Cut n' paste spacetimes

Mario Flory

Max-Planck-Institut für Physik

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

MAX-PLANCK-GESELLSCHAFT

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Einsteins Field Equations

Spacetime curvature $\sim$ distribution of matter and energy

Einsteins Field Equations

$$G = T$$

Spacetime curvature $\sim$ distribution of matter and energy

G: Einstein tensor

T: Energy-momentum tensor (a.k.a. stress energy tensor)

Einsteins Field Equations

$$G_{\mu\nu}[g] = R_{\mu\nu}[g] - \frac{1}{2}R[g]g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}[\phi, \psi, A_\nu, ...]$$

Spacetime curvature $\sim$ distribution of matter and energy

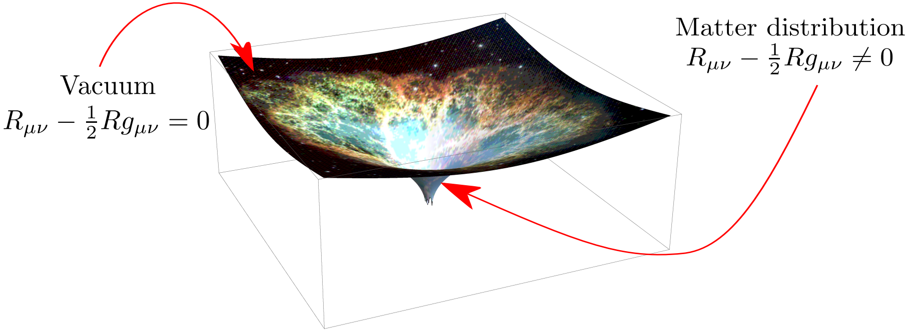
G: Einstein tensor

T: Energy-momentum tensor (a.k.a. stress energy tensor)

g: Metric tensor

$\phi, \psi, A_\nu, ...$: Matter content

Einsteins Field Equations



Not to scale:

- Crab Nebula (M1): diameter $\sim 11 ly$
- Crab Pulsar (PSR B0531+21): diameter $\sim 25 km$
- Schwarzschild radius: $r_s \sim 4 km$

Toy-models

Toy-models:

- simplified
- tractable
- yielding qualitative insights

Question: What is the backreaction of an infinitely thin (= codimension one) hypersurface carrying energy-momentum?

Answer: Israel junction conditions!

Israel junction conditions

In electromagnetism: To describe field around an infinitely thin charged surface Σ, integrate Maxwells equations in a box around Σ:

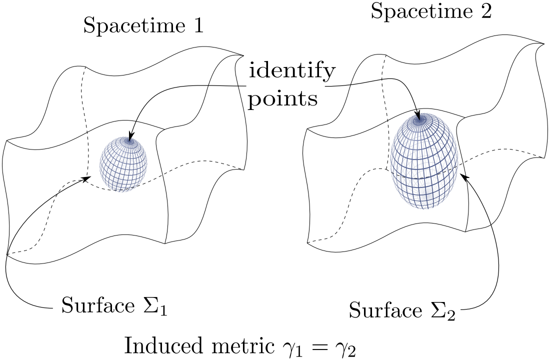
$$\Rightarrow \vec{E}_{||} \text{ continuous}, \;\; \vec{E}_{\perp} \text{ discontinuous on } \Sigma$$

In gravity: To describe backreaction of an infinitely thin massive surface, integrate Einsteins equations in a box
$\Rightarrow$ *Israel junction conditions* [Israel, 1966]:

$$(K^+_{ij} - \gamma_{ij} K^+) - (K^-_{ij} - \gamma_{ij} K^-) = -\kappa \, S_{ij}$$

S_{ij}: energy momentum tensor on the brane, γ_{ij}: induced metric,
$K^{\pm}$: extrinsic curvatures depending on embedding.

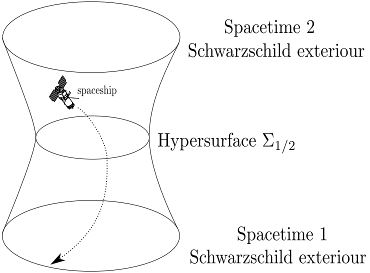
Israel junction conditions



Take two spacetimes (1 & 2) and define codimension one hypersurfaces $\Sigma_{1/2}$ such that they have the same topology. If the induced metric on $\Sigma_{1/2}$ is the same ($\gamma_1 = \gamma_2 \equiv \gamma$), the two spacetimes can be matched by identifying $\Sigma_{1/2}$ if the energy-momentum on Σ satisfies

$$(K_{ij}^{+} - \gamma_{ij} K^{+}) - (K_{ij}^{-} - \gamma_{ij} K^{-}) = -\kappa\, S_{ij}.$$

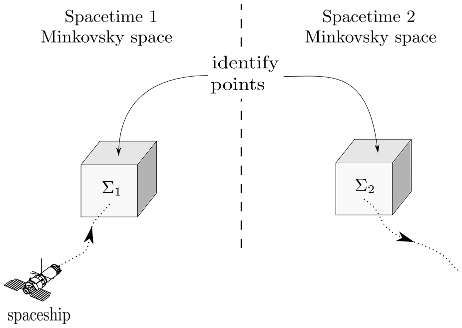
Examples I: spherical wormholes



- Wormhole connecting two Schwarzschild *exteriours*, no event horizons.
- Timedependent case analytically tractable.
- Thin shell of matter must carry *exotic* matter with negative energy.
- Energy conditions are violated: $S_{ij}k^i k^j < 0$ somewhere at shell.

[Visser: *Lorentzian Wormholes*]

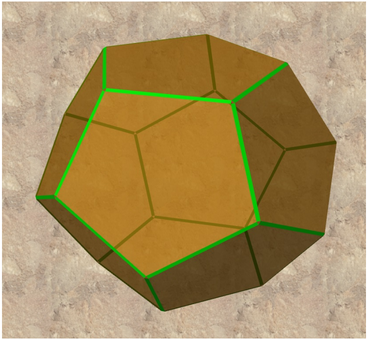
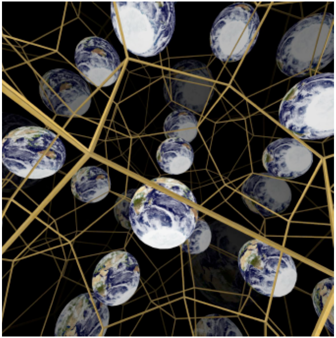
Examples II: polyhedral wormholes



- Analytically tractable even without spherical symmetry.
- Exotic matter is concentrated at edges of cube, wormhole can be transversed without encountering it.

[Visser: *Lorentzian Wormholes*]

Examples III: polyhedral universes

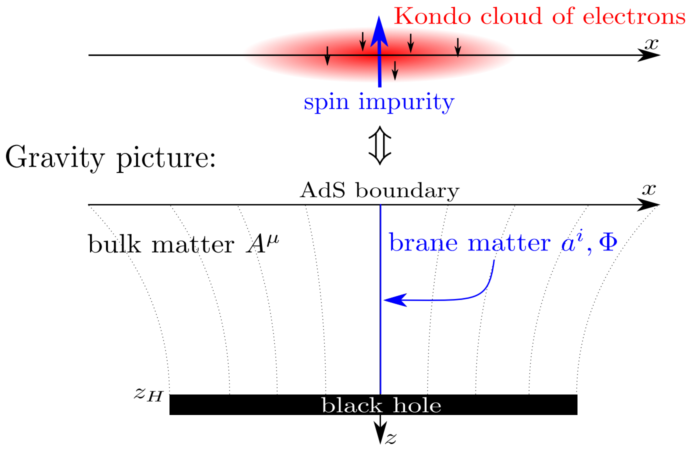
 

- Take a *compact* (possibly polyhedral) part of Minkovsky spacce (or dS/AdS).
- Glue it's faces onto *each other* to obtain a compact universe.
- In some cases, this works with $S_{ij} = 0$, i.e. naturally without matter present as "spacetime glue".

[Visser: *Lorentzian Wormholes*, Levin: *How the Universe got its spots*]

Examples IV: holographic Kondo model

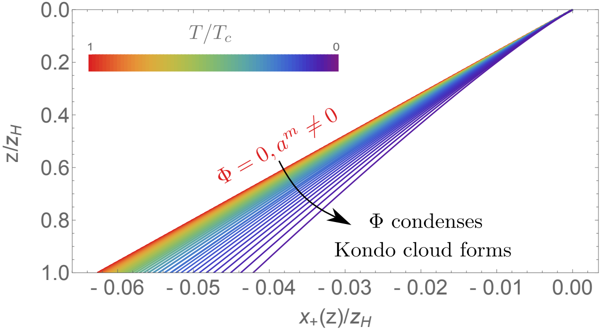
Field theory picture:



$$S = S_{CS}[A] - \int d^3x \delta(x) \sqrt{-g} \left(\frac{1}{4} f^{mn} f_{mn} + \gamma^{mn} (\mathcal{D}_m \Phi)^\dagger \mathcal{D}_n \Phi + V(\Phi^\dagger \Phi) \right)$$

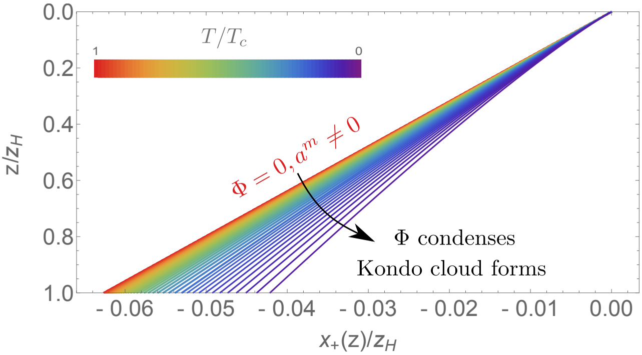
Examples IV: holographic Kondo model

- Due to Yang-Mills field a^m, SEC is violated everywhere in the bulk.
- Hence brane starts at boundary and falls into black hole, does *not* turn around and bend back to boundary.
- Numerical results for brane embedding w.r.t. right-hand side of bulk:



[Erdmenger et. al.: 1410.7811, 1511.03666]

Examples IV: holographic Kondo model



- Boundary RG flow ⇔ Reduction of spacetime volume.
- Defect entropy ⇔ Additional strip of event horizon.

[Erdmenger et. al.: 1410.7811, 1511.03666]

Thank you for your atention