D=5 Helical black holes

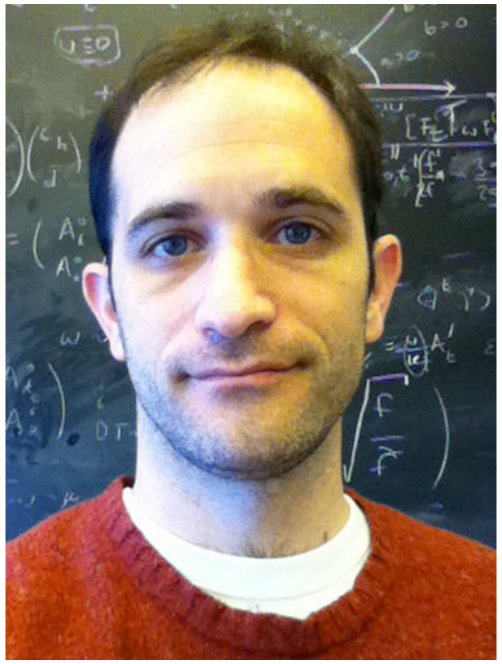
Stability analysis and higher derivative corrections

Abhiram Mamandur Kidambi

Max Planck Institute for Physics & ASC - LMU Munich

33rd IMPRS Workshop - Max Planck Institute for Physics

In collaboration with



Michael Haack
(ASC-LMU Munich)

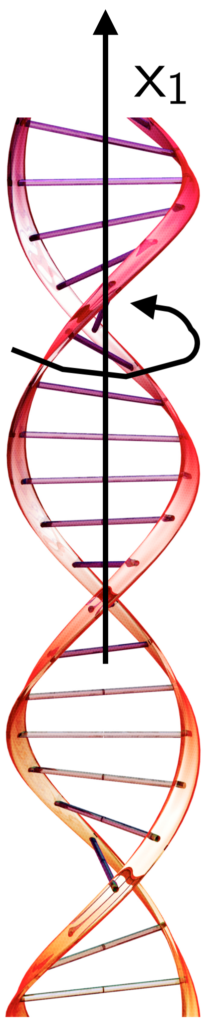
Amos Yarom
(Technion, Haifa)

Daniel Brattan
(Technion, Haifa)

arXiv:1511.*****

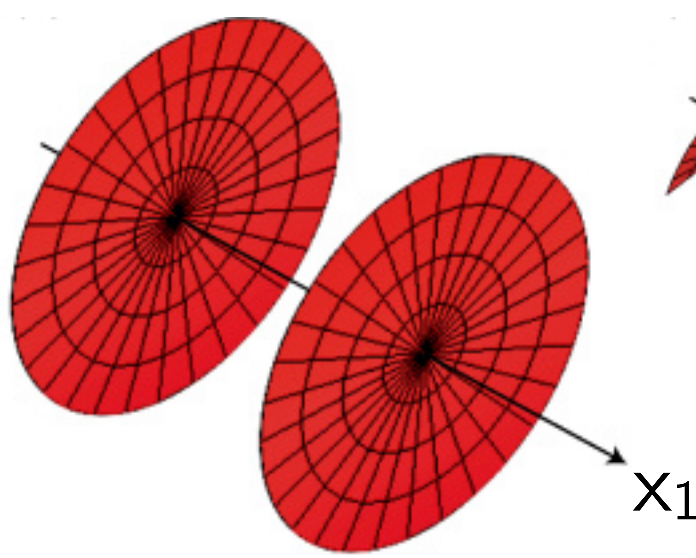
Helical Phases

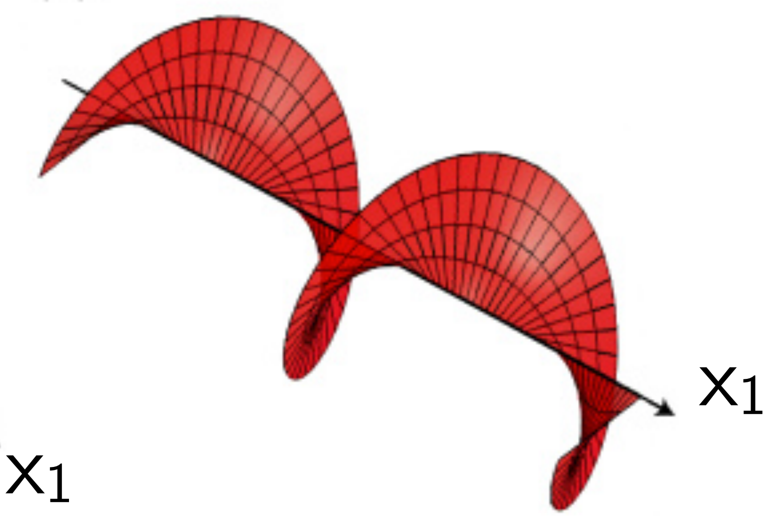
Helical Phase: A phase in which the **Euclidean symmetry is spontaneously broken into something called helical symmetry**.



Euclidean symmetry: Invariance under $\partial_{x_1}, \partial_{x_2}, \partial_{x_3}$

Helical symmetry: Invariance under $\partial_{x_1} - k(x_2\partial_{x_3} - x_3\partial_{x_2}), \partial_{x_2}, \partial_{x_3}$





What is gauge/gravity duality?

String theory/ Gravity at weak coupling and low curvature in D dim	$\longleftrightarrow$	Strongly coupled (conformal) QFT in D-1 dim

[Maldacena '97]
[Polyakov et.al '98]
[Witten '98]

The original AdS_5/CFT_4 conjecture

Type IIB supergravity on AdS_5 x S_5 $\longleftrightarrow$ $\mathcal{N} = 4$ SU(N) SYM in d $= 4$

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For this talk

Gravity (D dim)		Gauge theory (D-1 dim)
Black hole solution	$\longleftrightarrow$	Finite temperature field theory

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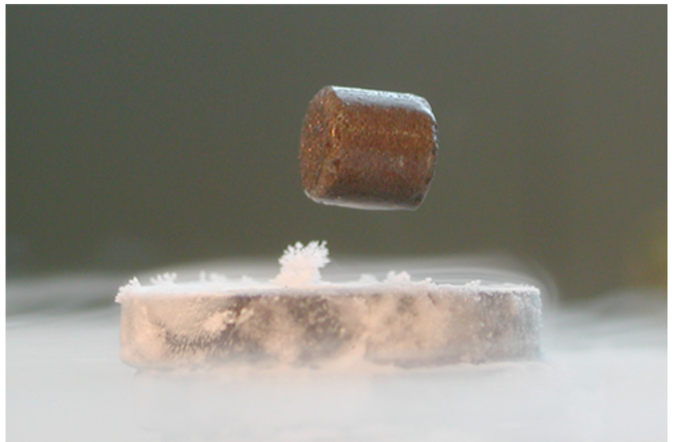
Electrically charged solution	$\longleftrightarrow$	Non-zero chemical potential
Instability of black hole solution	$\longleftrightarrow$	Phase transition

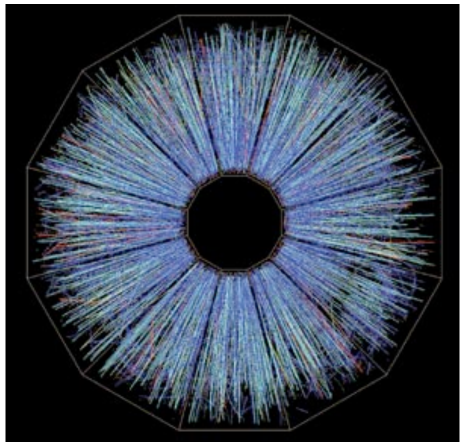
Motivation

AdS/CFT predicts cases in field theory with broken Euclidean symmetry (Spatially Modulated Phases)

*[**Erdmenger** et.al; Ooguri et.al; Gauntlett et.al, Fukushima; Hartnoll; Kiritsis et.al...]*



Superconductors



QCD @ high baryon density

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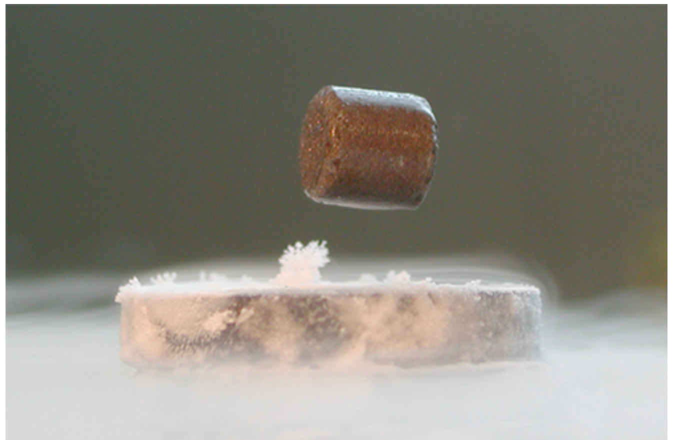

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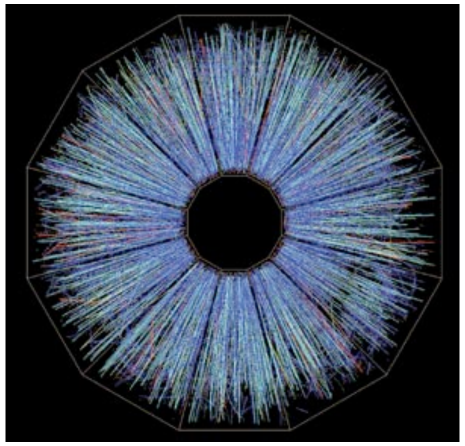

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Exist **exotic black hole solutions in supergravity (SUGRA)** dual to QFT's with broken Euclidean symmetry. *[Ooguri et.al 2010]* *[Donos, Gauntlett '12]*

Stability of black holes is essential

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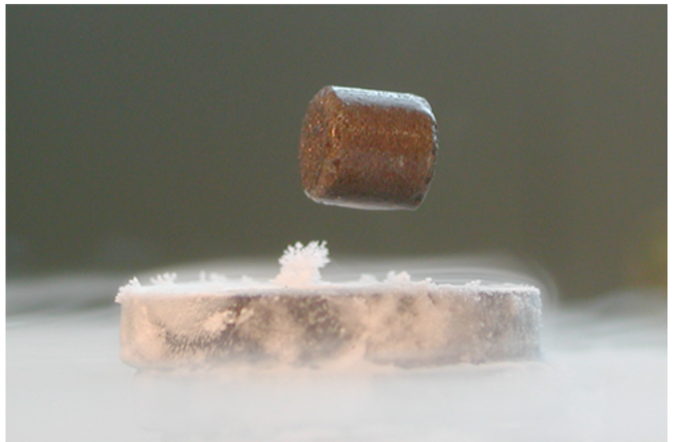

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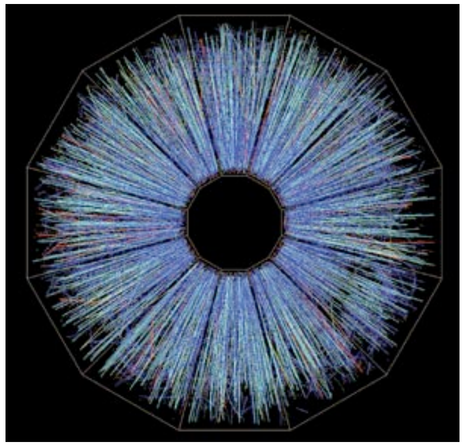

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1) Stability of **Reissner-Nordström black hole (RNBH)** in (super)gravity **w.r.t helical perturbations**.

2) **We analyse the stability criteria in presence of higher derivative corrections (h.d.c's).**

Overview

1. **Stability of black holes in the Einstein-Maxwell Chern-Simons theory in 5 dimensions w.r.t. helical phases**

2. **Higher derivative corrections and stability analysis**

3. **Overview of results**

4. **Outlook and further directions of research**

Einstein-Maxwell Chern-Simons (EMCS) Theory in $D = 5$

[Ooguri et.al 2010]

$$S_{EMCS} = \underbrace{\int d^5x\sqrt{-g}\left((R+12) - \frac{F^{ab}F_{ab}}{4}\right)}_{\text{Einstein-Maxwell}} - \underbrace{\frac{2}{3}\alpha\int F\wedge F\wedge A}_{\text{Chern-Simons}}$$

If $\alpha = \alpha_s = \frac{1}{2\sqrt{3}} \approx 0.2886$ $\longrightarrow$ N $=$ 2 minimal gauged supergravity.

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Step 1: Add "*fluctuations Q(r) and b(r)* & *helical terms*" to homogeneous solution

Near horizon part of extremal RN black hole:

$$ds^2 = \frac{-dt^2 + dr^2}{12r^2} + dx^2$$

$$A = \frac{E}{12r}dt$$

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$$ds^2 = \frac{-dt^2 + dr^2}{12r^2} + dx^2 + Q(r)dt^2 + 2Q(r)\omega_2 dt$$

$$A = \frac{E}{12r}dt + b(r)\omega_2$$

$\omega_2 = \cos(kx_1)dx_2 - \sin(kx_2)dx_3$ ⟶ helical symmetry

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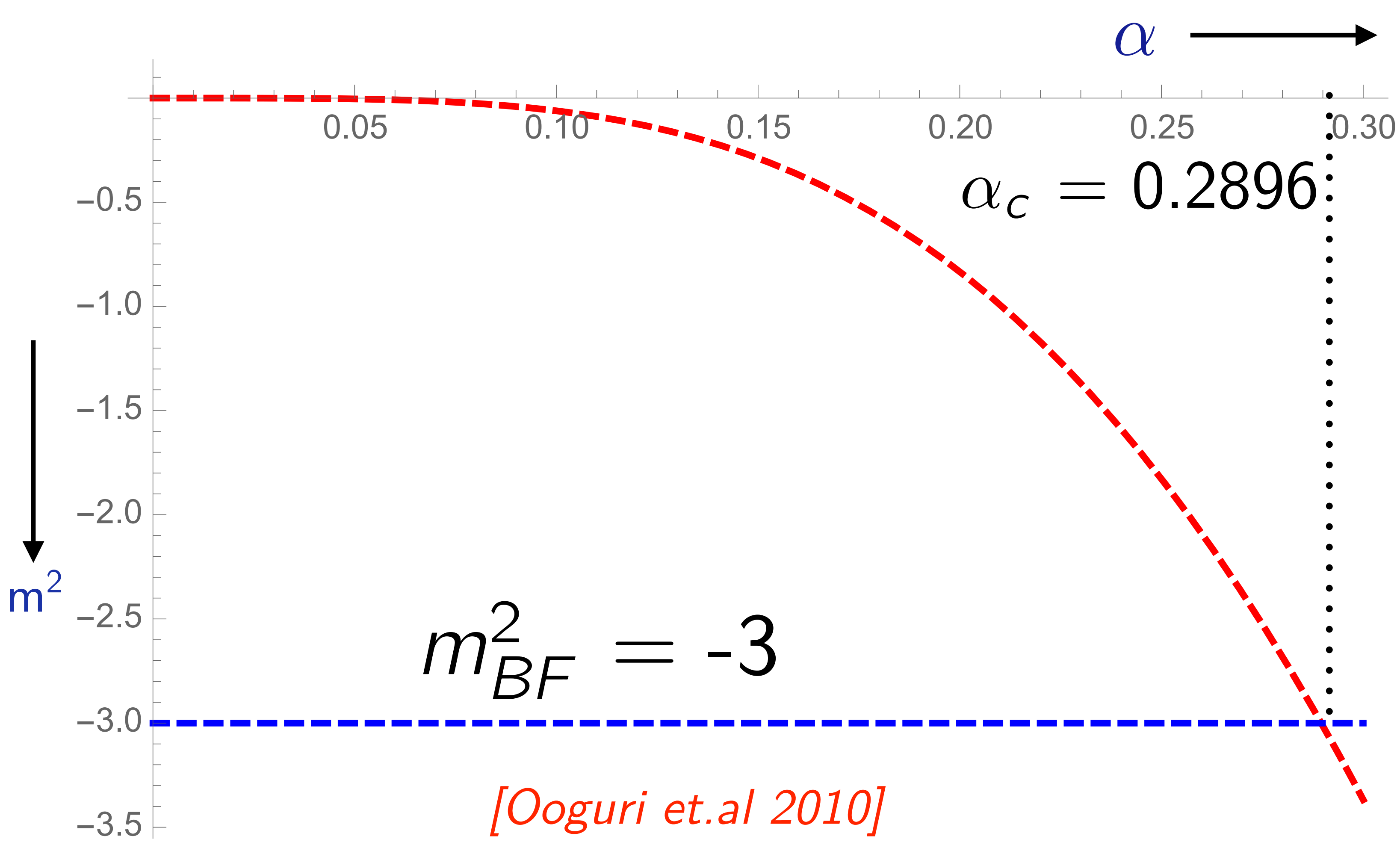

$\frac{\alpha_s}{\alpha_c} \approx 0.997$

Instability if $\alpha > \alpha_c = 0.2896$

Higher derivative corrections

The supersymmetric value of CS coupling just above the critical value.

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Add higher derivative corrections *[Myers, Sinha et. al '09]*

$$S = S_{EMCS} + c_1\left(R_{abcd}R^{abcd}\right) + c_2\left(R_{abcd}F^{ab}F^{cd}\right) + c_3\left(F_{ab}F^{ab}\right)^2$$
$$+ c_4\left(F^a_b F^b_c F^c_d F^d_a\right) + c_5\epsilon^{abcde}A_a R_{bcfg}R_{de}{}^{fg} + \cdots$$

$$c_i \ll 1,\ \ i = 1, \cdots, 5$$

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$c_i \ll 1,\ i = 1, \cdots, 5$

How does criticality change for higher derivative correction?

1) Compute corrections to m_{BF}, A *[Myers, Sinha et. al '09]*

2) Repeat analysis: Make ansatz, find E'sOM, obtain mass of perturbation.

3) Find critical values of α , c_i for which $m^2 < m_{BF}{}^2$.

Stability Analysis & Results

We still N = 2 minimal SUGRA

"The supersymmetric c_i's"

$$c_2 = -\frac{c_1}{2},\ c_3 = \frac{c_1}{24},\ c_4 = -\frac{5c_1}{24},\ c_5 = \frac{c_1}{2\sqrt{3}}$$

where

$$c_1 = \frac{1}{8}\frac{c-a}{c}$$

and ***c*,*a*** are the **central charges of the dual N = 1 SCFT.**

[Myers, Sinha et. al '09]

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Q: How do the c_i affect the critical and supersymmetric CS coupling?

With h.d.c's for EMCS with the "*supersymmetric c's*" relation

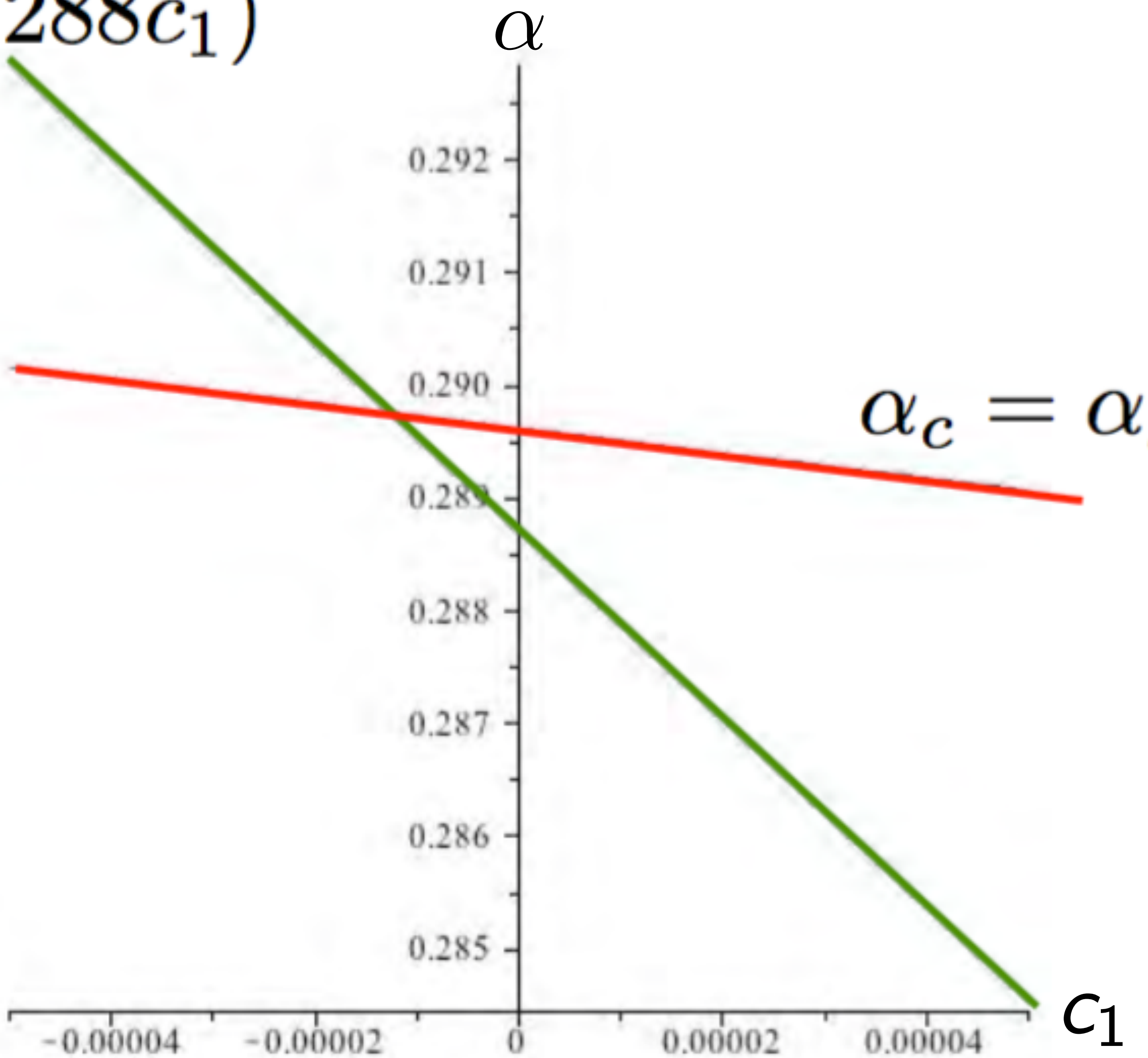
$$\alpha_c = \alpha_c^{(0)} - 14.16c_1$$

$$\alpha_s = \frac{1 - 288c_1}{2\sqrt{3}}$$

$\alpha_c^{(0)}$ = critical value of EMCS theory

Critical coupling corrections

$$\alpha_s = \alpha_s^{(0)}(1 - 288c_1)$$



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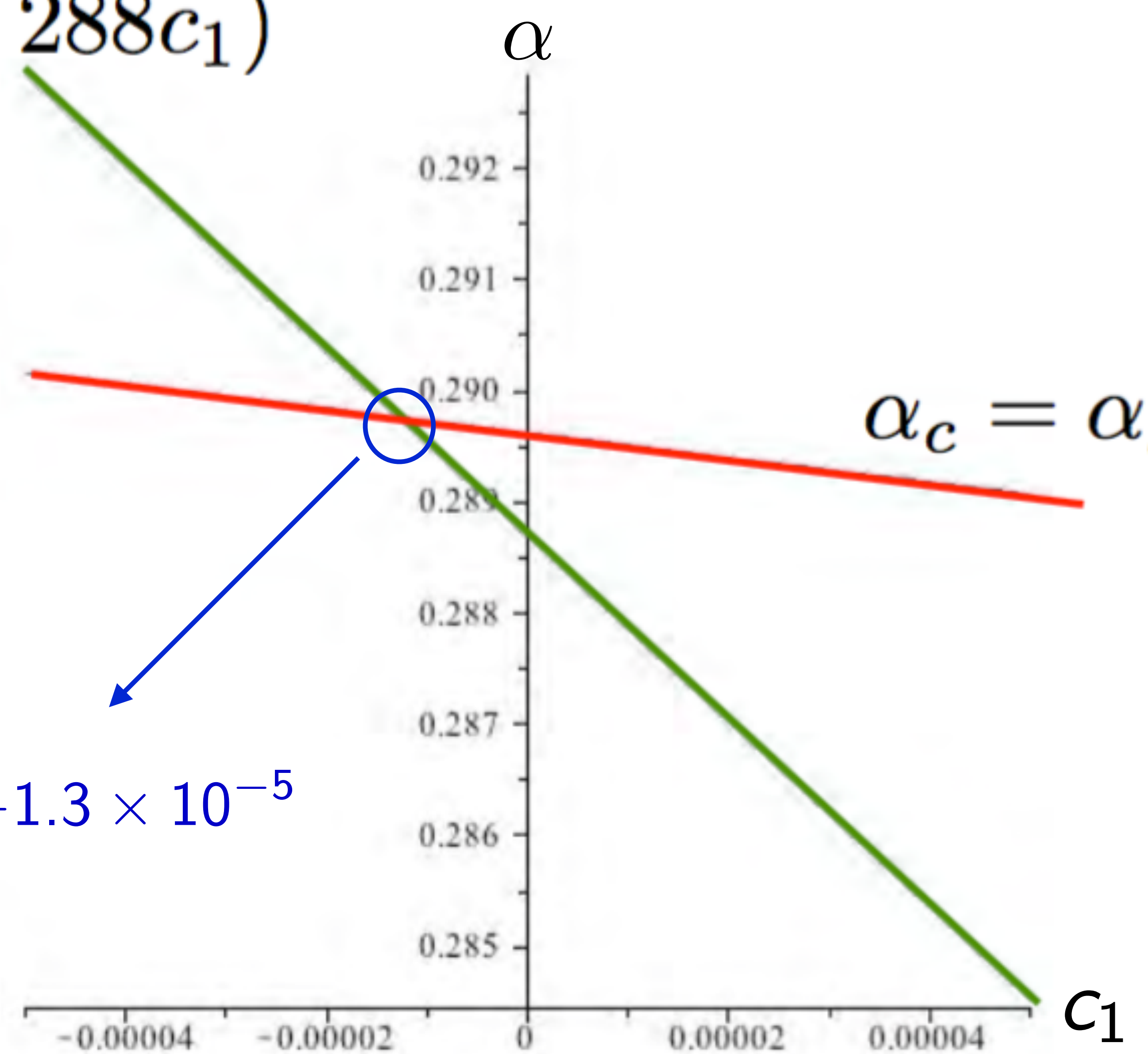

$$\alpha_c = \alpha_c^{(0)} - 14.16c_1$$

$$c_1 = \frac{1}{8}\frac{c-a}{c} = -1.3 \times 10^{-5}$$

c_1 corrections to α_s and α_c

If $c_1 > -1.3 \times 10^{-5}$, the RNBH in N = 2 minimal gauged SUGRA is stable w.r.t helical phases.

Comment on (c-a)

AdS/CFT has been used to conjecture the ratio of shear to entropy for theories with a gravity dual.

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

[Starinets, Son, Policastro, Buchel, Liu, Myers, Sinha...]

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With h.d.c's, this bound is not satisfied:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{c-a}{c}\right) = \frac{1}{4\pi}(1 - 8c_1) \ \text{ if } c_1 > 0$$

Normal theories (Lagrangian theories in the large N limit) have $c > a$ and therefore do not satisfy this bound. (Ex: Large N *SU(N), Sp(N) theories*)

[Shenker, Myers et.al '07], [Buchel, Myers, Sinha '12]

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Our analysis shows that there exist stable black hole solutions under h.d.c's which do not satisfy this shear-entropy bound.

This supports the need to modify this bound.

Review & Key results

1. RNBH solutions in EMCS theory is be **unstable** to helical phases **when CS coupling is greater than the critical coupling**.

2. **RNBH solutions in supersymmetric theories are barely stable**. Add higher derivative terms to analyse critical and supersymmetric CS couplings.

3. **For N = 2 minimal gauged SUGRA**, we see that the **RNBH solutions are stable if *(c-a)* $\propto$ $c_1 > -1.3 \times 10^{-5}$**.

4. **Most of these solutions do not obey the shear-entropy "bound"**. This violation is expected in theories with h.d.c's. This provides more reason to suggest that the **bound must be corrected.**

Further Developments and Outlook

1. Extend results to include full BH geometry (work in progress)

2. We have performed a linear analysis in order by order expansion of the CS coupling and expect our final results to be analytically computable in the limit that $\alpha \to \infty$.

3. We would like to describe the endpoint of the phase transition including higher derivative corrections, similar to *[Ooguri, Park '10]*.

4. Further analysis into (c-a): Is it possible to have $c < a$ i.e. $c_1 < 0$ in interacting Lagrangian theories? *[Maldacena, Hofmann '08]*

Thank you for your attention!

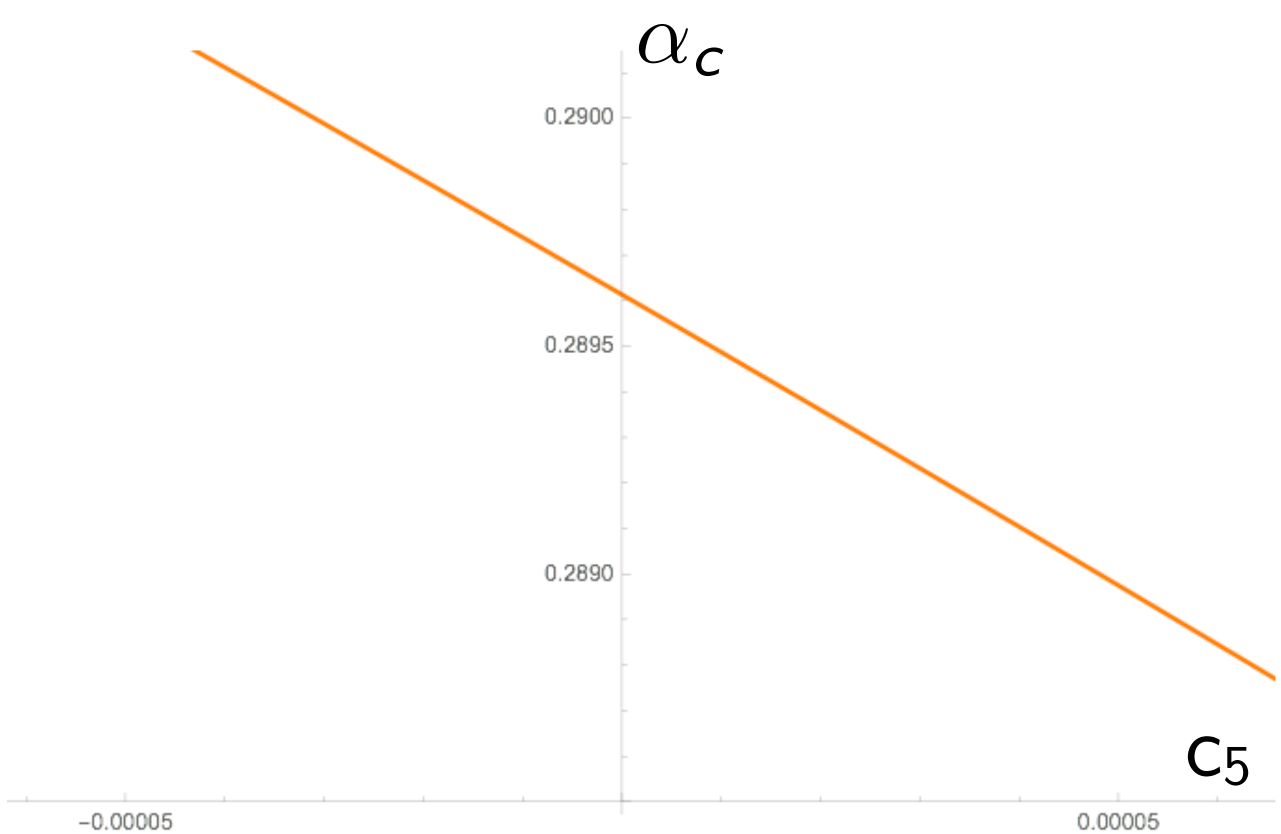
Generic conditions on stability

Corrections to the BF bound: $m_{BF}^2 = -(3 - 144c_2 - 576c_3 - 288c_4)$

Corrections to A: $A = \left(\frac{2\sqrt{6}}{12r} - \frac{4\sqrt{6}}{r}(c_1 + 2c_2 + 4c_3 + 2c_4)\right)$

$$\alpha_c = \alpha_c^{(0)} + 11.82c_1 + 37.06c_2 + 183.67c_3 + 55.01c_4 - 12.61c_5$$

There could non-supersymmetric solutions for which there are no stable helical BH solutions



c_5 decreases the critical CS coupling

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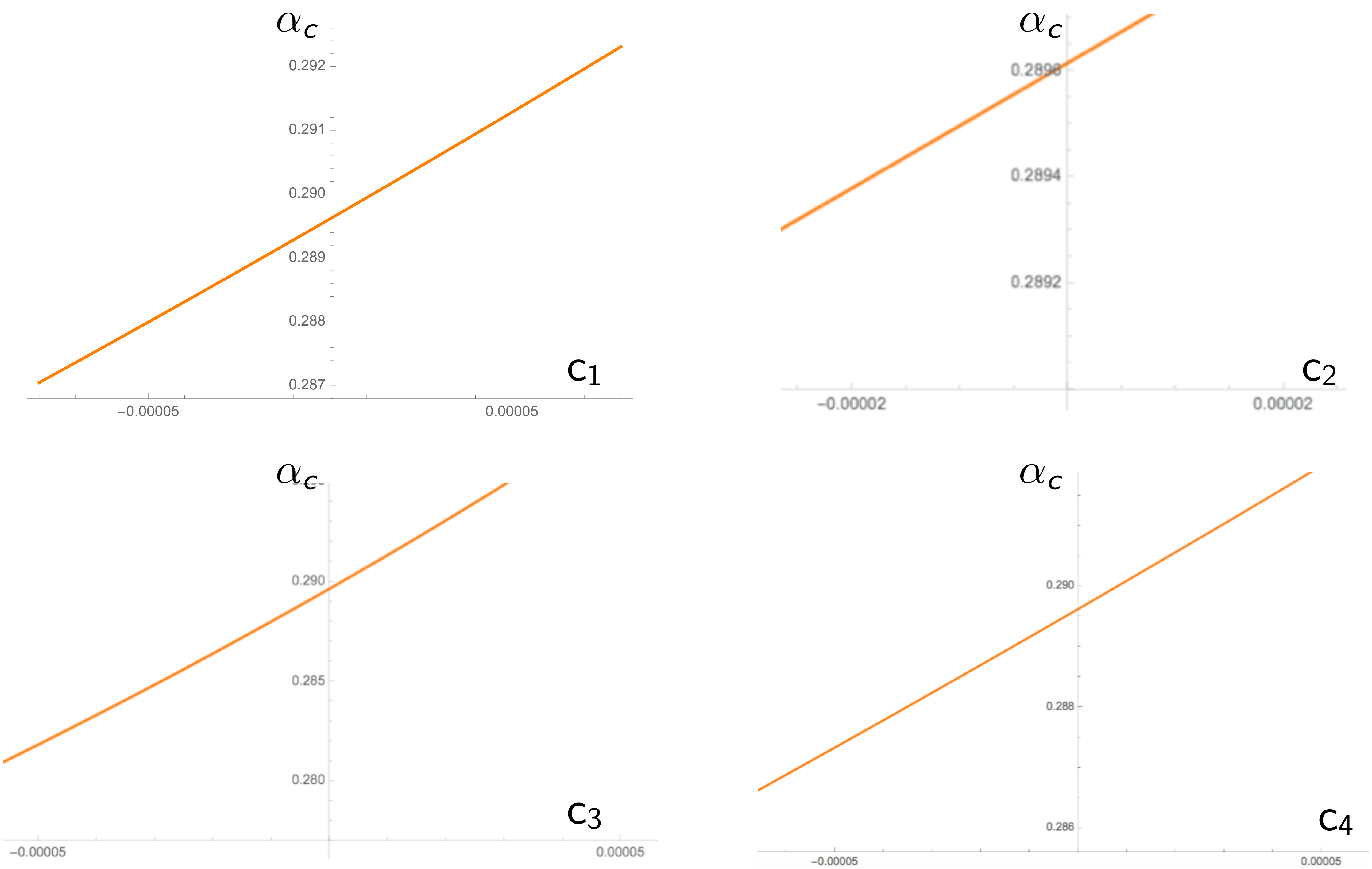
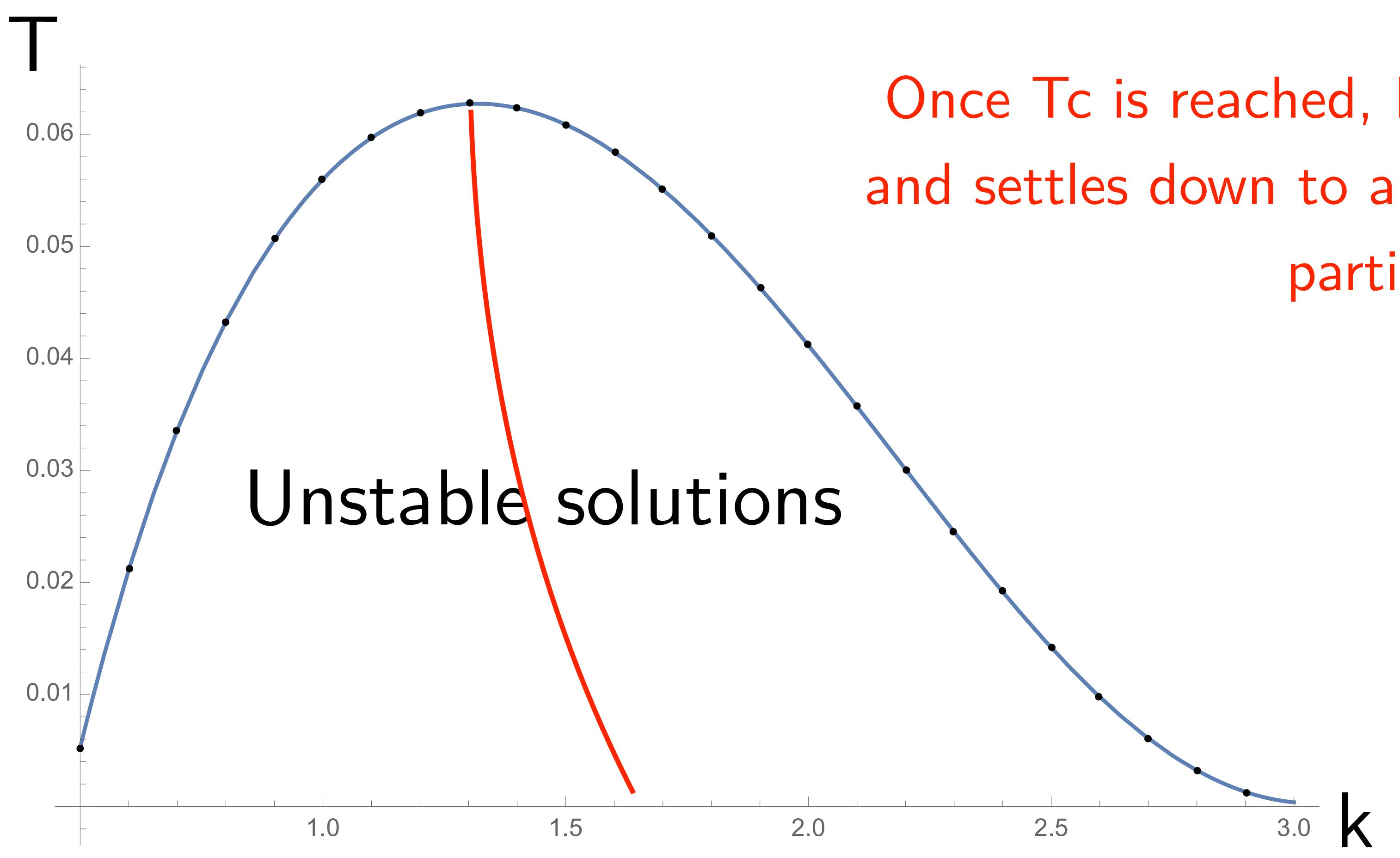


$c_{1,..,4}$ corrections increase the critical Chern-Simons coupling

Why extremal RN black holes?

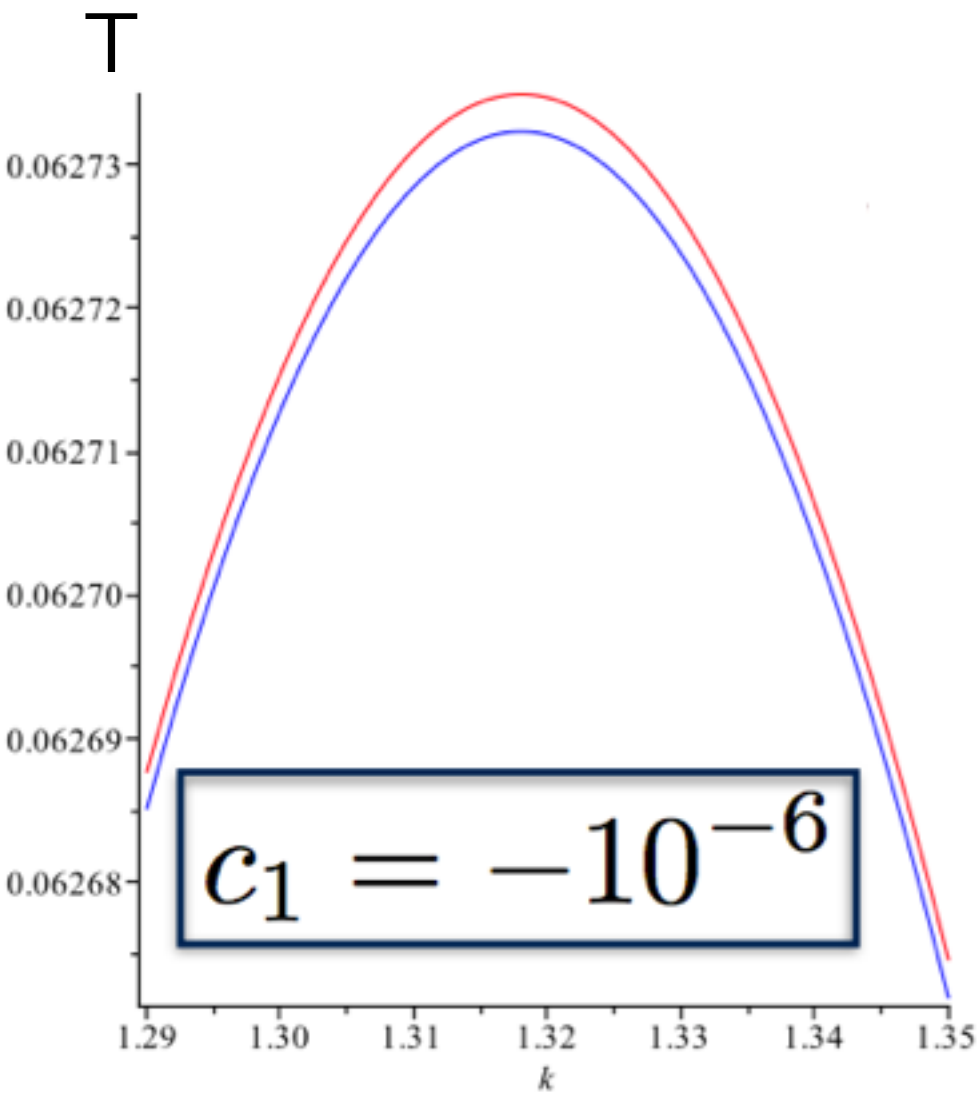


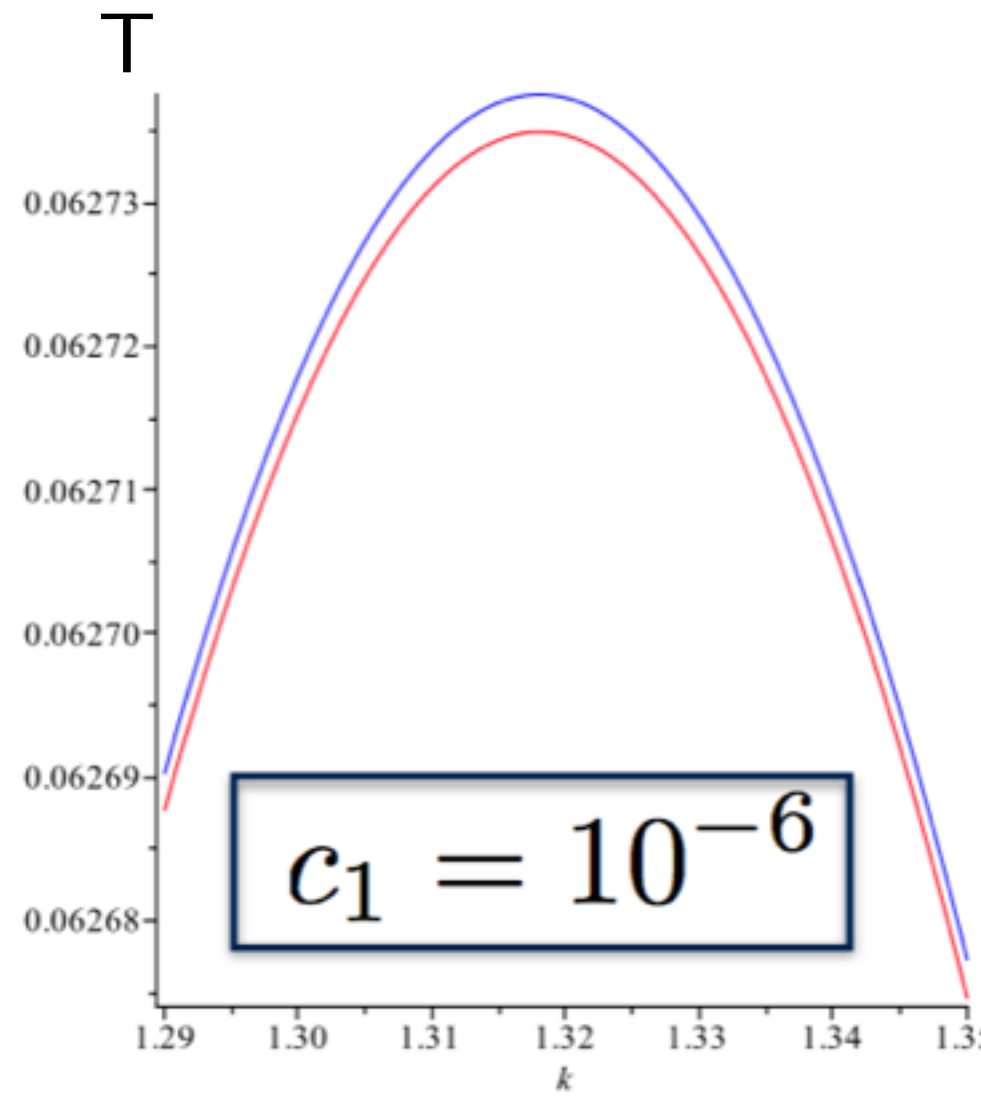
$\alpha = 0.425$

Once Tc is reached, BH undergoes a phase transition and settles down to an extremal, helical BH with some particular value of k.

Study black hole thermodynamics to find out that value of k.

Corrections with c_1





With and without c_1 correction

More on CS, N = 2 SUGRA and SCFT

For any supersymmetric solution of $D = 10$ or $D = 11$ supergravity that consists of a warped product of $d+1$ dimensional anti-de-Sitter space with a Riemannian manifold M, $AdS_{d+1} \times_w M$, there is a consistent Kaluza-Klein truncation on M to a gauged supergravity theory in $d+1$-dimensions for which the fields are dual to those in the superconformal current multiplet of the d-dimensional dual SCFT.

[Gauntlett, Varela '07]

To get **c** and ***a***

$$T^m_m = \frac{c}{16\pi^2} \underbrace{W^{abcd}}_{\text{Weyl Tensor}} W_{abcd} - \frac{a}{16\pi^2} \underbrace{\left(R^{abcd} R_{abcd} - 4 R^{ab} R_{ab} + R^2\right)}_{\text{4D Euler Density}}$$