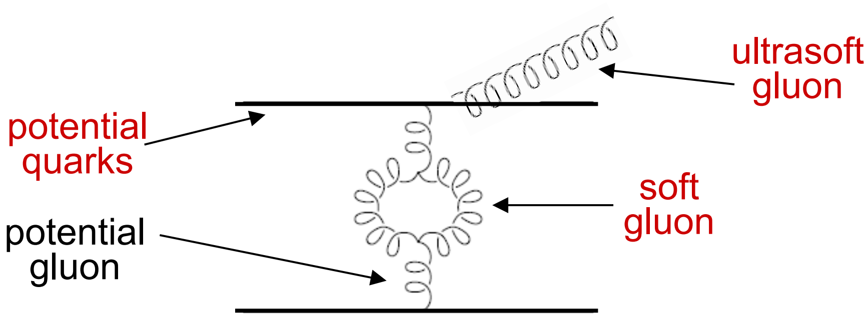
$$\sigma_{\text{tot}} \sim \text{Im}\left[\int d^4x e^{-iqx}\langle 0|T j(x) j(0)|0\rangle\right]$$

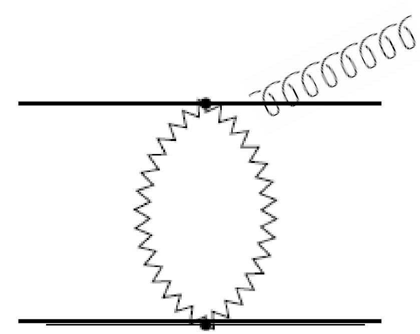
“hard”:	$(k^0, \boldsymbol{k})$	$\sim$	(m, m)
“soft”:	$(k^0, \boldsymbol{k})$	$\sim$	$(m\,v, m\,v)$
“potential”:	$(k^0, \boldsymbol{k})$	$\sim$	$(m\,v^2, m\,v)$
“ultrasoft”:	$(k^0, \boldsymbol{k})$	$\sim$	$(m\,v^2, m\,v^2)$





potentials are Wilson coefficients



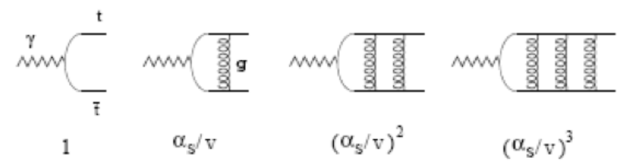


Top Reconstruction + Total Cross Section

Schematic:

$$\sigma_{\rm tot} \;\propto\; \mathrm{Im}\left[\int d^4x\, e^{-i\hat{q}x} \left\langle 0 \left| T\, \mathbf{O}_{\mathbf{p}}^{\dagger}(0)\, \mathbf{O}_{\mathbf{p}'}(x) \right| 0 \right\rangle\right]$$

$$\propto\; \mathrm{Im}\left[\left(C_A(\nu)^2 + C_V(\nu)^2\right) G(0,0,\sqrt{s})\right]$$



$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

- Dynamics described by non-relativistic Schrödinger equation
- Hard contributions: Wilson coefficient (static top anti-top pair)
- Imaginary part of Wilson coefficient: interference (e.g. single top background)

Top Reconstruction + Total Cross Section

Theory Status:

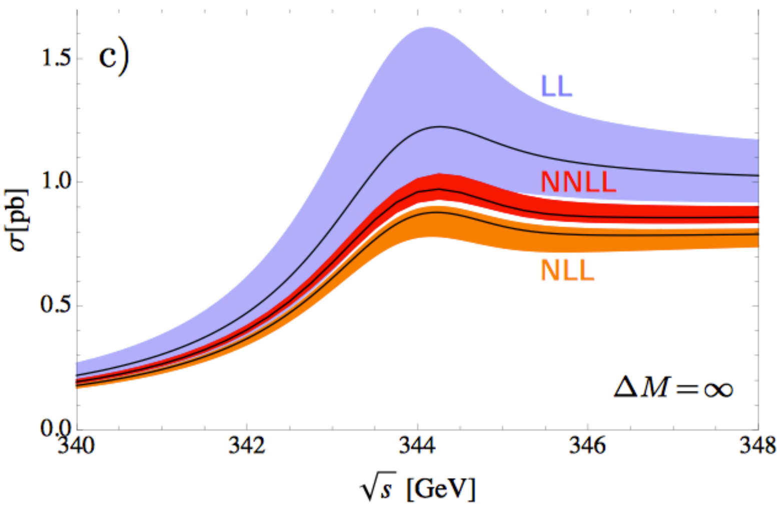
- NNLL renormalization group improved
- Current uncertainty: $\frac{\delta\sigma}{\sigma} = \pm 5\%$

- Recently: NNNLO fixed order calculation

Beneke etal. $\frac{\delta\sigma}{\sigma} = \pm 3\%$

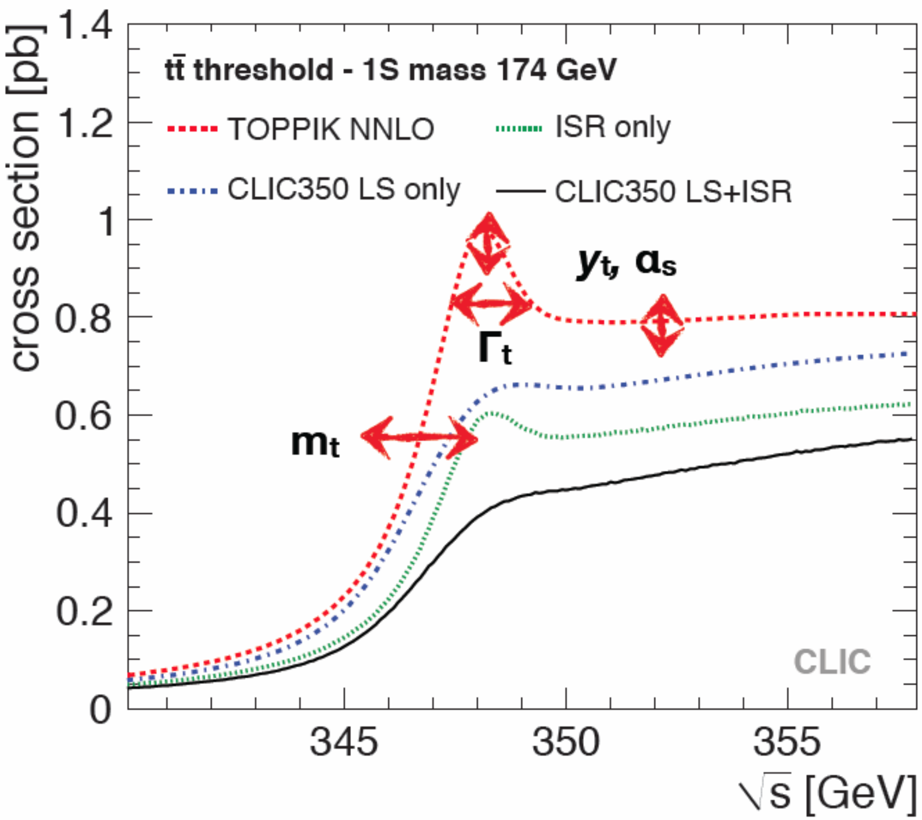
Theory error larger than experimental errors !

Hoang, Stahlhofen (2013)



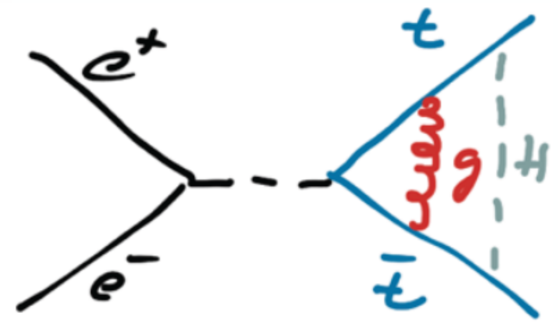
Top Reconstruction + Total Cross Section

Experimental Studies:



The cross-section around the threshold is affected by several properties of the top quark and by QCD

- Top mass, width, Yukawa coupling
- Strong coupling constant



- Effects of some parameters are correlated; dependence on Yukawa coupling rather weak - precise external α_s helps

Frank Simon

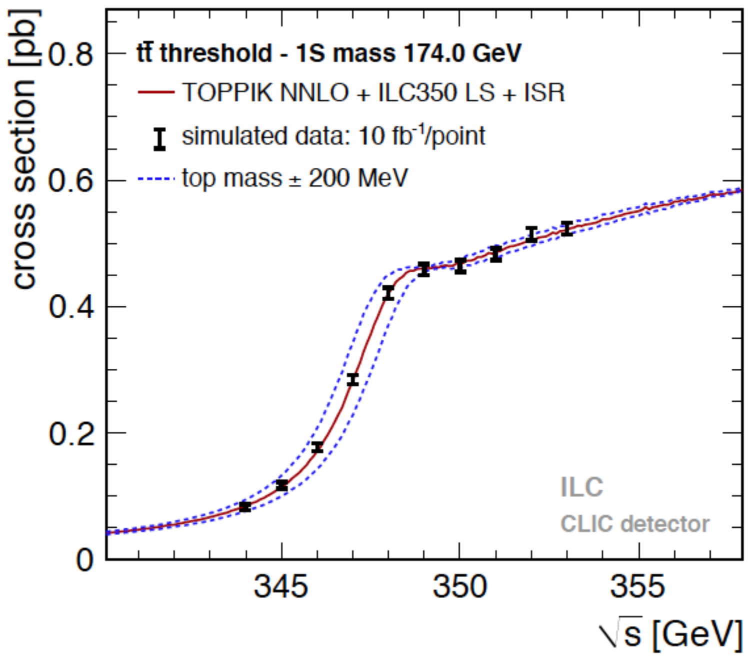
Top Reconstruction + Total Cross Section

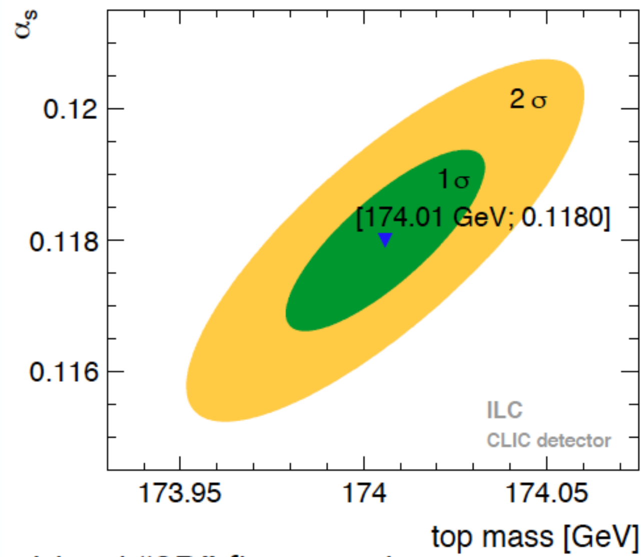
Frank Simon

Experimental Studies:

- The sensitivity of a ttbar threshold scan at Linear Colliders to top quark properties has been studied be a variety of groups at different points in time - with different degrees of realism
  - Here: Focus on the most recent studies

EPJ C73, 2540 (2013)





Combined "2D" fit: m_t and α_s:
Δm_t = 27 MeV (stat), $\Delta\alpha_s$ = 0.0008
Mass alone: 18 MeV (stat)

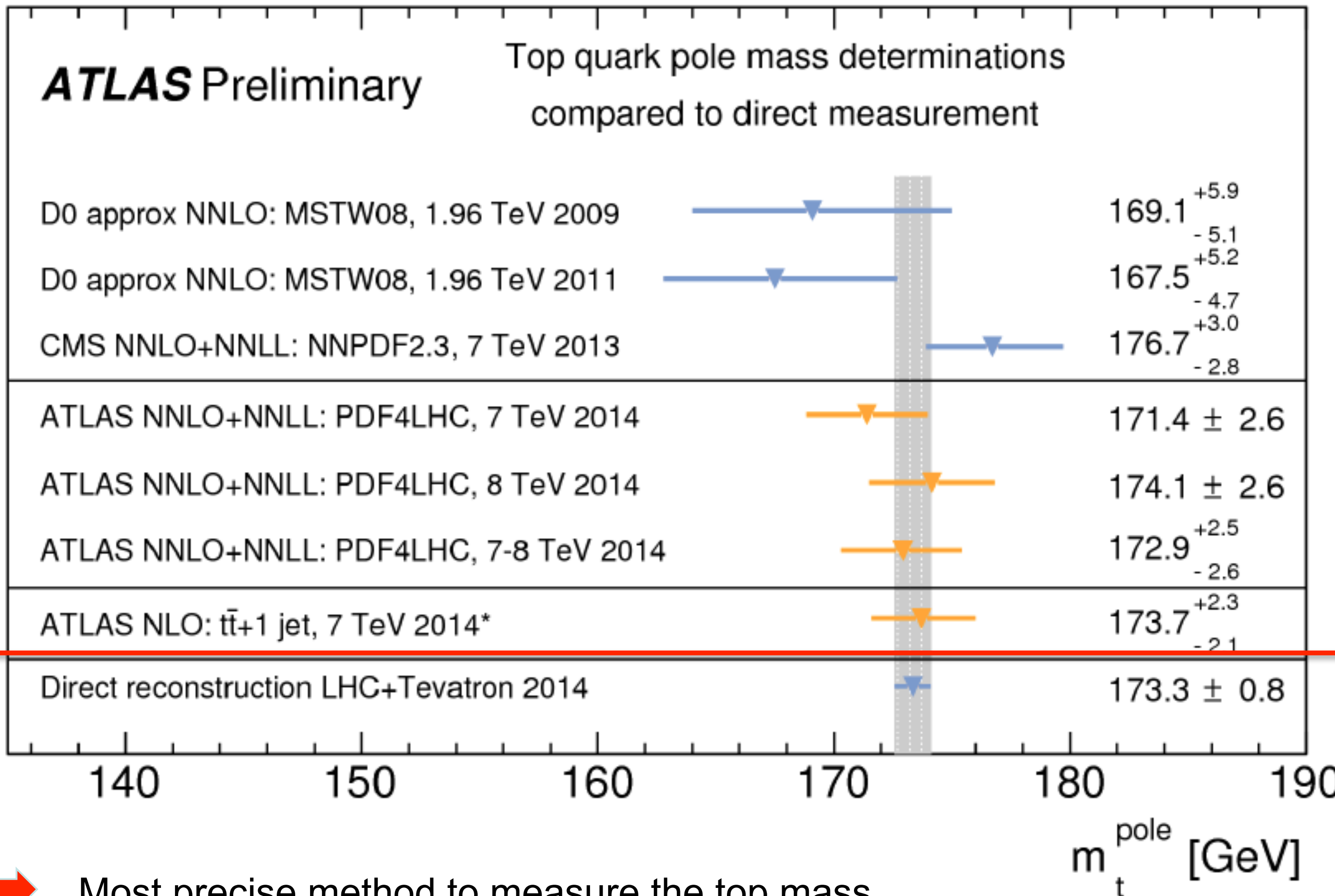
$$\left(\delta\overline{m}_t(\overline{m}_t)\right)^{\text{total}} \approx 40\ \text{MeV}$$

Todays Conclusions

- Precise measurements of the top quark important input for other precision predictions.
- Mass determinations are non-trivial because top is colored.
- Hadron collider measurements: complicated due to hadronic environment
- Ultimate precision can be obtained at a future lepton collider: toponium resonance can be computed precisely in pQCD.

End of Part 1

Direct Reconstruction of the Top Quark Mass

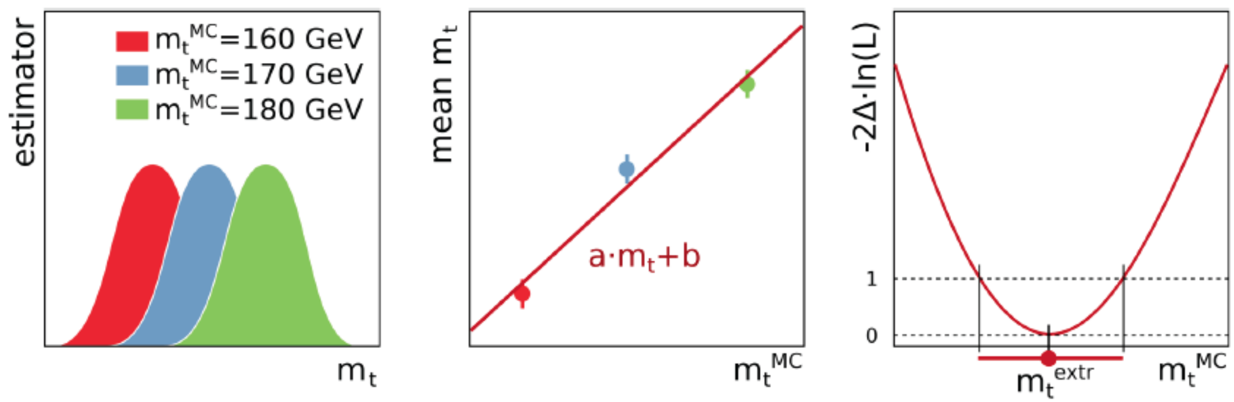


➡ Most precise method to measure the top mass

Direkt Reconstruction Method

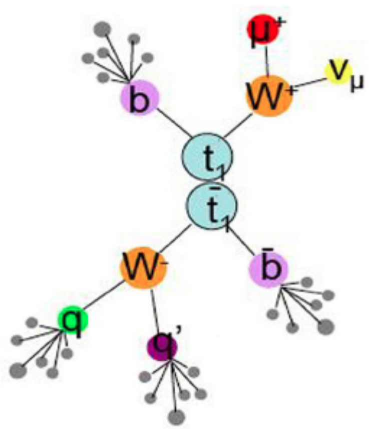
Measurement Method

- Build estimator for m_t (e.g. inv. mass of decay products)
- Parametrize estimator as function of m_t^{MC} (and possible other parameters)
- Possible per event combination of multiple estimators
- Ideogram method, CMS all-jets and l+jets
- Template method, all other measurements
- Perform maximum likelihood fit to data



Direkt Reconstruction Method

Inferring the top quark mass from the kinematic properties of its decay products.



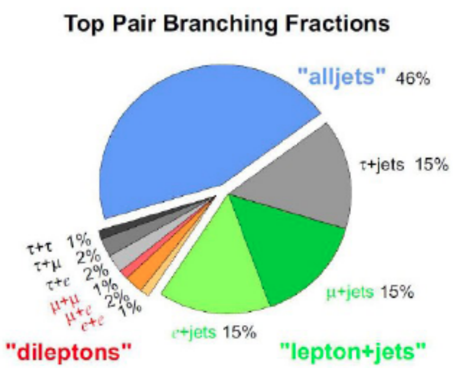
Matrix Element method (ME):

Calculates event probability densities from differential cross sections and detector resolutions.

- Maximizes the statistical information.
- Current implementations at LO.

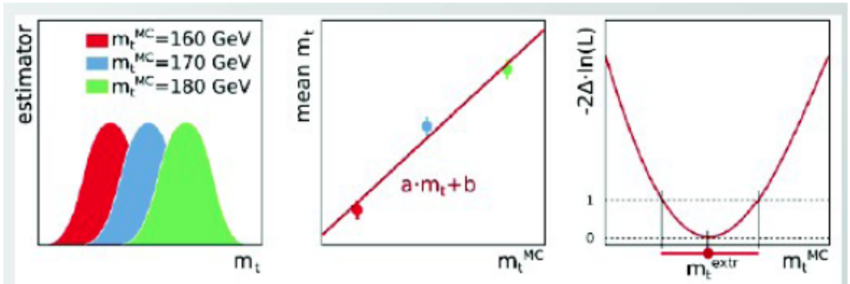
Ideogram method:

Event likelihood function to test compability of event kinematics with the decay hypothesis convoluted with detector resolutions.



Template method:

Compare histograms in data to simulations.

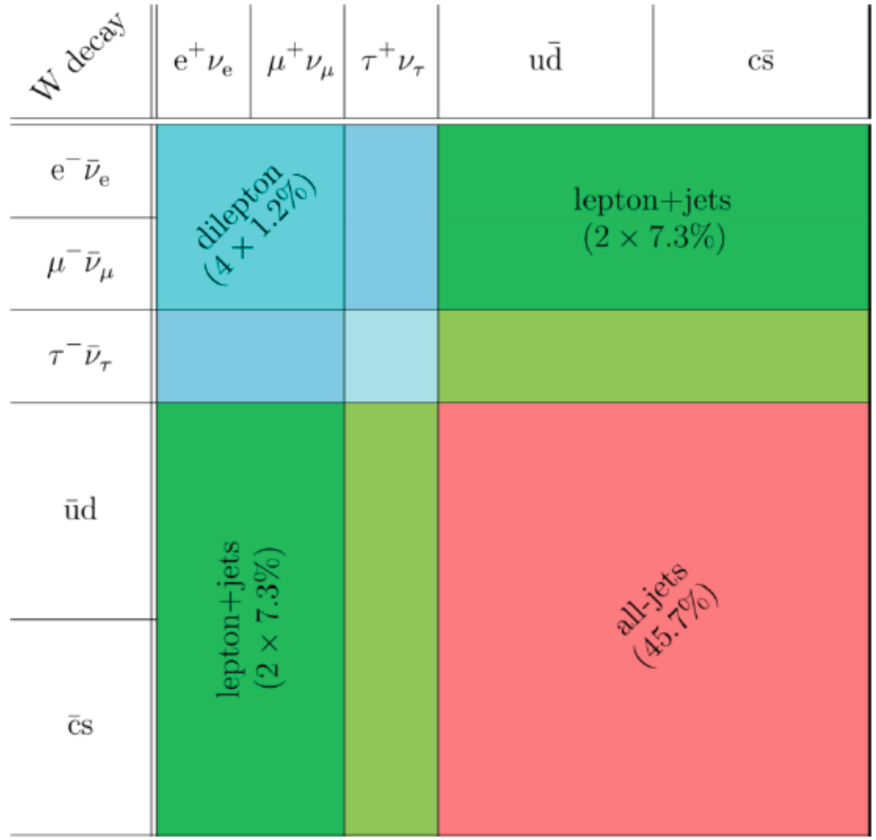


These methods measure the kinematic MC mass

Overview of Measurements

- Alljets channel:
  - ATLAS @ 7 TeV
  - CMS @ 7 & 8 TeV
- Lepton+Jets channel:
  - ATLAS @ 7 TeV
  - CMS @ 7 & 8 TeV
- Dilepton channel:
  - ATLAS @ 7 TeV
  - CMS @ 7 & 8 TeV

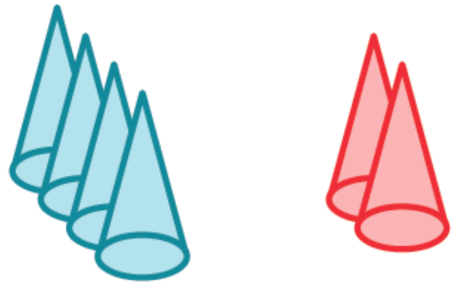
- No measurements in final states with τ

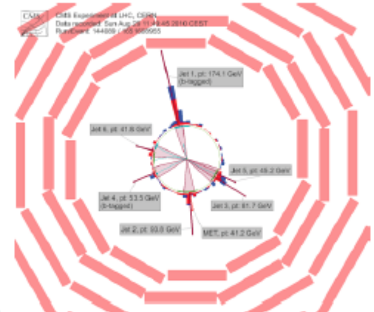


Direkt Reconstruction Method

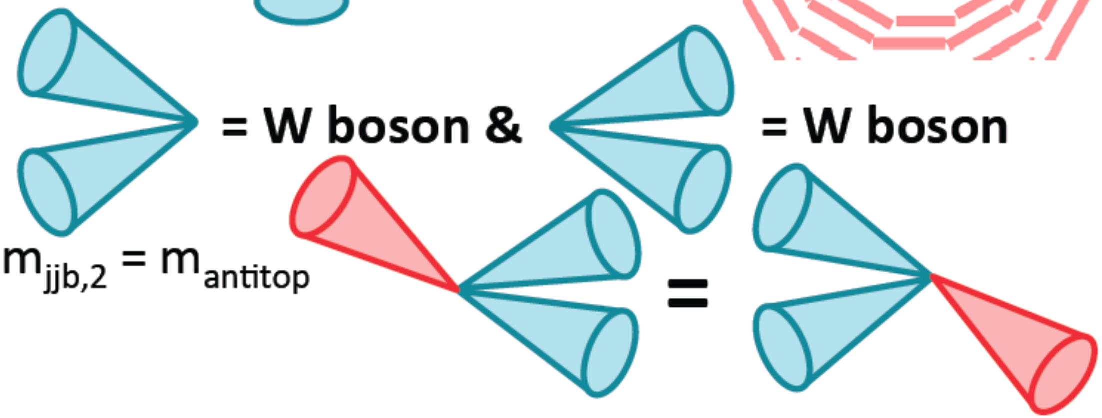
Kinematic Fit

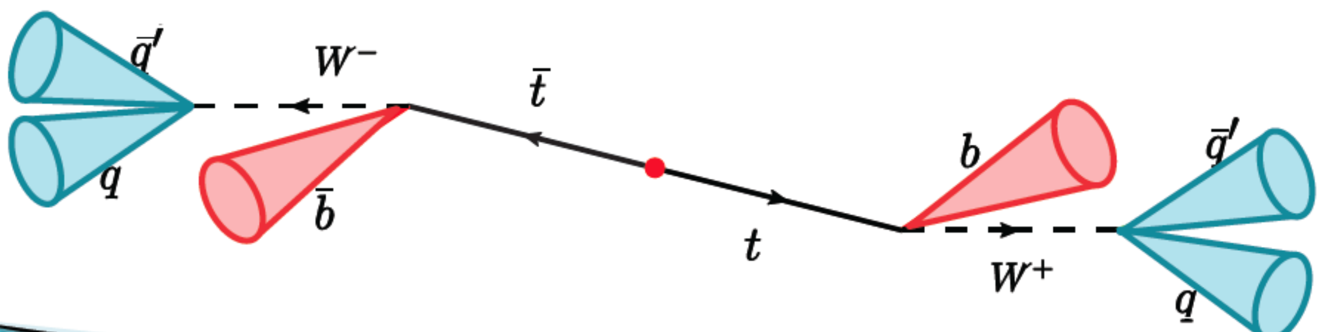
- Selected objects:
  - 4 untagged jets
  - 2 b-tagged jets





- Constraints:
  - 2x $m_{jj} = m_W$
  - $m_{top} = m_{jjb,1} = m_{jjb,2} = m_{antitop}$





Direkt Reconstruction Method

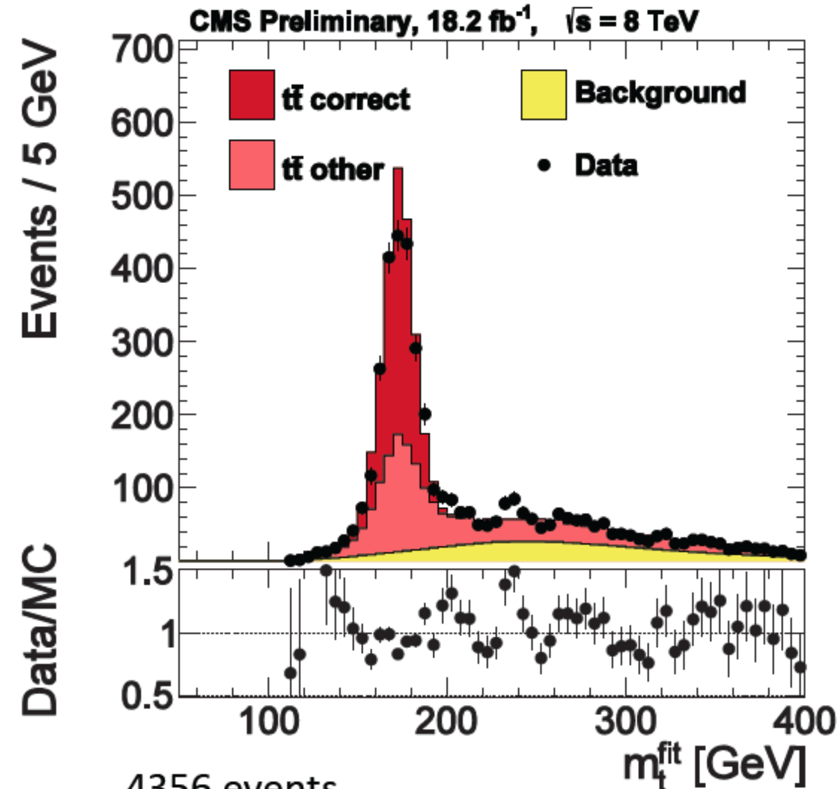
All-Jets Top-Quark Mass @ CMS 8 TeV

- Improved reconstruction
- Switch to 2D fit with JES scale factor (JSF)
- Fit signal and correct permutation fractions

JES+PU: 0.42
JSF: 0.24

Source	Unc. [GeV]
JES+PU+JSF	0.48
bJES+Had	0.39
Detector modelling	0.21
Signal modelling	0.52
Background	0.22
Method	0.06
Syst.	0.86
Stat. (m_t only)	0.27
Total	0.90

CMS-PAS-TOP-14-002

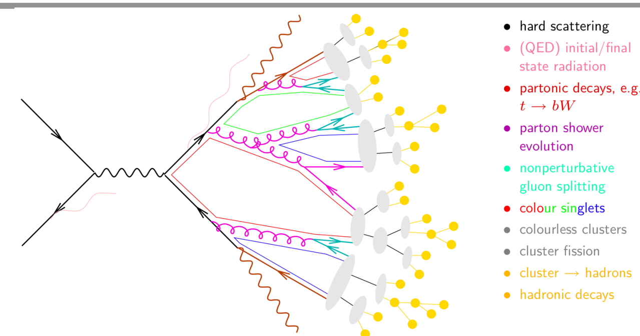


4356 events

$m_t = 172.08 \pm 0.90$ GeV (0.53%)

JSF = 1.007 ± 0.011

Monte-Carlo Event Generators



Monte-Carlo event generator:

- Hard matrix element:

  Initial parton annihilation and top production plus additional hard partons from pQCD.

- Parton shower evolution:

  Splitting into higher-multiplicity partonic states (plus top decay) with subsequently lower virtualities until shower cut Λ_s. NO top mass self-energy contributions (absorbed into mass).

  Splitting probabilities from pQCD (approx LL accuracy, soft-collinear limit).

  Can be viewed as a way to sum dominant perturbative corrections down to Λ_s = 1 GeV.

- Hadronization model:

  Turns partons into hadrons.

  Tune strongly dependent on parton shower implementation.

  Description of data (frequently) much better than the conceptual (LL) precision of parton evolution part.

- MC mass:

  Mass of top propagator prior to top decay.

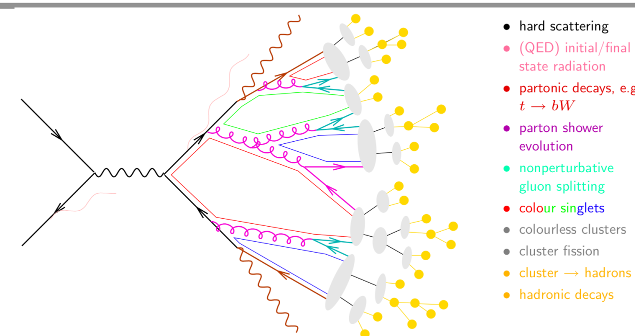
Pythia

Herwig

Sherpa

→ Interpretation of $m_t^{\rm MC}$ dependent on view whether MC is more model or or more first principles QCD.

Monte-Carlo Event Generators



Monte-Carlo event generator:

- Hard matrix element:

  Initial parton annihilation and top production plus additional hard partons from pQCD.

- Parton shower evolution:

  Sp … irtualities until
  sh
  Sp
  Ca

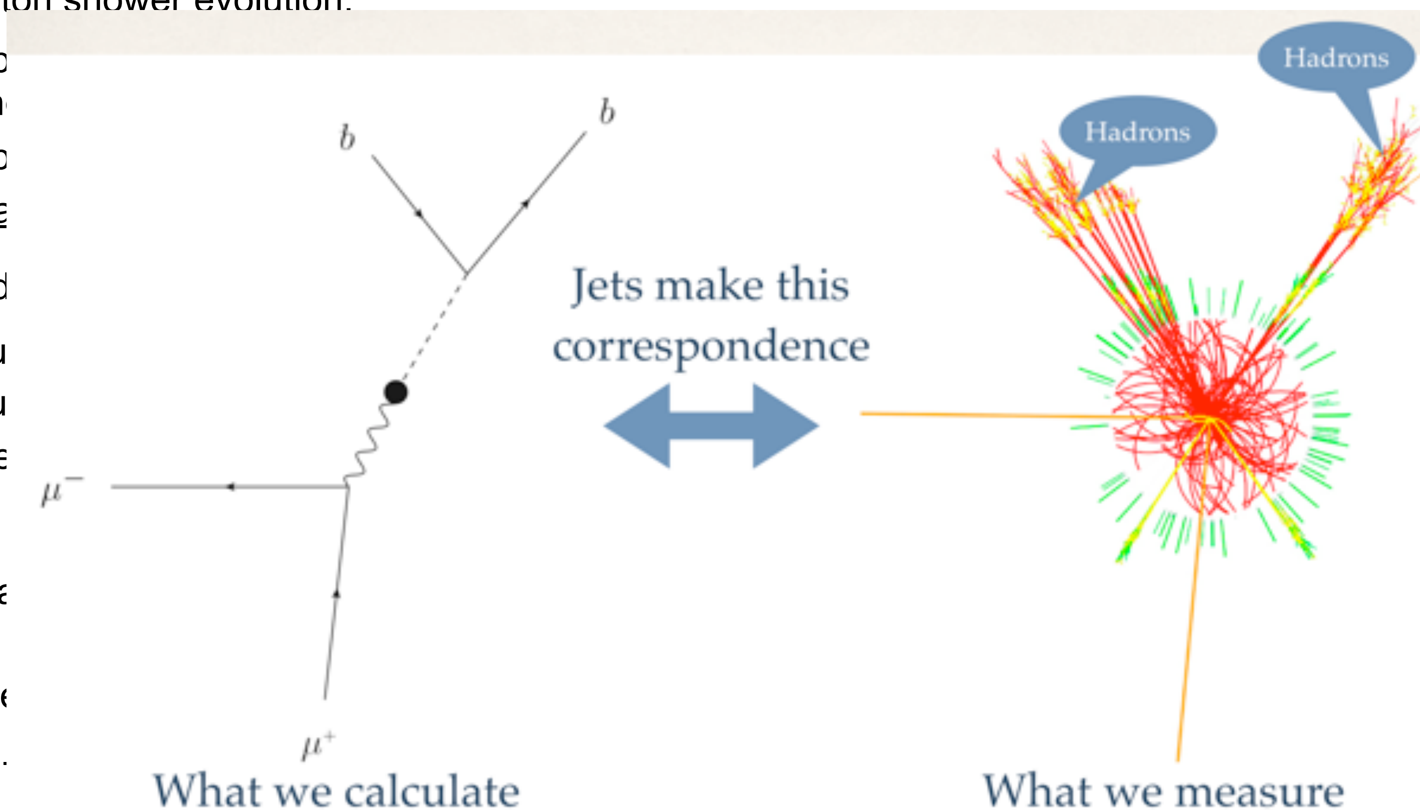
- Had

  Tu
  Tu
  De … evolution part.

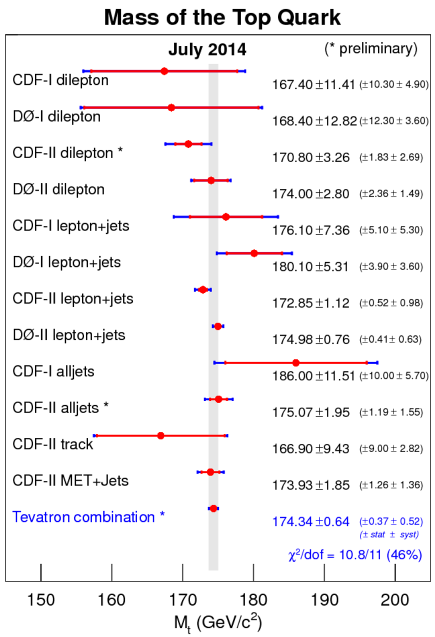
- MC

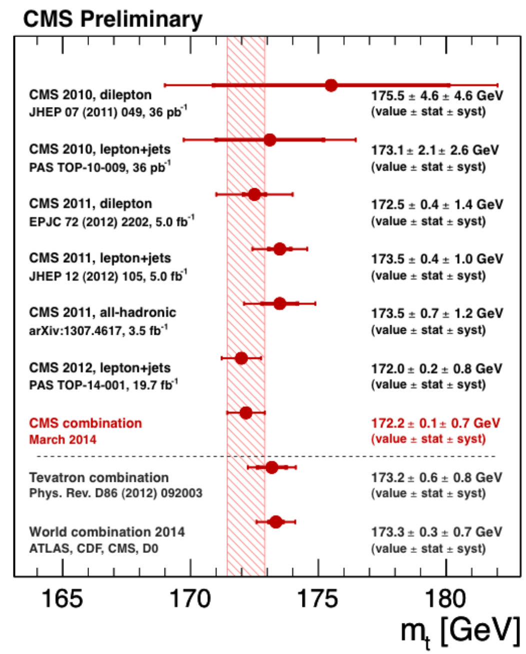
  Ma … ig

→ Inte
or … erpa



Direkt Reconstruction Method





→ Most precise mass from direct reconstruction: $m_t^{\mathrm{MC}} = 173.34 \pm 0.76\,\mathrm{GeV}$

→ m_t^{MC} cannot be used as direct input into NLO/NNLO calculations since it is not a field theoretic mass.

→ However: for the “observables” that dominate the matrix element and template at least an approximate relation to field theory masses should exist.

→ Currently: an additional error has to be accounted for when m_t^{MC} is used in pQCD.

Outline

Part 2a: $\rightarrow$ Theoretical thoughts on m_t^{MC}

- How m_t^{MC} is related to field theoretic masses.

"Which top mass definition(s) are likely numerically close to m_t^{MC}."

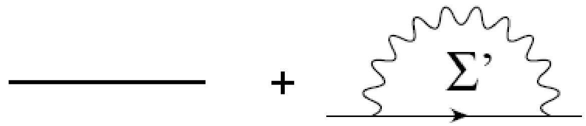
See: "The Top Mass: Interpretation and Theoretical Uncertainties", arXiv:1412.3649

Same conclusions: AH, Stewart: arXive:0808.0222

Part 2b: $\rightarrow$ Towards a determination of m_t^{MC}

- Variable Flavor Number Scheme for final state jets.
  Full massive event shape distribution
- Applicable to lepton colliders
- Status & preliminary results

Top Quark Mass



$$= p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \boxed{\Sigma^{\text{fin}}(m^0, m^0, \mu)}$$

$\overline{\text{MS}}$ scheme: $$m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$$

→ $\overline{m}(\mu)$ is pure UV-object without IR-sensitivity

→ Useful scheme for $\mu > m$

→ Far away from a kinematic mass of the quark

Pole scheme: $$m^0 = m^{\text{pole}} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\text{fin}}(m^{\text{pole}}, m^{\text{pole}}, \mu)$$

→ Absorbes all self energy corrections into the mass parameter

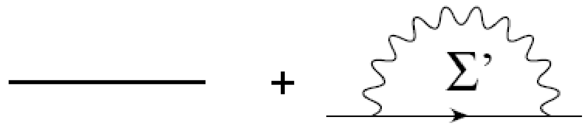
→ Close to the notion of the quark rest mass (kinematic mass)

→ Renormalon problem: infrared-sensitive contributions from < 1 GeV that cancel between self-energy and all other diagrams cannot cancel.

→ Σ^{fin} has perturbative instabilities due to sensitivity to momenta < 1 GeV (Λ_{QCD})

Should not be used if uncertainties are below 1 GeV !

Top Quark Mass



$$= p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \boxed{\Sigma^{\rm fin}(m^0, m^0, \mu)}$$

$\overline{\text{MS}}$ scheme:

$$m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$$

Pole scheme:

$$m^0 = m^{\rm pole} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\rm fin}(m^{\rm pole}, m^{\rm pole}, \mu)$$

MSR scheme:

$$m^{\rm MSR}(R) = m^{\rm pole} - \Sigma^{\rm fin}(R, R, \mu) \qquad \text{for } R < m$$

Jain, AH, Scimemi, Stewart (2008)

→ Like pole mass, but self-energy correction from scales < R are not absorbed into mass

→ Interpolates between MSbar and pole mass scheme

$$m_t^{\rm MSR}(R = 0) = m^{\rm pole}$$

$$m_t^{\rm MSR}(R = \overline{m}(\overline{m})) = \overline{m}(\overline{m})$$

→ More stable in perturbation theory.

→ $m_t^{\rm MSR}(R = 1\,{\rm GeV})$ close to the notion of a kinematic mass, but without renormalon problem.

MSR Mass Definition

MSbar Scheme: $(\mu > \overline{m}(\overline{m}))$

$$\overline{m}(\overline{m}) - m^{\text{pole}} = -\overline{m}(\overline{m})\,[0.42441\,\alpha_s(\overline{m}) + 0.8345\,\alpha_s^2(\overline{m}) + 2.368\,\alpha_s^3(\overline{m}) + \ldots]$$

Now known to $\mathcal{O}(\alpha_s^4)$!

Marquard, Smirnov, Smirnov, Steinhauser

MSR Scheme: $(R < \overline{m}(\overline{m}))$

$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R\,[0.42441\,\alpha_s(R) + 0.8345\,\alpha_s^2(R) + 2.368\,\alpha_s^3(R) + \ldots]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \overline{m}(\overline{m})$$

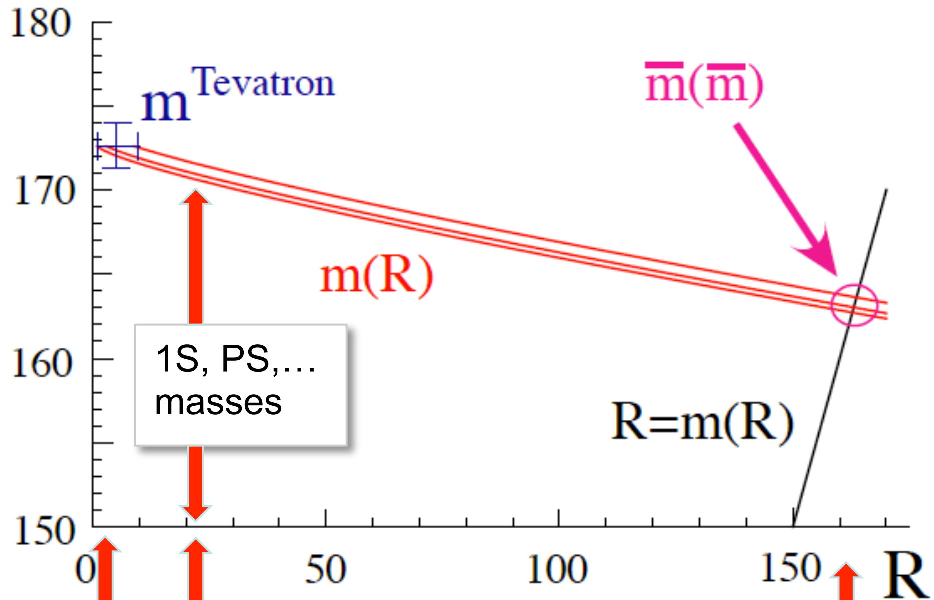
$\Longrightarrow$ $m_{\text{MSR}}(R)$ Short-distance mass that smoothly interpolates all R scales

- Excellent convergence of relation between MSR masses at different R values
- Excellent convergence of relation between MSR masses and other short-distance masses
- Smoothy interpolates to the MSbar mass.
- Pole mass problems: related to Landau pole in limit of vanishing R

MSR Mass Definition

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(3^{+6}_{-2}\ \mathrm{GeV}) = m_t^{\mathrm{MSR}}(3\ \mathrm{GeV})^{+0.6}_{-0.3}$$

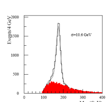
AH, Stewart: arXive:0808.0222



1S, PS,… masses

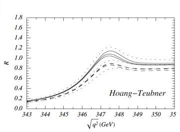
Good choice for R:

Of order of the typical scale of the observable used to measure the top mass.



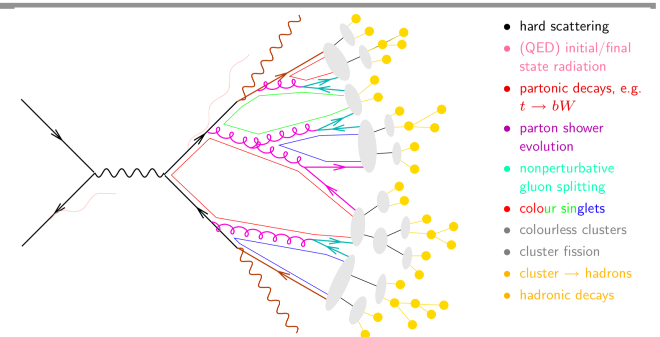
Peak of invariant mass distribution, endpoints

Top-antitop threshold at the ILC



Total cross section, e.w.precsion obs., Unification, MSbar mass

Heavy Quark Mass in the MC



Monte-Carlo event generator:

- Hard matrix element:

  Initial parton annihilation and top production plus additional hard partons from pQCD.

- Parton shower evolution:

  Splitting into higher-multiplicity partonic states (plus top decay) with subsequently lower virtualities until shower cut Λ_s. NO top mass self-energy contributions (absorbed into mass).

  Splitting probabilities from pQCD (approx LL accuracy, soft-collinear limit).

  Can be viewed as a way to sum dominant perturbative corrections down to Λ_s = 1 GeV.

- Hadronization model:

  Turns partons into hadrons.

  Tune strongly dependent on parton shower implementation.

  Description of data (frequently) much better than the conceptual (LL) precision of parton evolution part.

- MC mass:

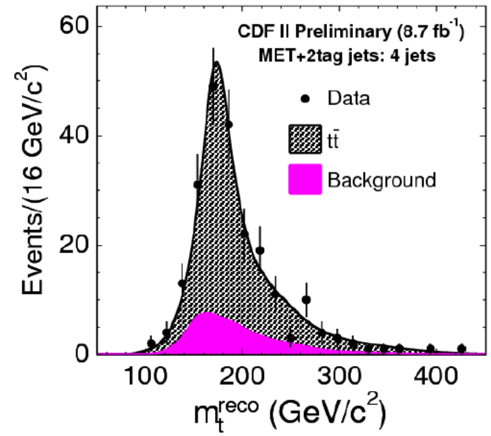
  Mass of top propagator prior to top decay.

→ Interpretation of m_t^{MC} dependent on view whether MC is more model or or more first principles QCD.

We have to assume this in order to go on.

Heavy Quark Mass in the MC

Let's take the reconstructed top invariant mass distribution as a concrete example to see how the MC components enter the templates and the MC mass fitting.



- Hard matrix element:

  Essentially only affects the norm

- MC mass:

  Determines overall location of mass range where distribution is peaked.

- Parton shower evolution + Hadronization model:

  Modify shape and distribution further.
  PS: perturbative part - <u>self-energy contributions absorbed into mass above</u> Λ_s = 1 GeV
  HM: non-perturbative part below Λ_s

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(R = 1\,\mathrm{GeV}) + \Delta_{t,\mathrm{MC}}(R = 1\,\mathrm{GeV})$$

$$\Delta_{t,\mathrm{MC}}(1\,\mathrm{GeV}) \simeq \mathcal{O}(1\,\mathrm{GeV})$$

Contains perturbative and non-perturbative contributions.
Conceptual reliability related to how precisely $\Delta_{t,\mathrm{MC}}$ can be determined.

Heavy Quark Mass in the MC

Analogy: Meson masses

$$m_B = m_b^{\mathrm{MSR}}(1\,\mathrm{GeV}) + \Delta_{b,B}(R = 1\,\mathrm{GeV})$$

$$\Delta_{b,B}(1\,\mathrm{GeV}) \simeq \mathcal{O}(1\,\mathrm{GeV})$$

Table 1. Some B mesons masses, MSR masses $m_b^{\mathrm{MSR}}(1\,\mathrm{GeV})$ and $m_b^{\mathrm{MSR}}(2\,\mathrm{GeV})$ from $m_b^{1S} = 4780 \pm 66\,\mathrm{MeV}$ [18], and corresponding values for $\Delta_{b,B}$. All in units of MeV, $\alpha_s(m_Z) = 0.1184$.

$m_b^{\mathrm{MSR}}(1\,\mathrm{GeV})$	$m_b^{\mathrm{MSR}}(2\,\mathrm{GeV})$	$m(B^0)$	$m(B^*)$	$m(B_1^0)$	$m(B_2^*)$
4795 ± 69	4571 ± 69	5279.58 ± 0.17	5325.2 ± 0.4	5724 ± 2	5743 ± 5
$\Delta_{b,B}(1\,\mathrm{GeV})$		485 ± 69	530 ± 69	929 ± 69	948 ± 69
	$\Delta_{b,B}(2\,\mathrm{GeV})$	709 ± 69	754 ± 69	1153 ± 69	1172 ± 69

Additional Comments

- Using NLO vs. LO matrix elements does not affect the interpretation of the MC mass (as dominated by soft-collinear approximation).
- Different parton evolution implies in principle a different MC mass.
- Relation of MC to MSR mass can be used to deal with mass dependent efficiencies for total cross section measurements.
- MC mass should be independent of the process and kinematic region used for fitting. (This statement should be tested in experimental analyses.)
- Future Linear Collider: (much) more experimental precision for top reconstruction, but conceptual issue of what the MC mass is remains

Theory Tools to Measure the MC mass

Part 2

The relation between MC mass and field theoretical mass can be made more precise by measuring the MC mass using a completely independent hadron level QCD prediction of a mass-dependent observable.

Need:

- Accurate analytic QCD predictions beyond LL/LO with full control over the quark mass dependence
- Theoretical description at the hadron level for comparison with MC at the hadron level

- Implementation of massive quarks into the SCET framework
- **VFNS for final state jets (with massive quarks)***

* In collaboration with: B. Dehnadi, V. Mateu, I. Stewart

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
arXiv:1405.4860 (PRD 90 114001 (2014))
More to come …

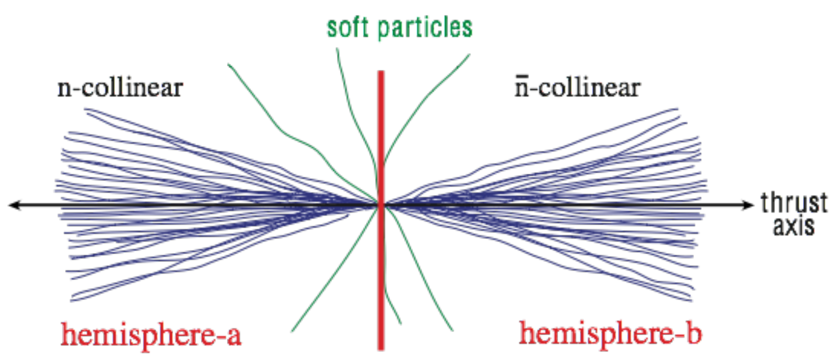
Theory Tools to Measure the MC mass

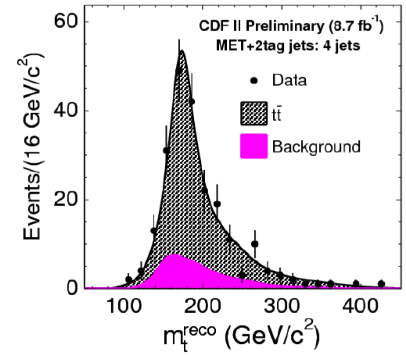
Observable: Thust in e+e-

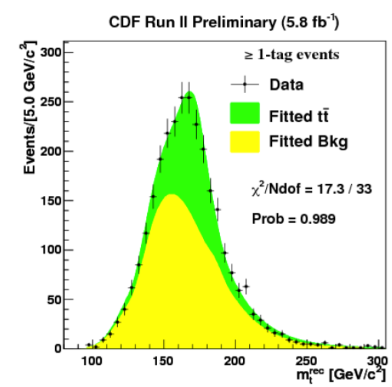
$$\tau \;=\; 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{Q}$$

$$\overset{\tau\to 0}{\approx} \;\; \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region !



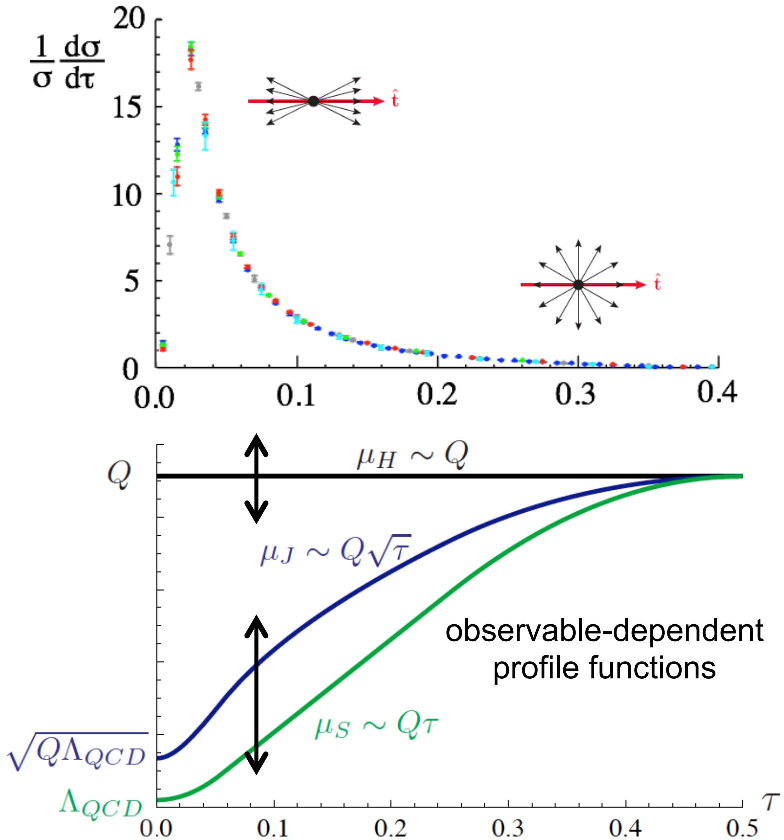




Factorization for Massless Quarks

$$\frac{d\sigma}{d\tau} = Q^2\sigma_0 H_0(Q,\mu)\int d\ell\, J_0(Q\ell,\mu)\, S_0\,(Q\tau-\ell,\mu)$$

Korshemski, Sterman
Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman



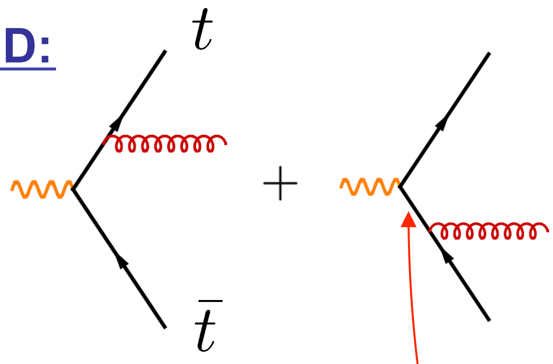
Abbate, AH, Fickinger, Mateu, Stewart

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q)U_H(Q,\mu_Q,\mu_s)\int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

$$k_+ = k_0 - k_3$$
$$k_- = k_0 + k_3$$

How jets emerge in theory:

full QCD:



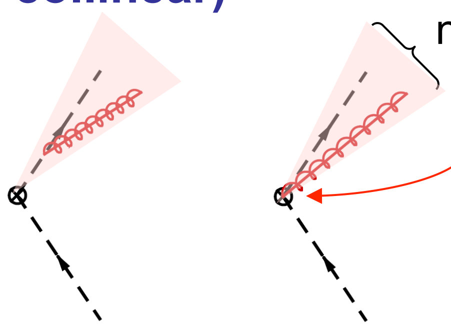
3 phase space regions: $\lambda \sim m_t/Q$

- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- $\bar{\text{n}}$-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$\frac{1}{(p_{\bar{t}}+k)^2 - m_t^2} \qquad (p_{\bar{t}}^2 \approx m_t^2\,,\ \bar{n}^2 = 0)$$

$$\frac{1}{Q\ \bar{n}.k}$$

Gluon collinear to the top:
(n-collinear)



$$W_n^\dagger(\infty, x) = \mathrm{P}\ \exp\left(ig\int_0^\infty ds\bar{n}\cdot A_+(ns+x)\right)$$

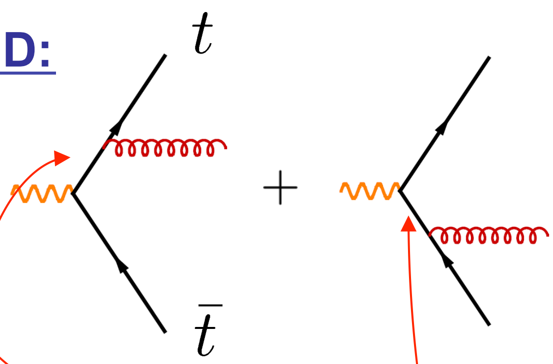
$h_{v_+}(x)$ → **gauge dependent**

$W_n^\dagger(\infty, x)\, h_{v_+}(x)$ → **gauge independent**

$$B_+(\hat{s}, \Gamma_t, \mu) = \mathrm{Im}\left[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\,\langle 0|\, T\,\{\bar{h}_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\,\}\,|0\rangle\right]$$

QCD Factorization

full QCD:

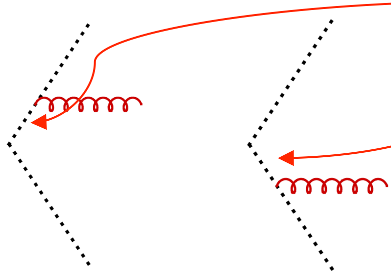


3 phase space regions: $\lambda \sim m_t/Q$

- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- n̄-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$\frac{1}{(p_{t,\bar t}+k)^2 - m_t^2} \qquad (p_{t,\bar t}^2 \approx m_t^2\,,\;\; n^2 = 0\,,\;\; \bar n^2 = 0)$$

Gluon soft:



$$\frac{1}{Q\;\; n.k} \qquad \frac{1}{Q\;\; \bar n.k}$$

$$Y_n(x) = \overline{\mathrm{P}}\,\exp\Big(-ig\int_0^\infty ds\, n\cdot A_s(ns+x)\Big)$$

$$\overline{Y_{\bar n}}(x) = \overline{\mathrm{P}}\,\exp\Big(-ig\int_0^\infty ds\, \bar n\cdot \overline{A}_s(\bar n s+x)\Big)$$

$$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c}\langle 0|\,(\overline{Y}_n)^{cd}\,(Y_n)^{ce}(0)\,\delta(\ell^- - (\hat P_a^+)^\dagger)\,\delta(\ell^- - \hat P_b^-)\,(Y_n^\dagger)^{ef}\,(\overline{Y}{}_n^\dagger)^{df}(0)\,|0\rangle$$

VFN Scheme for Final State Jets

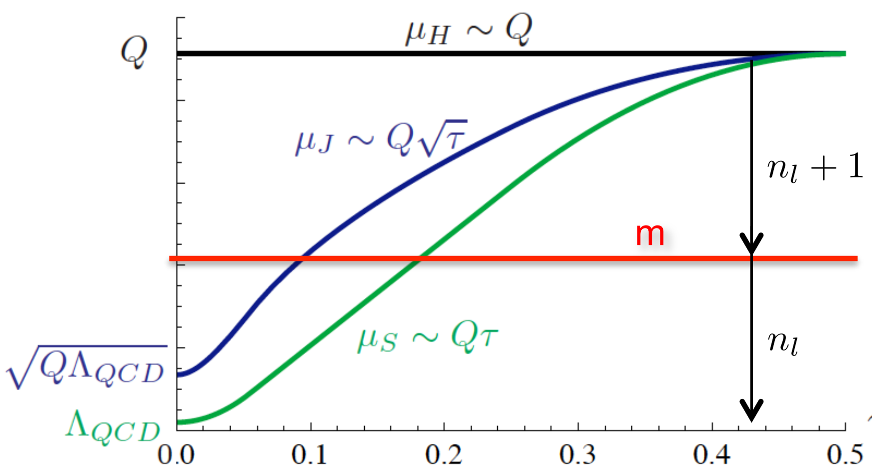
Gritschacher, AH,
Jemos, Pietrulewicz

→ consider: dijet in e^+e^- annihilation, n_l light quarks ⊕ one massive quark

→ obvious: (n_l+1)-evolution for $\mu \gtrsim m$ and (n_l)-evolution for $\mu \lesssim m$

→ obvious: different EFT scenarios w.r. to mass vs. Q – J – S scales

"profile functions"



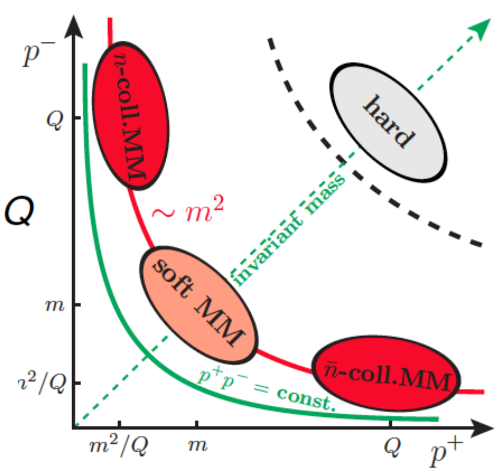
Aims:

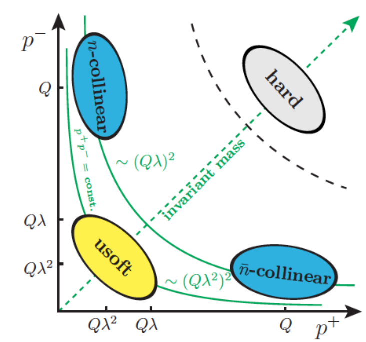
- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of $SCET_2$-type rapidity divergences

→ Deal with collinear and soft "mass modes"

→ Additional power counting parameter $\lambda_m = m/Q$

mode	$p^\mu = (+, -, \perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2





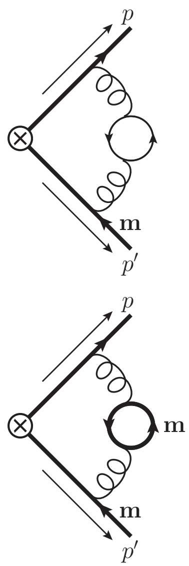
VFN Scheme: Primary Massive Quarks

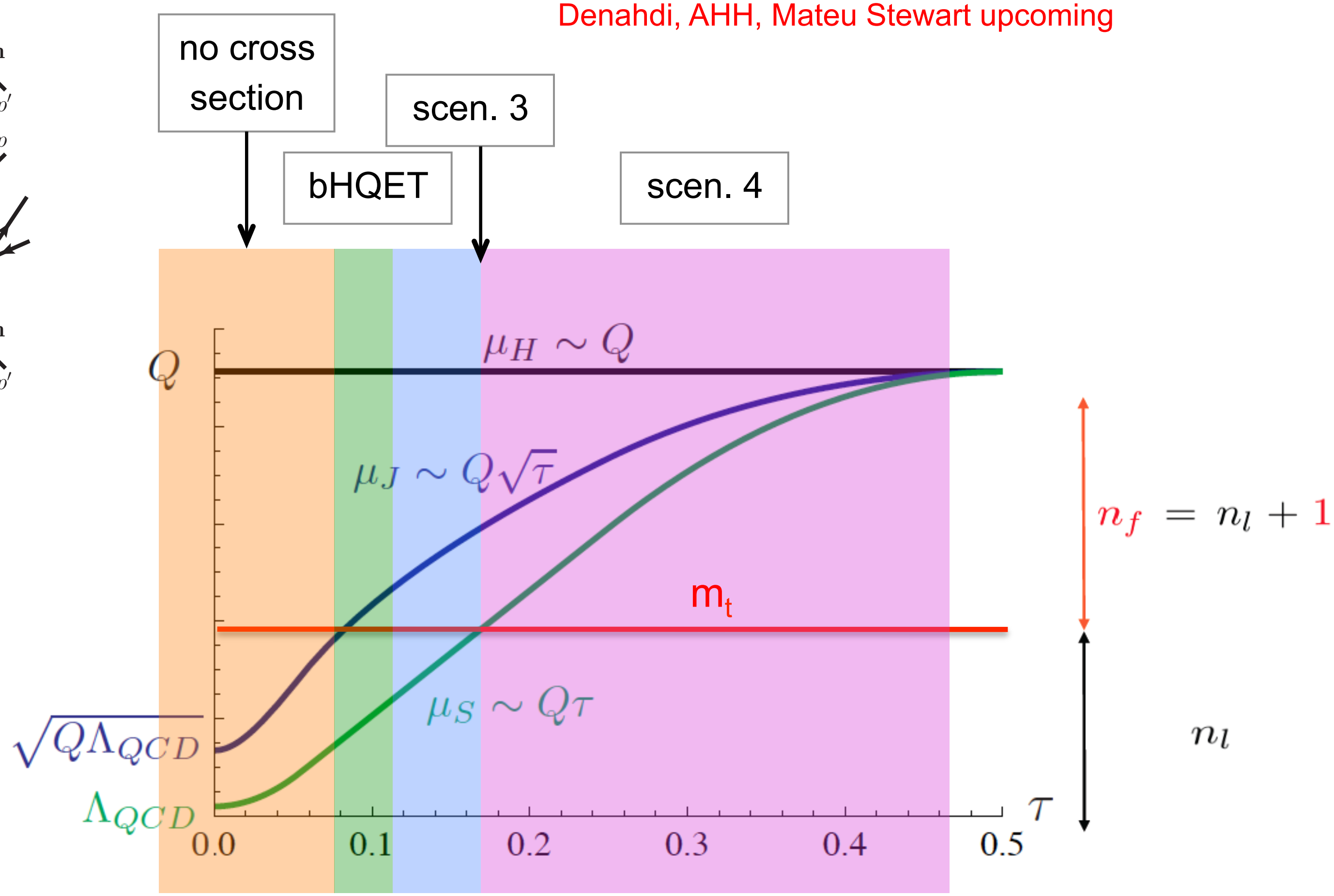
$\rightarrow$ bHQET-type theory when the jet scale approaches the quark mass

Fleming, AHH, Mantry, Stewart 2007

$\rightarrow$ two SCET-type theories

Denahdi, AHH, Mateu Stewart upcoming



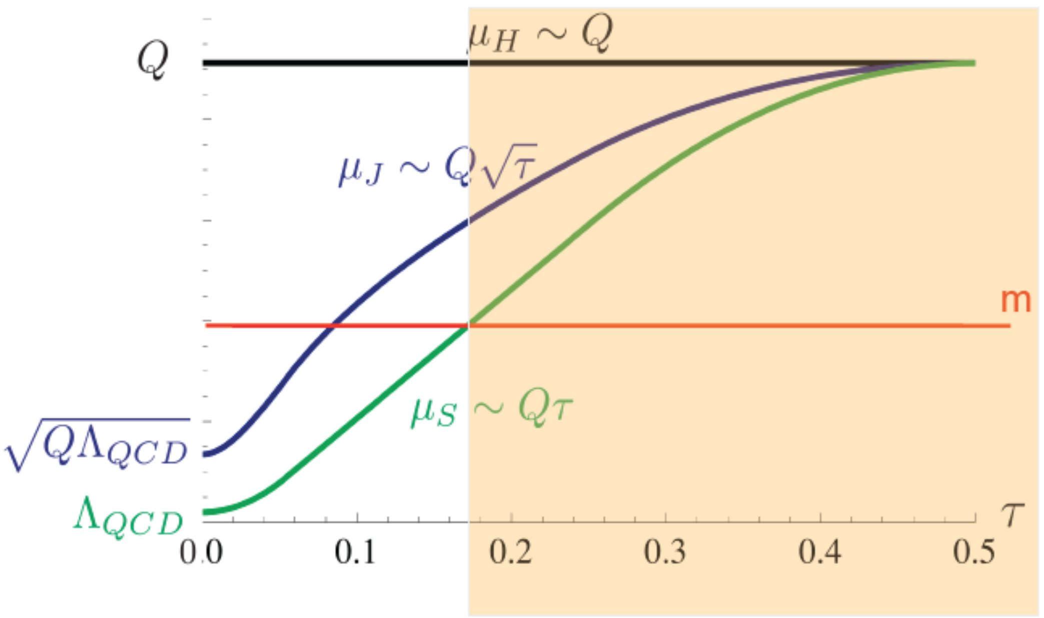


Scenario IV (SCET)

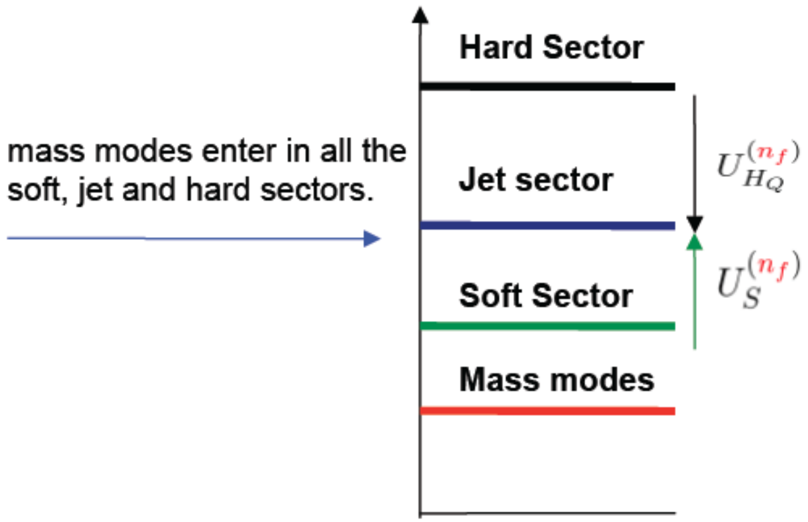
$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-IV}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int \mathrm{d}s \int \mathrm{d}k\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))$$

$$U_S^{(n_f)}(k,\mu_J,\mu_S)\, S_{\mathrm{part}}^{(n_f)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k,\mu_S\right)$$ **+ (QCD) Non-Singular**

$n_f = n_l + 1$



mass modes enter in all the soft, jet and hard sectors.



- (QCD) No-singular → Non-singular + Sub-leading singular contributions

Scenario III (SCET)

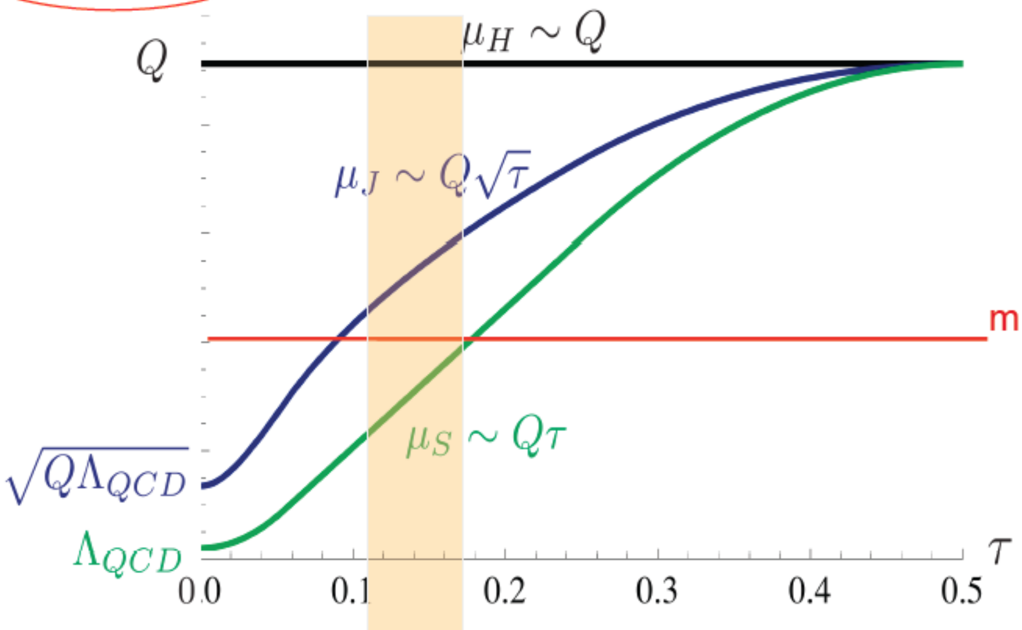
$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-III}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J) \int \mathrm{d}s \int \mathrm{d}k\, \mathrm{d}k'\, \mathrm{d}k''\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_m)$$

$$n_f = n_l + 1$$

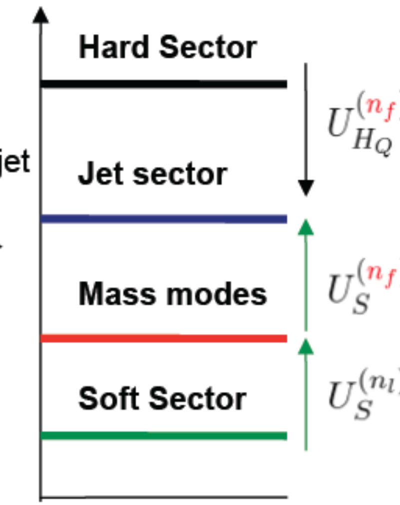
$$\boxed{\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s)}\, U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\, S_{\mathrm{part}}^{(n_l)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k'',\mu_S\right)$$

large rapidity logs
$\alpha_s^2 \log \sim \alpha_s$

+ (QCD) Non-Singular



mass modes enter in the jet and hard sectors.



- Soft mass-mode matching: **integrating in** the **mass-mode (secondary)** effects in the **evolution of the soft function** (top-down resummation). $\mathcal{O}(\alpha_s^2)$

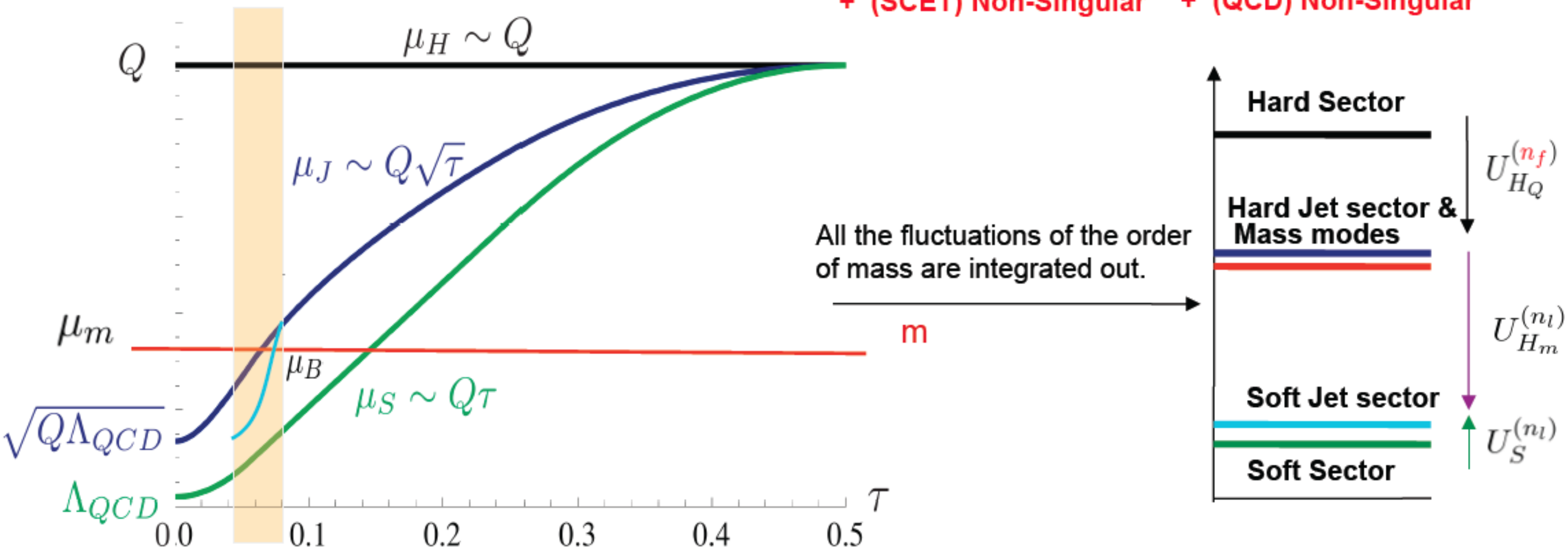
b(oosted)HQET

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\text{bHQET}} = Q\,H_Q^{(n_f)}(Q,\mu_Q)\,U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\,\boxed{H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m)}\,U_{H_m}^{(n_l)}\Big(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\Big)$$

$$\int \mathrm{d}s \int \mathrm{d}k\; B^{(n_l)}\Big(\frac{s}{m_J^{(n_l)}},\mu_B, m_J^{(n_l)}\Big)\,U_S^{(n_l)}(k,\mu_B,\mu_S)\,S_{\text{part}}^{(n_l)}\Big(Q\tau - Q\tau_{\text{MIN}} - \frac{s}{Q} - k,\mu_S\Big)$$

$n_f = n_l + 1$

+ (SCET) Non-Singular + (QCD) Non-Singular

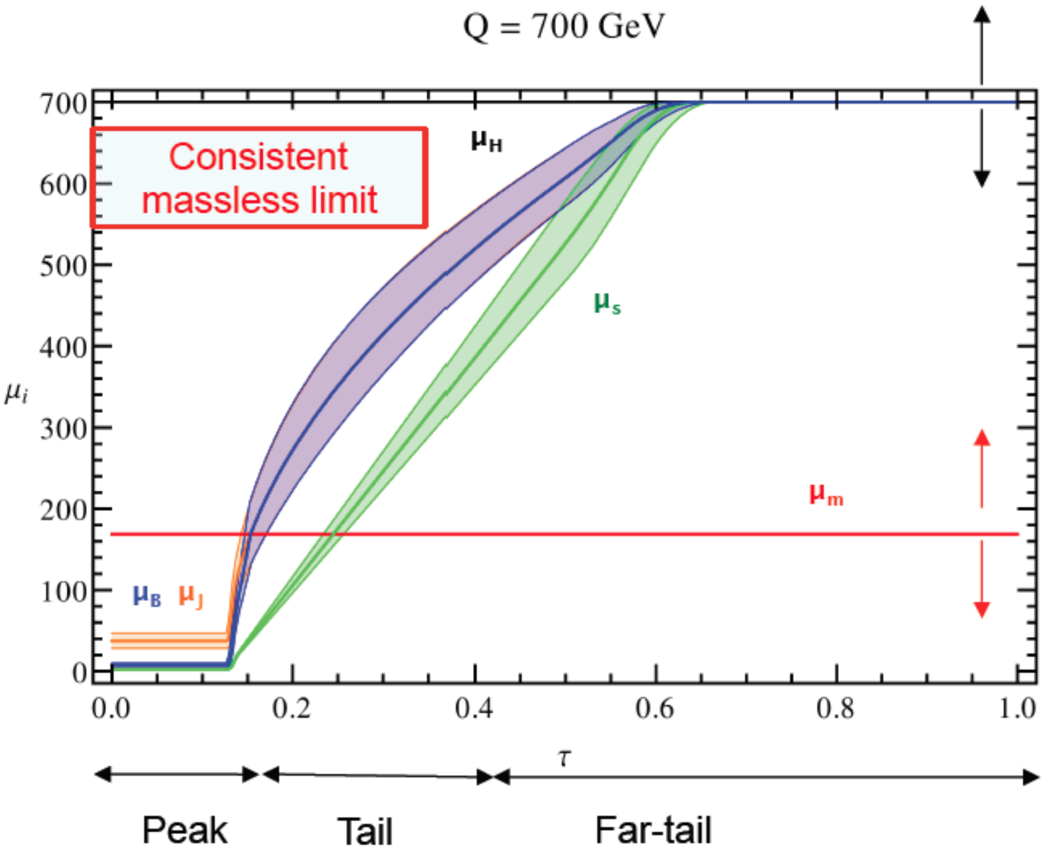


- Matching coefficient of SCET and bHQET have a large log from secondary corrections.

Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau-\tau_{\min})+2\Lambda_{\mathrm{QCD}}}{\mu_s}\right)$$



Scales Variation

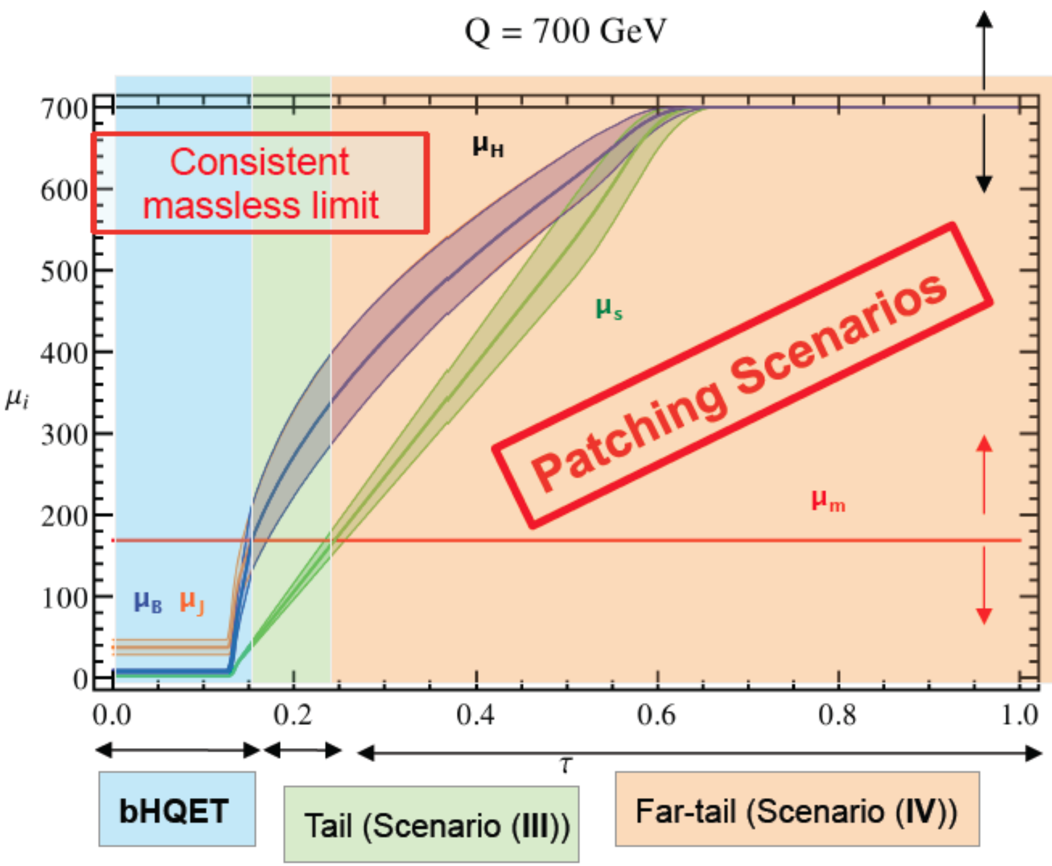
- **Generalized to arbitrary mass values**
- **Compatible with massless profiles**

Proper scale variations are essential in reliable estimation of missing higher order terms.

Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau-\tau_{\min})+2\Lambda_{\rm QCD}}{\mu_s}\right)$$



Scales Variation

- ✔ **Generalized to arbitrary mass values**
- ✔ **Compatible with massless profiles**

Proper scale variations are essential in reliable estimation of missing higher order terms.

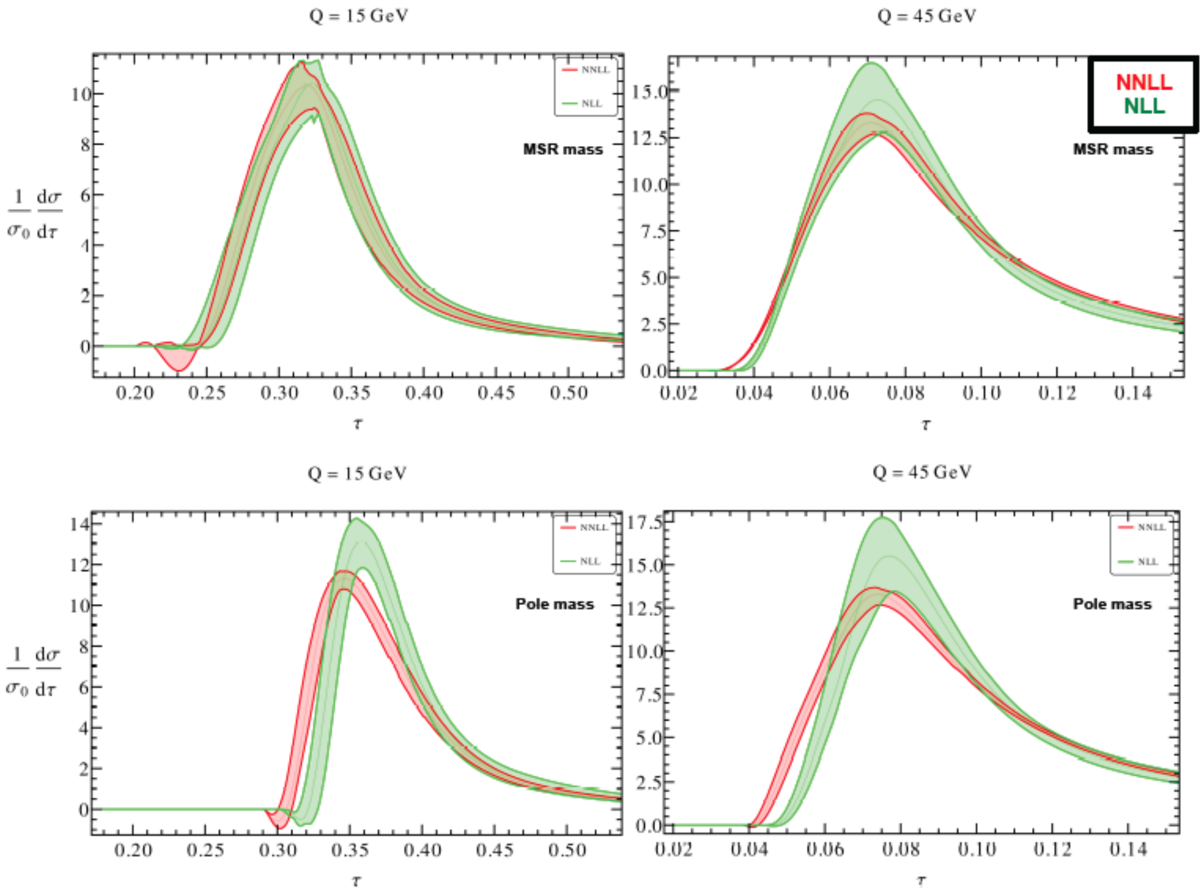
Thrust for Bottom Production

NNLL/NLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

$\overline{m}_b(\overline{m}_b) = 4.2$ GeV

Theory uncertainty (bottom)

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



- **Theory uncertainty is under control for bottom thrust distribution.**
- **Convergence of perturbation theory.**
- **The peak position in thrust is very sensitive to the mass.**
- **Stability of peak position with short distance mass scheme w.r.t. the pole mass scheme.**

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\text{GeV}$$

Preliminary Results

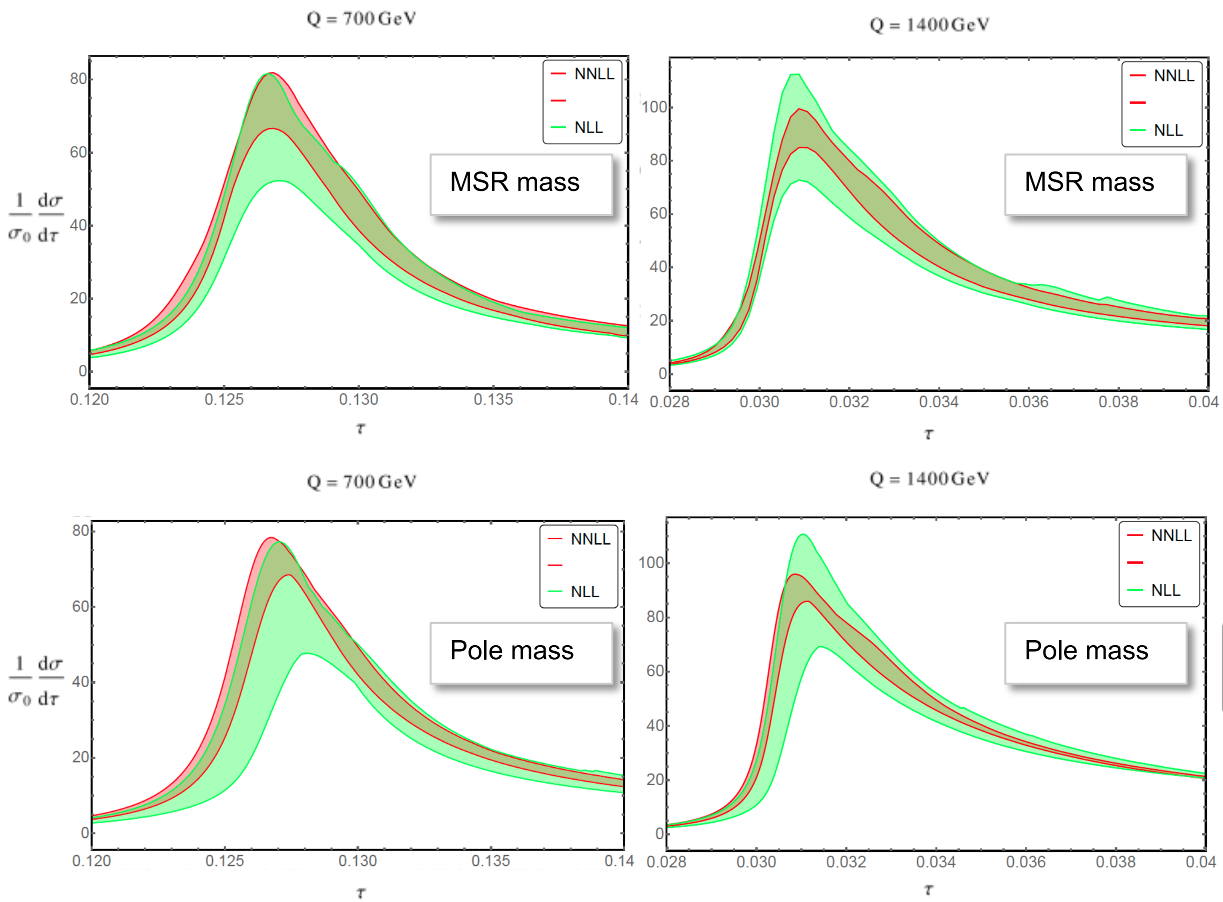
Thrust for Top Production

NNLL/NLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

$\overline{m}_t(\overline{m}_t)$ = 160 GeV

Theory uncertainty (top)

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



- Theory uncertainty in peak + tail is under control for top thrust distribution
- Convergence of perturbation theory
- Stability of peak position with short distance mass scheme w.r t. the pole mass scheme

$\alpha_s(m_Z) = 0.1184$
$\Omega_1 = 0.5\,\mathrm{GeV}$

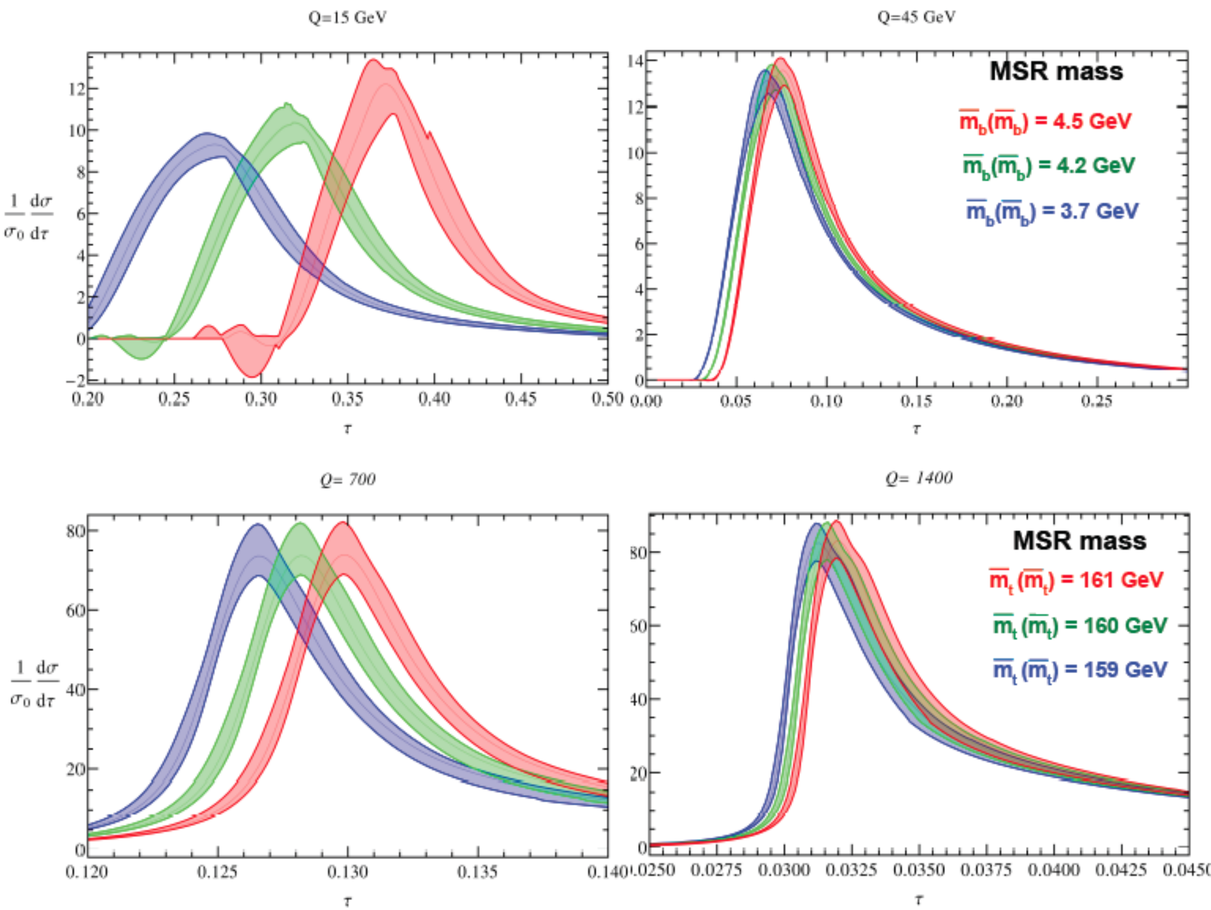
Preliminary Results

Thrust for Top Production

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

Peak sensitivity to mass

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



- **Peak position is more sensitive to the mass at low energies**
  - **bottom with better than 0.5 GeV**
  - **top with better than 1 GeV**

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\mathrm{GeV}$$

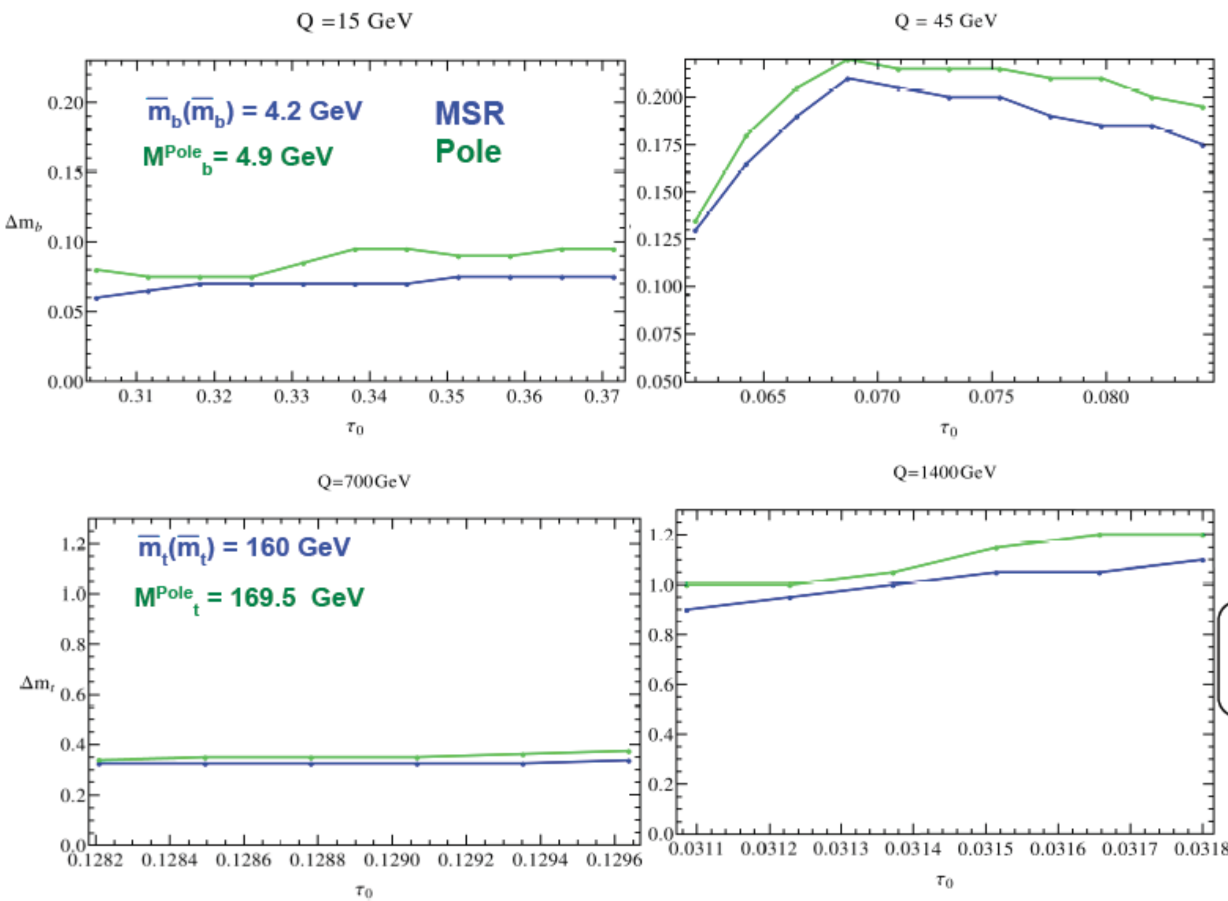
Preliminary Results

Theory Errors: Bottom and Top Mass

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

Theory uncertainty in mass

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



Error band method: Fitting the mass parameter of default cross section to the error band of default cross section.

- **bottom mass uncertainty < 0.5 GeV**
- **top mass uncertainty ~ 0.5 GeV**
- **Pole mass extractions are less precise.**

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\text{GeV}$$

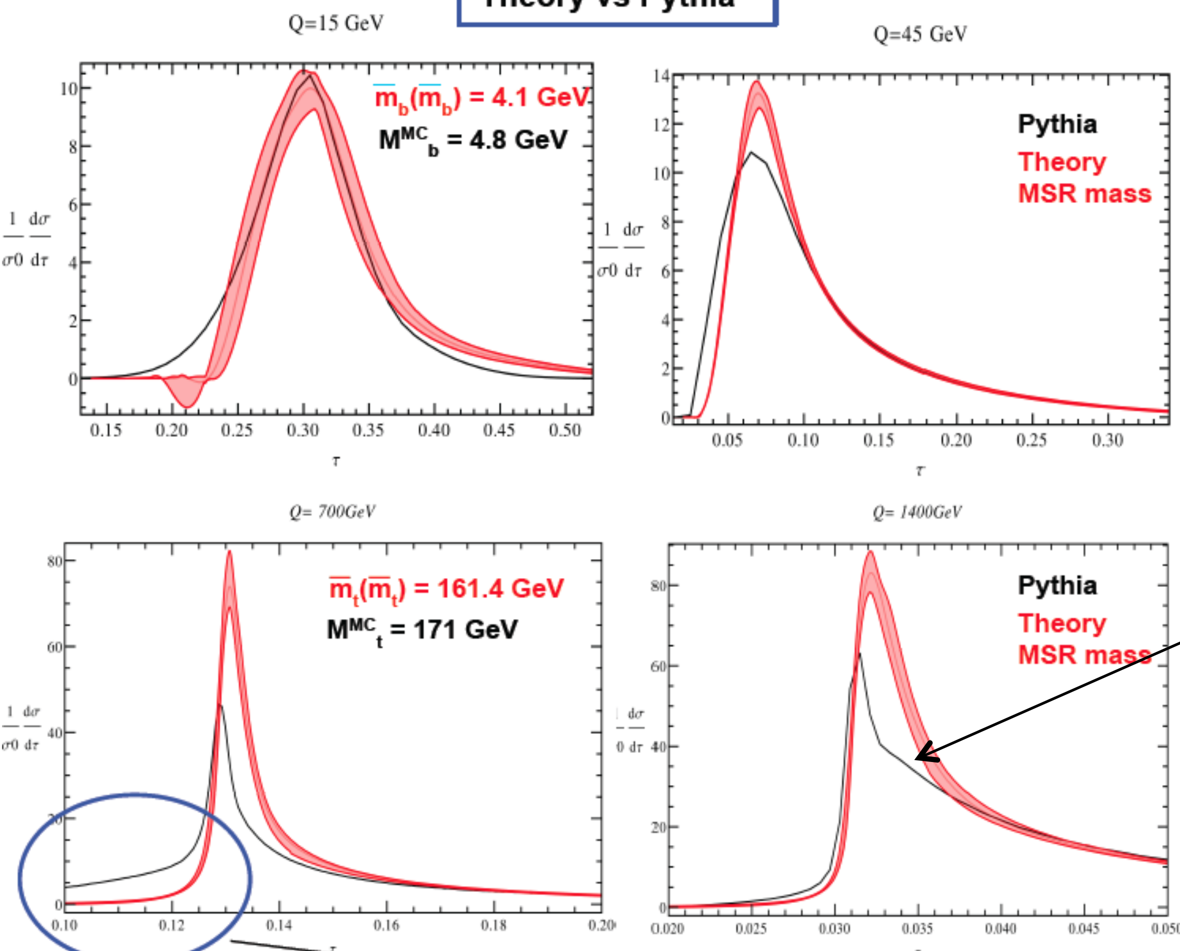
Preliminary Results

Theory vs. Pythia

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

Theory vs Pythia

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



- **Agreement of theory - Pythia :**
  - ✔ **Good for bottom**
  - ✔ **Some effect are likely missing (shoulder region) → off shell top + electroweak effects**

Tail disagreement (missing NLO)

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\text{GeV}$$

Shoulder region

Preliminary Results

Conclusions

Conclusions

- Complete description of the entire thrust distribution for boosted heavy quarks achieved with the formalism of VFNS for final-state jets and a sequence of effective field theory setups.

- The peak position in thrust is very sensitive (particularly at low energies) to the mass.
- Estimating theory errors is challenging
  - ✔ Under control directly at peak and tail.

- Our theory uncertainty for the mass extraction is reasonable and encouraging
  - ✔ Bottom $\rightarrow$ less than 0.5 GeV
  - ✔ Top $\rightarrow$ almost 0.5 GeV

- Simultaneous fit for α_s and Ω_1 is difficult, particularly for top $\rightarrow$ could be fixed externally
- Agreement between theory and Pythia:
  - ✔ Good for bottom
  - ✔ Some effects are likely missing for top (shoulder region) $\rightarrow$ off shell top + electroweak effects

Outlook

- Improving the precision to N^3LL seems mandatory.
- Off-shell top production + electroweak effects.

Backup Slides

Sensitivities: Bottom and Top Production

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

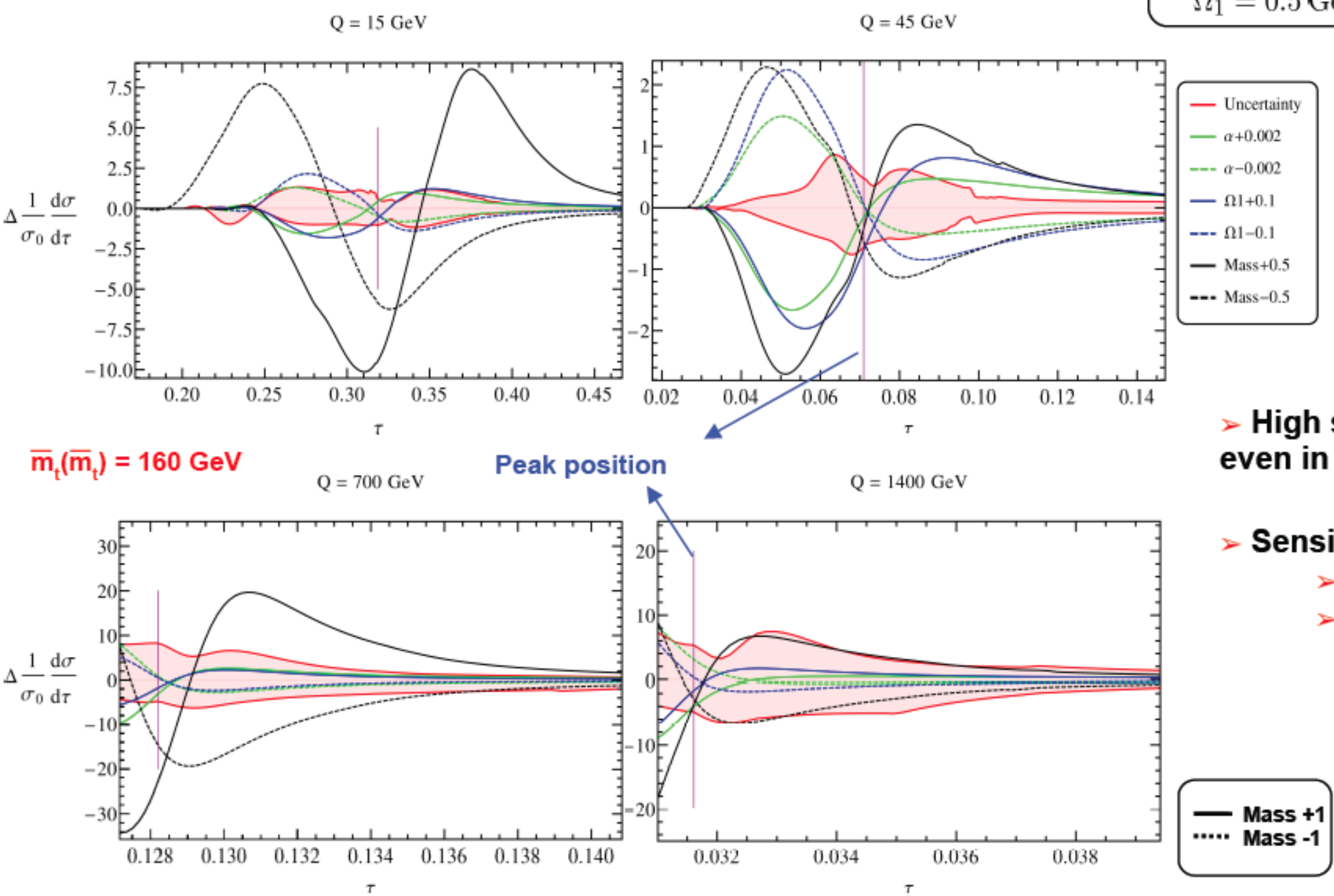
$\overline{m}_b(\overline{m}_b)$ = 4.2 GeV

Sensitivity of theory parameters

$\alpha_s(m_Z) = 0.1184$

$\Omega_1 = 0.5\,\mathrm{GeV}$

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



Comparison of the difference between the default cross section and the cross section varying only one parameter of theory w.r. to the theory uncertainties.

- **High sensitivity to (top & bottom) mass even in tail regions at low energies.**
- **Sensitivity to α_s and Ω_1**
  - **bottom → low sensitivity**
  - **top → negligible sensitivity (should be fixed externally)**

Preliminary Results

Masses Loop-Theorists Like to use

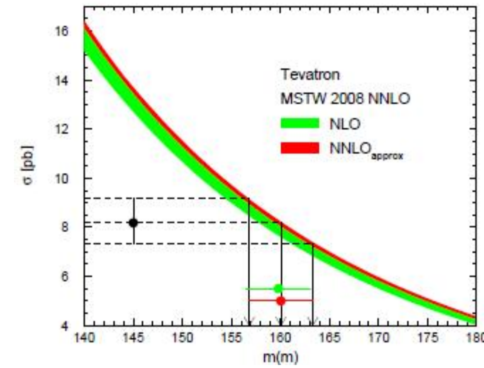
Total cross section (LHC/Tev):

$$m_t^{\rm MSR}(R=m_t)=\overline{m}_t(\overline{m}_t)$$

$$M_t \;=\; M_t^{(O)} \;+\; M_t(0)\alpha_s \;+\;\ldots$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

Langenfeld, Moch, Uwer



Mass schemes related to different computational methods

Relations computable in perturbation theory

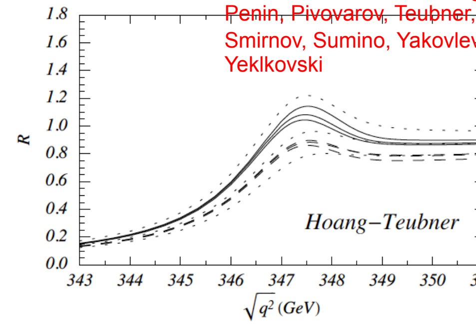
Threshold cross section (ILC):

$$m_t^{\rm MSR}(R\sim 20\ {\rm GeV})\,,\;\; m_t^{\rm 1S}\,,\;\; m_t^{\rm PS}(R)$$

$$M_t \;=\; M_t^{(O)} \;+\; \langle p_{\rm Bohr}\rangle\alpha_s \;+\;\ldots$$

$$\langle p_{\rm Bohr}\rangle \;=\; 20\,{\rm GeV}$$

Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Inv. mass reconstruction (ILC/LHC):

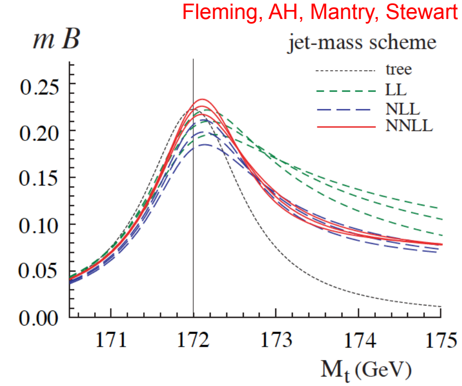
$$m_t^{\rm MSR}(R\sim \Gamma_t)\,,\;\; m_t^{\rm jet}(R)$$

$$M_t \;=\; M_t^{(O)} \;+\; \Gamma_t\alpha_s \;+\;\ldots$$

$$\Gamma_t \;=\; 1.3\,{\rm GeV}$$

- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections

Fleming, AH, Mantry, Stewart

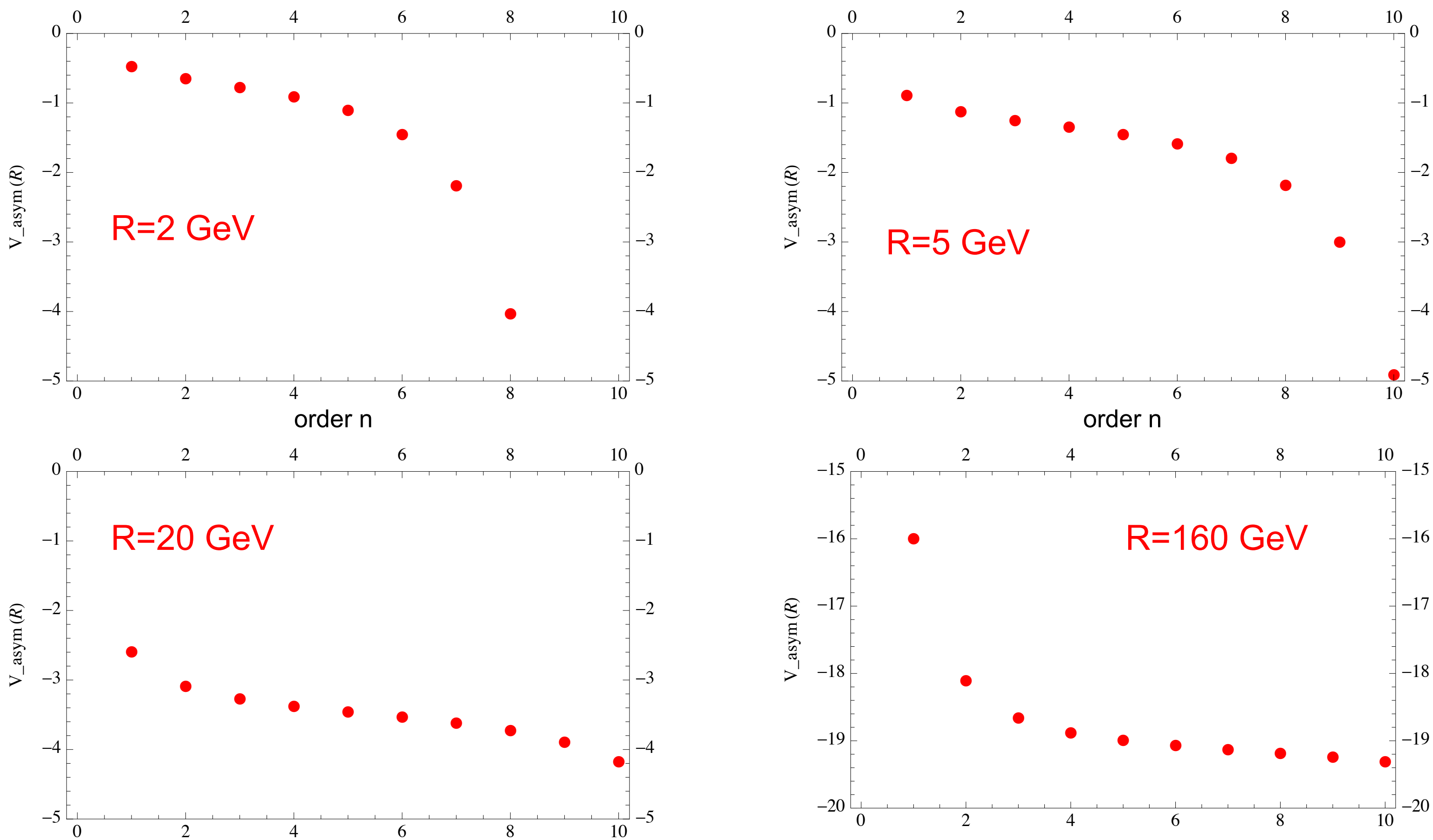


Series with a Renormalon

→ Behavior depends on the typical scale R of the observable ?

→ Series for large R converge longer, but size of corrections at lower orders are large

→ Formal ambiguity always the same: $\Lambda_{\mathrm{QCD}} \approx 0.5\ \mathrm{GeV}$



NLL Numerical Analysis

Double differential invariant mass distribution:

$$Q = 5 \times 172 \text{ GeV}$$

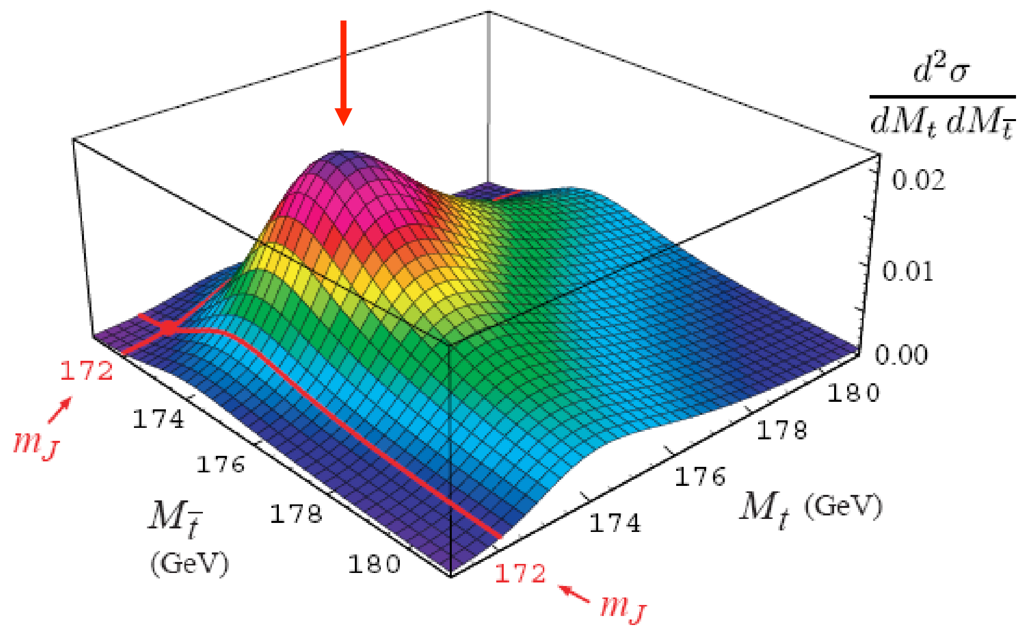
$$\Gamma = 1.43 \text{ GeV}$$

$$m_J(2\,\text{GeV}) = 172 \text{ GeV}$$

$$\mu_\Gamma = 5 \text{ GeV}$$

$$\mu_\Lambda = 1 \text{ GeV}$$

$$a = 2.5\,, \quad b = -0.4$$
$$\Lambda = 0.55 \text{ GeV}$$



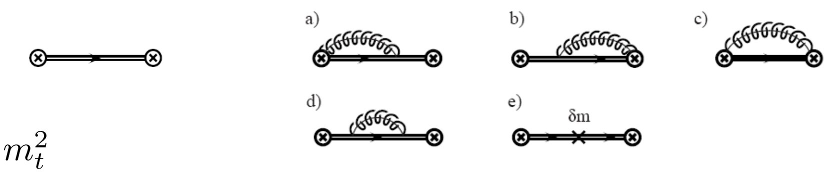
Non-perturbative effects shift the peak by <u>+2.4 GeV</u> and broaden the distribution.

Reconstructed Top Jets (ILC)

bHQET jet function:

$$B_+(2v_+\cdot k) \;=\; \frac{-1}{8\pi N_c m}\,\mathrm{Disc}\int d^4x\, e^{ik\cdot x}\,\langle 0|\mathrm{T}\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle$$

- perturbative, <u>any mass scheme</u>
- depends on m_t, Γ_t
- Breit-Wigner at tree level



$$B_\pm(\hat{s},\Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat{s}^2+\Gamma_t^2} \qquad\qquad \hat{s} = \frac{M^2-m_t^2}{m_t}$$

- Describes soft cross talk of the top (and its decay b quark) with the anti-top (and its decay anti-b quark) in the <u>top rest frame</u>
- Soft function describes soft radiation in the <u>lab frame</u>

Issues sorted out for the first time.

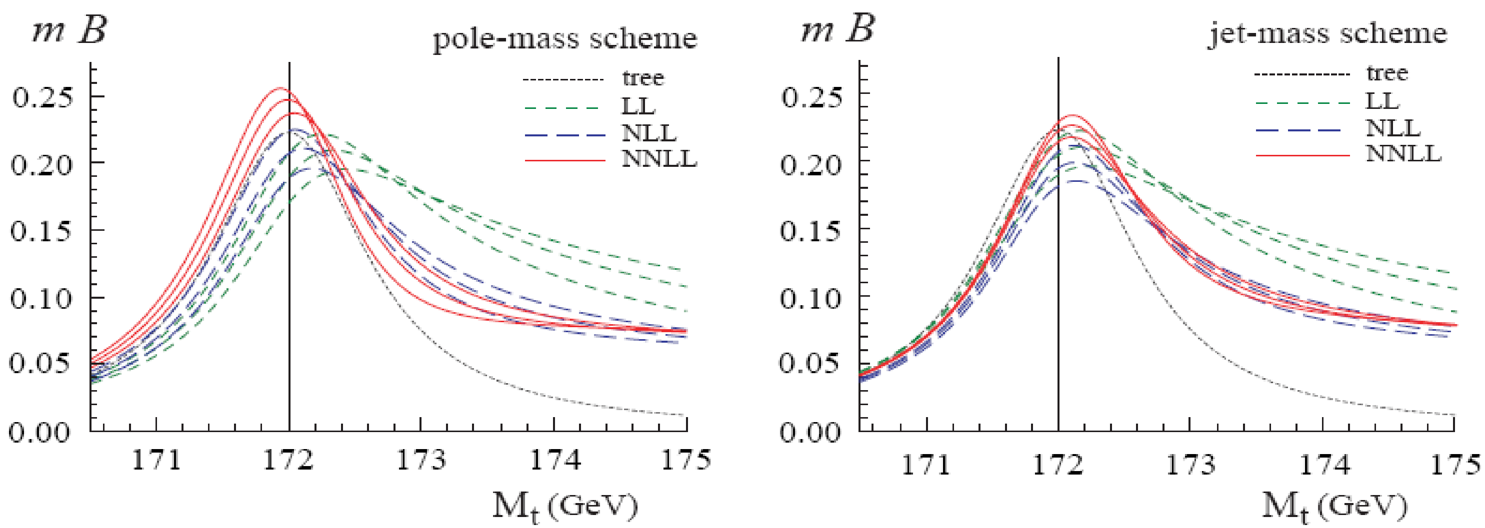
Results still true for LHC (but additional issues to resolved there)

Reconstructed Top Jets (ILC)

→ Jet function has an $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon in the pole mass scheme

$$\mathcal{B}_\pm(\hat{s},0,\mu,\delta m) = -\frac{1}{\pi m}\,\frac{1}{\hat{s}+i0}\left\{1+\frac{\alpha_s C_F}{4\pi}\left[4\ln^2\left(\frac{\mu}{-\hat{s}-i0}\right)+4\ln\left(\frac{\mu}{-\hat{s}-i0}\right)+4+\frac{5\pi^2}{6}\right]\right\} - \frac{1}{\pi m}\,\frac{2\delta m}{(\hat{s}+i0)^2}$$

$$\delta m = m_t^{\rm scheme} - m_t^{\rm pole}$$



Jain, Scimemi, Stewart PRD77, 094008(2008)

$$m_{\rm pole} = m_J(\mu) + e^{\gamma_E} R\,\frac{\alpha_s(\mu)\,C_F}{\pi}\left[\ln\frac{\mu}{R}+\frac{1}{2}\right] + \mathcal{O}(\alpha_s^2)$$

$$\boxed{R \sim \Gamma_t}$$

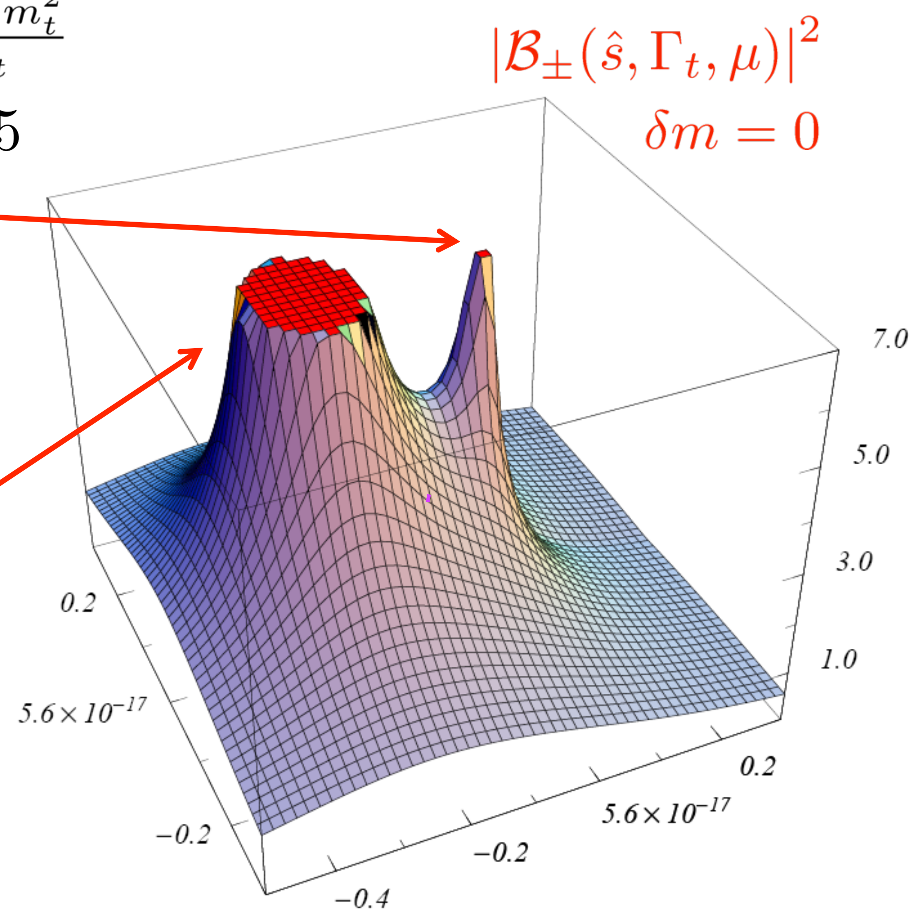
Reconstructed Top Jets (ILC)

Why is the pole mass not visible?

$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t}$$

$$\Gamma = \Gamma_t/5$$

pole mass peak



observable peak

→ jet mass is observable

- Located at the visible peak
- Short-distance mass

QCD Factorization

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Jet functions: $$B_+(\hat s, \Gamma_t, \mu) \;=\; \text{Im}\left[\frac{-i}{12\pi m_J} \int d^4x\, e^{ir.x} \langle 0|\, T\,\{\bar h_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\,\}\,|0\rangle\right]$$

- perturbative
- dependent on <u>mass, width, color charge</u>

$$B_\pm^{\text{Born}}(\hat s, \Gamma_t) = \frac{1}{\pi m_t} \frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function: $$S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a}) \delta(\ell^- - k_s^{-b}) \langle 0| \overline{Y}_{\bar n}\, Y_n(0) |X_s\rangle \langle X_s| Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

- non-perturbative
- analogous to the pdf's
- dependent on <u>color charge, kinematics</u>

Independent of the mass !

MC Mass

- Concept of mass in the MC depends on the **structure and reliability of the perturbative part** and the **interplay of perturbative and nonperturbative part** in the MC.

Parton shower (kinematic expansion)

Perturbative corrections

Hadronization Models

top mass definition decided here !

scheme-dependent separation (shower cut)

affect all top mass dependent observables

- Assume that the MC is a good QCD box (LO of s.th. more precise): How can one pin down the relation between m_t^{Pythia} and the Lagrangian mass ?
- Is the MC really a good QCD box ? Is the MC more a model or more QCD ?

Answer for m_t^{Pythia} might be process- and observable-dependent if the MC is not a good QCD box !