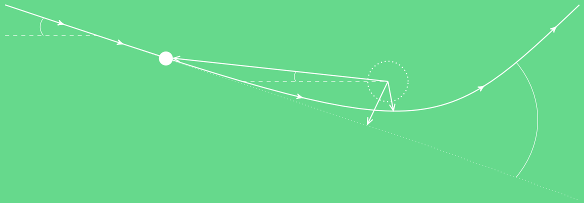
Quantum corrections to the neutrino and photon deflection in a classical background



Marta Dell'Atti – Università del Salento

IMPRS Workshop Munich – 26th October 2015

Introduction

- The Lagrangian of the Standard Model is described in a curved spacetime, with particular attention to the photon and neutrino sector
- Introduction of the semi-classical approach to study the deflection of massless particles
- Computation of the next-to-leading-order corrections
- Application: Gravitational lensing

Standard Model in a classical background

- The gravitational field source is a Schwarzschild black hole

$$d\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2\vartheta\, d\phi^2$$

for which the event horizon is the Schwarzschild radius $r_S = \dfrac{2GM}{c^2}$. From now on $c = 1 = \hbar$

- The Standard Model Lagrangian is embedded in a curved spacetime with metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ and action

$$\mathcal{S} = \underbrace{\int d^4x \sqrt{-g}\, \mathcal{L}_M}_{\text{Matter}} - \underbrace{\frac{1}{\kappa^2} \int d^4x \sqrt{-g}\, R}_{\text{Einstein-Hilbert}} + \underbrace{\chi \int d^4x \sqrt{-g}\, R\, h^\dagger h}_{\text{Improvement}} + \ldots$$

Standard Model in a classical background

Gravitational interactions of matter fields are introduced by minimal coupling

$$\mathcal{L}_{\text{int}} = -\frac{\kappa}{2} T^{\mu\nu} h_{\mu\nu}$$

Bosonic fields

- $\eta_{\mu\nu} \to g_{\mu\nu}$
- $d^4x \to \sqrt{-g} d^4x$
- $\partial_\mu \to D_\mu$, involving affine connection $\Gamma^\lambda_{\mu\nu}$

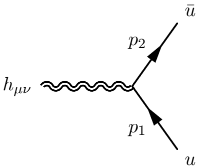
Fermionic fields

- $\eta_{\mu\nu} \to g_{\mu\nu}$
- $d^4x \to \sqrt{-g} d^4x$
- $\partial_\mu \to D_\mu$
- vierbein e^a_μ and spin connection ω^{ab}_μ

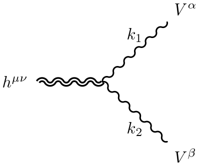
Leading order cross section

The $S-$matrix element is the starting point for the computation of the cross section

$$S_{fi} = -\frac{i\kappa}{2}\int d^4x\; h_{\mu\nu}(x)\,\langle f|T^{\mu\nu}(x)|i\rangle$$



(a)



(b)

Neutrino cross section

$$\frac{d\sigma}{d\Omega}^{(0)}_{\nu} = \left(\frac{GM}{\sin^2(\vartheta/2)}\right)^2 \cos^2\vartheta/2$$

Photon cross section

$$\frac{d\sigma}{d\Omega}^{(0)}_{\gamma} = (GM)^2\cot^4(\vartheta/2)$$

Massless particle deflection: classical problem

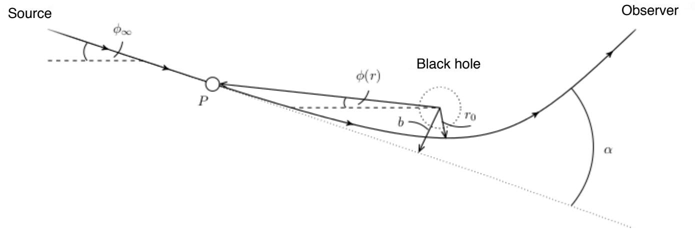


Figure: Deflection of massless particle P due to the black hole gravitational field

- The deflection angle is $\alpha(r_0) = 2\int_{r_0}^{\infty} \left|\frac{d\phi}{dr}\right| dr - \pi$
- The impact parameter is given by $b = \dfrac{r_0}{\sqrt{1 - \frac{1}{r_0}}}$

For $r_0 \gg r_S$ $\alpha(b) \sim \dfrac{4GM}{b}$

Semiclassical approach

From the classical study of a massless particle beam scattered by a central potential

$$b^2(\alpha) = 2\int_\alpha^\pi \frac{d\sigma}{d\Omega}\sin\vartheta\, d\vartheta$$

- In the neutrino case

$$b^2_{h,\nu}(\alpha) = -1 + \csc^2\frac{\alpha}{2} + 2\log\left(\sin\frac{\alpha}{2}\right)$$

  and for small deflection angles

$$b_{h,\nu}(\alpha) = \frac{2}{\alpha} + \frac{\alpha}{6}(1 + \log 8 - 3\log\alpha) + \mathcal{O}(\alpha^2)$$

- For the photon case

$$b^2_{h,\gamma}(\alpha) = \frac{1}{2}\left(-1 + \cos\alpha + 2\csc^2\frac{\alpha}{2} + 8\log\left(\sin\frac{\alpha}{2}\right)\right)$$

  and for small deflection angles

$$b_{h,\gamma}(\alpha) = \frac{2}{\alpha} + \alpha\left(\frac{1}{12} - \log 2 + \log\alpha\right) + \mathcal{O}(\alpha^3)$$

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For $\alpha \ll 1$ we get

$$b_\nu = b_\gamma \sim \frac{4GM}{\alpha}$$

Comparison between classical and semiclassical approach

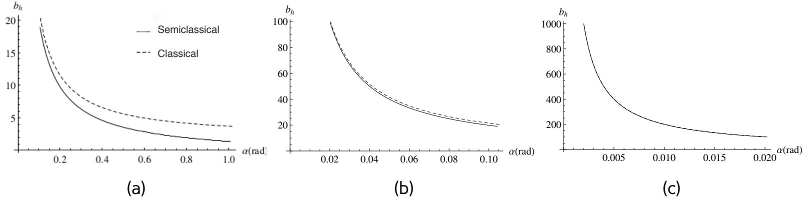

Figure: Impact parameter in horizon units b_h as a function of deflection angle for the leading order cross section for photon. Analogous results for the neutrino case.

One-loop: neutrino

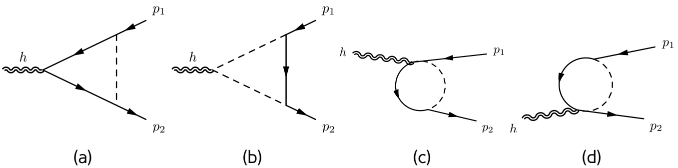

Figure: Neutrino diagrammatic expansion

The result of the Feynman diagrams computation can be organized as

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{Z} + \hat{T}^{\mu\nu}_{W} + \hat{T}^{\mu\nu}_{\text{ct}}$$

The next-to-leading-order cross section is, then,

$$\frac{d\sigma}{d\Omega}^{(1)}_{\nu} = (GM)^2 \frac{\cos^2(\vartheta/2)}{\sin^4(\vartheta/2)} \left\{ 1 + \frac{4G_F}{16\pi^2\sqrt{2}} \mathrm{Re}\left[F^{W}_{\nu}(E,\vartheta) + F^{Z}_{\nu}(E,\vartheta) - \frac{1}{4}\Sigma^{W}_{L} - \frac{1}{4}\Sigma^{Z}_{L} \right] \right\}$$

One-loop: photon

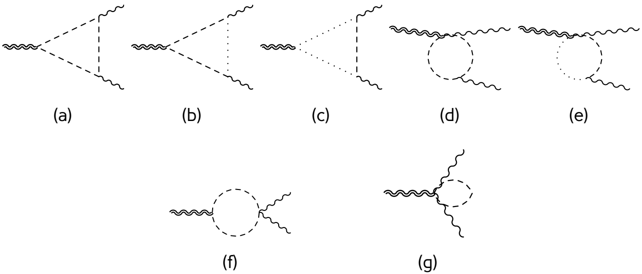

Figure: Internal lines are massive Z and $W^{\pm}$ gauge bosons, $\phi^{\pm}$ Nambu-Goldstone bosons, ghosts and fermions. Diagrams (a), (b) and (c) are triangular, (d) and (e) are *t-bubble*. Diagram (f) is a *s-bubble*, (g) is a *tadpole*.

The photon cross section is $\left.\frac{d\sigma}{d\Omega}\right._{\gamma}^{(1)} = (GM)^2 \cot^4(\vartheta/2)\left[1 + \mathrm{Re}\,\overline{\Phi}_3\right]$

One-loop cross section

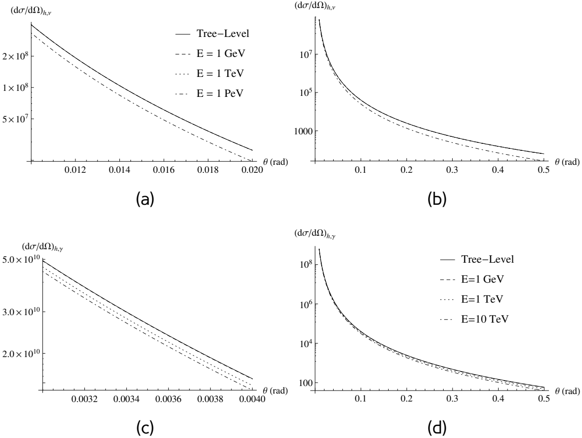

Figure: Differential neutrino and photon cross section in r_S units to the leading order (tree-level) and next-to-leading-order (one-loop).

Gravitational lensing

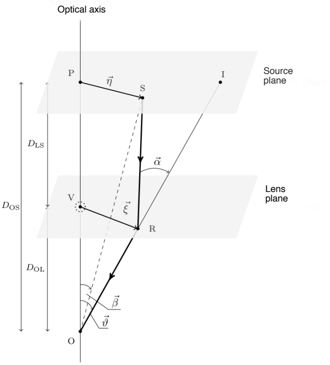

Figure: Deflection of the trajectory of a particle subjected to a gravitational lens. Arrows represent directions of the optical ray, the approximation of the real curved ray.

Reference

C. Corianò, A. Costantini, M. Dell'Atti, and L. Delle Rose. "*Neutrino and Photon Lensing by Black Holes: Radiative Lens Equations and Post-Newtonian Contributions*" *Journal of High Energy Physics* 2015:160 (Jul. 2015).

With the thin lens approximation the lens equation reads

$$\beta = \vartheta_{\mathrm{I}} - \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\alpha(b)$$

The Virbhadra-Ellis lens equation is

$$\tan\beta = \tan\vartheta - \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\left(\tan\vartheta_{\mathrm{I}} + \tan(\alpha(b) - \vartheta_{\mathrm{I}})\right)$$

Radiative lens equation

In the neutrino case, to the leading order, we have

$$\alpha = \frac{2}{b_{h,\nu}} - \frac{2}{b_{h,\nu}^3}\left(\log b_{h,\nu} + \frac{1}{3}\right) + \frac{3}{b_{h,\nu}^5}\left(\log^2 b_{h,\nu} - \frac{1}{5}\right) + \mathcal{O}(1/b_{h,\nu}^7)$$

while for the photon

$$\alpha = \frac{2}{b_{h,\gamma}} + \frac{1}{b_{h,\gamma}^3}\left(\frac{1}{3} - 4\log b_{h,\gamma}\right) + \frac{1}{b_{h,\gamma}^5}\left(-\frac{17}{20} - 10\log b_{h,\gamma} + 12\log^2 b_{h,\gamma}\right) + \mathcal{O}(1/b_{h,\gamma}^7)$$

- The classical deflection angle can be expanded for $b_h \gg 1$

$$\alpha = \frac{a_1}{b_h} + \frac{a_2}{b_h^2} + \frac{a_3}{b_h^3} + \frac{a_4}{b_h^4} + \frac{a_5}{b_h^5} + \cdots$$

- The general expression is given by

$$\alpha = \frac{2}{b_h} + \sum_{k=1} \frac{a_{2k}}{b_h^{2k}} + \sum_{k=1} \frac{1}{b_h^{2k+1}}\left(a_{2k+1} + d_0 \log b_h + d_1 \log^2 b_h + \cdots\right)$$

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Including quantum corrections $\alpha = \alpha(E)$ produces a gravitational rainbow.

Photon source: images

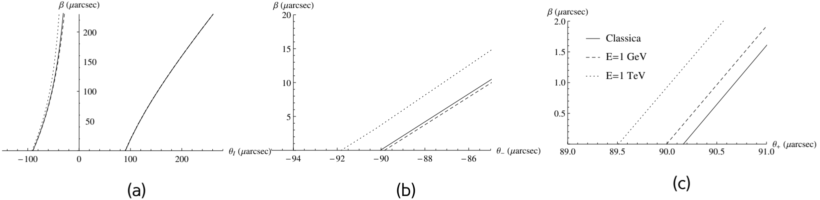
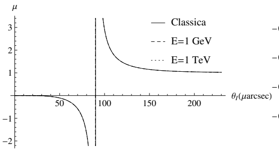
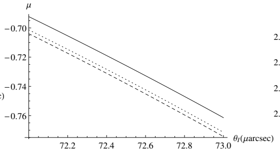


Figure: Source angular position β as a function of image angular position ϑ_I for a photon source at $D_S = 11$ kpc far from the observer in presence of a gravitational lens with mass $M = 10^6 M_\odot$ and $D_L = 10$ kpc. In (a) the behaviour for both primary and secondary images is shown, (b) and (c) refer respectively to secondary and primary image.

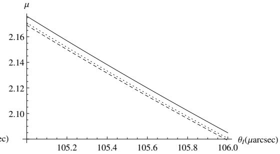
Photon source: magnification



(a)



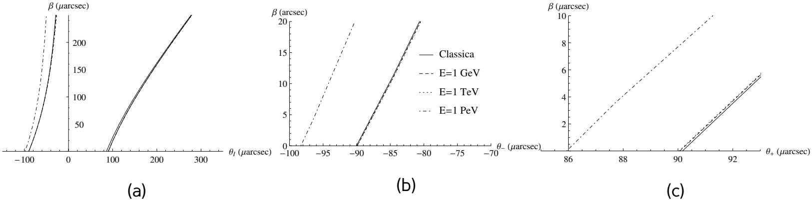
(b)



(c)

Figure: Magnification μ as a function of image position ϑ_I for a photon source at $D_S = 11$ kpc far from the observer in presence of a gravitational lens with mass $M = 10^6 M_\odot$ and $D_L = 10$ kpc. In (a) for primary and secondary images, (b) and (c) focus on secondary and primary image respectively.

Neutrino source: images



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Neutrino source: magnification

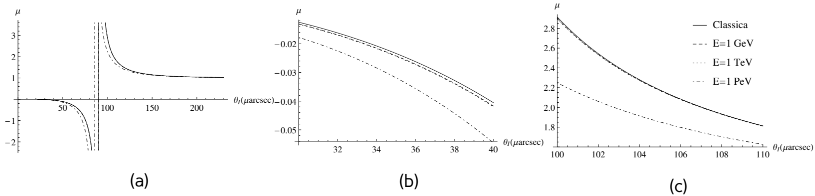

Figure: Magnification μ as a function of image position ϑ_I for a neutrino source at $D_L = 10$ kpc in presence of a gravitational lens with mass $M = 10^6 M_{\odot}$ and $D_L = 10$ kpc.

Modified Virbadhra-Ellis lens

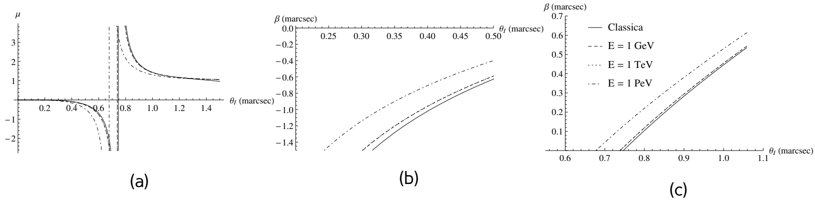

Figure: Magnification μ and β as functions of image position ϑ_I in presence of a gravitational lens with mass $M = 4 \times 10^6 M_\odot$ at $D_\mathrm{L} = 8$ kpc, and the distance between the lens and the source of $D_\mathrm{LS} = 1$ mpc.

Post-newtonian corrections

The generic form of the metric with isotropic coordinates is

$$d\tau^2 = \tilde{A}(\tilde{r})d\tilde{t}^2 - \tilde{B}(\tilde{r})\left[d\tilde{r}^2 - \tilde{r}^2\left(d\tilde{\vartheta}^2 + \sin^2\tilde{\vartheta}\,d\tilde{\phi}^2\right)\right]$$

and the post-newtonian expansion for the coefficients is given by

$$\tilde{A}(\tilde{r}) \simeq 1 + 2\tilde{\Phi} + 2\tilde{\Phi}^2 + \frac{3}{2}\tilde{\Phi}^3$$
$$\tilde{B}(\tilde{r}) \simeq 1 - 2\tilde{\Phi} + \frac{3}{2}\tilde{\Phi}^2 - \frac{1}{2}\tilde{\Phi}^3$$

The leading order cross section reads

$$\left.\frac{d\sigma}{d\Omega}\right._{2\mathrm{PN}}^{(0)} = \mathcal{K}(E,\vartheta)_{2\mathrm{PN}}\left.\frac{d\sigma}{d\Omega}\right._{0\mathrm{PN}}^{(0)}$$

The impact parameter to the 1PN order for the neutrino is

$$b^2(\alpha)_{\nu\,1\mathrm{PN}} = 4(GM)^2\left(-1 + \csc^2\frac{\alpha}{2} + 2\log\sin\frac{\alpha}{2}\right) + \pi E(GM)^3\left(4 + (\cos\alpha - 3)\csc\frac{\alpha}{2}\right) - \frac{\pi^2}{32}E^2(GM)^4\left(1 + \cos\alpha + 4\sin\frac{\alpha}{2}\right)$$

Conclusions and perspectives

- The semiclassical approach allows to give a first sight of quantum gravity effects
- Possible extension to different metrics, more general than the Shwarzschild metric
- An interesting feature is the gravitational rainbow and as a consequence the modified lens equations
- Using the modified Virbhadra-Ellis lens equation one could deepen the study of relativistic images

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