A taste of SUSY

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Motivation

In a classical field theory Lagrangian only the bare mass m_0 of a field ϕ enters

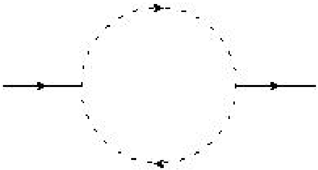
$$\mathcal{L} = \ldots + m_0\,\phi^2 + \ldots$$

However, in an interacting QFT the physical mass m depends on the energy scale Λ we are considering

$$m_0 \quad \Rightarrow \quad m(\Lambda)\,.$$

Reason: mass renormalization!

$\Rightarrow$

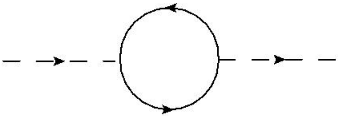


Motivation

The standard model (SM) is an effective theory, i.e. it is only valid up to a certain energy scale Λ and the masses of the particles depend on the physical mass m_h of the Higgs-particle H^0. This physical mass m_h depends heavily on the scale Λ of the theory. The strongest contribution to the mass correction δm_h arises from the coupling to the top-quark as

$$\mathcal{L}_{Yukawa} = -\frac{y_t}{\sqrt{2}} H_0 \bar{t}_L t_R + h.c. \quad \Rightarrow \quad m_t = \frac{y_t \langle H_0 \rangle}{\sqrt{2}}\,.$$

The one-loop correction to the Higgs-mass arises from the diagram



$$\Rightarrow \delta m_h^2|_{top} = -\frac{N_c |y_t|^2}{8\pi^2} \left(\Lambda^2 - 3 m_t^2 \log\left(\frac{\Lambda^2 + m_t^2}{m_t^2} \right) + \ldots \right)$$

Motivation

$$\delta m_h^2|_{top} = -\frac{N_c|y_t|^2}{8\pi^2}\left(\Lambda^2 - 3m_t^2 \log\left(\frac{\Lambda^2 + m_t^2}{m_t^2}\right) + \dots\right)$$

Problem: Already at $\Lambda \approx 1 TeV$ this contribution is not neglectable and the SM breaks down!

To extend the SM to higher energy scales we want to get rid of these divergent corrections.

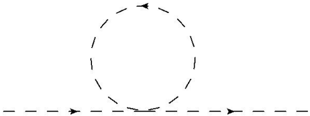
Idea: Scalar loops contribute corrections of different sign!

$\Rightarrow$ Introduce N new scalar fields ϕ_L, ϕ_R that interact with the Higgs field!

$$\mathcal{L}_{scalar} = -\frac{\lambda}{2}(h^0)^2(|\phi_L|^2+|\phi_R|^2)-h^0(\mu_L|\phi_L|^2+\mu_R|\phi_R|^2)-m_L^2|\phi_L|^2-m_R^2|\phi_R|^2$$

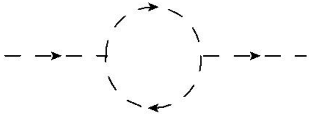
Motivation

One-loop correction of quartic coupling



$$\delta m_h^2|_{quartic} = \frac{\lambda N}{16\pi^2}\left(2\Lambda^2 - m_L^2 \log\left(\frac{\Lambda^2 + m_L^2}{m_L^2}\right) - m_R^2 \log\left(\frac{\Lambda^2 + m_R^2}{m_R^2}\right) + \dots\right).$$

One-loop correction of cubic coupling



$$\delta m_h^2|_{cubic} = -\frac{N}{16\pi^2}\left(\mu_L^2 \log\left(\frac{\Lambda^2 + m_L^2}{m_L^2}\right) + \mu_R^2 \log\left(\frac{\Lambda^2 + m_R^2}{m_R^2}\right) + \dots\right).$$

Motivation

Adding up the corrections we find for the quadratic divergences

$$\delta m_h^2 = \frac{\lambda N - |y_t|^2 N_c}{8\pi^2}\left(2\Lambda^2 + \dots\right).$$

$\Rightarrow$ We can cancel the quadractic divergence by introducing a scalar ϕ_L, ϕ_R for every top-quark t_L, t_R!

$$N = N_c, \quad \lambda = |y_t|^2$$

If we also set

$$m_t = m_L = m_R, \quad \mu_L^2 = \mu_R^2 = 2\lambda m_t^2$$

we cancel the logarithmic divergences in Λ!

$\Rightarrow$ Supersymmetry!

Superspace

We found a symmetry between scalars ϕ and fermions ψ of our theory!

Noether theorem: A symmetry leads to a conserved charge that generates the symmetry

Example: Translation-invariance $\Rightarrow$ Momentum conservation!

$$P_m = i\partial_m, \quad \delta_\epsilon F(x) = -i\epsilon^m P_m F = F(x) - F(x + i\epsilon) + \mathcal{O}(\epsilon^2)$$

We have supersymmety

$$\delta_\epsilon B = \epsilon \cdot F, \quad \delta_\epsilon F = \epsilon B$$

Note: $B \cdot B = B$, $B \cdot F = F$, $F \cdot F = B$, $\delta_\epsilon = B \Rightarrow \epsilon$ fermion!

The conserved charge is a fermion! $\quad \Rightarrow \quad$ Supercharges $Q_\alpha, \bar{Q}_{\dot{\alpha}}$

$$Q\phi \sim \psi, \quad \bar{Q}\psi \sim P\phi$$

Superspace

We want to interpret the supercharges $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ as translations.

Extend space-time x^m to superspace $(x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$!

θ^α are Grassmann variables $\alpha = 1, 2$,

$$\theta_\alpha \theta_\beta = -\theta_\beta \theta_\alpha, \quad \Rightarrow \quad f(\theta) = f_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_1 \theta_2 f_3,\ f_i \in \mathbb{C}$$
$$\frac{\partial}{\partial \theta_1} f(\theta) = f_1 + \theta_2 f_3, \quad \int d\theta_1 f(\theta) = f_1 + \theta_2 f_3 = \frac{\partial}{\partial \theta_1} f(\theta)$$

Result:

$$\delta_\epsilon \theta^\beta = (\epsilon_\alpha Q^\alpha) \theta^\beta = \epsilon^\beta$$

$\Rightarrow$ represent Q_α by $\partial_\alpha + \ldots$

To close the algebra, i.e. $B \to F \to B$, we impose

$$\{Q, \bar{Q}\} \sim P\,.$$

Superspace

Define covariant derivatives D_α on superspace, i.e.

$$\{D_\alpha, Q_\beta\} = 0, \quad \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = 0, \quad [D_\alpha, P_m] = 0,$$
$$P_m = i\partial_m, \quad Q_\alpha = \partial_\alpha - i(\sigma^m\bar{\theta})_\alpha\partial_m, \quad D_\alpha = \partial_\alpha + i(\sigma^m\bar{\theta})_\alpha\partial_m$$

and also an extended integration measure

$$\int d^4x\, f(x) \quad \longrightarrow \quad \int d^4x \int d^4\theta\, F(x,\theta,\bar{\theta}),$$

with $F(x,\theta,\bar{\theta}) = \ldots + \theta_1\theta_2\bar{\theta}_1\bar{\theta}_2 f(x)$. $\Rightarrow$ Pick out θ^4 term!

From the theory we constructed, one can show

$$\int d^4\theta \quad = \quad \left(D^2\bar{D}^2 + \partial_m(\ldots) \right)\Big|_{\theta=0},$$

i.e. integration is equivalent to differentiation! (after setting all $\theta = 0$.)

Superspace

Consider a superfield: a complex function of superspace

$$\Phi(x,\theta,\bar\theta) = \phi(x) + \theta\psi(x) + \bar\theta\bar\psi(x) + \theta^2 F(x) + \theta\sigma^m\bar\theta A_m(x) + \ldots + \theta^2\bar\theta^2 D(x)\,.$$

$$\begin{aligned} \Rightarrow \quad & \Phi(x,\theta,\bar\theta)|_{\theta=0} = \phi(x)\,, \\ & Q_\alpha\Phi(x,\theta,\bar\theta)|_{\theta=0} = \partial_\alpha\Phi(x,\theta,\bar\theta)|_{\theta=0} = \psi_\alpha(x) \end{aligned}$$

$\Rightarrow$ Q maps ϕ to its superpartner ψ!

Too many fields in Φ (reducible representations) $\Rightarrow$ Impose constraints!

Example: Chiral superfield $D_{\dot\alpha}\Phi = 0$,

$$\Rightarrow \quad \phi(x), \quad \psi_\alpha(x), \quad F(x) \quad \text{independent}\,,$$

$$A_m \sim \partial_m\phi, \quad D \sim \Box C \quad \Rightarrow \text{derivative terms!}$$

Superspace

Use these derivative terms to construct a kinetic term for our SUSY Lagrangian!

Simplest example: free Wess-Zumino model (on-shell)

$$\mathcal{L}_{kin} = \int d^4\theta \, \Phi\bar{\Phi} = -\partial_m \phi \partial^m \bar{\phi} - i\bar{\psi}\sigma^m \partial_m \psi \,.$$

$\Rightarrow$ Kinetic terms for free complex scalar ϕ and Weyl-fermion ψ

Advantage of superspace formalism:

For every real function $K(\Phi^i, \bar{\Phi}^j)$ and Φ^i chiral superfields we obtain a manifestly supersymmetric theory

$$\mathcal{L}_{kin} = \int d^4\theta \, K(\Phi, \bar{\Phi}) = -K_{i\bar{j}} \, \partial_m \phi^i \partial^m \bar{\phi}^j + (\text{fermi}) \,.$$

$\Rightarrow$ K Kählerpotential (metric on target space)

Superspace

Is this everything we can construct? No!

Integrate over half the superspace (Pick out θ^2- and $\bar{\theta}^2$-terms)

$$\mathcal{L}_{pot} = \int d^2\theta \, W(\Phi) + \int d^2\bar{\theta} \, \bar{W}(\bar{\Phi}) \, ,$$

the function $W(\Phi)$ is called superpotential and needs to be holomorphic, i.e.

$$W(\Phi) = \sum_{n=-\infty}^{\infty} a_n \Phi^n , \quad a_n \in \mathbb{C} \, .$$

Example: Add superpotential $W(\Phi) = \frac{1}{2} m \Phi^2 + \frac{1}{6} \lambda \Phi^3$

$$\begin{aligned} \mathcal{L} &= \int d^4\theta \, \Phi \bar{\Phi} + \int d^2\theta \, W(\Phi) + \int d^2\bar{\theta} \, \bar{W}(\bar{\Phi}) \\ &= -\partial_\mu \phi \partial^\mu \bar{\phi} - i \psi \sigma^\mu \partial_\mu \bar{\psi} + |m\phi + \frac{1}{2}\phi^2|^2 + (m + \lambda\phi)\psi^2 + (\bar{m} + \bar{\lambda}\bar{\phi})\bar{\psi}^2 \, . \end{aligned}$$

Superspace

Let us consider renormalization by loop-diagrams (perturbative quantum corrections)!

Idea: write down the most general theory that is consistent with the symmetries of our classical theory

We are interested in the mass correction, so we consider the renormalizable classical potential

$$W_{tree}(\Phi) = \frac{1}{2} m \Phi^2 + \frac{1}{6} \lambda \Phi^3$$

and compatibility with SUSY, locality, weak-coupling limit etc. restrict the effective potential

$$W_{tree}(\Phi) + (\text{non-perturbative}) = W_{eff}(\Phi)\,.$$

$\Rightarrow$ No loop-corrections!

Superspace

These non-renormalization theorems can be generalized, for example

$$W_{tree} = \sum_{n=1}^{\infty} \mu_n \Phi^n$$

does not receive perturbative corrections!

$\Rightarrow$ SUSY might be a first step to UV-complete theories!

Note that the theory with $W(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{6}\lambda\Phi^3$ describes the supersymmetrisation of the top-quark we had at the start.

ψ	left-handed top-quark t_L
C	super-partner of the quark t_L, ϕ_L (squark)
$\bar{\psi}$	right-handed top-quark t_R
$\bar{C}$	super-partner of the quark t_R, ϕ_R (squark)

Summary and Outlook

Message: Supersymmetry emerges naturally when we want to enhance the behavior under quantum corrections of a theory

Supersymmetry has far reaching consequences (non-renormalization theorems)

We can learn a lot about gauge-theories in general (super-QCD)

How we actually use it:

In gauged supergravity we encounter a huge amount of terms.
It suffices to consider only the purely bosonic terms of an action, since the rest is fixed by SUSY.

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L}_{eff} = \frac{1}{2}R - K_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^j - \frac{1}{4}Re(f(z))F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}Im(f(z))F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Thank you for your attention!

Questions?

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