Flavor entanglement in gauge/gravity duality

Nina Miekley

Master thesis supervised by Prof. Johanna Erdmenger

Ludwig Maximilian University of Munich
Max Planck Institute for Physics

October 26, 2015

Introduction

- gauge/gravity duality:
  conformal field theory $\longleftrightarrow$ gravity

- new insights into strongly coupled field theories

- entanglement entropy:
  - order parameter
  - related to degrees of freedom
  - difficult to calculate in field theory

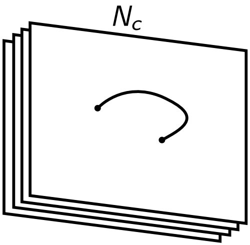
Structure

1. Gauge/gravity duality
2. Entanglement
3. Results
4. Outlook

Gauge/gravity duality

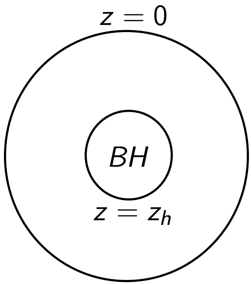
two perspectives of N_c D3-branes

- $\mathcal{N} = 4$ supersymmetric Yang–Mills theory
  - massless gauge multiplet in adjoint representation of $SU(N_c)$
  - Yang-Mills coupling constant g_{YM}
    t'Hooft coupling constant $\lambda = g_{YM} N_c$



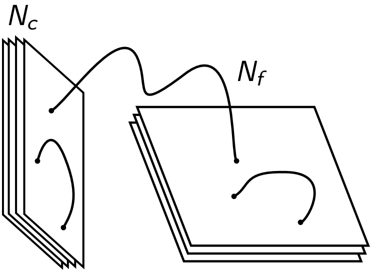
$\Updownarrow$

- supergravity on $\mathrm{AdS}_5 \times S^5$
  - black hole solution
  - z-coordinate $\leftrightarrow$ energy scale



Gauge/gravity duality

fundamental degrees of freedom:



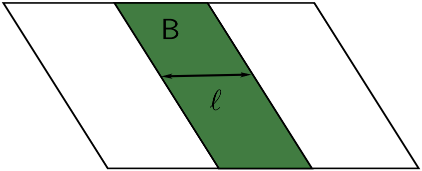
- add N_f D7-branes
- D7-branes span S^3 subspace of S^5
- introduce flavor mass by separating D7- and D3-branes
- probe approximation: neglect backreaction

$$S_{\mathrm{DBI}} \propto \int d^8 w \sqrt{-\det(P[g]_{ab} + 2\pi\alpha' F_{ab})}$$

Entanglement

entanglement entropy:

$$\rho_B = \text{Tr}_C(\rho)$$
$$S_{EE}(B) = -\,\text{Tr}_B\,(\rho_B \log \rho_B)$$



- measures correlation between region and complement
- non-local property
- effective degrees of freedom at the energy scale $\Lambda \propto 1/\ell$
- complicated quantum calculation $\rightarrow$ not practicable in $d > 2$

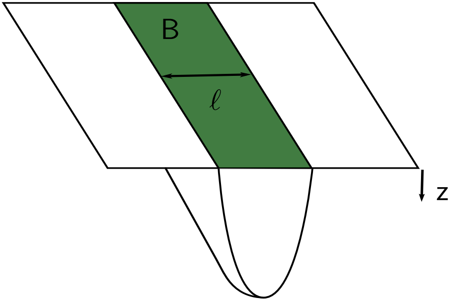
Entanglement

gravity dual
[hep-th/0605073]:

$$S_{EE}(B) = \min_{\gamma_B | \partial\gamma_B = \partial B} \frac{A(\gamma_B)}{4G_N}$$

$$\updownarrow$$

$$S_{BH}(B) = \frac{A_{Horizon}}{4G_N}$$



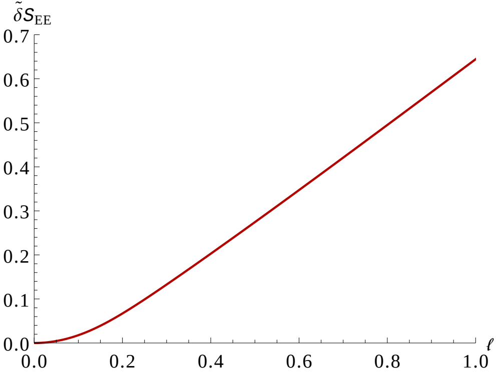
flavor contribution: due to backreaction on metric [1307.5325]

Results - Thermal entropy contribution

- $g_{\rm cft} \neq \eta$, mass dependent
  $\rightarrow$ rescaling of time coordinate to obtain Minkowski metric
- temperature shift $T \rightarrow T + \delta T$

contribution:

$$\frac{\delta S_{EE}^{(0)}}{\delta T} \delta T \propto \tilde{\delta} S_{EE}$$



Results - Thermal entropy contribution

linear for large region
$\rightarrow$ volume term, thermal entropy

$$S_{th} = \frac{\pi^2}{2} T^3 Vol \cdot \left(N_c^2 + N_f N_c \lambda f(m_q) \right)$$

$$F = -\int dT \; S_{th}, \quad \langle E \rangle = F + S_{th} T$$

Energy momentum tensor in field theory

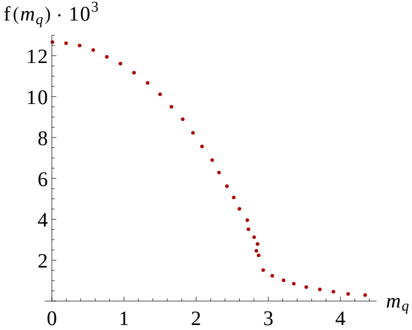
$$\langle T_{\mu\nu} \rangle = \frac{\pi^2}{8} T^3 \left(N_c^2 + N_f N_c \lambda f(m_q) \right) \begin{pmatrix} -3 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Results - Thermal entropy contribution

linear for large region
$\rightarrow$ volume term, thermal entropy

$$S_{th} = \frac{\pi^2}{2} T^3 Vol \cdot \left(N_c^2 + N_f N_c \lambda f(m_q) \right)$$

$$F = -\int dT \; S_{th}, \quad \langle E \rangle = F + S_{th} T$$

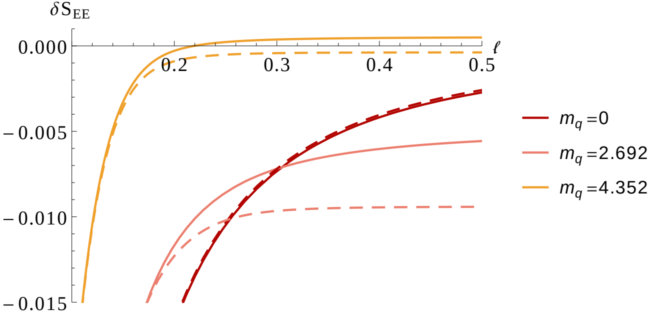


Energy momentum tensor in field theory

$$\langle T_{\mu\nu} \rangle = \frac{\pi^2}{8} T^3 \left(N_c^2 + N_f N_c \lambda f(m_q) \right) \begin{pmatrix} -3 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Results

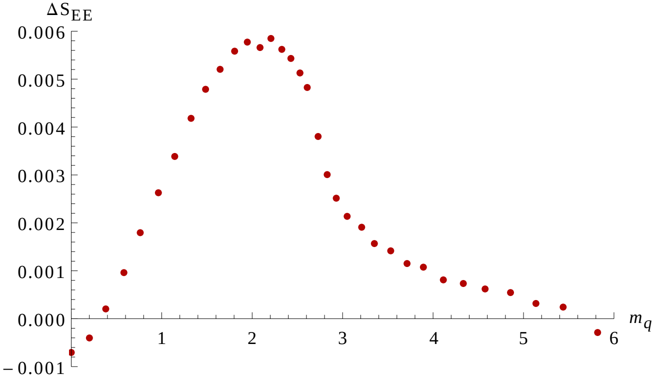
- δS_{EE} without thermal contribution
- similar results to $T = 0$



– – – – zero-temperature results [1505.07697]

Results

- long-range behavior modified
- mass-dependent shift ΔS_{EE} of asymptotic value



Outlook

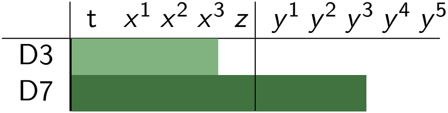
- analyze behavior near the phase transition (meson melting)
- include magnetic field, charge density
  $\rightarrow$ variety of phase transitions, chiral symmetry breaking
- method also applicable to D5-probe branes
  $\rightarrow$ defect field theory

Thank you for your attention

Metric

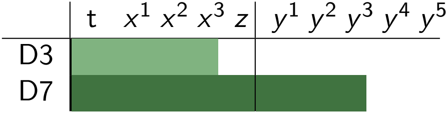
$$
\begin{aligned}
ds^2_{AdS} =& \frac{L^2}{z^2}\left(\frac{dz^2}{b(z)} - b(z)dt^2 + d\vec{x}^2\right)\\
b(z) =& 1 - z^4/z_h^4\\
\delta g =& t_0\frac{L^2}{z^2}\left(f(z)\frac{dz^2}{b(z)} - b(z)h(z)dt^2 + j(z)d\vec{x}^2\right)
\end{aligned}
$$

D7 embedding

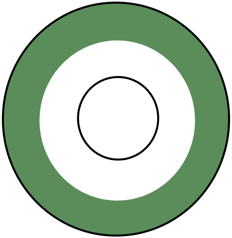


- D7-branes span S^3 subspace of S^5

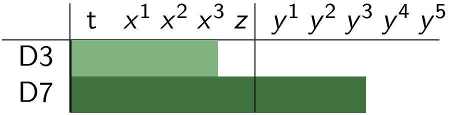
D7 embedding



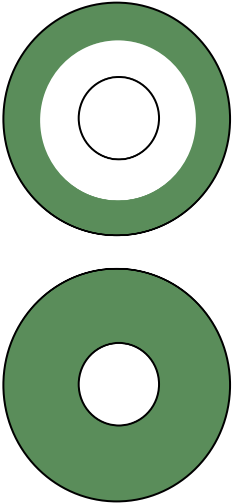
- D7-branes span S^3 subspace of S^5
- radius $r_{S^3} = \cos\theta(z)$ depends on z
  two qualitatively different
  embeddings
  - Minkowski embedding
    S^3 collapses outside horizon
  - Black hole embedding
    D7-branes reach horizon



D7 embedding



- D7-branes span S^3 subspace of S^5
- radius $r_{S^3} = \cos\theta(z)$ depends on z
  two qualitatively different embeddings
  - Minkowski embedding
    S^3 collapses outside horizon
  - Black hole embedding
    D7-branes reach horizon



$\mathcal{N} = 4$ supersymmetric Yang–Mills theory

$\mathcal{N} = 4$ gauge multiplet	
A_μ	gauge field
λ^a, $a \in \{1, 2, 3, 4\}$	Weyl fermions
ϕ^i, $i \in \{1, \dots, 6\}$	(real) scalars

$\rightarrow$ group into $\mathcal{N} = 1$ vector multiplet and three chiral multiplets

$\mathcal{N} = 4$ supersymmetric Yang–Mills theory

$\mathcal{N} = 4$ gauge multiplet	
A_μ	gauge field
λ^a, $a \in \{1, 2, 3, 4\}$	Weyl fermions
ϕ^i, $i \in \{1, \ldots, 6\}$	(real) scalars

$\rightarrow$ group into $\mathcal{N} = 1$ vector multiplet and three chiral multiplets

introduce flavor $\rightarrow$ breaks two of the supersymmetries
particles split into $\mathcal{N} = 2$ multiplets: vector multiplet and hypermultiplet